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PUBLIC DEBT MANAGEMENT, MONETARY POLICY AND
FINANCIAL INSTITUTIONS

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Abstract

Despite the large size of the Brazilian debt market, as well the large diversity of its bonds, the picture that emerges is of a market that has not yet completed its transition from the role it performed during the megainflation years, namely that of providing a liquid asset that provided positive real returns. This unfinished transition is currently placing the market under severe stress, as fears of a possible default from the next administration grow larger. This paper analyzes several aspects pertaining to the management of the domestic public debt. The causes for the extremely large and fast growth of the domestic public debt during the seven-year period that President Cardoso are discussed in Section 2. The main culprit is the very high and risky interest rate, with the recognition of old debts (hidden liabilities and state debt renegotiation) coming in second. Section 3 computes Value at Risk and Cash Flow at Risk measures for the domestic public debt. These risk measures show that the current composition of the public debt is very risky. The rollover risk is introduced in a mean-variance framework in Section 4, and the maturity structure evolution is discussed. The increased riskness was the cost to improve the maturity structure. Section 5 discusses a few issues pertaining to the overlap between debt management and monetary policy. Finally, Section 6 wraps up with policy discussion and policy recommendations.

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1- Introduction

Despite the large size of the Brazilian debt market, as well the large diversity of its bonds, the picture that emerges is of a market that has not yet completed its transition from the role it performed during the megainflation years, namely that of providing a liquid asset that provided positive real returns. This unfinished transition is currently placing the market under severe stress, as fears of a possible default from the next administration grow larger.

This paper analyzes several aspects pertaining to the management of the domestic public debt. The causes for the extremely large and fast growth of the domestic public debt during the seven-year period that President Cardoso are discussed in Section 2. The main culprit is the very high and risky interest rate, with the recognition of old debts (hidden liabilities and state debt renegotiation) coming in second. Although there are no reasons to suppose that the future will replicate the past, simulations show that the maintenance of substantial primary surplus is essential to keep the debt path sustainable.

Section 3 computes Value at Risk and Cash Flow at Risk measures for the domestic public debt. These risk measures show that the current composition of the public debt is very risky. Volatility increases precisely when the economy weakens, draining fiscal resources to serve the debt precisely when they are less plenty.

The rollover risk is introduced in a mean-variance framework in Section 4, and the maturity structure evolution is discussed. The increased riskness was the cost to improve the maturity structure, i.e., market risk was increased while refinancing risk was reduced. The recent emergence of large discounts on the zero-duration bonds, requiring the Treasury to shorten the debt seems to be a further unfortunate step. Not only the debt structure is extremely risky from the market risk perspective, but also the Treasury is being increasingly pressured to shorten the debt, thereby increasing the refinancing risk, because of the perception that credit risk has increased.

Section 5 relates debt management to monetary policy. It reviews the origins of the current problems in the debt market, and explains how the two policies operate in Brazil. It shows that reserve requirements serve many purposes. Because of this, they are likely to remain around for a long time. The current difficulties in rolling over long debt are analyzed. Finally, Section 6 concludes and presents policy discussion and policy recommendations.

2- Decomposing the Public Debt Growth

During the period 1995-2001, the Brazilian federal domestic public bonded debt more than quadrupled in real (percentage of GDP) terms. This Section decomposes the domestic federal bonded debt growth, searching for the macroeconomic causes of the very large growth that took place during the last seven-year period. We attempt to quantify the contraction and expansion sources of the federal bonded debt. The methodology used was developed in Bevilaqua and Garcia [2002], where it is thoroughly explained.

Table 1.1 displays the factors of expansion and contraction of the federal public bonded debt (in **nominal** terms). One must keep in mind that, since we are working with nominal values over a seven-year-period, the values presented on this table can be misleading.¹ The most important individual factor for debt growth was interest payments (61.04% of the total variation of R\$535.343.38), followed by the accumulation of the states' debt (32.60%). This is a debt that several Brazilian state governments owe to the federal government, the actual repayment of which will remain an open question in the next years. These two items alone add up to 93.64% of the total variation in the domestic federal bonded debt.

Table 1.2 displays the factors of expansion and contraction of the federal public debt (as percentage of GDP). The analysis in real terms is the most relevant to the current economic situation. The interest rate share increased even more in real terms: interest payments (32.43% of GDP) alone exceeded the full variation of the federal net debt (20.19% of GDP) and was almost equal to the total variation of the domestic federal bonded debt (36.13% of GDP). If we compute the implicit real interest rate on the net debt, by dividing the nominal interest payments by the preceding net debt stock, we get the following figures:

Table 2.3 Excess Interest Rates on the Net Debt

	1995	1996	1997	1998	1999	2000	2001
I(t)/D(t-1)	28.45%	25.28%	15.99%	32.43%	38.43%	17.37%	18.82%
Y(t)/Y(t-1)	1.3401	1.1982	1.1091	1.0292	1.1311	1.0923	1.0841
(1+I(t)/D(t-1))/(Y(t)/Y(t-1))	-4.22%	4.55%	4.58%	28.68%	22.38%	7.45%	9.60%

The line I(t)/D(t-1) contains the implicit net debt nominal interest rate, obtained through the division of the nominal interest payments in year t by the debt stock at the end of the previous year. Subtracting from this nominal interest rate the GDP growth rate (Y(t)/Y(t-1)), we obtain a measure of the excess of the nominal rate in relation to nominal GDP growth, which is the relevant variable to access how important the interest payments are in the growth of the debt to GDP ratio.² Note that these implicit interest rates measure a lagged average of the current market rates. The lag length depends on the debt average remaining life, and its composition. For nominal debt, an interest rate increase would only show up in the figures above when the existing bonds at the time of the interest rate increase started to mature, and new ones were issued with a higher interest rate. However,

¹ We preferred to present first the nominal values so that the total value to be explained was equal to that published by the Central Bank.

² The following equation represents the simplest debt dynamics, where D is the total debt, i is the interest rate and X is public deficit: $D_t = D_{t-1}(1+i_t) + X_t$

Dividing by the GDP (Y), we obtain: $\frac{D_t}{Y_t} = \frac{D_{t-1}}{Y_{t-1}} \cdot \frac{Y_{t-1}}{Y_t} (1+i_t) + \frac{X_t}{Y_t} \Rightarrow d_t = d_{t-1} \cdot \frac{(1+i_t)}{(1+g_t)(1+\pi_t)} + x_t$, where

d and x are the total debt and the public deficit over GDP, g is the growth rate of real GDP and π is the rate of inflation.

Therefore, if the fiscal deficit is zero, the debt to GDP ratio will grow whenever the nominal interest rate in excess of nominal GDP growth is positive. This is equivalent to the real interest rate being higher than the real GDP growth rate.

if the debt is indexed to the interest rate or to other indices positively correlated with it, then the effect of the interest rate increase is either immediate or occurs sooner.

With this in mind, we may interpret the figures. The implicit excess interest rates were negative or low until 1997, and jumped upwards after the start of the crises period in that year with the Asian crisis. 1998 and 1999 were years of extremely high interest rates, both because of the very high interest rates, and because of the devaluation, which impacted the US\$-linked debt. In 2000, the implicit interest rate fell, and in 2001 it increased somewhat. Given that 2001 was also a crisis year, with successive interest rate increases in Brazil (the basic interest rate was raised from 15.25% in January to 19% in July), we may conclude that in the floating exchange rate regime, international crises no longer have such a heavy impact on the debt growth. Nevertheless, the figures for 2000 and 2001 are still cause for concern, since they are not as close to zero as required to provide stability to the debt to GDP ratio.

The recognition of existing debts (skeletons) added up to 12.87% of GDP, with the bulk of it occurring during the 1999-2001 period. One would hope that most of the skeletons would already be out of the closet. However, bad surprises still occur quite often, and it would be an excellent measure if the government could do an exhaustive job of opening every and each closet to convey to the market what the bad shocks in the future will be. More important, it should make sure that the new skeletons are not currently being manufactured. The fiscal responsibility law is a major deterrent against the creation of unfunded liabilities. However, the inventiveness of some public officials in bypassing the spirit of the law is always amazing, as it can be seen by the mushrooming of state and municipal pension plans. It is quite likely that as these insufficiently funded pension plans start to have more retirees, a new skeleton will come out of the closet.

Privatizations revenues accounted only to less than half of the recognition of existing debts (6.09% of GDP). As far as privatizations are concerned, the performance of the period 1999-2001 in comparison to the previous four-year period is not so good, reflecting the general slow-down in economic reforms that marked the second term of president Cardoso.

The asset accumulation (16.24% of GDP) was almost completely accounted for by the increase in domestic assets (16.02%), many of which contain large credit risk. The state debts that were renegotiated constituted the bulk of the domestic assets (14.09% of GDP).³ Foreign Reserves were kept almost constant as percentage of GDP, thereby making the

³ It is claimed that the recent history provides evidence showing that the Central Government (CG) has the means to enforce the states to fulfill their obligations. In fact, CG has withheld transfers to Minas Gerais in the famous episode of January, 1999, forcing the state to pay before sitting to the table to renegotiate. While this has certainly been true during the second Cardoso administration, many doubts remain on what will be the stance of the next federal administration. For example, the politician that has always complained about the burden imposed by the debt renegotiation and has, so far unsuccessfully, tried to renegotiate the debt under more favorable terms, is the mayor of São Paulo, who belongs to the same party of the front runner in the October presidential election. It seems only natural to doubt whether the current stance of keeping the original terms of the debt renegotiation will be kept in the event that the main opposition party winning. And, if the debt is once again renegotiated with one debtor, all others will want (and probably obtain) the more favorable terms. Given that the terms of the currently valid renegotiation are a key factor for the primary surplus maintenance, this is an extremely important risk factor for the fiscal stance of the next administration.

whole Asset Accumulation much less attractive as an indicator of solvency. This is one of the reasons why many analysts prefer to look at the gross debt, instead at the net debt as a measure of fiscal solvency.

Other debts also remained fairly stable, while the foreign debt increased 2.12% of GDP. This increase reflects basically the change in the real exchange rate after the 1999 devaluation, and not an increase in the foreign debt in US dollars.

Therefore, the picture that emerges from the analysis of the very large increase in the domestic bonded debt is one where the privatization revenues were insufficient to counteract the appearance of lagged fiscal deficits, in the form of the renegotiation of the state debts and other liabilities (skeletons), as well as the large interest payments. Those figures highlight the importance of avoiding the creation of new skeletons which may haunt the debt figures in the future, and of lowering the still very high interest rates.

3. Risk Measures for the Public Debt

Before introducing the risk measures, let us first complement the previous Section with a description of the debt composition during the Real Plan. Figure 1 displays the evolution of the federal domestic bonded debt structure since the start of the Real Plan, in July, 1994. Besides the very large and fast growth after mid-1995, whose determinants were analyzed in Section 2, the change in debt structure is also remarkable. Figure 2 displays the same data in a different graph format, so that the changes in composition are more clearly visible.

The US\$-linked bonds have grown during the whole period, both in real terms, as well as in percentage of the total domestic bonded debt. After an initial growth in the years following the beginning of the Real Plan, the share of pre-fixed (nominal) bonds have decreased dramatically, since the Russian crisis, being of small importance in the recent years, despite the iterated official intentions of issuing a larger share of nominal bonds.⁴ The place of the nominal bonds was taken by the zero-duration (Selic) bonds.⁵ These bonds constitute nowadays the majority of the domestic public bonded debt. Although the price-level-linked debt has increased its share in the total domestic bonded debt, its importance is still quite small. Other indexes account for some 5% of the debt.

⁴ See STN [2001], where the Treasury intended to have 22% of the total debt as nominal bonds until December 2001. The document is available at:

http://www.tesouro.fazenda.gov.br/hp/downloads/plano_financiamento_divida_8.4.pdf

⁵ The bond indexed to the short-run interest rate (Selic) is a security sold at a discount which had its face value corrected daily by the average daily interest rates during its term. It is a floating interest rate, adapted to the high frequency required by the high inflation and daily indexation conditions prevalent when it was created (1985). It would be equivalent to a bond whose nominal value is accrued every day by the daily accrual of the Libor. This is the closest one can get to perfect indexation in fixed income markets. It corresponds to a bond of duration zero (that being the reason why we call this type of bond **zero-duration bond**), since it practically does not suffer any price fall when interest rates go up. These bonds were widely used in times of high uncertainty, as, for example, the crossover to the Cruzado Plan in 1986. On the other hand, monetary policy has a very limited **wealth** effect, as far as public is concerned, since rises in interest rates do not affect the value of the private financial wealth in these fixed income securities.

As far as average remaining life is concerned, the debt has been lengthened quite substantially, as the nominal (short-term) bonds were replaced by the zero-duration bonds. However, average duration has not increased nearly quite as much. This discrepancy shows that refinancing (rollover) risk was given higher priority than market risk, as will be further discussed when we introduce a few risk measures.

The right risk measure to use depends on the agent's characteristics. The Brazilian Treasury defines in its Annual Borrowing Plan (Brazilian Treasury, 2001) the objective of debt management as to minimize long-term funding costs of the government under prudent risk limits. The risks comprise refinancing risk, market risk, foreign exchange event risk, and concentration risk (Fratzscher and del Valle, 2002). Private agents care about a different set of risks, e.g., market, credit and liquidity risks. Depending on the agent, the focus on one risk or the other may be different. Pension funds have a much longer horizon than money-market funds, which must provide daily liquidity to their participants. This means that not only liquidity risk is much more important to money-market funds, but also that the relevant volatilities used to measure market risk must be computed differently for these two agents, due to the different time horizon of each of them.

From the point of view of the debt issuer (the Brazilian Treasury) the goal, of course, is to use the different risk measures to provide policy recommendations on which securities to issue given the market conditions. The best way to construct such encompassing risk measure is probably through a multiperiod simulation model such as the CAR (Cost at Risk) used by several countries. The construction of such model is a task beyond the scope of this paper, and should be undertaken by the Brazilian Treasury in the near future. Last year, I supervised a student in constructing a prototype simulation model focused on debt composition (Azeredo, 2002). Here, a simple simulation model emphasizes the effects of shocks on the primary budget. These two aspects should be combined in a much more detailed model to serve as a policy tool to the Brazilian Treasury. The maturity structure, which is the key determinant for the rollover or refinancing risk, should also be modeled. Here, we provide a few measures of the recent evolution of the maturity structure, and discuss its relation to the debt composition, as well as the risks.

From the point of view of the debt holder, several different risk measures are available. The most widely used risk measure in the Brazilian markets is the [V@R](#) (value at risk). Although this measure focus exclusively on the short-term market risk, it is widely used, and it has become more so as regulation has required even investors with a longer time horizon to daily mark-to-market their portfolios. We provide here a measure of the [V@R](#) of the debt. Although the V@R does not adequately gauge the risk from the Treasury's perspective, it is a useful tool to measure the risk relevant to most market players, and, therefore, could be used as an ancillary tool to debt management.

[V@R](#) is defined as ... *the worst expected loss over a given horizon under normal market conditions at a given confidence level* (Jorion, 2001, p. xxii). The basic idea is to have one number that summarizes the risk involved in the overall portfolio of a financial institution. There are several different methodologies to generate this number. The easiest one is the delta-normal. It assumes that the returns of the different assets and liabilities of a portfolio

are multinormally distributed with zero mean.⁶ Since the portfolio's return is a weighted sum of the multinormally distributed individual returns, it is also normally distributed. Therefore, if we take the difference between the portfolio value today, and the 5th percentile of its distribution tomorrow, we obtain the worst expected loss over a day under normal market conditions at a 95% confidence level, i.e., it is expected that only in 5% of the times, the loss will be over the V@R limit.

In spite of not being the ideal risk measure from the government point of view, the public bonded debt V@R may provide many interesting insights. For example, the large nominal deficits registered in 1999 and 2001 were in great measure caused by the increase in value of the domestic debt due to the indexation clauses present in several bonds, both to the exchange rate and to the short term interest rate. Therefore, it is very important that the risk involved in the debt structure be adequately assessed, so that policy makers and the public can evaluate the true risk/reward tradeoff involved in public debt management.

The task of providing an assessment of the risks hidden in the indexed debt has become even more important after the second revision of the agreement with the IMF⁷ on 3/26/2002, when the Brazilian Central Bank was allowed to resume some trading in derivatives markets to rollover the existing exchange rate linked debt. Derivatives are off-balance-sheet items, usually with purchasing prices far below (zero for futures, forwards and swaps) the potential loss that they may entail. Therefore, the only way one may appraise the potential loss involved in these items is through a risk measure as the V@R.

In Section 3.1 we spell out the methodology to construct the public debt V@R, and show its evolution since the devaluation in 1999. Section 3.2 contains the description of the CF@R methodology, as well as the relevant numbers. Finally, Section 3.3 puts the two methodologies together to construct a single measure for risk assessment.

3.1. Value-at-Risk (V@R)

3.1.1. Nominal Bonds

The methodology to compute the V@R for nominal bonds is quite standard. Jorion [2001] is a good reference, and Appendix 1 details the *formulae* used and data sources.

TheV@R is constructed accordingly to the following steps. At any given date, we depart from the redemption schedule of the nominal bonds. Therefore, for each day, we have a list of future dates when coupons and/or the principals are repaid, with the present value (evaluated with the yield curve of that day) of those cash flows.

⁶ The multinormality assumption is clearly an heroic one that is undertaken because it greatly simplifies the computation, by avoiding the need to simulate.

⁷ See <http://www.imf.org/external/np/sec/pr/2001/pr0138.htm>

Given the history of interest rate variations, our goal is to compute the worst plausible outcome, defined as the 95th percentile⁸ of the distribution of the possible (stochastic) values of the total nominal debt in the following day. The standard V@R methodology assumes that the returns are multinormally distributed with zero means and standard deviations and covariances to be estimated from the data. Therefore, the distribution of the overall portfolio return, by virtue of being a linear convex combination of the multinormal returns (weighted by the respective present values) is also normal, allowing us to compute the 95th percentile with the help of a standard normal distribution table.

Since the redemption schedule contains payments arbitrarily spread over future dates, we must choose a few dates (time vertices) to concentrate the payments, so that we can compute variances (and covariances) for those yields (interest rates). Having done that, for each day, we must also compute the variance covariance matrix for the time vertices. This is done through the exponentially weighted moving average (EWMA) model. [Jorion (2001), pages 193-196]

Figure 3 shows the evolution of the volatilities (standard-deviations) of the daily interest rates for the following time vertices: 5 days, 20 days, 40 days, 60 days, 80 days, 100 days, 150 days, 200 days, 250 days. To compute those volatilities, we were forced to use data from the derivatives markets at the BM&F—The Brazilian Commodities and Futures Exchange—, since there are no liquid secondary markets for government bonds in Brazil. Therefore, our calculations must be interpreted as an approximation that excludes liquidity risk, and most of the credit risk. This is because lack of liquidity may cause the actual value loss when trying to sell a government security in any given day to be larger than the one implied by the movements in interest rates. Furthermore, non-deliverable derivatives, as the ones used at the BM&F, carry much less credit risk than government bonds, because only the changes in yields generate cash flows that are backed by collateral (mostly in government bonds). In other words, in both cases, the credit risk is associated with public debt default. However, in such a credit event, public debt holders would lose 100%, while derivatives holders would only lose what they have gained, which is usually much less than the derivatives notional value.

Figure 3 makes clear that the longer the period, the higher the volatility, so that the volatility yield curve would always be positively sloped. It also displays a pattern where spikes in all volatilities are followed by a decrease until another spike is reached. This pattern is due to the EWMA model.

For example, look at the beginning of the period, January 13, 1999, when the Real was floated. When that happened, interest rates for all vertices shot up, and that shows up in the increased volatility. The same data point of 1/13/99 also appears in the computation of the following day volatility, but with a lower weight (we used 0.95 as the decay factor). Therefore, until another shock makes interest rates increase a lot, the volatilities display a long-term mean reversion pattern. In that respect, it is interesting to note that until the end

⁸ We use here the 95th percentile instead of the 5th percentile because for the government what constitutes risk is a higher debt, while for banks' portfolios, the risk is a lower portfolio value. Given the symmetry of the normal distribution, the result is exactly the same.

of 2001 the long-term averages have not reverted to the lower level that prevailed until the first quarter of 2001, when the situation in Argentina worsened substantially.

Figure 4 shows both the total value of the nominal bonds (RHS scale), as well as the V@R (LHS scale), defined as the difference between the 95th and the 50th percentiles, corresponding to the worst plausible daily increase in the nominal debt value. The V@R increases fivefold after the floating (due to the increase in the volatilities), reaching over half billion R\$ falling afterwards until the end of 1999. In the periods when the volatilities are falling, the V@R responds more clearly to the change in the total value of the debt. After March, 2001, when Brazil was hit by the Argentina contagion, the V@R shot back to half billion R\$, although the total value was three times larger than in January, 1999.

Figure 5 computes the [V@R](#) as a percentage of the total nominal debt. The picture shows that the immediately after the devaluation, the interest rate variations were so high that the daily [V@R](#) was around 1.5%, an extremely high figure, even for a variable income portfolio, let alone a fixed income one denominated in the domestic currency. Other peaks occurred, as analyzed in the previous paragraphs, but the % [V@R](#) never went above half of the initial peak. The main reason for the decrease in the maximum [V@R](#) is that, in the floating exchange rate regime, the impact of crises (negative external shocks) are jointly shared by the interest and the exchange rate. For example, in March 2001, not only interest rates were increased, but also the exchange rate depreciated.

Notwithstanding the decrease in the interest volatility, it remains quite high, being a fundamental deterrent to the lengthening of the nominal debt. A back-of-the-envelope calculation help clarify the point. In Brazil, the interest rate volatility remains high enough even for the short maturities traded nowadays. Since the sensitivity of bond prices to the interest rate may be well approximated by the duration, which is similar to the maturity, we may conclude that if there were markets for longer term nominal bonds (say, five or ten years), the [V@R](#) would be much higher than variable income markets, defeating the very purpose of investing in fixed income securities. Therefore, it remains a tough, if not impossible, challenge to lengthen the debt with nominal bonds in the current Brazilian macroeconomic conditions.

3.1.2. Exchange-Rate-Linked Bonds

The computation of the [V@R](#) for US dollar-linked bonds is similar to the one for nominal bonds. Appendix 1 contains a detailed description of the calculation process. Here we shall emphasize the intuition. Suppose we were working with returns in US dollars. If that were the case, the calculation would be the same as for the nominal bonds, explained before. However, we are working with returns in R\$. Therefore, we should also consider the volatility of the exchange rate. Since [V@R](#) is about computing variances, we have to take account of the covariances between the returns of the exchange rate (the rate of depreciation) and the yields of the dollar-linked bonds for the several vertices (maturities).

Figure 6 displays the volatility yield curve for the returns of the dollar-linked bonds. Since there are no liquid secondary markets for these bonds in Brazil, the volatilities are inferred from the prices of derivative securities at the BM&F—The Brazilian Commodities and

Futures Exchange—, and, as before, must be interpreted as approximations that exclude the liquidity risk.

The extremely high volatilities at the beginning of the sample were caused by the very wild fluctuations of the US dollar during the first weeks of the depreciation, when it overshot from 1.21 R\$ to 2.12 R\$. In the derivatives market we used to get the data, the interest rate paid by a hypothetical dollar-linked-bond is determined by subtracting the forward premium (the expected depreciation plus the exchange rate risk) from the domestic interest rate. When the exchange rate is varying a lot (see Figure 7), that causes these implied rates to move a lot.⁹ Figure 7 shows the volatility of the spot US\$/R\$ daily exchange rate. Incidentally, these figures do not seem to corroborate any “fear of floating” in Brazil.

Figure 8 displays the [V@R](#) for the dollar-linked-bonds in % of the total dollar-linked debt. It is substantially higher than the nominal bond [V@R](#). This is a natural corollary of the fact that the risk here is measured against the basic interest rate in R\$ (the Selic), and not against a return in US\$.

Although the period covered in this study does not include the resumption of the Central Bank trading in foreign exchange derivatives, the [V@R](#) measure just described is a very natural way to convey the increase in risk when the Central Bank issues foreign exchange swaps.

3.1.3. Zero-Duration Bonds

Zero-duration bonds, by definition, bear no market risk. This is because this bond is redeemed by the initial value capitalized by the accumulation of the daily basic interest rate (Selic). Since the present value of any asset is computed by discounting its redemption value by the accumulation of the daily Selic rates, the present value of this asset is the same, no matter what is the path of the Selic rate.¹⁰ Therefore, this bond bears no market risk.

The reader familiar with the macroeconomic literature on optimal taxation may find quite strange to attribute zero risk to bonds that immediately start paying higher interest rates when there is an interest shock. After all, a positive interest shock coupled with a debt structure heavily weighted in zero-duration bonds is bound to generate a heavy burden to the fiscal budget, negatively impacting the necessary taxation. This only highlights the fact that the perception of risk to market players is different from the one relevant to the government. Nevertheless, it is possible to adapt the threshold against which the [V@R](#) is measured. Government revenues growth would be an ideal however infeasible threshold. The same applies to nominal GDP growth. Inflation would probably be the best candidate,

⁹ In spite of this plausible explanation, we are trying to get hold of an alternative dataset to check the numbers.

¹⁰ Of course, if the Selic rate were to be kept at a very low level, incompatible with the interest parity conditions, it would lose its place as a market benchmark, and the zero duration bond would no longer be considered devoid of interest rate risk. That would be a different story from the recent increase in the perception of the credit risk of these bonds, which are analyzed later.

given the availability of price level data. However, returns on price-level-linked bonds are very hard to come by, as explained next.

3.1.4. Price-Level-Linked Bonds

To compute the risk of price-level-linked bonds is a really difficult task. This is because on top of the inexistence of a liquid market for price-level-linked bonds, the derivatives market for the yields on those bonds is not only very illiquid, but is also very new. Therefore, we have very little data to work with. Appendix 1 describes all the hypotheses made to achieve proxies for the $V@R$ of the price-level-linked bonds.

Figure 9 displays the first approximation (proxy 1) to the price-level-linked bonds $V@R$. It considers only the inflation volatility, neglecting the possible yield (real interest rate) variation. Given the low inflation volatility, proxy 1 leads to very low $V@R$ measures.

Proxies 2, 3 and 4 try to take into account the volatility of the yields, which is much higher than inflation volatility. As a result, the $V@R$ figures grow more than tenfold.

The $V@Rs$ as percentages of the total price-level-linked debt are displayed in Figure 11, together with the $V@R$ for the nominal and exchange-rate-linked bonds. We see that the proxies 2, 3 and 4 for the price-level-linked bonds are much higher than the other two for the months after the devaluation, while proxy 1 is lower. Probably, the true risk measure is somewhere in between proxy 1 and the other three proxies.

3.1.5. Total $V@R$

To compute the total $V@R$ we must basically compute a variance of the overall return. That requires accounting for the diversification effects that arise within the portfolio. For that, we would need all the covariances between all the risk factor considered previously, e.g., the covariance between the yield of a dollar-linked bond of 250-day maturity and the yield of a nominal bond of 20-day maturity. The data required for some of those covariances are not available. Furthermore, by assuming extreme assumptions, i.e., correlations equal to +1 and -1 between all variables, we may get the lower and upper bounds of the total $V@R$.

Figure 12 displays the total $V@R$ together with the debt figures (the zero-duration debt is excluded because it bears no market risk). Figure 13 displays the $V@R$ as % of the debt. We see that after being very high immediately after the devaluation, the $V@R$ decrease during 2000, and rose again in 2001, hovering below 1% per day at that year-end.

Figure 14 displays the $V@R$ as a percentage of GDP. We see the same pattern, with the $V@R$ decreasing after the devaluation from 0.6% to 0.1% of GDP by the end of 2000. During 2001, the $V@R$ increases again, doubling by the third quarter, when it flattened and fell a little. Figure 15 displays the (daily) $V@R$ as a % of (monthly) treasury revenues. The lines follow the same pattern, but the magnitudes become more telling. After reaching

almost 30% of the revenues, the V@R was, at the end of the sample, at the 10% of monthly revenues level. That means that the daily V@R was almost three times the daily treasury revenue, which is probably a very large magnitude.

As commented before, this measure is probably misleading as the relevant measure for the government. This is because when the interest rates rise, the interest payments related to the zero-duration bonds also rise, but the present value of these bonds do not, implying a zero market risk. The V@R is a good measure of the risk born by the private sector in holding the domestic public debt. Therefore, during 2001, not only the debt size grew substantially (see Figure 1), but the V@R as a percentage of the total debt also doubled, thereby increasing a lot the risk born the private sector in holding it. This is compatible with the increase in the implicit excess interest rates computed in Section 2 (Table 2.3).

3.2. Cash-Flow at Risk (CF@R)

The risk factors analyzed in the last subsections impact also the government's cash flows. For example, fluctuations in risk factors as exchange and interest rates affect other components of the fiscal accounts besides the public debt. For example, the gains obtained by private agents that purchased exchange rate linked bonds in times of depreciation are partially taxed away through the income tax. This appears as an increase in tax revenues in times of depreciation. A V@R for the public debt would overestimate the negative impact of exchange rate depreciation on the fiscal accounts, because it would miss the increase in income tax. To correct this flaw, we borrow the concept of Cash Flow At Risk from the corporate literature.

Non-financial enterprises are not so well characterized by their portfolios. To assess the risk, it is more important to quantify the impact of the risk factor in the profits and losses of the firms. For example, a large depreciation is bound to have a substantial negative effect in the future profits of an import firm, notwithstanding the fact that the immediate impact on its portfolio could be a positive one. The Cash Flow at Risk (CF@R) takes account of the impact on the firm's cash flow (Jorion, 2001, p. 366). The CF@R methodology (see Jorion, 2001, p. 366) requires the following steps:

- 1) Compute the exposures of the cash flows to the risk factors;
- 2) Model the behavior of the risk factors; and
- 3) Simulate the risk factors and get the distribution of the resulting cash flows. The CF@R will be the difference between the 50th and the 5th percentile of that distribution.

In order to get item (1) above, we ran a VAR (Vector Auto-Regression) in proxies of the following five variables: real exchange rate, real interest rate, inflation, GDP growth, and primary surplus to GDP ratio. Non-stationary behavior was identified in a few of the series. This is probably due to the significant changes that fiscal and exchange rate policies underwent within the period. After the floating of the exchange rate in January, 1999,

inflation shot up and has decreased afterwards, in line with a falling schedule of inflation targets. To account for the negative trend while inflation was converging to the new lower level, we constructed a variable—the **inflation gap**—that measures the deviation of actual inflation from the target, which is computed by interpolating the two targets for adjacent years. A similar thing was done with the fiscal surplus to GDP ratio. Since the last quarter of 1998, Brazil has an agreement with the IMF that promises to fulfill certain targets for the primary surplus, among other requirements. During 1999, the primary fiscal surplus to GDP ratio increased from zero to around the 3.5% level where it has been kept until today. Also, to account for the trend, we constructed a variable—the **fiscal gap**—that measures the deviation of actual surplus from the target, which is computed by interpolating the two targets for adjacent years. From the definitions used, the larger the inflation gap, the higher the inflation; and the larger the fiscal gap, the lower the primary surplus. In other words, positive values for the inflation and fiscal gaps mean that the targets are not being fulfilled. The results are in Table **3.1**. Appendix **2** contains the time-series variables charts.

The risk factors in this case are the contemporaneous shocks to the variables. We assume they are multnormally distributed with variances and covariances equal to those estimated through the VAR.

Finally, we use Monte Carlo simulation to get item (3) and compute the [CF@R](#). The results are drawn in Figures **16** and **17**.

Figure 16 displays the histogram of the variable used to proxy for the primary surplus—the fiscal gap. As explained before, when the fiscal gap is negative, the primary surplus target (currently at 3.5% of GDP) is surpassed. We simulated 1,000 one-month-ahead scenarios. The little dots represent the 5th and the 95th percentiles. We are interested in the latter, since higher results mean lower primary surpluses. We see that the primary surplus target is not at all in jeopardy when one considers the shocks to the exchange rate, the interest rate, the GDP growth, and inflation. On the contrary, those shocks tend to increase the primary surplus (the starting value was zero).

If we keep shocking the system for 12 months, we get a slightly different result. Figure 17 shows this case. The distribution of the fiscal gap is more spread, as expected, but the 95th percentile is still negative, meaning that obtaining the fiscal target is not a problem if the past performance is kept. Since the fiscal performance for the period analyzed, 1999-2001, has been impeccable, this is not a surprise. Also, since we do not analyze the fiscal accounts, but only the impact of the shocks to the exchange rate, the interest rate, the GDP growth, and inflation on the primary surplus, neither serves the result as an assurance that the fiscal stance will be kept in the future.

Table 3.1
Vector Auto-Regression Estimation

Sample: 1999:05 2002:01
Included observations: 33
Standard errors & t-statistics in parentheses

	DRER	Real Interest Rate	Inflation Gap	Output Growth	Fiscal Gap
DRER(-1)	0.433689 (0.17612) (2.46246)	0.152833 (0.14628) (1.04479)	0.028182 (0.02919) (0.96540)	-0.262320 (0.09512) (-2.75767)	-0.039040 (0.00962) (-4.05936)
Real Interest Rate (-1)	-0.109766 (0.17517) (-0.62663)	0.710234 (0.14549) (4.88164)	0.008366 (0.02903) (0.28815)	-0.062864 (0.09461) (-0.66445)	-0.010766 (0.00957) (-1.12557)
Inflation Gap(-1)	-0.399522 (0.66246) (-0.60309)	0.528311 (0.55022) (0.96017)	0.959459 (0.10980) (8.73800)	-0.582926 (0.35780) (-1.62919)	-0.085243 (0.03617) (-2.35644)
Output Growth(-1)	-0.140137 (0.28852) (-0.48571)	0.027067 (0.23964) (0.11295)	-0.014530 (0.04782) (-0.30384)	-0.559655 (0.15583) (-3.59140)	-0.020766 (0.01575) (-1.31804)
Fiscal Gap(-1)	3.699937 (2.56685) (1.44143)	3.047467 (2.13196) (1.42942)	-0.619063 (0.42545) (-1.45506)	-2.087710 (1.38638) (-1.50588)	0.287218 (0.14017) (2.04914)
C	0.030247 (0.02199) (1.37525)	0.030481 (0.01827) (1.66861)	-0.001198 (0.00365) (-0.32867)	0.011345 (0.01188) (0.95505)	-0.000124 (0.00120) (-0.10354)
R-squared	0.322041	0.568699	0.877845	0.404351	0.637631
Adj. R-squared	0.196493	0.488829	0.855224	0.294045	0.570525
Sum sq. resids	0.024557	0.016941	0.000675	0.007164	7.32E-05
S.E. equation	0.030158	0.025049	0.004999	0.016289	0.001647
F-statistic	2.565080	7.120266	38.80624	3.665736	9.501918
Log likelihood	72.02900	78.15509	131.3391	92.35659	167.9802
Akaike AIC	-4.001758	-4.373035	-7.596311	-5.233732	-9.816979
Schwarz SC	-3.729665	-4.100943	-7.324218	-4.961640	-9.544887
Mean dependent	0.007124	0.100634	0.008873	0.003310	-0.002881
S.D. dependent	0.033644	0.035035	0.013137	0.019386	0.002513
Determinant Residual Covariance	1.79E-21				
Log Likelihood	554.1592				
Akaike Information Criteria	-31.76722				
Schwarz Criteria	-30.40676				

3.3. V@R and CF@R Together

We may now consider the two measures together, the [V@R](#) and the [CF@R](#). As explained in Section 2.2, considering the impact of the shocks to the exchange rate, the interest rate, the GDP growth, and inflation tends to improve the primary surplus. Therefore, it would tend to lower the budgetary impact of negative shocks that increase the debt. In other words, the primary surplus tends to act as a shock absorber (albeit a weak one) to the increase in the debt stemming from shocks to the exchange rate, the interest rate, the GDP growth, and inflation. This may be explained, for example, by the extra income tax that the recipients of the higher interest rates that are paid on government debt when the exchange rate depreciates or the basic interest rate (Selic) is raised must pay. However, to determine exactly where this increase in the primary surplus comes from it would be required a study of the fiscal accounts, which is beyond the scope of this paper.

A very interesting complement to this study would be to consider a [V@R](#) measure through Monte Carlo simulation. This would allow us to simulate together both the primary surplus, as done through the [CF@R](#), and the debt. This procedure would provide a consistent joint measure of total risk implied to the fiscal accounts by the shocks to the exchange rate, the interest rate, the GDP growth, and inflation.

4. Rollover Risk

The policymaker's decision of what kind of debt to float (denomination, indexation, and maturity) may be described as follows. Given the government's objective function, the debt manager has to decide which bonds and in what quantities to float. The debt manager maximizes the government's objective function based on the history of the rates of return of the several bonds and their statistical properties (expected return, variance, etc.).¹¹

This maximization problem may be interpreted as the symmetric of the portfolio allocation decision, in which the investor decides his portfolio composition by maximizing his utility function defined over wealth or consumption. Several models of portfolio allocation are available, the most famous being the Mean-Variance analysis of Markowitz [1952].¹²

Only very strict hypotheses may justify that expected utility be defined exclusively over expected returns and variances for arbitrary distributions of returns and utility functions.¹³ Nevertheless, Mean-Variance (MV) analysis, since its development by Markowitz fifty years ago, has become by far the most widely known principle of portfolio allocation.

Here we adapt the MV analysis for the public debt manager problem. Two features of this adaptation are worth of noting. First, the expected return for the bond holder is converted in expected cost for the debt manager. Therefore, the debt manager dislikes higher expected return. Second, the safest asset for the bondholder (let's say, the asset perfectly indexed to consumption) is the riskiest for the government. What happens here is that the risk is shifted from one side to the other.

Another important risk source that is considered by the debt manager is the rollover risk. Several studies emphasize the importance of not allowing large portions of the public debt to mature at the same time, since that may expose the government to pay abnormally high rates of return to roll the debt over, or even be forced to monetize the domestic debt or default on the foreign debt. Therefore, lengthening the debt maturity is also an objective of the debt manager in order to avoid the rollover risk. We posit a very simple way to model

¹¹ Missale [1999] describes several approaches to the debt management problem.

¹² The model requires very restrictive assumptions, which are contradicted by realities in Brazil: markets are not perfect (taxes, regulations, short selling), investors are not rational price takers, there is no risk-free asset with unlimited borrowing, assets are not fully liquid and divisible.

¹³ As shown by Huang and Litzenberger [1988], pp. 60-62, there are basically two ways to justify the mean-variance approach. First, for arbitrary distributions of returns, quadratic utility would suffice for expected utility to be defined only over the mean and the variance of the rates of return. Unfortunately, quadratic utility also implies satiation and increasing absolute risk aversion, which are undesirable properties, since most individuals are believed to prefer more wealth to less and to treat risky investments as normal goods. Second, for arbitrary preferences, the mean-variance model would also follow from the assumption that the rates of return on risky assets are multivariate normally distributed (this is a sufficient, not a necessary condition). Normal distributions are unbounded from below, which is inconsistent with limited liability and economic theory, which attributes no meaning to negative consumption. Also, rates of return are known to possess skewed and leptokurtic (fat tails) distributions, which is not the case of the normal distribution (Campbell, Lo, and Mackinlay [1997], pp. 16-19).

this rollover risk that is compatible with MV analysis, so that we can still rely on its well-known mathematics to develop the policy implications.

4.1. Mean-Variance with Rollover Risk

The goal is to adapt the widely used MV analysis of Markowitz [1952] to the debt management problem, also incorporating the rollover risk. In order to do that, we will resort to an example with two assets, and then will generalize the problem to three or more kinds of bonds.

4.1.1. An example with two assets

Suppose there are only two bonds. The nominal bond is a regular zero-coupon bond. Its return in domestic currency, R\$, is known in advance. The other kind of bond is the floating bond, whose return varies with the interest rate. For the sake of this example, we assume the following parameter values:

TABLE 4.1: PARAMETERS VALUES

Bond kind	Expected Return (negative)	Standard Deviation	Maturity (months)
Nominal	-10%	1%	1
Floating	-8%	5%	36

We also assume that the rates of return on both assets have zero correlation. Note that the governments dislikes expected return, therefore, *ceteris paribus*, the debt manager would like to maximize the negative of the expected return, which corresponds to cost minimization. This, of course, is the symmetric of the standard investor attitude, which seeks to maximize the expected return.

Note also that the floating bond volatility (standard deviation) is higher than the nominal bond's, which may sound counterintuitive despite the difference in maturities. The explanation, besides the longer maturity of the floating bond, is that for the government what counts is the volatility of the **deflated future value at the maturity**, not the volatility of the marked-to-market price (the volatility of the present discounted value of the bond). For example, take a floating bond perfectly indexed to the interest rate, as the zero-duration bond. When the interest rate rises, the bond's present discounted value does not change. However, the amount in R\$ to be disbursed at maturity increases substantially, and, if

inflation remains stable, so does the real value of the disbursement. This is assumed to be the volatility (market risk) that matters for the government.¹⁴

Again, as in the case of expected return, the government objective function is the opposite of the investor. However, since variance is independent of the deviation from the mean sign, it is nonsense to change the sign, as we did with the expected return. The adaptation that makes sense is to realize that an indexed bond, i.e., a bond whose present discounted value varies very little, thereby being a safe investment for the holder, is very risky for the issuer, i.e., the government. That is what is accomplished by measuring the standard deviation in the way sketched above.

Therefore, one could approximate the standard deviation of the nominal bond by the standard deviation of monthly inflation, and the standard deviation of the floating bond by the standard deviation of the three-year **real** interest rate. The numbers in the example are merely for illustrative purposes.

With these adaptations, the MV diagram for the government is displayed in Chart 1. The governments' indifference curves should be positively sloped and convex, with the government's objective function increasing as the curves move toward the northwest. Therefore, the efficient set is formed by all combinations (portfolios) of bonds that are above the minimum-variance portfolio, as it is the case in standard MV analysis.

4.1.2. Rollover Risk and the Minimum Degree of Indexation

So far the adaptations made in MV analysis are fairly mild. Now, we introduce a new source of risk, the refinancing (rollover) risk. It is the risk that the debt manager may be placed in a corner when she needs to rollover a large portion of the debt, thereby having to offer extremely high yields (low bond prices). Ideally, such risk should show up in the rate of return distributions, i.e., the probability distribution of bond returns should incorporate these "corner" events. Here, we take a short-cut, that may correspond to a distribution which incorporates these corner events.

The rollover risk depends positively on how well spread through time the bonds' maturities are. The more spread apart they are, the lower the risk that the debt manager be placed in a corner. A proxy for how well spread the bonds' maturities are is the average maturity of the bond. The issuance of very short-maturity bonds tends to concentrate the bonds' redemption, e.g., if only one-month bills were issued, the whole public debt would eventually mature within the following month.

Therefore, one possibility for modeling the rates of return distribution of public bonds is that its variance varies according to the average maturity of the debt stock. The higher the debt average maturity, the lower the rollover risk, and the lower the variance of the bonds' returns at the placement auctions. We could call this model HCDM, for Heteroskedasticity Conditional on Debt Maturity.

¹⁴ We could alternatively, adapt the model to other risk factors by measuring the volatility of the ratio of the bond price to the GDP, or even the ratio of the bond price to the fiscal revenues.

With this interpretation in mind, we go one step further and model this conditional heteroskedasticity by adding to the variance a quadratic term that accounts for the debt maturity. This quadratic term is decreasing on debt maturity, being zero when maturity is the highest possible. Therefore, for our two-bond example, the negative of the expected cost and the modified variance are:

$$-\text{Expected Cost} = -[\alpha E(R_n) + (1-\alpha)E(R_f)] \quad ; \quad (1)$$

$$\begin{aligned} \text{Modified Variance} = & \left\{ \left[\alpha^2 s^2(R_n) + (1-\alpha)^2 s^2(R_f) + 2\alpha(1-\alpha)rs(R_n)s(R_f) \right] \right. \\ & \left. + h \left[\alpha M_{R_n} + (1-\alpha)M_{R_f} - M_{R_f} \right]^2 \right\} \quad ; \quad (2) \end{aligned}$$

where,

R_N = nominal bond return

R_F = floating bond return

α = portfolio weight on the nominal bond

$(1-\alpha)$ = portfolio weight on the floating bond

M_{R_N} = nominal bond maturity

M_{R_F} = floating bond maturity

η = rollover risk weight

$E(\cdot)$ = return's expected value

$F(\cdot)$ = return's standard deviation

D = returns' correlation coefficient

The parameter η is the weight that incorporates to the variance the effect of the rollover risk. Chart 2 shows how the incorporation of rollover risk affects the MV analysis. The curve labeled $\eta=0$ is the one of Chart 1. As η increases, the risk, as measured by the modified standard deviation, also increases for all portfolios but the one with 100% allocated in the longest maturity bond. For $\eta=0.000002$, the risk of the 100% short-term portfolio equals the risk of 100% long-term portfolio. For $\eta>0.000002$, the risk of the 100% short-term portfolio exceeds the risk of 100% long-term portfolio. As η keeps increasing, the rollover risk becomes completely dominant, as shown in Chart 3.

Chart 4 displays the same data in the space Maturity vs. Standard Deviation. Since the portfolio maturity is also a linear convex combination of the two bonds' maturities, as it is the case of the expected returns, Chart 4 has the same shape as Chart 2.

One interesting feature displayed by Charts 2 and 4 is that the efficient set decreases as η increases. That means that the number of acceptable debt structures decreases as the rollover risk becomes increasingly important for the government, as, for example, it would be the case in a contagion model where a neighbor country got hit by a negative shock.

This seems to be precisely what happened in Brazil during the Russian crisis, when the government became increased fearful of not being able to roll over its very short maturity

debt. Figures 1 and 2 show that after May, 1998, the nominal (shorter maturity) bonds were replaced by the zero-duration (longer maturity) bonds. This caused the average maturity to increase, lowering the rollover risk. The market risk, however, increased substantially. When the interest rates were raised—firstly to counteract the succession of speculative attacks that eventually led to the devaluation of January, 1999, and secondly to avoid the exchange rate overshooting that followed—, the risk turned into reality, and the debt increased much more than it would have increased were it composed in its majority by nominal (nominal) bonds.

TABLE 4.2: MINIMUM INDEXATION AND MINIMUM MATURITY

η	Minimum Portfolio Weight On Floating Bonds	Minimum Maturity (months)
0.000000000	4%	2.4
0.000001000	35%	13.3
0.000002000	50%	18.5
0.000005000	72%	26.2
0.000009000	82%	29.7
0.000010000	83%	30.1
0.000100000	98%	35.3
0.000406041	100%	36

What happened in Brazil may be modeled by an increase in the parameter η . The debt structures that leave the efficient set as η increases are the ones with more nominal and less floating bonds. As the rollover risk becomes more important, the “optimal” debt structure tends to display a longer maturity, precisely to diffuse the rollover risk. Table 2 shows this characteristic of the model. With η sufficiently high (equal to 0.000406041 and above), only 100% floating bonds portfolios are acceptable.

4.2. The Problem with Multiple Bonds

In the two-bond case, we saw that the "modified" variance was:

$$\begin{aligned}
 \text{Modified Variance} &= \left\{ \left[\alpha^2 \sigma^2(R_N) + (1-\alpha)^2 \sigma^2(R_F) + 2\rho\sigma(R_N)\sigma(R_F)\alpha(1-\alpha) \right] + \right. \\
 &\quad \left. + \eta \left[(\alpha M_{RN} + (1-\alpha)M_{RF} - M_{RF}) \right] \right\} = \\
 &= \left\{ \left[\alpha^2 \sigma^2(R_N) + (1-\alpha)^2 \sigma^2(R_F) + 2\rho\sigma(R_N)\sigma(R_F)\alpha(1-\alpha) \right] + \right. \\
 &\quad \left. + \eta \left[\alpha^2 (M_{RN} - M_{RF})^2 \right] \right\} = \\
 &= \left\{ \alpha^2 \left[\sigma^2(R_N) + \eta (M_{RN} - M_{RF})^2 \right] + (1-\alpha)^2 \sigma^2(R_F) + 2\rho\sigma(R_N)\sigma(R_F)\alpha(1-\alpha) \right\} \quad (3)
 \end{aligned}$$

With 3 bonds, the third bond being the longest, the term that is added to the variance is:

$$\begin{aligned}
 & \eta [w_1 M_1 + w_2 M_2 + (1 - w_1 - w_2) M_3 - M_3]^2 = \\
 & = \eta [w_1 M_1 + w_2 M_2 - (w_1 + w_2) M_3]^2 = \\
 & = \eta [w_1 (M_1 - M_3) + w_2 (M_2 - M_3)]^2 = \\
 & = \eta \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} (M_1 - M_3)^2 & (M_1 - M_3)(M_2 - M_3) \\ (M_1 - M_3)(M_2 - M_3) & (M_2 - M_3)^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (4)
 \end{aligned}$$

Therefore, it is easy to see that, in order to compute the "modified" variance, all one needs to do is:

- 1 - reorder the n bond kinds, so that the last one is the longest;
- 2 - take the original variance-covariance matrix, Ω ($n \times n$), and consider the principal minor ($(n-1) \times (n-1)$) formed by elimination of the last row and column;
- 3 - to each cell ($\Omega_{i,j}$) of the principal minor add $[\eta(M_i - M_n)(M_j - M_n)]$, $i < n, j > n$;
- 4 - put back the n^{th} row and n^{th} column that had been previously eliminated to get the modified variance-covariance matrix, Ω_{MOD} ;

- 5 - the modified variance is simply

$$\text{Modified Variance} = \mathbf{l}' \Omega_{\text{MOD}} \mathbf{l}$$

where,

\mathbf{l} = vector ($n \times 1$) of portfolio weights;

Ω_{MOD} = modified variance-covariance matrix.

Now, all we need to do is to prove that the modified variance-covariance matrix is positive definite. This is easily accomplished by noting that the modified variance is obtained by adding to the original variance (itself a quadratic form with a positive definite matrix) a quadratic term that is greater than zero whenever all bonds are not of the same (i.e., the longest) maturity. Therefore, the modified variance-covariance matrix must also be positive definite.

With the modified variance-covariance matrix being a legitimate positive-definite variance-covariance matrix, all the MV results go through as if we were dealing with a standard

variance-covariance matrix. We may, thus, rely on the large set of results concerning portfolio allocations for multiple assets with Mean-Variance preferences.

4.3. Maturity Structure and Rollover Risk: Recent Evolution

Until May 2002, the Treasury was following a strategy of mitigating the rollover risk by lengthening the debt through the use of indexed bonds. Figure 18 shows the federal bonded debt redemption schedules in January 2000; January 2001; January 2002; and April 2002 (the last available at the time of writing).

Figure 19 shows the same data in a friendlier format. It accumulates the debt of Figure 18, and represents it as a percentage of the total debt. Therefore, it can be read as a cumulative distribution function. If one wants to know what is the share of the debt that matures until six months, all one has to do is to read the value on the y-axis that corresponds to the 6 Months on the x-axis. A longer and better-distributed debt profile is represented in figure 19 by a curve that lies below the other one. Therefore, as mentioned before, one can see that until April 2002, successes were being achieved in lengthening the debt. This relative success was paid with the increase in risk measured in section 3. However, all this effort is currently being lost with the increase in macroeconomic risk that started to worsen considerably in May, 2002.

5. Monetary Policy and Public Debt Management

In this Section we analyze a few issues pertaining to the overlapping of monetary policy and public debt management. In every country both policies are related. However, this is more so in Brazil, because of the domestic currency substitution process that characterized the megainflation of the 80s and the first half of the 90s.

Regular currency substitution was avoided through the provision of regular bank deposits that were protected from inflation. Those deposits, which were considered as money and had daily liquidity, were backed by government debt. Monetary policy became completely passive because it could not jeopardize the domestic currency substitution mechanism by raising interest rates. Although this state of affairs has changed substantially after the Real Plan, a few characteristics of today's monetary operation mechanism are inherited from that period.

5.1. Monetary Policy Regimes and the Demand for Debt

As analyzed elsewhere,¹⁵ Brazil was able to retain a fairly stable demand for its national currency during the megainflation years through the provision of (domestic) currency substitutes protected from inflation erosion. In those years, the Central Bank monetary policy was restricted to provide a positive and not very volatile real interest rate. Financial intermediaries would carry government bonds in their balance sheets and provide money market accounts that were widely perceived as being protected from inflation, unlike the regular currency. Were the Central Bank to raise interest rate to deter the inflation, it would impinge large losses to the financial intermediaries, thereby jeopardizing their ability to provide inflation protected domestic currency substitutes. Not surprisingly, the monetary policy was completely accommodative as inflation drifted upwards until it was successfully stopped by the Real Plan of July 1994.

Since monetary policy was *de facto* precluded from exerting its main goal, i.e., to fight inflation through the interest rate management, debt managers engineered the zero-duration bonds (see Section 3) to save the volatility premium that appeared in the bonds' auctions. In other words, financial institutions would purchase short-term nominal debt with a sizeable discount because of the interest rate risk. Note that the interest rate risk during megainflation is essentially driven by the jumps in inflation expectations, which are much higher than the changes in the real interest rate. With zero-duration bonds the interest rate risk was eliminated, and the government could sell bonds at a higher price.

However, with the zero-duration bonds, monetary policy becomes completely devoided of any wealth effect. Interest rates may rise or fall, and the present value of the zero-duration debt will remain constant. Of course, this (tautological) statement has to be taken with a grain of salt. After all, if the domestic interest rate were to fall too much, violating the bounds

¹⁵ See Garcia [1996].

imposed by interest parity conditions, a capital outflow would result, since the domestic interest rate would no longer serve as trusted benchmark.

After the Real Plan, financial intermediaries remained addicted to government bonds whose prices have very low of zero volatility. Until 1997 (see Figure 1), the lengthening through nominal debt proceeded, only to be interrupted by the Asian crisis. Increases of more than 2000 basis points in the basic interest rate happened a few times until 1999, all but killing the prospects of a demand for long nominal bonds. Although in the current floating rate regime the exchange rate also serves as a shock absorber, thereby decreasing the interest rate volatility, the lengthening of the nominal debt has yet to reach the two-year maturity that was being auctioned just before the Asian crisis.

Financial intermediaries used to look for the zero duration bonds that have no market risk so that they could provide money market funds whose yields track the basic interest rate benchmark (the Selic rate). Quite recently, however, given the introduction of stricter rules forcing the fund industry to observe mark-to-market practices, as well as the uncertainty pertaining to the electoral process (will the next president tamper with the public debt?), even the zero-duration debt has been trading with a sizeable discount (sometimes above 200 basis points). This discount reflects jointly liquidity and (mostly) credit risks, and has been causing losses for many market funds, forcing them to offer their clients negative yields. Negative yields were considered an anathema in the fund industry, and it is still unclear what this new state of affairs—where the agents no longer have (at least the feeling of) a complete safe haven from liquidity and credit risks—will entail.

5.2. Reserve Requirements

Reserve requirements were always very large during the megainflation years, and are still quite high. Figure 20 shows the reserve requirements evolution, as well the ratio of total reserve requirements to M4 (RHS scale). When the Real Plan started, in July 1994, the reserve requirements were raised because of fears that the increase in money demand could be confounded with inflationary money printing, and to deter excess credit expansion that could jeopardize the initial phase of the plan. As the plan became more and more successful, the reserve requirements were further raised to prevent excessive growth of the aggregate demand. Even a reserve requirement of 15% on bank loans was imposed.¹⁶

High reserve requirements serve not only as a deterrence against excessive credit expansion—always a danger in a country with such a low total credit to GDP ratio as Brazil (less than 30%)—, but also to a very convenient and cheap way of rolling over the debt (part of the reserve requirements are to deposited in government bonds). Since inflation targeting was adopted as the monetary policy framework in May 1999, the Central Bank has tried to lower the reserve requirements. However, last year, to preclude banks from speculating in the exchange rate markets (buying dollars), the Central Bank decided to raise

¹⁶ See Garcia [1995].

reserve requirements on time deposits. This year, it did it again in June, with the same objective of deterring exchange rate overshooting. Therefore, this tool seems to still be used for many different purposes.

Recently, with the introduction of the real-time-gross-settlement payment system, the large reserve requirements have proven to be very useful. This is because the Central Bank allows banks to use their reserve requirements during the day to settle transactions, hereby providing enough extra liquidity to meet the extra liquidity needs that arose from the passage of a net-deferment system to a real-time-gross-settlement payment system.

In summary, it seems that the large reserve requirements that were inherited from the megainflation years will prove to be very difficult to be reduced to the very low levels currently in place in most OECD countries, since they have a very high “opportunistic” value as a tool to obtain several different objectives.

5.3. The Financial Transactions Tax (CPMF)

Since it has been reinstated in 1997, the tax on financial transactions (CPMF) has become a major revenue source for the budget, as shown in by the numbers below. Currently, it also serves as a means to find tax evaders, by picking up those with little (reported) income and high payments of financial tax. Recently, Congress has exempted stock market operations from the tax, a measure long due. Banks are also exempted in their activities. The financial tax is, thus, much more important for the fiscal policy than for monetary policy or for debt management.

The financial tax acts as a deterrent to the increase of liquidity of public debt secondary markets. Since only financial institutions are exempted from this tax, all other possible players in the secondary debt market have to bear this extra cost. Therefore, short-term operations involving debt (as repos) become very expensive. The current preferred tax vehicle seems to be the “exclusive fund”.

Table 5.1

Constant R\$ Million (Dec/2001)

	CPMF Revenue	Total Treasury Revenue	CPMF's share of the Total
1997	8,842.45	139,838.13	6.32%
1998	10,096.10	171,230.49	5.90%
1999	9,299.88	249,866.88	3.72%
2000	16,121.53	261,813.03	6.16%
2001	17,804.25	282,150.22	6.31%

5.4. Open Market Operations and the Provision of Liquidity to Banks

Open market operations represent the main operating channel linking monetary policy to debt management policy. Given the institutional idiosyncrasy that tax and loans accounts must be in the Central Bank (this is a constitutional clause), the Central Bank has a lot of work deriving from the administration of the Treasury's accounts. In days when the civil servants get paid, the Treasury first transfers the funds to the banks, and the Central Bank must conduct contractionary open market operations to mop up the banks' excess liquidity until actual payments are made. Conversely, in days where the banks are due to transfer to the Treasury the taxes they have collected, the Central Bank must conduct expansionary open market operations to replenish the banks with reserves. If it did not act in this way, the basic interest rate would fluctuate wildly. This is a very interesting feature of the Brazilian monetary system: because the interest rate would fluctuate too much if the Central Bank did not intervene often, it ends up intervening so strongly as to shut off completely any intra-day variability in the basic interest rate.

In Figure 21 we look at monthly averages of the lack (positive) and the excess (negative) of bank reserves of the entire banking sector. Basically it reflects the amounts the Central Bank has to mop up or to replenish in order to clear the market for bank reserves. A positive figure means that the aggregate of banks have less reserves than what is required to fulfill the reserve requirements. Since the Central Bank is a monopolist in this market, if it did not intervene by conducting purchase of government bonds with repurchase (by the banks) agreements, the basic interest rate would rise enormously. The reverse would happen if banks had reserves in excess of the amount needed to fulfill the reserve requirements. The only difference is that in the latter situation, banks may decide not to loan the idle funds (out of fear of credit risk, for example) and keep the excess reserves.

As it can be seen from the last part of Figure 21, since the end of 2001, the lack of bank reserves has been gradually turned into excess of reserves. This is because the banks have been fearsome of purchasing government bonds fearing the credit risk of a possible debt default of the next government. Therefore, when the old debt matures, many prefer not to purchase new bonds and loan those funds through the open market. Since they get from these daily loans to the Central Bank just a few basis points below the interest rate paid in the bonds that will mature after the change in administration, they prefer to remain liquid. Therefore, it is a case where debt management becomes a case of monetary policy. The Central Bank and the Treasury have been trying many different approaches as shortening the debt and letting the intra-day Selic rate vary more to entice the banks to lock up the higher rates by purchasing the bonds at the auctions. Nevertheless, it remains a distinct possibility that the excess reserves position will grow larger as the elections approach, and the Central Bank will have to conduct larger operations to mop up liquidity in a daily basis. The debt shortening that started in May 2002, was a measure to avoid the increase in the excess reserves position. The rationale seems to be that it is preferable to sell short term debt than to be forced to roll the same debt overnight.

6. Conclusion and Policy Discussion

The management of the domestic public debt is perhaps the single most important issue currently in the economic policy agenda, as well as in the presidential candidate's economic programs. This is due to its large size (above 50% of GDP), as well as the extremely high and counter-cyclical interest rates, which inflicts a higher toll on the budget precisely when the economy is weak. Both factors jointly threaten to put the debt in an unsustainable path. Simulations¹⁷ show that under reasonable assumptions, the tough fiscal stance, currently delivering a primary surplus of 3.5% of GDP, must be maintained in the next years in order to keep the debt to GDP ratio from growing further.

Here, we analyze several aspects pertaining to the management of the debt. Section 2 studies the causes for the extremely large and fast growth of the domestic public debt during the seven-year period that President Cardoso has been in power (until the end of 2001). The data show that interest payments were by far the largest culprit for the debt growth. Other components, as the accumulation of assets and hidden liabilities were also important. Furthermore, given that many of the assets, especially the state debt, are of doubtful value, the picture displayed by the net debt figures may underestimate the true situation.

The macroeconomic summary behind the data is the following. In the first years of the Real, the fiscal stance was quite lax. Given the weak fiscal stance, monetary policy was then used to prevent the excessive growth of aggregate demand that would threaten the main achievement of the plan, the low inflation. During most of this period, foreign capital was flowing in, forcing the government to impose controls in capital **in**flows to prevent the appreciation of the real.¹⁸

This state of affairs changed after the Asian crisis. Then, interest rates had to be raised to avoid capital outflows, which would threaten the managed exchange rate, and, therefore, also threaten inflation stability. This situation became prevalent until the beginning of 1999, when the real was floated, and the new monetary policy regime was created according to the new world paradigm of inflation targeting. Since 1998.3, a new, and much tougher, fiscal stance had been put in place, with ambitious targets for primary surpluses, which the Brazilian government has been fulfilling until present. However, the composition of the debt, roughly half indexed to the short term interest rate and a fourth indexed to the exchange rate, maintained the debt growth rate at high levels in face of external shocks that caused real depreciation and required higher interest rates to ensure that the inflation targets were not abandoned.

Section 3 implements risk measures for the domestic public debt. Value at Risk (**V@R**) measures are computed for the different debt components, as well as for the aggregate. Given the lack of liquidity, which prevented us from having the necessary bonds' prices, a few heroic assumptions had to be made to allow the computation of the **V@R**. The results

¹⁷ Several investment banks (JP Morgan, Deutsch Bank, etc) regularly produce debt sustainability simulations. See also Bevilaqua and Garcia [2002].

¹⁸ See Garcia and Valpassos [2000].

show that the risk shot upwards during the floating of the currency, when volatility grew a lot. After that, it decreased steadily until the beginning of 2001, when several shocks started hitting the economy (the recession in the US, the contagion from Argentina, the energy crisis and political problems among the government allies in Congress). All these increased volatility and risk, as measured through [the V@R](#).

The V@R measures the **worst plausible loss** of a portfolio present value. Present values in Brazilian domestic currency are usually computed by discounting the future values by the domestic interest rate (Selic). Therefore, the zero-duration bonds, whose stock amounts to more than 50% of the domestic public debt, bear no risk. This causes the [V@R](#) to underestimate the budgetary risk that is relevant for government decisions. After all, when interest rates are raised, the present value of the zero-duration debt does not change, while the real value of interest payments do increase, be they deflated by the price level, or computed as % of GDP. Even with this bias toward underestimation, when computed as a percentage of Treasury's revenues, the risk proves to be very high. Each day, the worst plausible loss (increase) in the value of the domestic debt corresponds roughly to three times the daily Treasury's revenue.

In order to incorporate the impact of the shocks to the budget, the concept of cash flow at risk ([CF@R](#)) was adapted to the government budget. The impacts of several variables as the interest rate, the exchange rate, the GDP growth rate and inflation on the primary balance were computed through a vector auto-regression (VAR). The results show that shocks to those variables have a positive impact on the primary surplus, although the magnitude is small when compared to the increase in the debt value that the same shocks would cause. It remains to be constructed a risk measure that jointly incorporates the effects of the shocks on both the budget and the debt portfolio. Nevertheless, the conclusion is that the debt current debt structure is extremely risky, because volatility increases precisely when the economy weakens, draining fiscal resources to serve the debt precisely when they are mostly needed.

Section 4 considers the rollover risk in the context of the widely known mean-variance analysis. It is shown that the decisions regarding the debt composition that were taken in May, 1998 could be interpreted as shocks that tilted the government's trade-off between market risk and rollover risk. Further work is necessary to implement the model with parameters that accurately represent the problem faced by the Brazilian public debt manager.

The data show that until April 2002, the maturity structure was being gradually lengthened. The increased risk ness was the cost to improve the maturity structure, i.e., market risk was increased while refinancing risk was reduced. The recent emergence of large discounts on the zero-duration bonds, requiring the Treasury to shorten the debt seems to be a further unfortunate step. Not only the debt structure is extremely risky from the market risk perspective, but also the Treasury is being increasingly pressured to shorten the debt, thereby increasing the refinancing risk, because of the perception that credit risk has increased.

Finally, Section 5 considers several important points pertaining to the overlapping of monetary policy and debt management. First it is shown that despite the fact that megainflation was defeated almost eight years ago, some of its effects are still present, as it happens with the domestic demand for public debt. During the megainflation years, the demand for the public debt and for the domestic currency were kept alive by preserving the value of the former in face of very high and volatile inflation, and by making the latter a vehicle to assess the inflation protected asset. Therefore, Brazilian firms and households with access to the banking sector got used to paying transactions with chunks of government bonds (although most ignored it). Positive real interest rates with daily liquidity were the reasons why Brazil did not undergo a currency substitution process as many of its neighbors. Monetary policy became completely passive to allow for this to happen.

This habit of having daily liquidity and high real interest rates did not subside with inflation stability. Money market funds are still obliged to offer positive real returns with daily liquidity, or so they feel. The problems that are currently surfacing in the domestic debt markets, as the presidential elections polls bring fears that a (partial) default may be favored by the next president, reflect to a great extent this habit that Brazilians grew accustomed to having. As even zero-duration bonds, which are free from interest rate (market) risk, begin to be traded at large discounts reflecting credit and liquidity risks, money market funds are no longer allowed to pretend that they are able to offer daily liquidity with no risk. This is bound to have an impact on the demand for domestic debt, although it is not currently clear to which extent.

The many different roles of reserve requirements were also reviewed. The general conclusion is that the high reserve requirements very often help the monetary authority to achieve certain ancillary goals that have nothing to do with what reserve requirements are for. For example, reserve requirements were used as a means to control credit expansion, speculation in the foreign exchange markets, and to provide intra-day limits for banks to operate in the newly released real-time-gross-settlement payment system. For that option value, it is likely that reserve requirements will be kept for much longer at the current very high levels.

The financial transaction tax detrimental role in preventing greater liquidity in the public debt secondary market is also mentioned. Finally, the management of bank reserves through open market operations is studied. The Brazilian Constitution mandates that all government bank accounts be kept at the Central Bank. This adds a seasonal pattern and a lot of noise to the daily work conducted by the Central Bank desk in setting the interest rate. It would be a good idea to allow the Treasury to keep its accounts in banks outside the Central Bank, since that would free the Central Bank from intervening so often and so strongly. We also show that the aforementioned problems in the debt markets are showing up in the monetary market as excess liquidity of the banks.

In summary, the overall message regarding the Brazilian domestic public debt market is that of an unfinished transition between the mechanism that made possible to prevent currency substitution during the megainflation years through the provision of a domestic currency substitute to a more standard debt market with different degrees of liquidity,

different maturities and different returns. Despite the large variety, most bonds are still seen as a means to provide positive real returns with daily liquidity. The separation of cash management from long-term savings was never completed. Nor will it be until the next president settles down. When he does, however, this is task of utmost importance.

For the lengthening process to proceed, some sort of indexation will likely be necessary. After all, eight years after the defeat of megainflation, the longest nominal bond that was floated was a two-year bond (in 1997). The inflation risk seems to be very high for a market in long nominal bonds to appear. Since the inflation risk is something under the control of the government, it is reasonable for the government to offer the private sector insurance against it. The provision of insurance against inflation (inflation indexation) serves as a commitment device.

However, the same argument does not apply to offering insurance against neither the nominal interest rate or the nominal exchange rate. After all, to index to the daily interest rate is akin to providing insurance not only against inflation risk, but also against real interest rate risk. Similarly, to index to the daily exchange rate is akin to providing insurance not only against inflation risk, but also against real exchange rate risk. In principle, there is no good argument for the government to be involved in providing insurance against these two real shocks. Therefore, the lengthening process must emphasize price-level-linked bonds in lieu of zero-duration or exchange-rate-linked bonds.

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TABLE I.1. FEDERAL DEBT USES: 1995 - 2001

In R\$ Million	Dec/94	Dec/95	Dec/96	Dec/97	Dec/98	Dec/99	Dec/00	Dec/01	Variation	Percentage Share
Federal Net Debt (+ Central Bank)	65,836.21	90,406.30	128,413.28	167,741.82	231,267.74	316,221.69	352,967.13	411,771.95	345,935.75	64.62%
Interest Payments (Federal Government + CB)	18,727.80	22,853.13	20,537.19	54,402.28	88,881.41	54,926.30	66,434.53	326,762.64	326,762.64	61.04%
Primary Deficit (Federal Government + CB)	-3,335.75	-2,907.57	2,374.56	-5,041.51	-22,672.11	-20,430.59	-21,979.78	-73,992.75	-73,992.75	-13.82%
Nominal Deficit minus Net Debt Variation of the States and Municipalities	1,681.52	214.31	3,823.79	3,402.93	-9,292.19	8,383.79	-19,984.34	-6,722.63	-6,722.63	-1.26%
Nominal Deficit minus Net Debt Variation of the State	-773.34	2,855.03	26,592.35	5,808.53	-6,514.65	-3,433.79	-1,398.89	23,135.24	23,135.24	4.32%
Owned Enterprises										
Balance Sheet Adjustment Variation		16,142.14	2,646.83	17,813.92	43,524.52	17,537.27	36,714.00	134,378.68	134,378.68	25.10%
Privatization Adjustment Variation		1,144.00	16,646.14	12,860.27	8,973.03	20,238.56	980.50	60,842.50	60,842.50	11.37%
Adjustment not Computed by the Central Bank		8,264.69	0.00	0.00	0.00	0.00	0.00	8,264.69	8,264.69	1.54%
Assets	106,558.55	147,888.21	192,459.55	257,349.80	270,151.58	343,338.99	377,796.92	462,963.12	356,404.57	66.57%
1. Domestic	73,806.29	97,534.72	130,029.31	199,145.60	216,332.39	278,352.22	313,246.24	379,738.69	305,932.40	57.15%
1.1. FAT	12,800.00	17,728.00	20,486.00	23,291.23	27,878.83	33,405.29	51,092.01	60,977.25	48,177.25	9.00%
1.2. CB's credits to financial institutions	20,561.00	34,577.00	67,648.00	68,920.00	48,490.18	40,812.82	37,341.00	21,573.00	1,012.00	0.19%
1.3. Federal Government's credits (Law 8727/93)	8,276.27	10,011.03	11,469.69	12,998.61	3,849.18	4,851.06	4,754.65	19,246.02	10,969.75	2.05%
1.4. Debt Renegotiations with the States	0.00	0.00	0.00	49,480.37	86,612.46	131,540.25	154,830.36	174,501.56	174,501.56	32.60%
1.5. Others	32,169.01	35,218.69	30,425.62	44,455.39	49,501.74	67,742.81	65,228.21	103,440.86	71,271.85	13.31%
2. Foreign Reserves	32,752.26	50,353.49	62,430.25	58,204.20	53,819.19	64,986.76	64,550.68	83,224.43	50,472.17	9.43%
Other Debts (-)	112,139.76	131,628.51	147,965.84	172,806.07	178,795.81	245,193.50	241,554.98	279,136.69	173,141.55	32.34%
1. Domestic	46,946.67	57,561.68	72,858.44	97,113.73	86,164.38	97,042.92	91,609.77	92,659.91	42,335.02	7.91%
1.1 Monetray Base	17,685.00	21,681.00	19,796.00	31,828.00	39,223.00	48,430.00	47,679.00	53,247.00	32,039.00	5.98%
1.2. Others	29,261.67	35,880.68	53,062.44	65,285.73	46,941.38	48,612.92	43,930.77	39,412.91	10,296.02	1.92%
2. Foreign	65,193.09	74,066.83	75,107.40	75,692.34	92,631.44	148,150.58	149,945.20	186,476.78	130,806.53	24.43%
TOTAL	60,255.00	106,666.00	172,907.00	252,285.55	322,623.50	414,367.18	489,209.07	595,598.38	535,343.38	100.00%

TABLE I.2. FEDERAL DEBT USES: 1995 - 2001

In Percent of GDP	Dec/94	Dec/95	Dec/96	Dec/97	Dec/98	Dec/99	Dec/00	Dec/01	Variation	Percent Variation
Federal Net Debt (+ Central Bank)	13.06%	13.37%	15.85%	18.67%	25.01%	30.23%	30.89%	33.24%	20.19%	154.58%
Interest Payments (Federal Government + CB)		2.77%	2.82%	2.29%	5.88%	8.50%	4.81%	5.36%	32.43%	
Primary Deficit (Federal Government + CB)		-0.49%	-0.36%	0.26%	-0.55%	-2.17%	-1.79%	-1.77%	-6.86%	
Nominal Deficit minus Net Debt Variation of the		0.25%	0.03%	0.43%	0.37%	-0.89%	0.73%	-1.61%	-0.70%	
States and Municipalities										
Nominal Deficit minus Net Debt Variation of the State		-0.11%	0.35%	2.96%	0.63%	-0.62%	-0.30%	-0.11%	2.79%	
Owned Enterprises										
Balance Sheet Adjustment Variation		0.00%	1.99%	0.29%	1.93%	4.16%	1.54%	2.96%	12.87%	
Privatization Adjustment Variation		0.00%	0.14%	1.85%	1.39%	0.86%	1.77%	0.08%	6.09%	
Adjustment not Computed by the Central Bank		1.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.22%	
Assets	21.14%	21.87%	23.76%	28.64%	29.22%	32.83%	33.07%	37.38%	16.24%	76.85%
1. Domestic	14.64%	14.43%	16.05%	22.17%	23.40%	26.61%	27.42%	30.66%	16.02%	
1.1. FAT	2.54%	2.62%	2.53%	2.59%	3.01%	3.19%	4.47%	4.92%	2.38%	
1.2. CB's credits to financial institutions	4.08%	5.11%	8.35%	7.67%	5.24%	3.90%	3.27%	1.74%	-2.34%	
1.3. Federal Government's credits (Law 8727/93)	1.64%	1.48%	1.42%	1.45%	0.42%	0.46%	0.42%	1.55%	-0.09%	
1.4. Debt Renegotiations with the States	0.00%	0.00%	0.00%	5.51%	9.37%	12.58%	13.55%	14.09%	14.09%	
1.5. Others	6.38%	5.21%	3.76%	4.95%	5.35%	6.48%	5.71%	8.35%	1.97%	
2. Foreign Reserves	6.50%	7.45%	7.71%	6.48%	5.82%	6.21%	5.65%	6.72%	0.22%	
Other Debts (-)	22.24%	19.47%	18.27%	19.23%	19.34%	23.44%	21.14%	22.54%	0.29%	1.32%
1. Domestic	9.31%	8.51%	8.99%	10.81%	9.32%	9.28%	8.02%	7.48%	-1.83%	
1.1. Monetray Base	3.51%	3.21%	2.44%	3.54%	4.24%	4.63%	4.17%	4.30%	0.79%	
1.2. Others	5.80%	5.31%	6.55%	7.27%	5.08%	4.65%	3.85%	3.18%	-2.62%	
2. Foreign	12.93%	10.96%	9.27%	8.42%	10.02%	14.16%	13.12%	15.06%	2.12%	
TOTAL	11.95%	15.78%	21.34%	28.08%	34.89%	39.62%	42.82%	48.09%	36.13%	302.34%

FIGURE 1
Federal Bonded Debt: Composition and Average Maturity

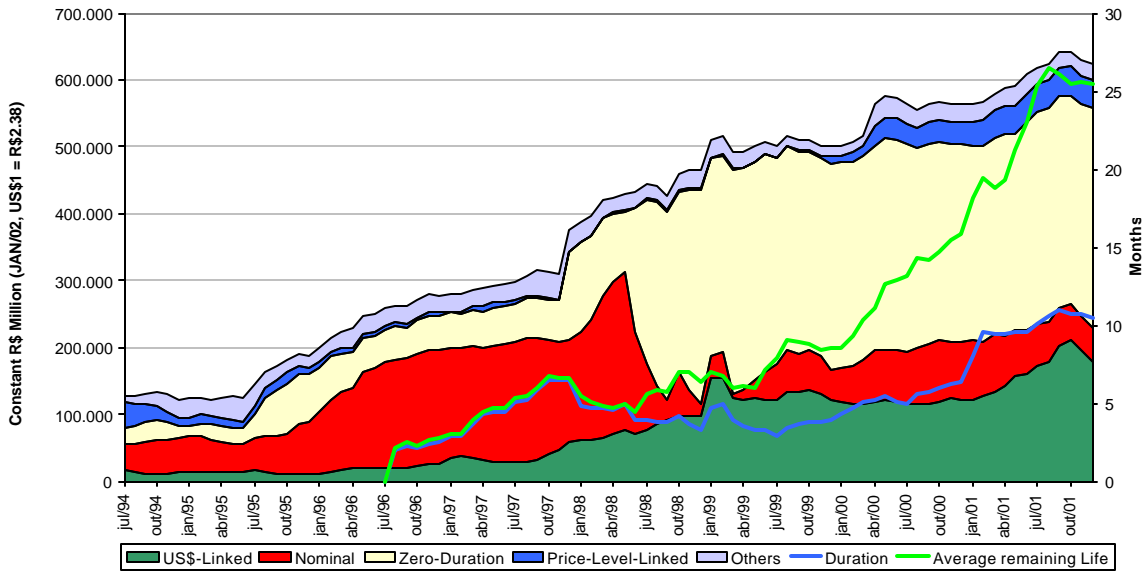


FIGURE 2
Federal Bonded Debt: Composition (%)

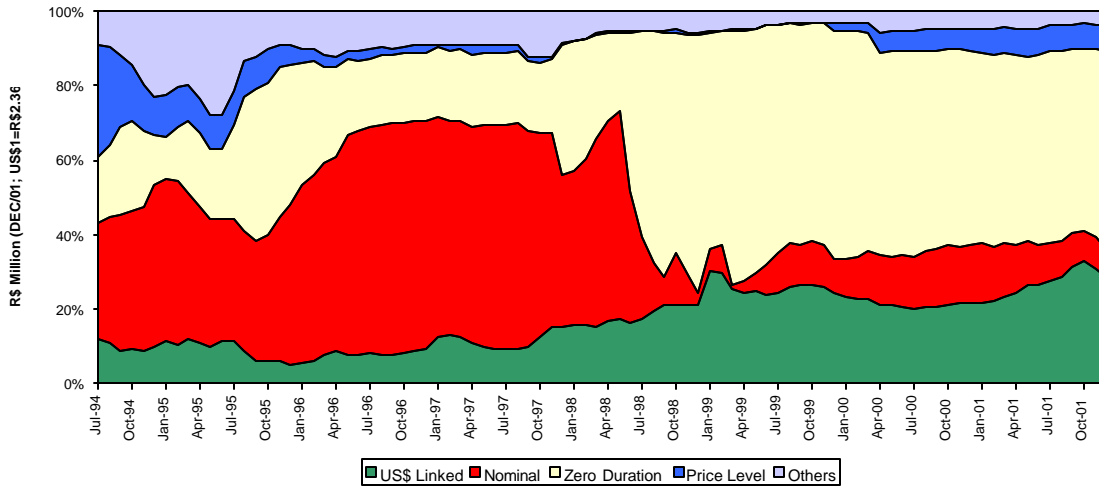


FIGURE 3
Daily Volatility Yield Curve - Nominal Bonds

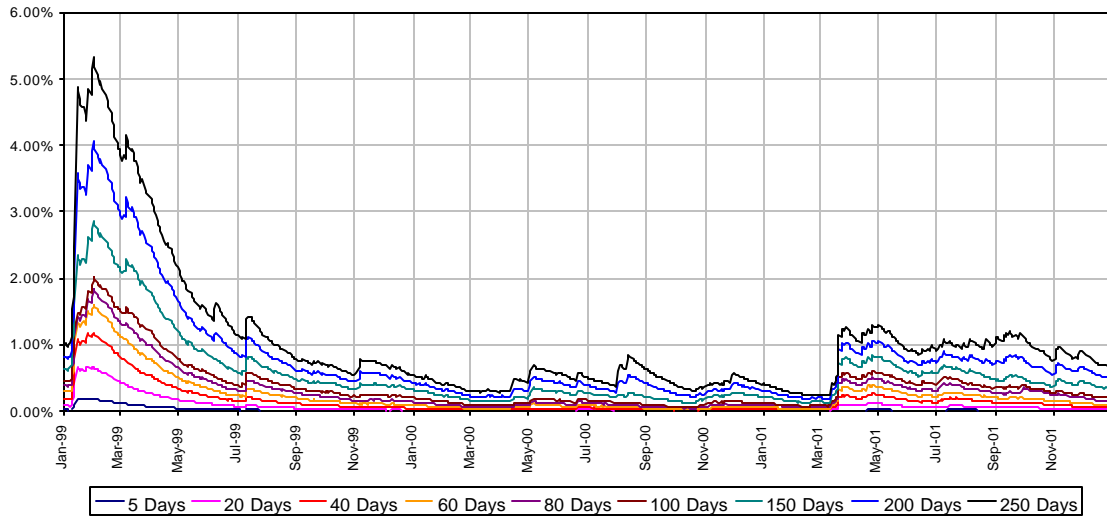


FIGURE 4
Daily V@R - Nominal Bonds

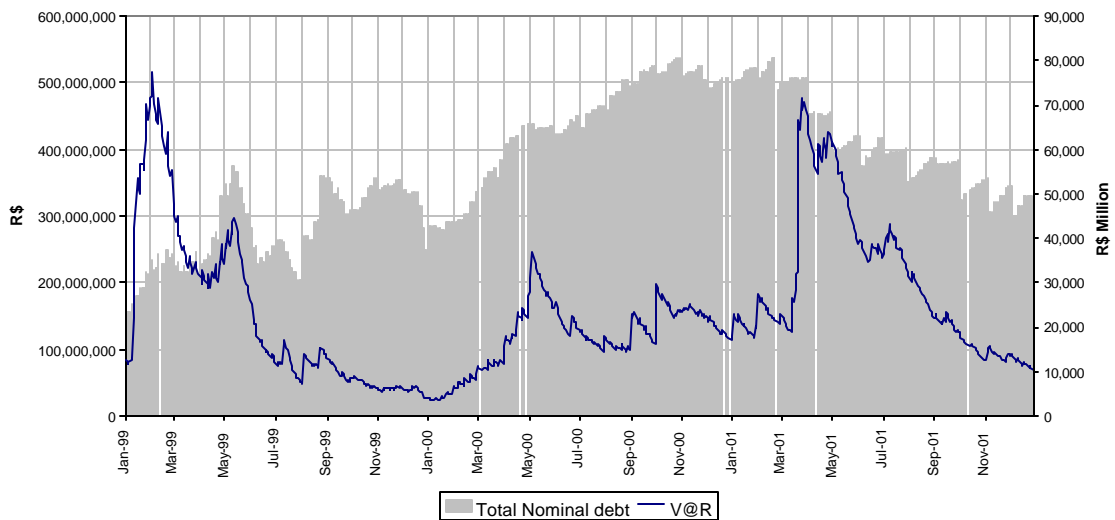


FIGURE 5
Daily V@R (%Total Nominal Debt)



FIGURE 6
Daily Volatility Yield Curve - Dollar Linked Interest Rate

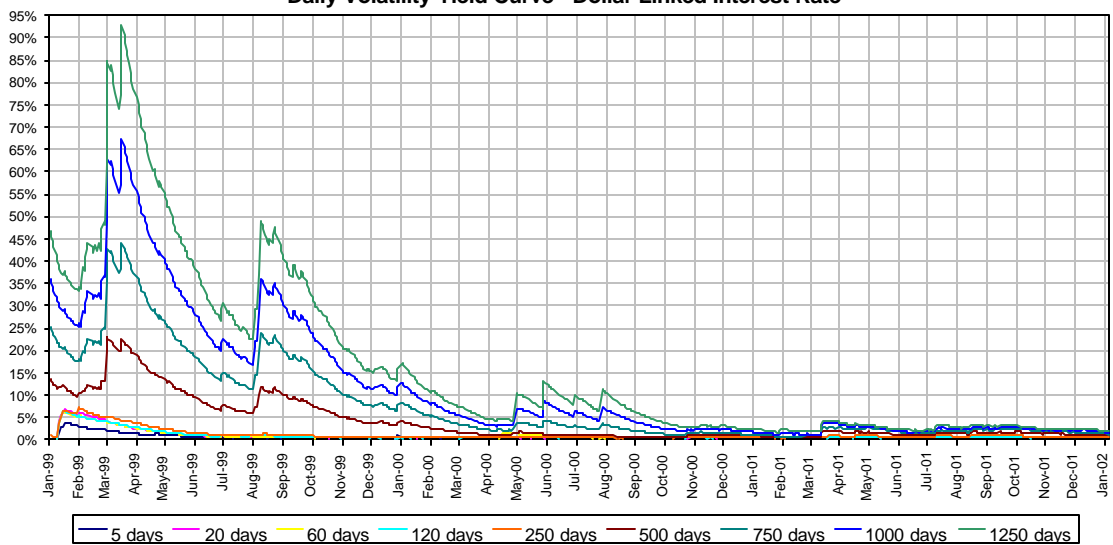


FIGURE 7
Spot Dollar Volatility

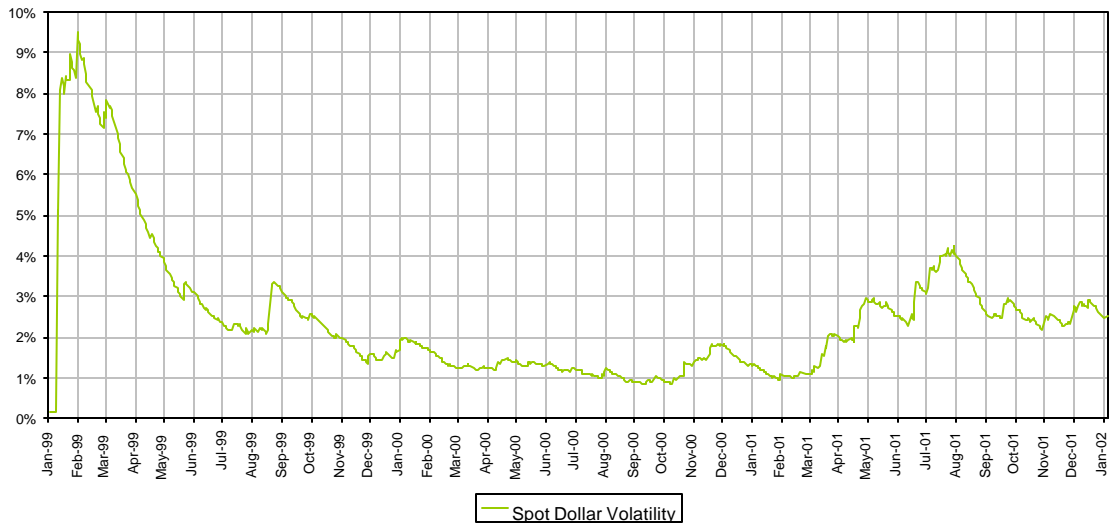


FIGURE 8
DAILY V@R - National Treasury Dollar Linked Bond (NTN-D)
(% Total NTN-D)

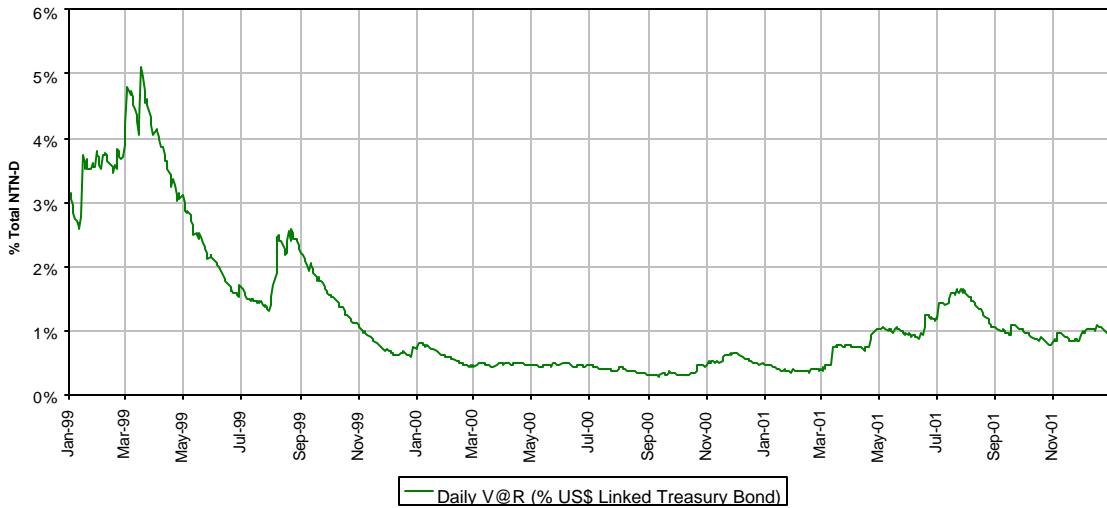


FIGURE 9
Daily V@R IGP-M Linked Debt
Proxy 1

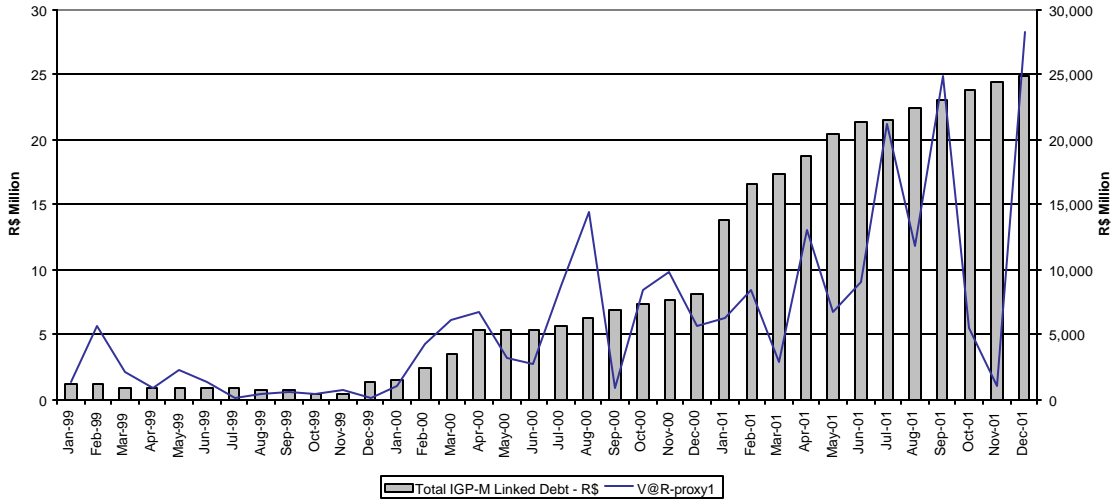


FIGURE 10
Daily V@R IGP-M Linked Debt
proxy 2 (correl = 0); proxy 3 (correl = 1); proxy 4 (correl = -1)
daily volatility Swap DI x IGP-M = 2.5%

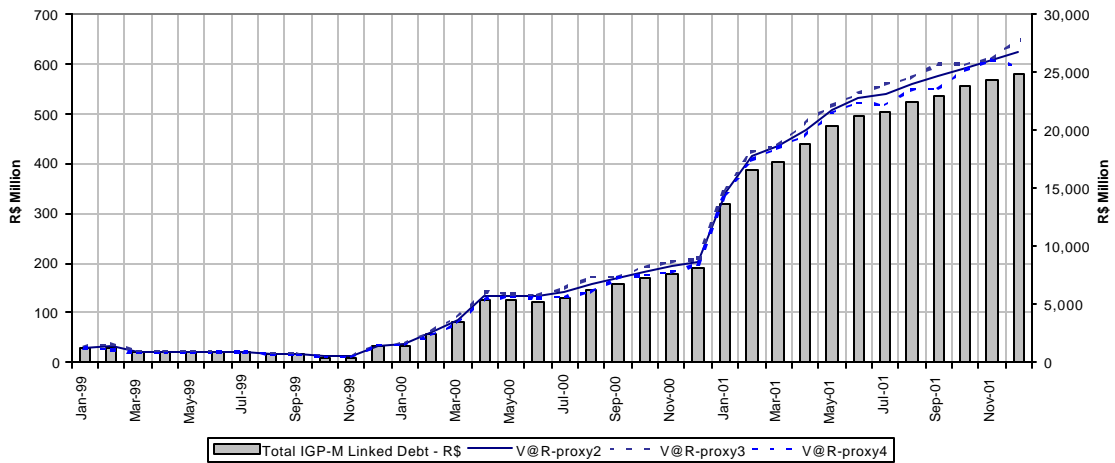


FIGURE 11
Daily V@R as % of total respective debts

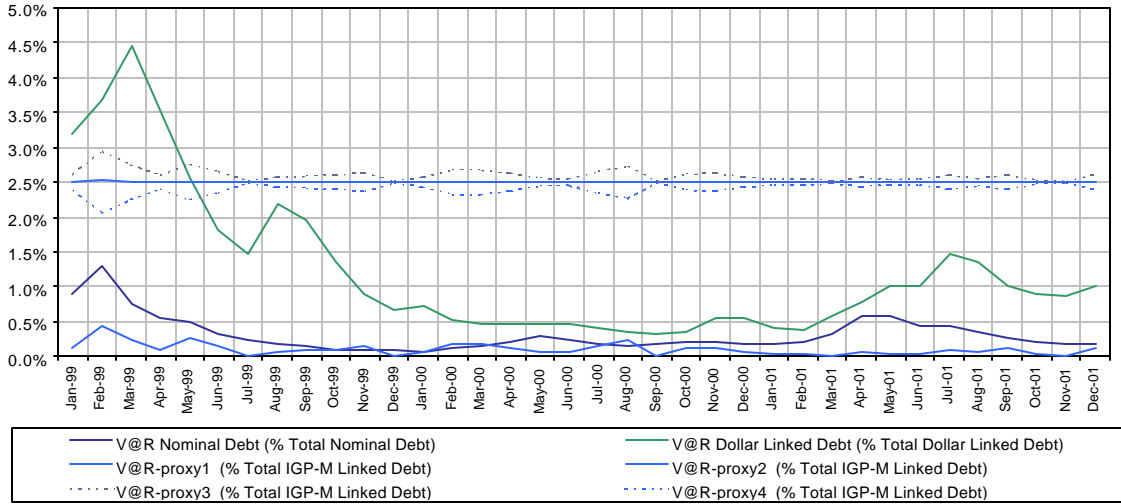


FIGURE 12
TOTAL DAILY V@R

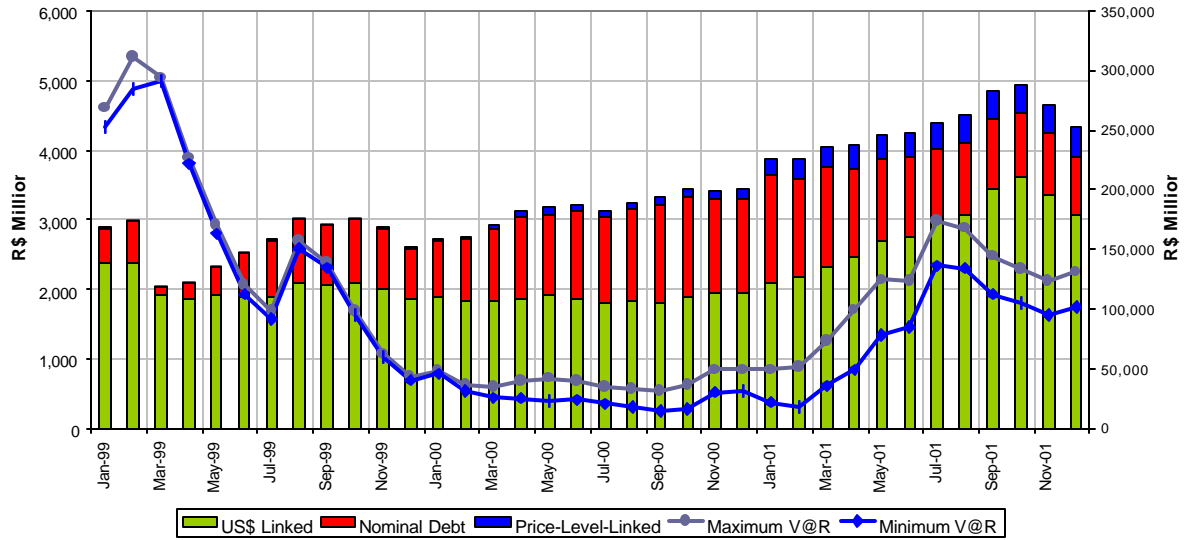


Figura 13
Total Daily V@R/Debt

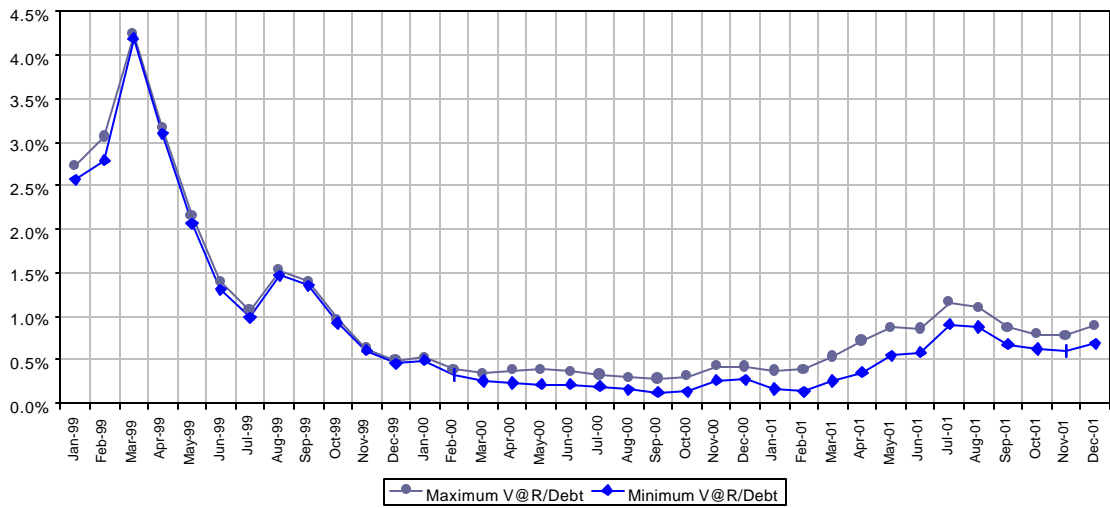


FIGURE 14
Total Daily V@R as percentage of Last 12 Months GDP

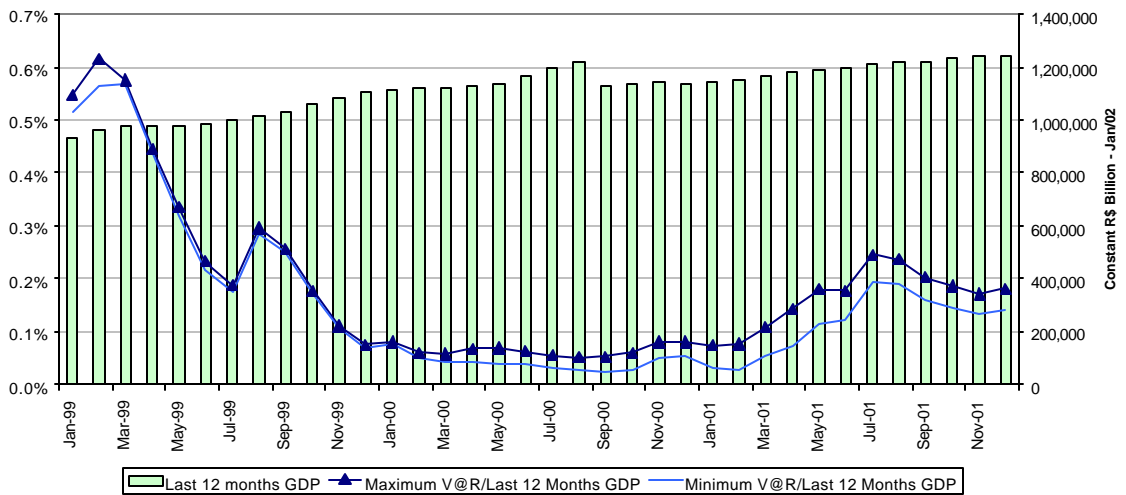


FIGURE 15
Total Daily Debt V@R as percentage of Monthly Treasury Revenue

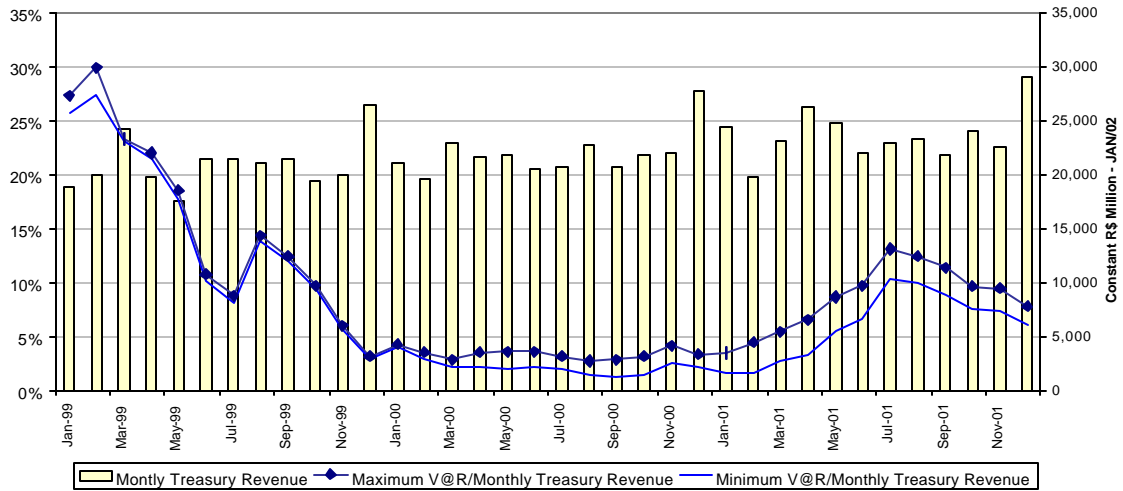


FIGURE 16
Fiscal Gap Distribution – One-month-ahead

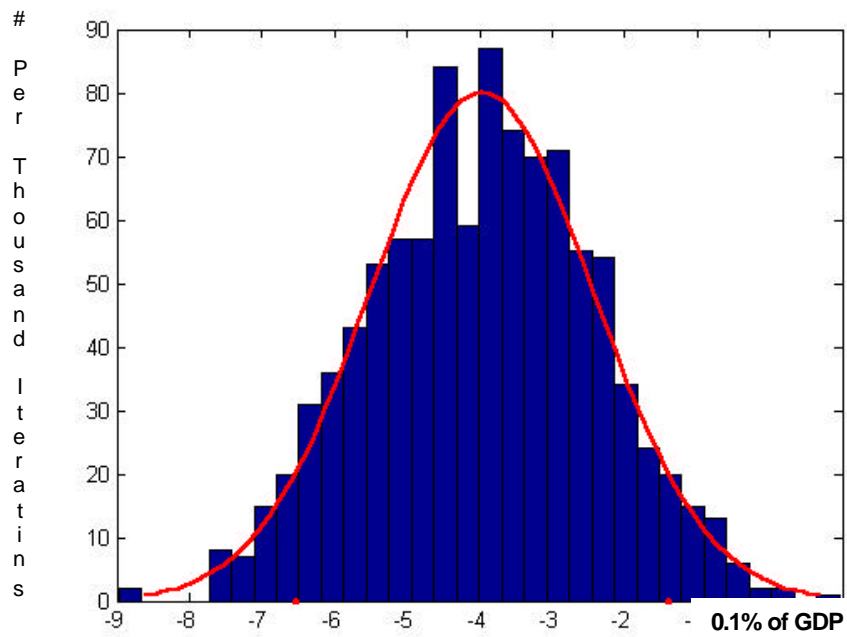


FIGURE 17

Fiscal Gap Distribution – 12-month-ahead

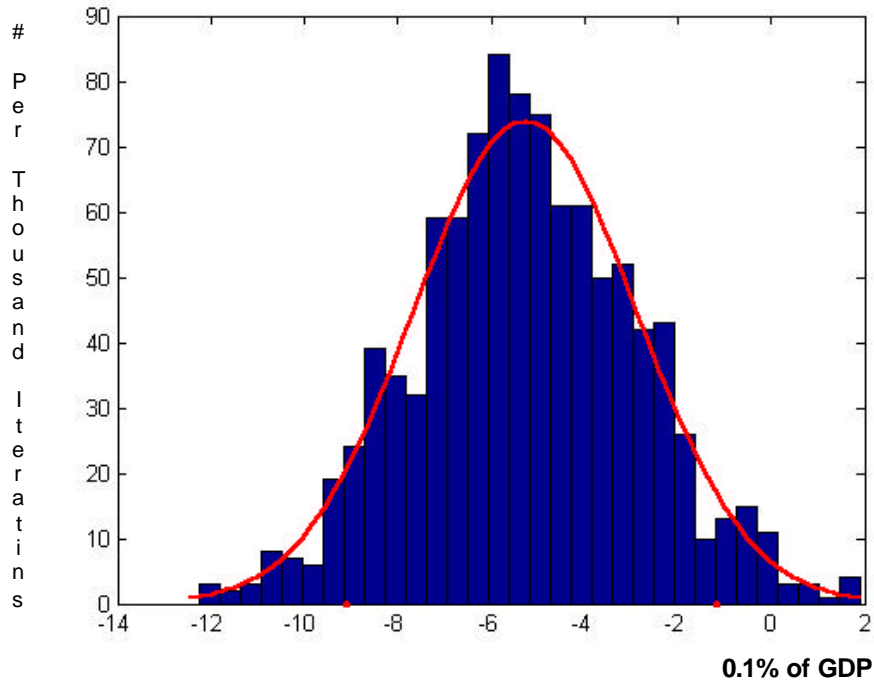


FIGURE 18
Maturity Structure

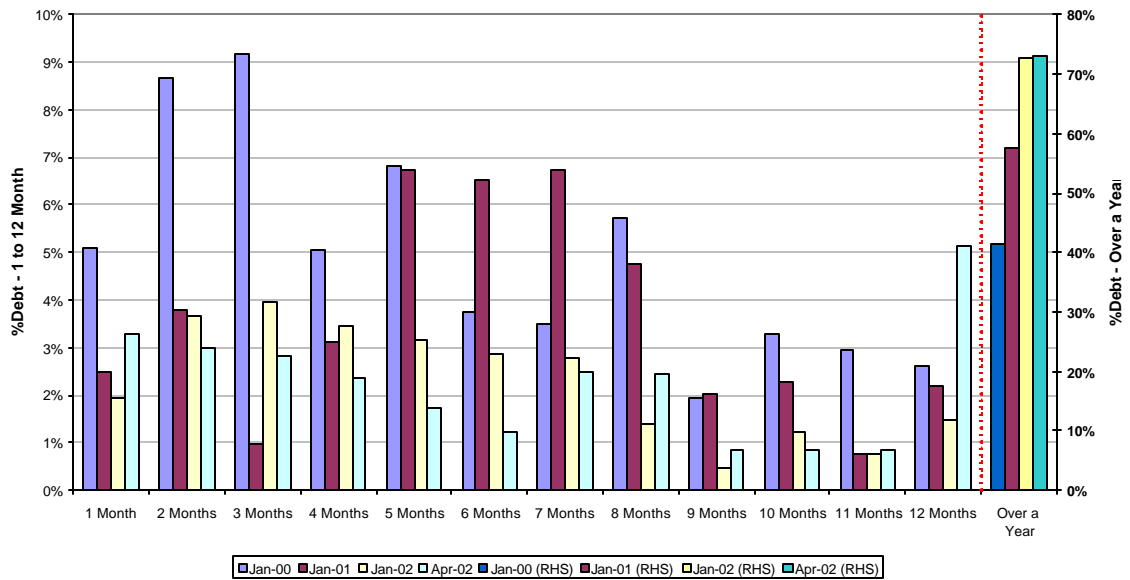


FIGURE 19
Maturity Structure

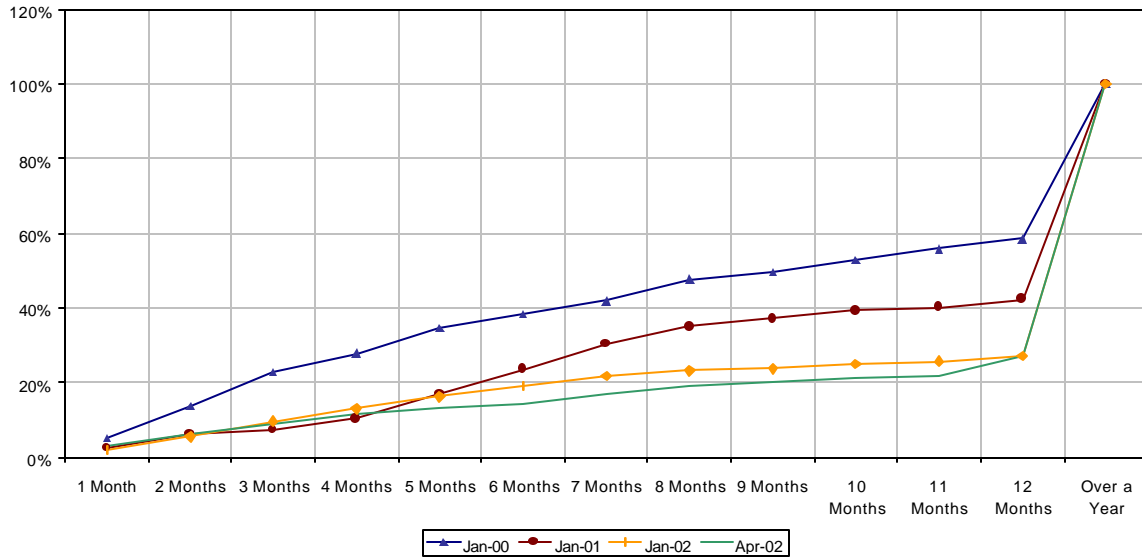


FIGURE 20
Reserve Requirements

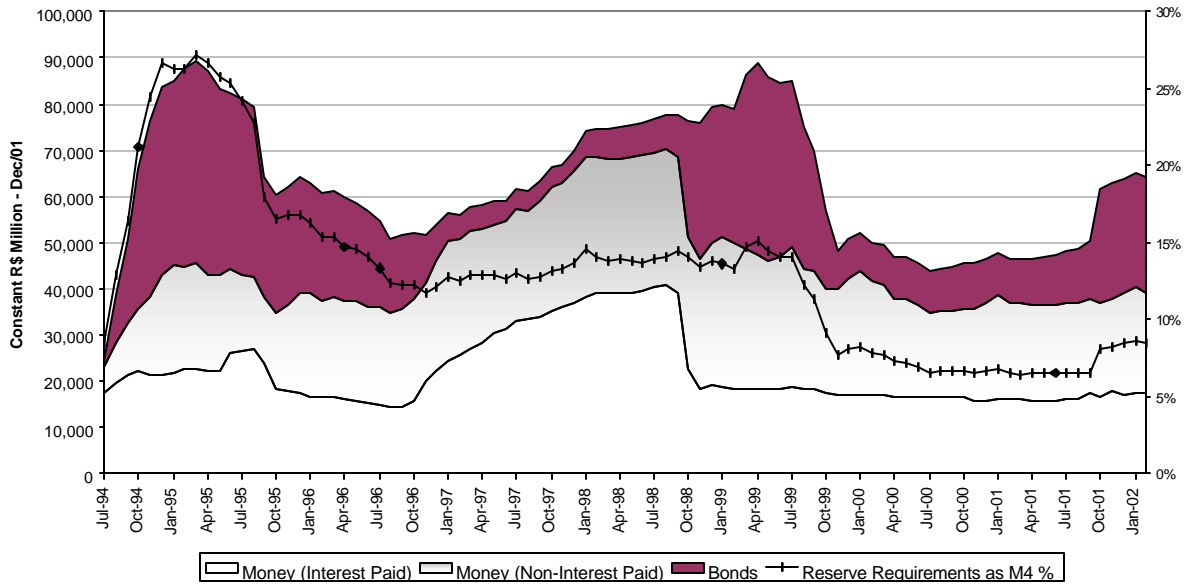


FIGURE 21

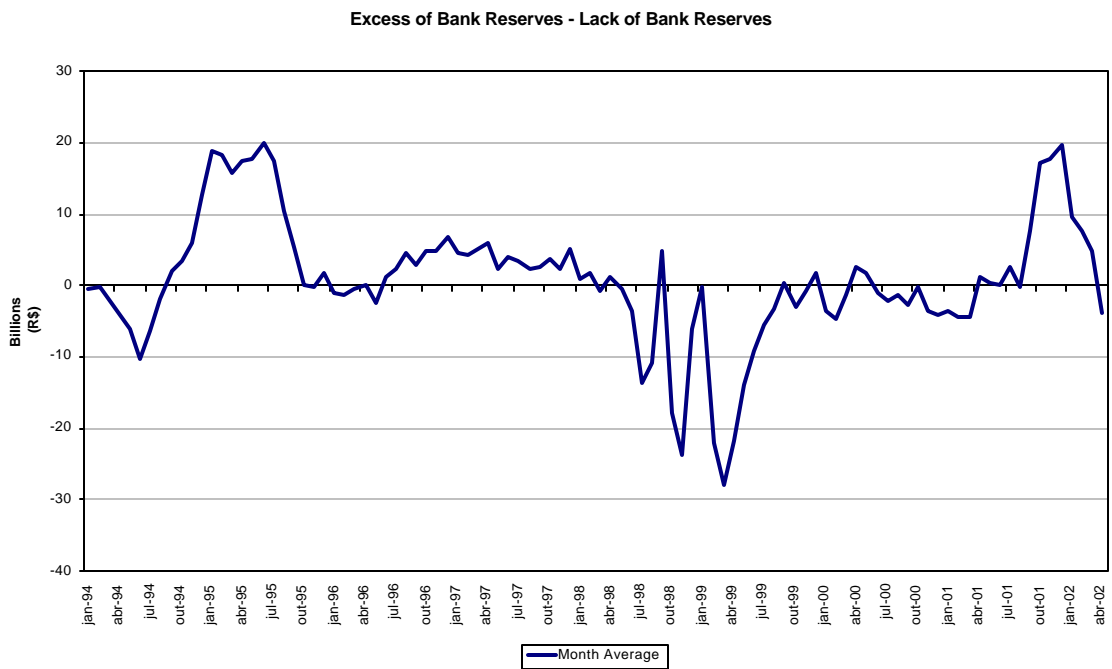


Chart 1
 Mean-Variance Analysis with Rollover Risk
 $h=0, r=0$

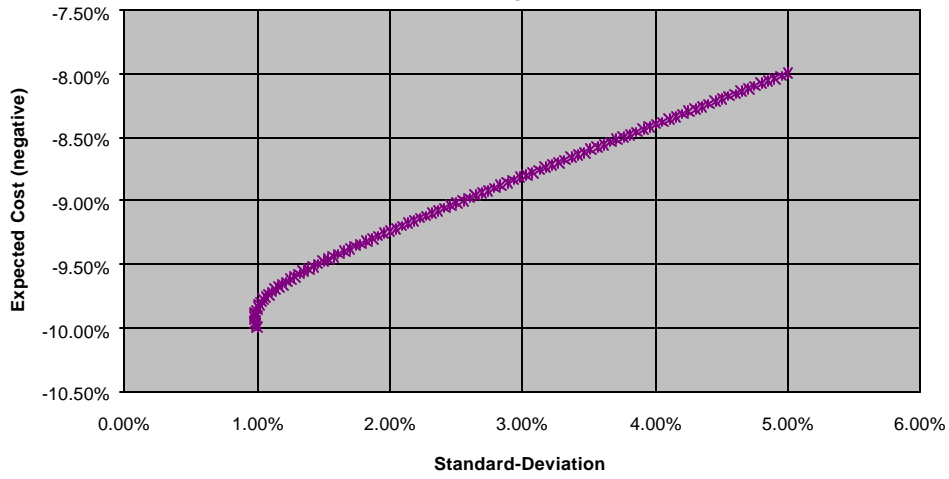


Chart 2
 Mean-Variance Analysis with Rollover Risk
 $r=0$

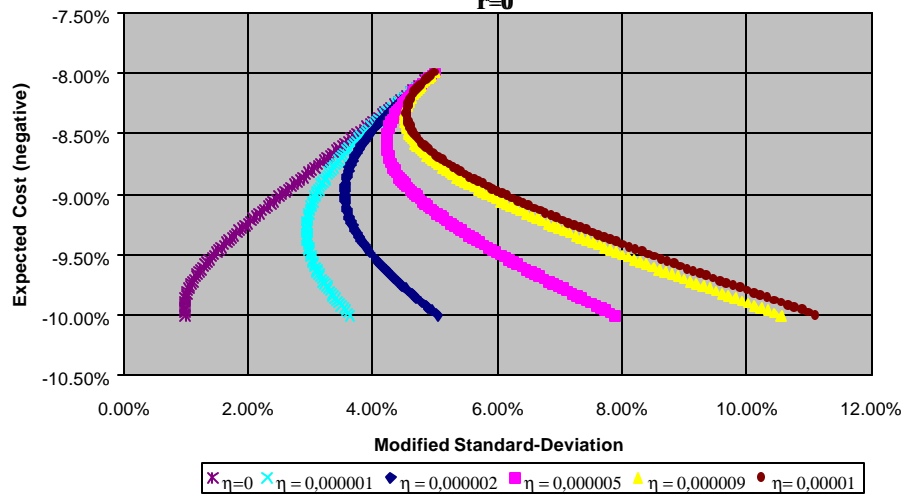


Chart 3
 Mean-Variance Analysis with Rollover Risk
 $h=0, r=0$

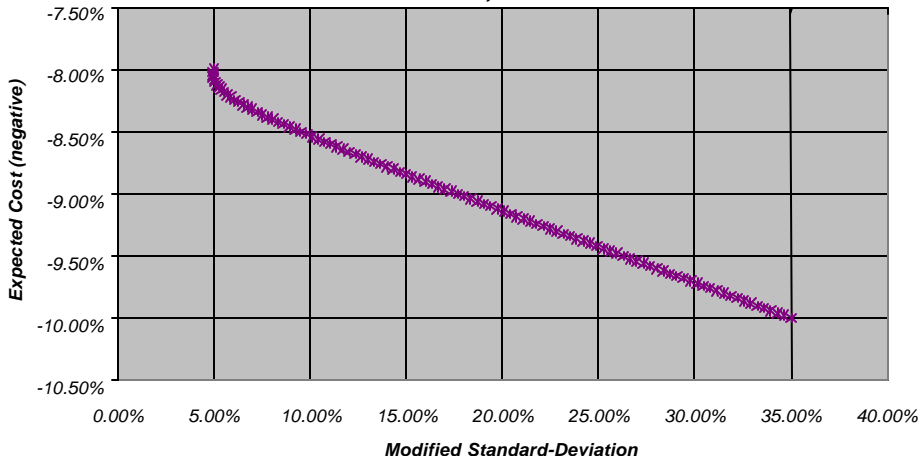
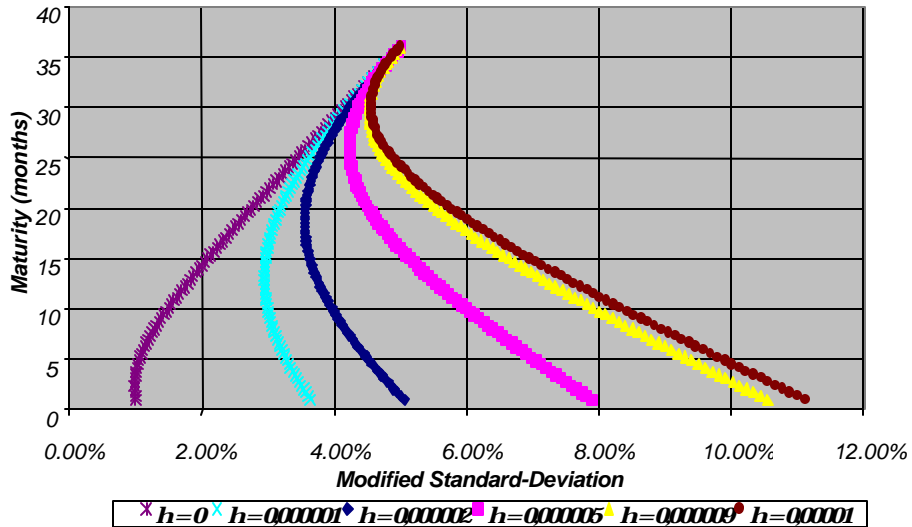


Chart 4
 Maturity with Rollover Risk
 $r=0$



APPENDIX 1: Value-at-Risk Methodology

The Methodology applied to estimate the V@R followed the *RiskMetrics*¹⁹ Model as described below. The data range is the workdays from January 4th, 1999 to December 31st, 2001.

1.1 – Nominal and Dollar Linked Bonds²⁰

1.1.1- Volatility Estimation

The Volatilities estimations were computed using the Exponentially Weighted Moving Average (EWMA) method as suggested in *RiskMetrics Technical Document* with a 160 days window. The decay factor (λ) used was 0,95 and the confidence level was 95%.

$$\sigma_t = \text{Volatility} = \sqrt{\frac{\sum_{t=1}^T \lambda^{t-1} (r_t - \bar{r})^2}{\sum_{t=1}^T \lambda^t}},$$

where \bar{r} can be considered null by virtue of being a financial long-period average return, as suggested by *Riskmetrics*.

1.1.2 - Mapping:

The total nominal bonds and total Dollar linked bonds daily value was mapped into selected vertices by the *RiskMetrics* methodology. The vertices chosen were 5, 20, 40, 60, 80, 100, 150, 200 and 250 days for the nominal bonds and 5, 20, 60, 120, 250, 500, 750, 1000, 1250 days for the Dollar linked bonds.

A total value with maturity x^* was decomposed in two different vertices, x_- and x_+ , respectively, precedent and posterior vertices of x^*

The decomposition methodology of *RiskMetrics* transforms the original flow into the selected vertices considering its historical volatilities and respecting the following basic rules: the present value of the decomposed flow must be equal to the original flow, the

¹⁹ <http://www.riskmetrics.com> - *RiskMetrics Technical Document*

²⁰ We used as the total stock of Nominal Bonds the total value of the bonds LTN and BBC. The majors Dollar linked bonds are the NTN-D and NBC-E, however, we calculated just the NTN-D V@R by virtue of facing problems in NBC-E auctions data.

market risk of the flow from the decomposition must be equal to the original flow and the decomposed flow must have the same sign of the original flow.

The method is based in a quadratic interpolation as described hereafter:

The factor of decomposition (α): It's the percentage of the original flow that will be decomposed into the precedent vertice. By virtue of respect the basic rules explained above, the parameter α is computed by the following quadratic equation:

$$\mathbf{s}_{x^*}^2 - \mathbf{a}^2 \mathbf{s}_{x^-}^2 + (1 - \mathbf{a})^2 \mathbf{s}_{x^+}^2 + 2\mathbf{a}(1 - \mathbf{a}) \mathbf{r}_{-+} \mathbf{s}_{x^-} \mathbf{s}_{x^+} = 0,$$

where we can denominate $a = \mathbf{s}_{x^+}^2 + \mathbf{s}_{x^-}^2 - 2\mathbf{r}_{-+} \mathbf{r}_{x^-} \mathbf{r}_{x^+}$

$$b = 2\mathbf{r}_{-+} \mathbf{s}_{x^-} \mathbf{s}_{x^+} - 2\mathbf{s}_{x^+}^2$$

$$c = \mathbf{s}_{x^+}^2 - \mathbf{s}_{x^*}^2$$

The factor of decomposition is then:

$$\mathbf{a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.1.3- Value-at-Risk:

The main target of the V@R is estimate a confidence interval to a portfolio's value in progress to assess a maximum likelihood loss in a period of time. The period considered in this document was 1 day.

The *RiskMetrics* methodology to compute the V@R of a portfolio with just two assets is explained below. In our case, were used nine vertices to the Nominal Bonds Estimation and *nine plus one* vertices for Dollar Linked Bonds²¹. Therefore no more than ten types of assets composed the daily portfolio. Thus, the estimation is analogous to the two assets case, when considering up to ten assets.

$$V @ R = \sqrt{\text{var } iance} = \sqrt{\text{var}(ax + bY)} = \sqrt{\text{var}(ax) + \text{var}(bY) + 2 \text{cov}(ax, bY)}$$

$$V@R = \sqrt{a^2 \mathbf{s}_x^2 + b^2 \mathbf{s}_y^2 + 2ab \mathbf{s}_{xy}} =$$

$$\sqrt{\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} \mathbf{s}_x^2 & \mathbf{s}_{xy} \\ \mathbf{s}_{xy} & \mathbf{s}_y^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}} = \sqrt{\begin{bmatrix} a \mathbf{s}_x & b \mathbf{s}_y \end{bmatrix} \begin{bmatrix} 1 & \mathbf{r}_{xy} \\ \mathbf{r}_{xy} & 1 \end{bmatrix} \begin{bmatrix} a \mathbf{s}_x \\ b \mathbf{s}_y \end{bmatrix}}$$

²¹ The Dollar Linked Bonds with coupon payments have two primitive risk factors: the Dollar linked Interest rate risk and the Spot Dollar risk. Hence, the flow was decomposed into nine vertices considering the Dollar linked Interest rate volatility and one vertice considering the Spot Dollar risk, summing up ten. The Nominal Bonds, however, have just the interest rate risk factor, and then demand just the time vertices.

1.2 – Inflation-Linked Bonds

The inflation-indexed bonds considered were the ones linked to the General Price Index (IGP-M).

1.2.1 – Market-to-Market Pricing

Their market-to-market price is given by the present value of the principal and the coupons payments based on the nominal value corrected by the accumulated inflation. Simplifying the discussion for a zero-coupon bond we have:

$$P_t^T = \frac{1 \cdot \frac{I_t^{T^e}}{I_0}}{(1 + i_t^T)^{\frac{(T-t)}{252}}},$$

where $I_t^{T^e}$ is the price index projected for the expiring date T, I_0 is the price index on the emission date and i_t^T is the future interest rate of maturity (T – t) annualized on a 252 days basis.

Rearranging, we obtain:

$$P_t^T = 1 \cdot \frac{I_t}{I_0} \cdot \left[\frac{1}{\frac{(1 + i_t^T)^{\frac{(T-t)}{252}}}{\frac{I_t^{T^e}}{I_t}}} \right],$$

where we distinguish $1 \cdot \frac{I_t}{I_0}$ as the corrected nominal value and $\frac{(1 + i)^{\frac{(T-t)}{252}}}{\frac{I_t^{T^e}}{I_t}}$ as the future real

interest rate. Therefore, we can consider the Inflation linked bond having two primitive risk factors: the inflation and the future real interest rate (which has two primitive risk factors of its own, the future nominal interest rate and the future inflation rate).

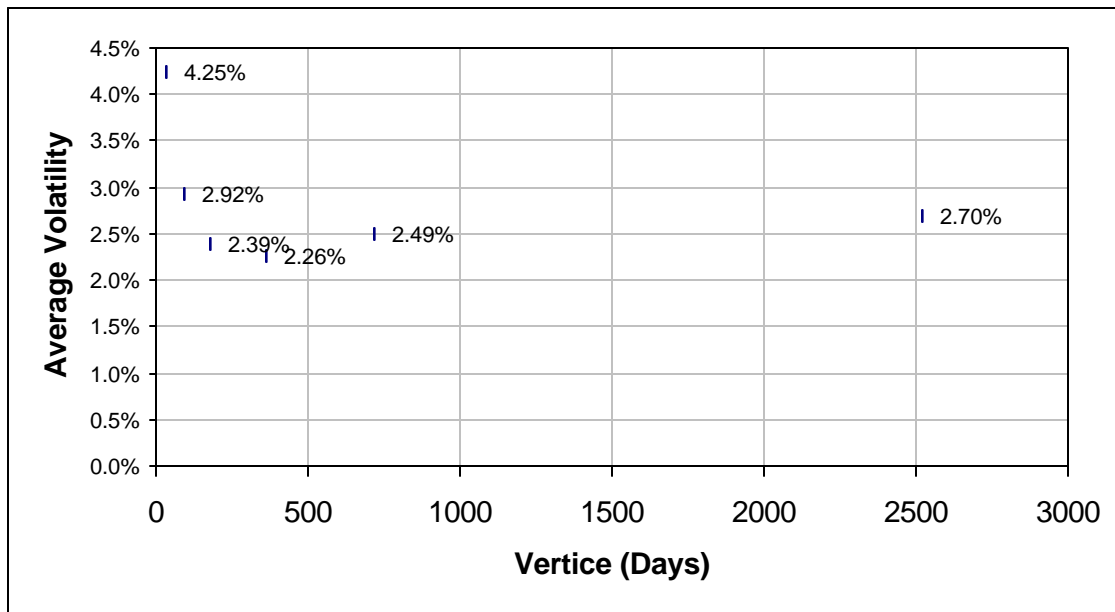
However, the data relating to the expected future inflation rate is not available and the data of the future real interest rate is available since just October 2001 (rate of the *Swap DI x IGP-M*). Hence, it is not possible to compute the market-to-market price of the inflation-linked bonds in Brazil for the period taken into account in this paper and therefore proxies had to be made.

1.2.2 – Volatility Estimation

The volatilities estimations were also computed by the EWMA method.

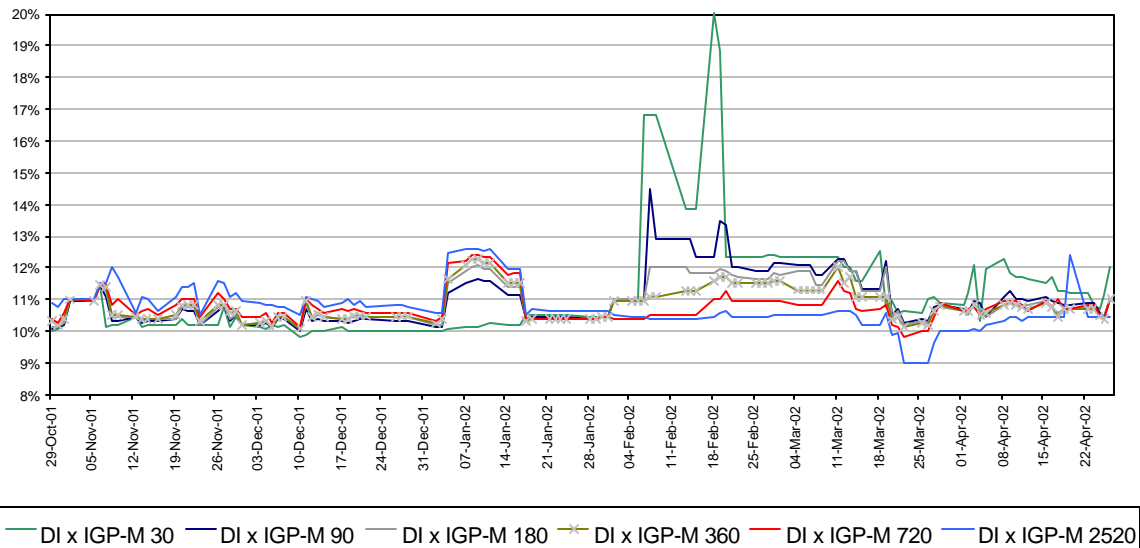
In the inflation monthly volatilities estimations were used a 6 month window, a decay factor for 0,9 and a 95% level of confidence. The daily volatility was approximated by dividing the monthly volatility by $\sqrt{22}$, referred to a workday basis.

The future real interest rate, i.e, the rate of the *Swap DI x IGP-M*, volatilities estimations were done just for the short period available of October 2001 to April 2002, summing up 127 daily observations. The methodology was again the EWMA procedure, with a 40 days window, decay factor of 0,95 and a confidence level of 95%. The average volatilities are shown below:



Thus, is observed an inverted yield curve of the rate of return volatility of the *Swap DI x IGP_M*. Moreover, looking at this Swaps yield curve, it can be seen that no pattern is distinguishable. Those issues are related to the liquidity problems of this Swap since it is a new instrument of Brazilian's financial market.

DI x IGP-M Yield Curve



1.2.3 – Value-at-Risk Estimation

The *formulae* to estimate the V@R are the same shown in the Section 1.1.3 of this appendix.

To estimate the value-at-risk of inflation-linked bonds, some simplifying assumptions were necessary:

Proxy 1:

We considered the rate of the *Swap DI x IGP-M* as a constant, and hence just the inflation primitive risk factor was taken into account.

Then the V@R formula was reduced to:

$$V@R = (\text{total value}) \times (\text{inflation volatility})$$

Proxies 2, 3 and 4:

A second type of proxy was generated assuming the rate of the *Swap DI x IGP-M* with a flat term structure (not needing to map the portfolio in time vertices) and assuming its rate of return with a constant volatility.

Nevertheless, the second proxy had another difficult point: the estimation of the covariances between the rate of inflation and the rate of return of the *Swap DI x IGP-M*, since the data for this *Swap* was not available for the period in question. Henceforward, we

used three proxies: one assuming the independence between these variables (**proxy 2**), another considering perfect positive correlation between them, i.e., the correlation coefficient equals to one (**proxy 3**), and a last proxy considering the hypothesis of a perfect negative correlation between them (**proxy 4**). Then, we could estimate a maximum and a minimum value for inflation-linked debt V@R.

1.3 - TOTAL V@R

The total V@R refers to the nominal, Dollar linked and IGP-M linked debt stocks. The methodology adopted was not conventional, however showed very good results. The modification in comparison to the *Riskmetrics* methodology surrounds the volatilities and the primitive risk factors considerations.

Since we did not have a daily data for the inflation linked V@R, we pointed out to the monthly average of the daily V@R of nominal and dollar linked debt. In the estimations we, then, considered the values of the V@R as a percentage of the total (type of) debt as the volatilities to be used in the total V@R computation. Then:

$$\text{TOTAL V@R} = \sqrt{N^2 \cdot \frac{V@R^2_{\text{LTN+BBC}}}{(\text{LTN} + \text{BBC})^2} + D^2 \cdot \frac{V@R^2_{\text{NTN-D}}}{(\text{NTN} - D)^2} + \pi_{\text{IGP-M}}^2 \cdot \frac{V@R^2_{\text{IGP-M}}}{\pi_{\text{IGP-M}}^2} + \text{"cov ariances"}},$$

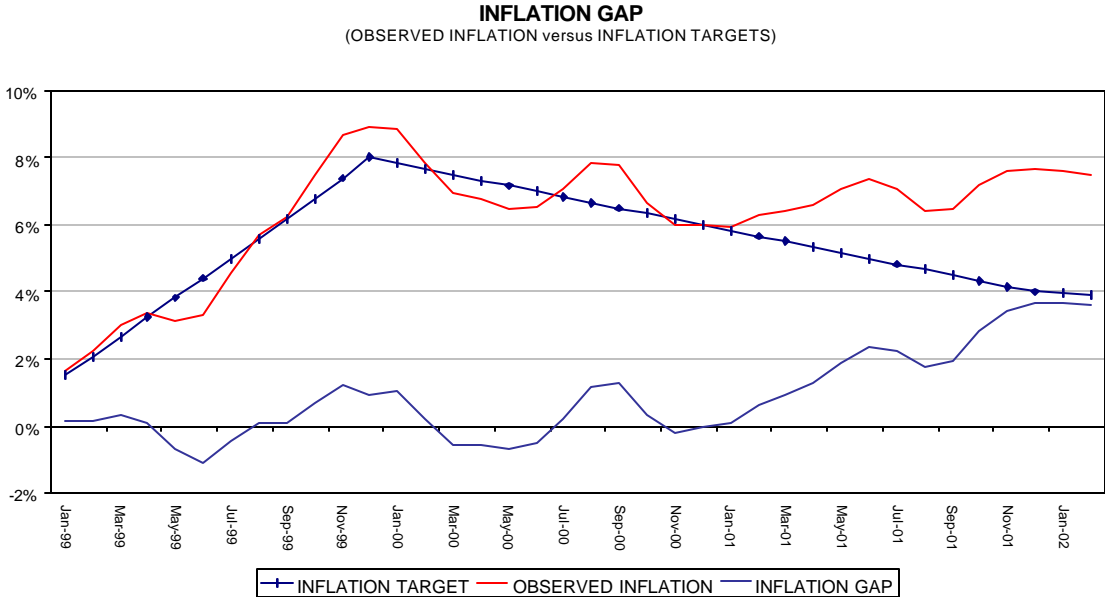
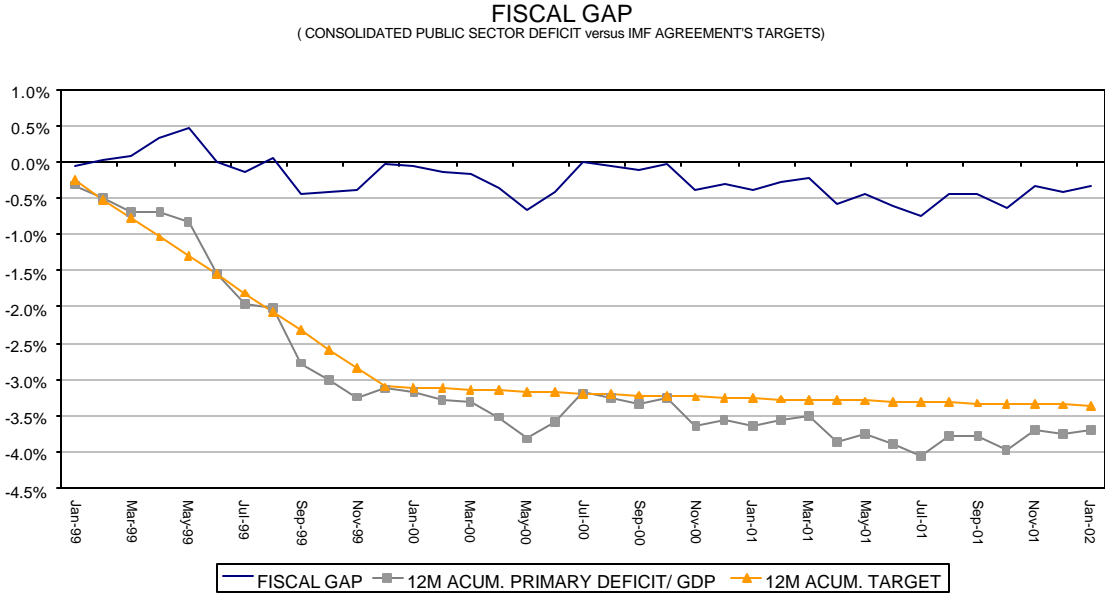
Where N is the total nominal bonds, D is the total dollar linked bonds and $\pi_{\text{IGP-M}}$ is the total IGP-M linked bonds.

For the “covariances” issue, we assumed for the **maximum value of the total V@R** as having all the correlation coefficients equal 1 and for the **minimum V@R** we assumed all correlation coefficients equal -1 and the rate of the *Swap DI x IGP-M* as constant.

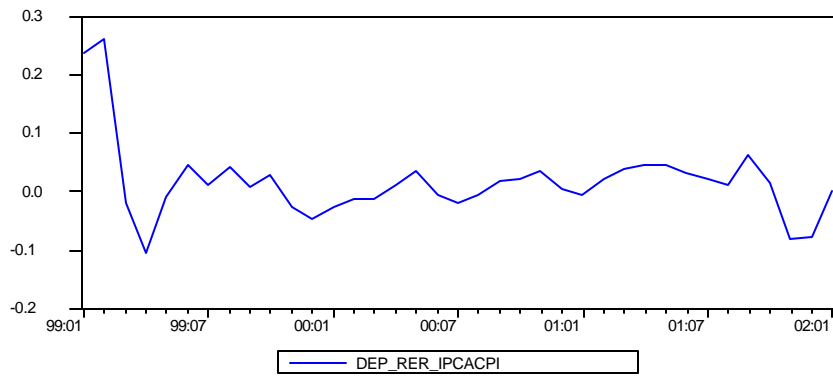
APPENDIX 2: Cash Flow at Risk Methodology

2.1 – Vector Auto regression estimation

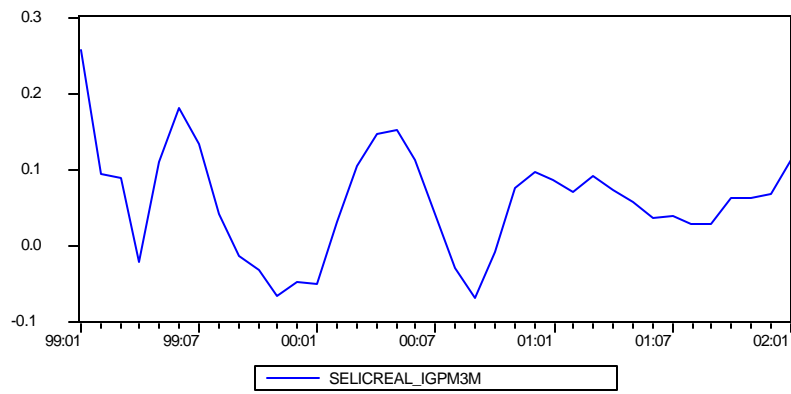
2.1.1 – Variables



REAL DEPRECIATION



REAL INTEREST RATE



OUTPUT GROWTH

