

Monetary Policy During Brazil's Real Plan: Estimating the Central Bank's Reaction Function *

Maria José S. Salgado, Márcio G. P. Garcia, and Marcelo C. Medeiros

Department of Economics, Pontifical Catholic University of Rio de Janeiro

September 4, 2001

Abstract

This paper uses a Threshold Autoregressive (TAR) model with exogenous variables to explain a change in regime in Brazilian nominal interest rates. By using an indicator of currency crises - which is chosen endogenously - the model tries to explain the difference in the dynamics of nominal interest rates during and out of a currency crises. The paper then compares the performance of the nonlinear model to a modified Taylor Rule adjusted to Brazilian interest rates, and shows that the former performs considerably better than the latter.

Keywords: Time series, interest rates, threshold models, nonlinearity, reaction function, Taylor rule.

JEL Classification Codes: C22, C51, C52, E52, E58

*This paper is based on the first author's Master Dissertation. The authors would like to thank Renato Flôres for his insightful comments. This research is partially supported by CNPq.

1 Introduction

In July 1994, Brazil launched the Real Plan, ending a long period of very high inflation rates that had started in the seventies. The very high domestic interest rates coupled with the favorable conditions of the international financial markets in the immediate months after the Plan prompted a nominal (and real) appreciation of the currency. However, the Mexican crisis of December, 1994 changed the benevolent scenario. In March, 1995, after suffering large losses of international reserves, interest rates rose again and a new exchange rate regime was introduced. The new regime was, *de facto*, a very narrow crawling band that amounted to a crawling peg, i.e., to a predetermined exchange rate. This regime lasted until January, 1999, when the Real was allowed to float after having survived previous speculative attacks.

Even during this *de facto* predetermined exchange rate regime, monetary policy played an active role during tranquil periods of the international financial markets. Given the inconsistent trinity principle – a country cannot retain both autonomous exchange rate and monetary policies under perfect capital mobility –, the use of active monetary policy required the use of capital controls on capital inflows.

Monetary policy was used during the tranquil subperiods to prevent aggregate demand from growing too fast, with its deleterious inflationary consequences. The extremely high interest rates practiced by the Brazilian central bank prompted capital inflows. To keep the exchange rate from appreciating, capital controls were imposed on capital inflows (Garcia and Valpassos 1998). In other words, during tranquil periods, domestic rates were set at a level above and beyond what international investors required to transfer short term funds to buy Brazilian bonds. During the crises periods, however, the interest rate required to keep these funds in Brazil rose substantially, prompting the central bank to increase interest rates even further.

Figure 1 shows the Brazilian nominal interest rates ¹. The shaded areas correspond to periods usually associated with the currency crises (Mexican, Asian and Russian) that preceded the floating of the Real. During the crises periods, the domestic interest rate increased remarkably. Both the mean and the variance of the crises subperiods are substantially higher than these of the tranquil periods.

Therefore, it is only natural to consider that different models, possibly with different variables, were used by the central bank to determine the interest rate, i.e., that the central bank reaction function changed between tranquil and crises periods. For example, during the crises subperiods, interest rates were in-

¹The rate used is the basic interbank rate, the Selic rate (equivalent to the FED funds rate).

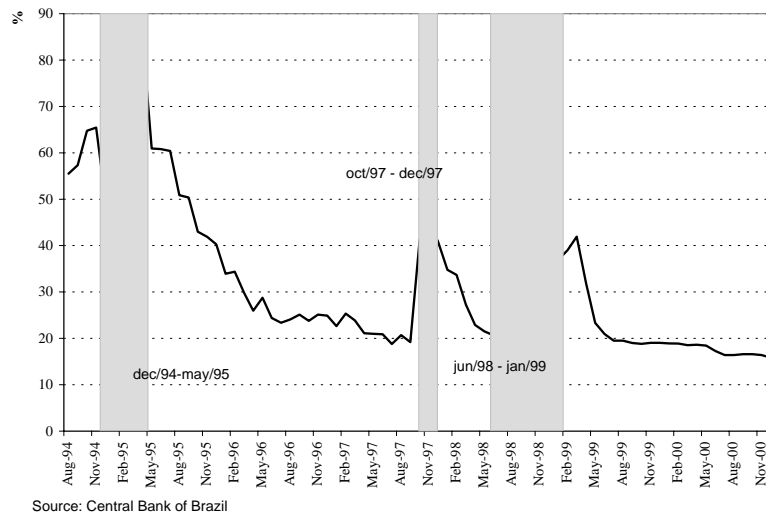


Figure 1: End of period Brazilian nominal interest rate (% year).

creased to avoid further loss of foreign reserves, quite independently of what was happening to inflation or to the output gap. The latter two variables, on the other hand, played a decisive role on the central bank reaction function during the tranquil subperiods.

Our main purpose is to estimate the Brazilian central bank reaction function during the Real Plan. The data covers the period from August, 1994 to December, 2000. Given the previous motivation, we will run a horse race between usual models for the reaction function, and a nonlinear alternative – a Threshold Autoregressive (TAR) model with exogenous variables – that contemplates the change between two different models: one for tranquil and another for crises subperiods. We will show that the latter model better fits the Brazilian data.

This paper is organized in four sections, including this introduction, which is the first. Section 2 discusses some monetary policy rules. Section 3 briefly describes the methodology used to estimate the non-linear model. Section 4 estimates a linear model and the model with two regimes for Brazilian nominal interest rates. Section 5 concludes the paper.

2 Monetary Policy Rules

In this section we review some monetary policy rules and suggest a functional form for the one followed by the Brazilian Central Bank. Probably the most well known reaction function is the Taylor Rule, proposed in Taylor (1993), by which the Central Bank uses the nominal interest rate to minimize the total variance of inflation and output. It has the following representation:

$$i_t = a + g\tilde{y}_t + h(\pi_t - \pi^*), \quad (1)$$

where i_t is the short-run nominal interest rate, π_t is the rate of inflation, π^* is the inflation target, \tilde{y}_t is the output gap and a , g and h are parameters.

Other reaction functions are forward looking, as they depend on the expectation of future inflation rates and output. For example, Clarida, Galí and Gertler (2000) propose the following rule:

$$r_t^* = \alpha + \beta [E(\pi_{t,k}|\Omega_t) - \pi^*] + \gamma E(x_{t,k}|\Omega_t), \quad (2)$$

where r_t^* is the nominal interest rate target determined by the Central Bank, $\pi_{t,k}$ is the inflation rate between t and $t + k$, π^* is the inflation target, $x_{t,k}$ is a measure of the output gap between t and $t + k$, $E(\cdot)$ is the conditional expectations operator and Ω_t is the information set available in t , when the interest rate is determined. The scalars α , β , and γ are parameters. The constant represents the desired nominal interest rate when inflation and output are equal to the targets. The authors argue, however, that there is a tendency for Central Banks to smooth changes in interest rates, so that they do not always achieve r_t^* . Therefore, the effective nominal interest rate would be:

$$i_t = (1 - \rho)r_t^* + \rho i_{t-1} + \nu_t, \quad (3)$$

where $\rho \in [0, 1]$ indicates the degree of smoothing of interest rates, ν_t is a zero mean external shock, and r_t^* is the interest rate target determined by (2).

Monetary policy rules can have many different instruments and objectives. McCallum (2000), for

example, suggests the following:

$$\Delta b_t = \Delta x^* - \Delta \nu_t^a + 0.5 (\Delta x^* - \Delta x_{t-1}), \quad (4)$$

where Δb_t is the change in the log of the monetary base, Δx^* is the target for nominal GDP growth and $\Delta \nu_t^a$ is the average growth rate of the monetary base velocity over the last sixteen quarters. The term $(\Delta x^* - \Delta x_{t-1})$ reflects long-run changes in the demand for monetary base.

We argue that the Brazilian Central Bank has used the nominal interest rate as a monetary policy instrument since the implementation of the Real Plan in July 1994. Its main objectives, aside from controlling inflation and output, are to prevent large changes in international reserves ² without, however, promoting drastic changes in the interest rate as in Clarida et al. (2000). Therefore we have the following rule:

$$i_t = \alpha + \beta i_{t-1} + \delta \pi_t + \kappa \tilde{y}_t + \theta \Delta R_t, \quad (5)$$

where ΔR_t is the change in international reserves and α , δ , κ , and θ are parameters. A similar rule is estimated for Brazil by Carneiro and Wu (2001) and is clearly supported by the Central Bank's actions which are documented in the reports made during the meetings of the Central Bank's Committee of Monetary Policy (COPOM) in which the nominal interest rate is determined .

3 The Threshold Autoregressive Model

The Threshold AutoRegressive (TAR) model was first proposed by Tong (1978) and further developed by Tong and Lim (1980) and Tong (1983). The main idea of the TAR model is to describe a given stochastic process by a piecewise linear autoregressive model, where the determination of whether each of the models is active or not depends on the value of a known variable, called the *threshold variable*.

A time series y_t is a *threshold process* with h regimes if it follows the model

$$y_t = \sum_{i=1}^h \left[\alpha_0^{(i)} + \sum_{j=1}^p \alpha_j^{(i)} y_{t-j} + \varepsilon_t^{(i)} \right] I_i(q_t), \quad (6)$$

²This extension is also used in Clarida, Galí and Certler (1997)

where $\varepsilon_t^{(i)} \sim \text{NID}(0, \sigma_i^2)$. The terms $\alpha_0^{(i)}, \dots, \alpha_p^{(i)}, i = 1, \dots, h$, are coefficients associated with each regime. $I_i(\cdot)$ is an indicator function, defined by

$$I_i(q_t) = \begin{cases} 1, & \text{if } q_t \in \mathbb{R}_i; \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

where \mathbb{R}_i is defined as $\mathbb{R}_i = (r_{i-1}, r_i]$ and $\{r_1, \dots, r_h\}$ is a linearly ordered subset of the real numbers, such that $-\infty < r_1 < r_2 < \dots < r_h < \infty$.

Model (6) is composed of h linear autoregressive models of order p , $\text{AR}(p)$, each of which will be active or not depending on the value of the threshold variable q_t . In this paper we generalize model (6) to include some exogenous variables. The model is thus defined as

$$y_t = \sum_{i=1}^h \left[\alpha_0^{(i)} + \sum_{j=1}^p \alpha_j^{(i)} y_{t-j} + \sum_{j=1}^q \beta_j^{(i)} x_{j,t} + \varepsilon_t^{(i)} \right] I_i(q_t). \quad (8)$$

The modelling procedure of TAR models consists of five steps:

1. Specifying a linear model.
2. Testing linearity against a TAR model and selecting the threshold variable.
3. Determining the number of thresholds.
4. Estimating the model.
5. Evaluating the estimated model.

The first step of the modelling cycle is carried out using standard linear time series and regression techniques. To carry out steps 2–5, Tsay (1989) proposed a simple model building procedure based on the residuals of an arranged regression. Suppose we have the following linear model for y_t :

$$y_t = \alpha_0 + \sum_{j=1}^p \alpha_j y_{t-j} + \sum_{j=1}^q \beta_j x_{j,t} + \varepsilon_t. \quad (9)$$

We refer to $[y_t, 1, y_{t-1}, \dots, y_{t-p}, x_{1,t}, \dots, x_{q,t}]$ as a *case* of data. An arranged regression is a regression with the cases reordered, based on the values of a particular variable. In the framework of the TAR

model, arranged regression becomes useful if we reorder the cases according to the threshold variable q_t . Tsay (1989) suggested a simple statistic to test for the threshold nonlinearity and to specify the threshold variable based on the recursive least squares estimates of the parameters of the arranged regression. He proposed running the linearity test for different choices of q_t , and selecting the one that minimize the p -value of the test. To identify the number and the candidate locations of the thresholds he also proposed some graphical techniques.

Due to the discontinuity at each threshold, the derivative based optimization techniques can not be applied to estimate the parameters of model (8). However, once the locations of the thresholds are determined, the least squares algorithm can be used to estimate each one of the h linear models separately.

The final step of the model building procedure is carried out by using well-known model misspecification tests, such as the ARCH Lagrange Multiplier (LM) ARCH test (Engle 1982) and the Lomnicki-Jarque-Bera test of normality.

4 Estimation Results

In this section we study whether there is evidence that the Brazilian nominal interest rate followed a nonlinear process between August 1994 and December 2000. The idea is that the Central Bank used the nominal interest rate as a monetary policy instrument, but its dynamic was different during currency crises, when compared to periods out of a crisis. In this section, a linear and a nonlinear model will be estimated and then compared. First, however, a brief description of the data set used will be made.

4.1 The Data

The nominal interest rate used is the monthly annualized *Taxa Selic*, controlled by the Central Bank. It will be used in levels despite the fact that usual unit root tests often reject the null of a stationary process. We argue that this happens because of the lack of power of these tests and also as a result of the relatively small sample used in the estimation (76 observations). Output is measured by monthly industrial production and the gap is measured as the residual from a Hodrick-Prescott filter (Hodrick and Prescott 1997). The inflation rate is calculated by a monthly wholesale index. The change in international reserves is used in the concept of international liquidity. These last variables are all considered stationary

by usual unit root tests. Figure 2 shows the time-series used. The data sources are Banco Central do Brasil, IBGE (Instituto Brasileiro de Geografia e Estatística), and IPEA (Instituto de Pesquisa Econômica Aplicada).

4.2 The Linear Model

In order to verify if the Brazilian nominal interest rate follows the Modified Taylor Rule described in Section 2, we estimate the following linear model:

$$i_t = \alpha + \beta i_{t-1} + \delta \pi_t + \kappa \tilde{y}_t + \theta \Delta R_t + \varepsilon_t, \quad (10)$$

where ε_i is assumed to be $NID(0, \sigma^2)$. We find the following results:

$$i_t = -0.02 + 0.93 i_{t-1} + 1.99 \pi_t - 0.43 \tilde{y}_t - 0.7 \times 10^{-3} \Delta R_t + \varepsilon_t \quad (11)$$

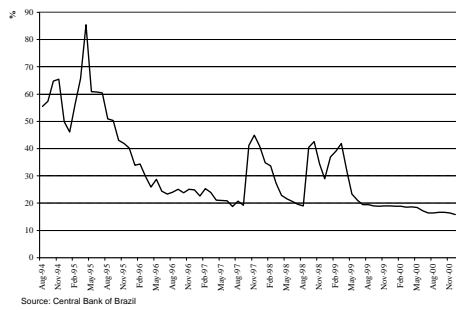
(1.44)
(0.04)
(0.69)
(0.19)
(0.1 \times 10^{-3})

$$\hat{\sigma} = 5.04 \quad R^2 = 0.89 \quad LJB = 1.1 \times 10^{-7} \quad SBIC = 3.52$$

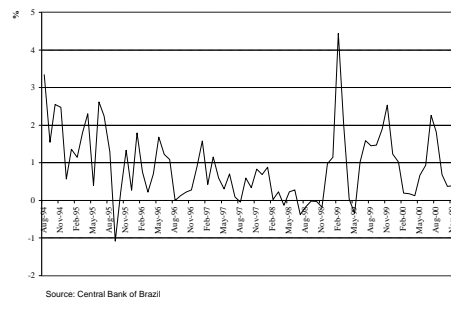
$$ARCH(1) = 0.12 \quad ARCH(2) = 0.20 \quad ARCH(3) = 0.36 \quad ARCH(4) = 0.1 \times 10^{-2}$$

$$LB(1) = 0.42 \quad LB(2) = 0.25 \quad LB(3) = 0.36 \quad LB(4) = 0.19 \quad LB(5) = 0.01 \quad LB(6) = 0.01,$$

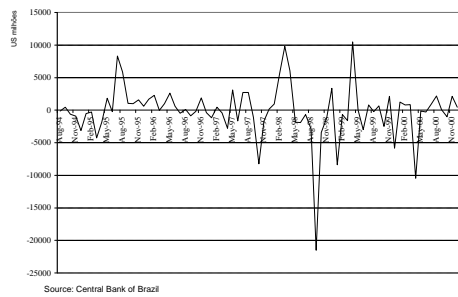
where $\hat{\sigma}$ is the residual standard deviation, R^2 is the determination coefficient, LJB is the Lomnicki-Jarque-Bera test of normality, $SBIC$ is the value of the Schwarz information criteria (Schwarz 1978), $ARCH(j)$, $j = 1, \dots, 4$, is the p -value of the LM test of no ARCH against ARCH of order j , and $LB(j)$, $j = 1, \dots, 6$, is the p -value of the Ljung-Box test of no serial correlation against serial correlation of order j . All coefficients have the desired signs and are statistically significant at 5% (except for the constant). The analysis of the regression residuals shows that the model may not be correctly specified, since an ARCH effect is present and the hypothesis of normally distributed residuals is strongly rejected. There is also evidence of serial correlation of orders 5 and 6. Figures 4 and 5 show, respectively, the estimated interest rates and deviations for the period considered. The very large positive errors outside the 5% confidence interval in periods usually associated to currency crises suggest that a model with more than one regime may be more adequate to represent the Brazilian nominal interest rate.



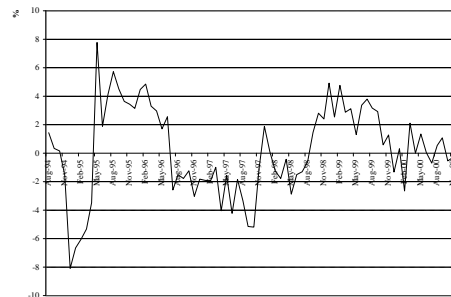
(a)



(b)



(c)



(d)

Figure 2: Time series. Panel (a) refers to nominal interest rate, *Taxa Selic*. Panel (b) refers to the monthly inflation rate - wholesale price index. Panel (c) refers to the monthly change in international reserves - US\$ million. Panel (d) refers to the output gap.

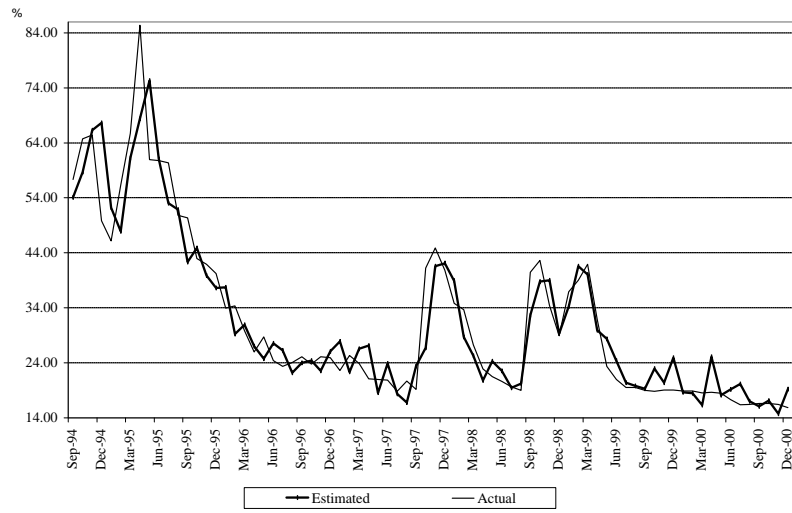


Figure 3: Nominal interest rate: actual and estimated by a linear model - Aug 1994 - Dec 2000.

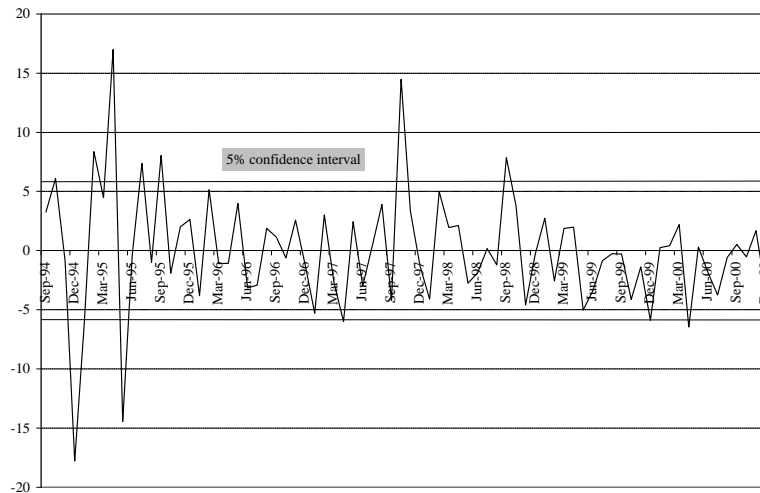


Figure 4: Regression residuals.

4.3 The Nonlinear Model

We now estimate a TAR model for Brazilian nominal interest rates. We want to investigate whether its dynamics are better represented by a two regime model, the first consisting of periods of currency crises and the second of periods out of crises (the tranquil periods). These will be determined endogenously by the model. The motivation for the estimation of a model with more than one regime is twofold: one theoretical, explained in the Introduction, which suggests that during currency crises the Central Bank is mainly concerned with preventing capital outflows; and an empirical factor which arises from the large deviations of the estimated interest rate from the actual rate when a linear model is estimated. Before estimating a model with more than one regime, however, it is necessary to test for nonlinearity in equation (10) and to select the more adequate threshold variable. As there is no evidence of any crises during the floating exchange-rate period considered in this paper (after January, 1999), we chose a threshold variable that is a good indicator of crises during the fixed-exchange rate period. The selected variable is the accumulated 3 month change in international reserves. The reason for constructing such a variable is that during the fixed exchange regime, which ended in January 1999, the change in international reserves was a good indicator of currency crises. However, when the exchange rate was allowed to float, the change in international reserves no longer reflected balance of payment pressures since the exchange rate is supposedly determined by the market. This can be seen from Figure 5, which shows that between August 1994 and January 1999 there are huge losses in international reserves in periods associated to currency crises, and large increases in international reserves at the end of crises, which in general results from the increase in interest rates which attracts short term capital inflows. On the other hand, the behavior of international reserves after January 1999 does not seem correlated to currency crises, especially because there were no such crises in the period considered. For this reason we considered the months of floating exchange-rate as belonging to the no crises period. The F statistic for the nonlinearity test associated with this threshold variable has a corresponding p -value of 1.58×10^{-5} .

Having found the threshold variable, the next step is to find the value of the threshold so as to minimize the total variance of the residuals. As can be seen in Figure 6, this variance is minimized when the accumulated 3 month change in international reservers is equal to -3.94 billions. Therefore, the

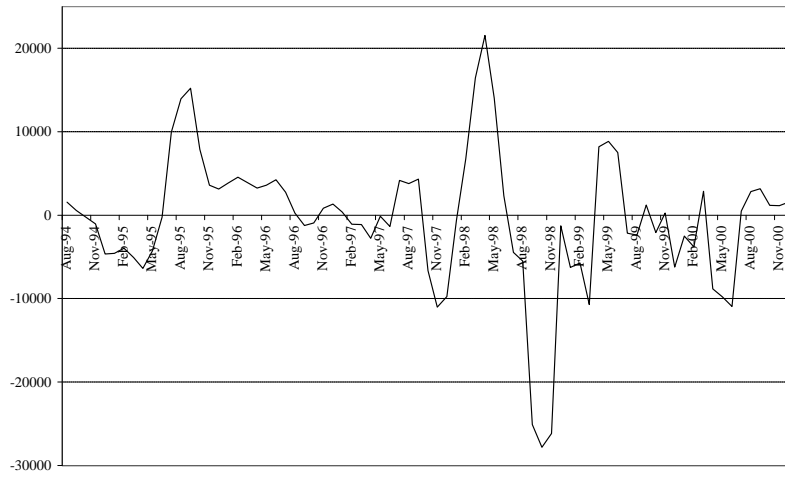


Figure 5: Accumulated 3 month change in international reserves - US\$ millions.

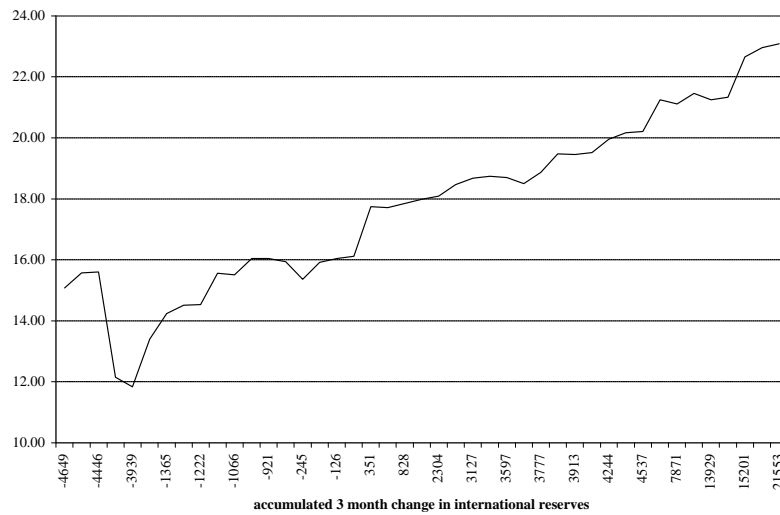


Figure 6: Residual variance of TAR model versus the accumulated 3 month change in international reserves.

estimated model has the following form:

$$i_t = \begin{cases} 11.48 + 0.54 i_{t-1} + 11.78 \pi_t - 0.9 \times 10^{-3} \Delta R_t + \varepsilon_t^{(1)}, & \text{if } I(\Delta R_{3,t}, t) = 1; \\ -1.69 + 0.98 i_{t-1} + 1.29 \pi_t - 0.33 \tilde{y}_t - 0.3 \times 10^{-3} \Delta R_t + \varepsilon_t^{(2)}, & \text{otherwise;} \end{cases} \quad (12)$$

$$\hat{\sigma} = 3.44 \quad R^2 = 0.94 \quad LJB = 1.1 \times 10^{-7} \quad SBIC = 3.04$$

$$ARCH(1) = 0.13 \quad ARCH(2) = 0.23 \quad ARCH(3) = 0.35 \quad ARCH(4) = 0.20$$

$$LM_{SI}(1) = 0.32 \quad LM_{SI}(2) = 0.38 \quad LM_{SI}(3) = 0.27$$

$$LM_{SI}(4) = 0.13 \quad LM_{SI}(5) = 0.14 \quad LM_{SI}(6) = 0.17,$$

where

$$I(\Delta R_{3,t}, t) = \begin{cases} 1, & \text{if } \Delta R_{3,t} < -3.94 \times 10^9 \text{ and } t < \text{Jan 1999;} \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

and $LM_{SI}(j)$ is the p -value of the LM test of no serial correlation against serial correlation of order j proposed by Eitrheim and Teräsvirta (1996). Although the test was originally proposed to evaluate Smooth Transition Autoregressive (STAR) models, it can be easily adapted to the TAR case as pointed out by Eitrheim and Teräsvirta (1996, p. 69). The standard deviation of the residuals associated with the crisis period (15 observations) is 5.81 and the one associated with the no crisis period (61 observations) is 2.54. An interesting result is the relatively small autoregressive coefficient in the regime with crises compared to the second regime. The size of the coefficient on the monthly change in international reserves is three times smaller in the first regime when compared to the second regime, which is consistent with the economic intuition that preventing capital outflows is more important during currency crises. The fact the output gap is statistically equal to zero in the first regime is also expected. The apparently large coefficient of the inflation rate in the crises regime is due to the fact that inflation is measured in % per month, while the interest rate is measured in % per year. The coefficients in the second regime all have the desired signs and expected magnitudes, except for the inflation rate coefficient, which is very small, after dividing by twelve. This is probably due to the fact that the coefficient of the lagged value of the interest rate is almost 1, explaining most of the variance of the series. The analysis of the residuals of the nonlinear model shows an improvement with respect to the linear model. It can be seen that the

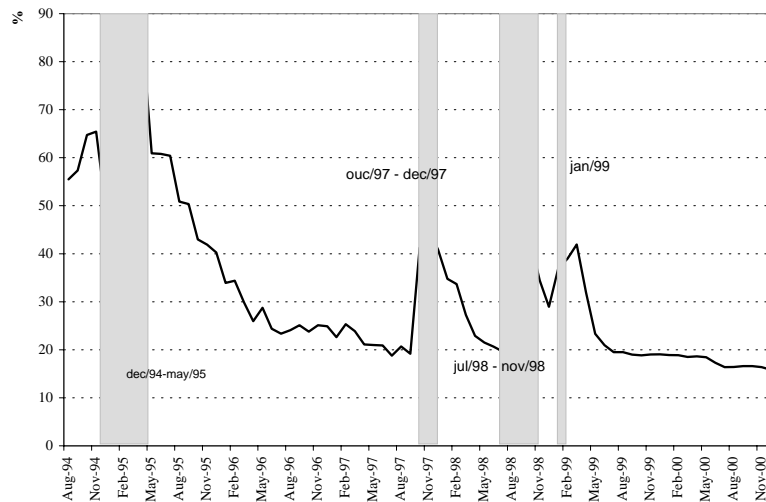


Figure 7: Nominal interest rates with the shaded areas representing crises periods.

residuals are normal at a 6% level of significance and that there is no ARCH effect when the residuals are standardized. The shaded area in Figure 6 are the months considered of crisis by the model.

Table 1 compares the linear and the TAR models. It can be seen that the latter performs better in most of the cases. In fact there are evidence of misspecification in the linear model which are not present in the nonlinear one. The SBIC is smaller in the nonlinear case, which is a result of the improvement in the fit when two regimes are considered. Therefore, we conclude that the TAR model is a better way to represent the reaction function of the Brazilian Central Bank.

5 Conclusions

The objective of this paper was to estimate the reaction function of the Central Bank of Brazil between August 1994 and December 2000. A linear model for the functional form for this reaction function was defined, where the nominal interest rate depends on its lagged value, the inflation rate, output gap and change in international reserves. Despite its high coefficient of determination and significant coefficients with the expected sign, there were some signs of misspecification, especially due to the large residuals

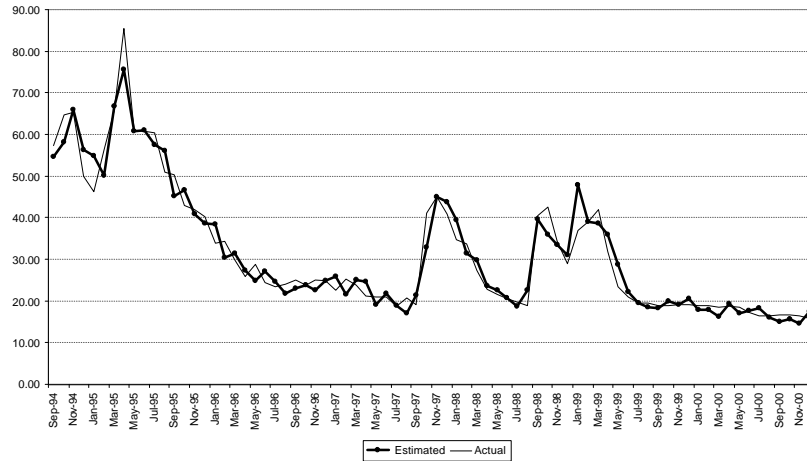


Figure 8: Nominal interest rate: actual and estimated by a TAR model - Aug 1994 - Dec 2000.

Table 1: Comparison of linear and nonlinear models.

Criterion	Linear	Nonlinear	Conclusion
SBIC	3.52	3.04	Very significant improvement in fit with the TAR model. Ratio TAR/Linear=0.86
Vres with Crises	84.53	33.76	Very significant improvement in fit during crises. Ratio TAR/Linear=0.40
Vres w/o Crises	10.63	6.44	Very significant improvement in fit during crises. Ratio TAR/Linear=0.61
Jarque-Bera	1.5×10^{-7}	5.8×10^{-2}	Both residuals are statistically not Normal
ARCH LM test	1.3×10^{-3}	0.40	Linear model has ARCH effect: evidence of nonlinearity
Serial correlation	Lags 5 and 6	No evidence	Linear model has serially correlated residuals

in periods associated to crises and to the ARCH effect.

As economic intuition also suggests a different behavior of interest rates during crises, the nonlinearity test proposed by Tsay (1989) was implemented, with the threshold variable being the accumulated three month change in international reserves for the period until January 1999. The model was found to be nonlinear with a significance level of 1.58×10^{-5} . By minimizing the total variance of the model, a threshold of US\$ -3.94 billions was chosen. When the threshold variable is less than this value, the economy is said to be in crisis, the opposite happening when the threshold is over US\$ -3.94 billions.

For the first regime, the coefficients on the lagged interest rate, output gap and international reserves have the expected signs and magnitudes. The large size of the inflation coefficient is probably a spurious effect that results from the small number of observations in the sample. For the second regime, all coefficients are as expected. It is interesting how the effect of the lagged interest rate is more important in this case, the opposite happening to the change in international reserves. Comparing the linear model with the TAR, there are significant advantages of the second approach. In the first place, the fit of the model is better when considering the SBIC. Analyzing only the periods determined to have crisis by the TAR model, the ratio of the residual variance of the nonlinear to linear model is equal to 0.40, and 0.61 when considering the periods without crisis. Second, the analysis of the residuals of the linear model shows that there may be a misspecification error, which does not happen with the TAR. As a result, we conclude that the non-linear model is a more convenient way to explain the reaction function of the Brazilian Central Bank between August 1994 and December 2000.

References

- Carneiro, D. and Wu, T. (2001). Contas externas e política monetária, *Texto para Discussão 442*, Pontifical Catholic University of Rio de Janeiro, Department of Economics.
- Clarida, R., Galí, J. and Gertler, M. (1997). Monetary policy rules in practice: Some international evidence, *NBER Working Papers 6254*, National Bureau of Economic Research.
- Clarida, R., Galí, J. and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory, *Quarterly Journal of Economics* **CXV - vol.1**: 147–180.

- Eitrheim, . and Teräsvirta, T. (1996). Testing the adequacy of smooth transition autoregressive models, *Journal of Econometrics* **74**: 59–75.
- Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* **50**: 987–1008.
- Garcia, M. and Valpassos, M. (1998). Capital flows, capital controls and currency crisis: The case of Brazil in the nineties, *Texto para Discussão 389*, Pontifical Catholic University of Rio de Janeiro, Department of Economics.
- Hodrick, R. and Prescott, E. (1997). Postwar u.s. business cycles: An empirical investigation, *Journal of Money, Credit and Banking* **29**: 1–16.
- McCallum, P. (2000). Alternative monetary policy rules: A comparison with historical settings for the united states, the united kingdom and japan, *NBER Working Papers 7725*, National Bureau of Economic Research.
- Schwarz, G. (1978). Estimating the dimension of a model, *Annals of Statistics* **6**: 461–464.
- Taylor, J. (1993). Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* **39**: 195–214.
- Tong, H. (1978). On a threshold model, in C. H. Chen (ed.), *Pattern Recognition and Signal Processing*, Sijthoff and Noordhoff, Amsterdam.
- Tong, H. (1983). *Threshold Models in Non-linear Time Series Analysis*, Vol. 21 of *Lecture Notes in Statistics*, Springer-Verlag, Heidelberg.
- Tong, H. and Lim, K. (1980). Threshold autoregression, limit cycles and cyclical data (with discussion), *Journal of the Royal Statistical Society, Series B* **42**: 245–292.
- Tsay, R. (1989). Testing and modeling threshold autoregressive processes, *Journal of the American Statistical Association* **84**: 431–452.