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DERIVED DEMAND AND CAPACITY PLANNING UNDER UNCERTAINTY
(revised edition)

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1. Introduction

In this paper we are primarily concerned with the derivation of demand functions for primary resources under uncertainty. Our framework posits an economic sector buying primary resources for the production of end-use goods characterized by an uncertain demand. Demand met through conversion and transmission of primary supplies in the energy sector is an example.

The literature on the theory of the firm facing uncertain demand 1/ has concentrated upon the impact on production decisions of alternative behavioral modes (price quoting and/or quantity setting) under different market structures (competitive and monopoly). However, little research effort has been devoted to modeling the effect on the derived demand functions for primary resources of the stochastic environment for the end-use products demand. This issue is critical when the resource markets have to decide in the face of uncertainty contracts, the expansion of capacity and the introduction of new technologies. Long-term contracts that take place in the energy sector primary fuel markets are an example (e.g., coal and uranium).

While the models we propose and the subsequent analysis are primarily data oriented, these can be directly related to alternative behavioral modes of action under uncertainty in the literature of stochastic programming. Following this introduction, we propose in section 2 a wait-and-see and a here-and-now 2/ approach for the derivation of the economic sector demand for primary resources under uncertainty. These are related by an equilibrium condition between "contingency" and "future" market prices for the resource. In section 3

1/ See for example Baron (1971), Dreze and Gabsewicz (1967), Sandmo (1971) and Leland (1970).

2/ This nomenclature was first suggested by Tintner (1955).

properties and implications of such an equilibrium are further explored, showing how the derivative of the expected value of perfect information, the price of uncertainty, is built into future prices. Section 4 presents a specific application to the U.S. energy sector demand for coal and the expansion of capacity of coal-fired electricity generation plants. Finally, in Section 5 the work is concluded highlighting policy implications and extensions.

2. Primary Resource Demand Under Uncertainty

In our framework of analysis we assume the existence of an economic sector for which a vector of end-use demands d, with components d_j, is given exogenously. Denoting by s the vector of primary supplies, with components s_i, available to the sector for the production of end-use goods, a model of the economic sector can be developed to calculate the cost function:

$$\Phi(s,d) = \text{cost of meeting the demand for end-use goods } d \text{ when } s \text{ is the amount of primary resources available to the sector.} \quad (1)$$

In our development below the cost function (1) is determined by

$$\Phi(s,d) = \text{minimum } c(x) \quad \text{s.t. } x \in X(s,d) \quad (2)$$

where the vector x denotes the level of operation of the sectoral activities and other decision variables restricted to lie within the set of feasible production possibilities X. The function c denotes the operating cost of the sector's activities.

The following assumptions guarantee desirable properties for the cost function Φ:

Assumption 1: i) c is convex

ii) X is a convex point-to-set mapping in the sense that if $x^1 \in X(s^1,d^1)$ and $x^2 \in X(s^2,d^2)$ then $\lambda x^1 + (1-\lambda)x^2 \in X(\lambda s^1 + (1-\lambda)s^2, \lambda d^1 + (1-\lambda)d^2)$ for all $0 \leq \lambda \leq 1$.

Assumption 2: For $d^1 \geq d^2$, $X(s,d^1) \leq X(s,d^2)$.

Assumption 3: For $s^1 \geq s^2$, $X(s^1,d) \geq X(s^2,d)$

Under the above assumptions it can be easily shown that Φ is convex, nondecreasing in d and nonincreasing in s. Furthermore, when Φ is differentiable[3/], the sectoral demand for primary supplies is given at positive resource levels[4/] by

$$p_i = -\frac{\partial \Phi}{\partial s_i}$$

which equates price with the marginal cost savings resulting from the resource availability.

In this study we are particularly concerned with the derivation of the sector demand for primary supplies under uncertainty in the end-use demands d. Letting d^θ denote the end-use demand vector associated with the state of the nature $\theta \varepsilon \Theta$, with probability density dF_θ, a cost function

$$\Phi^\theta(s) \equiv \Phi(s,d^\theta)$$

and a demand function

$$p^\theta(s) \equiv p(s,d^\theta)$$

contingent upon the state can be associated with each possible realization of the state of nature. This would be the case when all the sector's primary supplies are bought on contingency markets or posterior to observing the end-use demands. In other wordes, the sector can wait and see the resolution of uncertainty.

Alternatively for primary supplies that need to be contracted prior to the resolution of uncertainty, the sector has to be able to determine here and now its demand

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3/ In our application to the U.S. energy sector demand for coal in section 4 the resulting cost function will be convex but not everywhere differentiable. However, these technical difficulties can be overcome by the suitable use of subgradients (see Grinold(1975)).

4/ Formally as a result of minimizing $\Phi(s)$ + ps subject to $s \geq 0$, the optimality conditions require $p_i \geq -\partial\Phi/\partial s_i$ with equality holding for $s_i > 0$.

for primary resources. When simultaneously the operational levels or the capacity expansion of some resource using activities have to be decided prior to the resolution of uncertainty, we assume that the economic sector plans in two-stages 5/. In the first-stage the level of operations of the sectoral activities and other decision variables y that need to be decided here and now are determined. A second-stage then establishes decision rules contingent upon the first-stage decision, indicating the level of operation of the remaining activities z^θ for each and every possible realization of the state of nature $\theta \varepsilon \Theta$. A two-stage model of the economic sector can be developed to compute

$$\Psi(s) = \text{here-and-now expected cost of meeting the demand for end-use goods when s is the amount of primary resources available to the sector.} \quad (3)$$

In an analogous fashion to (2) the here-and-now expected cost function (3) is determined by

$$\Psi(s) = \text{minimum} \int c(y,z^\theta)dF_\theta \text{ s.t. } (y,z^\theta) \varepsilon X(s,d^\theta) \text{ for all } \theta\varepsilon\Theta. \quad (4)$$

Assumptions 1 and 2 guarantee that Ψ is convex and non--increasing in s. Furthermore, when Ψ is differentiable, the sectoral demand for primary resources is given at positive resource levels 6/ by

$$p_i = -\frac{\partial\Psi}{\partial s_i}.$$

Equilibrium in the future and contingency markets is established by the following condition:

"Given positive contingency prices p^θ for all $\theta\varepsilon\Theta$ and a future price p^f, the contingency and future markets are in equilibrium at $s=\hat{s}$ if

5/ See Dantzig (1955).

6/ The same observations as in footnote 3 and 4 apply to the minimization of two-stage costs.

$$p^f = - \frac{\partial\Psi}{s} \Bigg|_{s=\hat{s}}$$

5)

and,

$$p^\theta = - \frac{\partial\Phi_\theta}{\partial s} \Bigg|_{s=\hat{s}}$$

for all $\theta\varepsilon\Theta$"

The rationale for this equilibrium condition stems from the interpretation of the contingency prices p^θ. With (5) if this set of prices were to prevail in the resource markets the optimal resource consumption under the two-stage scheme would turn out to be optimal for each and every possible realization of the state of nature $\theta\varepsilon\Theta$ under the wait-and--see strategy.

3. The Cost of Uncertainty

It can be verified that for any fixed resource supply s, the solution to the two-stage problem (4) yields feasible solutions to the wait-and-see problems (2) for each every possible realization of the state of nature. Hence, the here--and-now cost function (3) and the wait-and-see cost functions (1) are related by

$$\Psi(s) \geq \int \Phi^\theta(s) dF_\theta. \tag{6}$$

Furthermore, we assume that $X(0,d^\theta)$ is non-empty and that the here-and-now and the wait-and see decisions coincide in the absence of primary resources[7/]. Under these conditions, we have

$$\Psi(0) = \int \Phi_\theta(0) dF_\theta. \tag{7}$$

which implies that the here-and-now and the expected wait--and-see costs are equivalent at s = 0.

The difference

$$\Delta(s) = \Psi(s) - \int \Phi^\theta(s)\ dF_\theta. \tag{8}$$

7/ This will be the case in section 4, for additional coal-fired electricity generating plants in the absence of coal, when $\gamma = 0$ or $s = 0$.

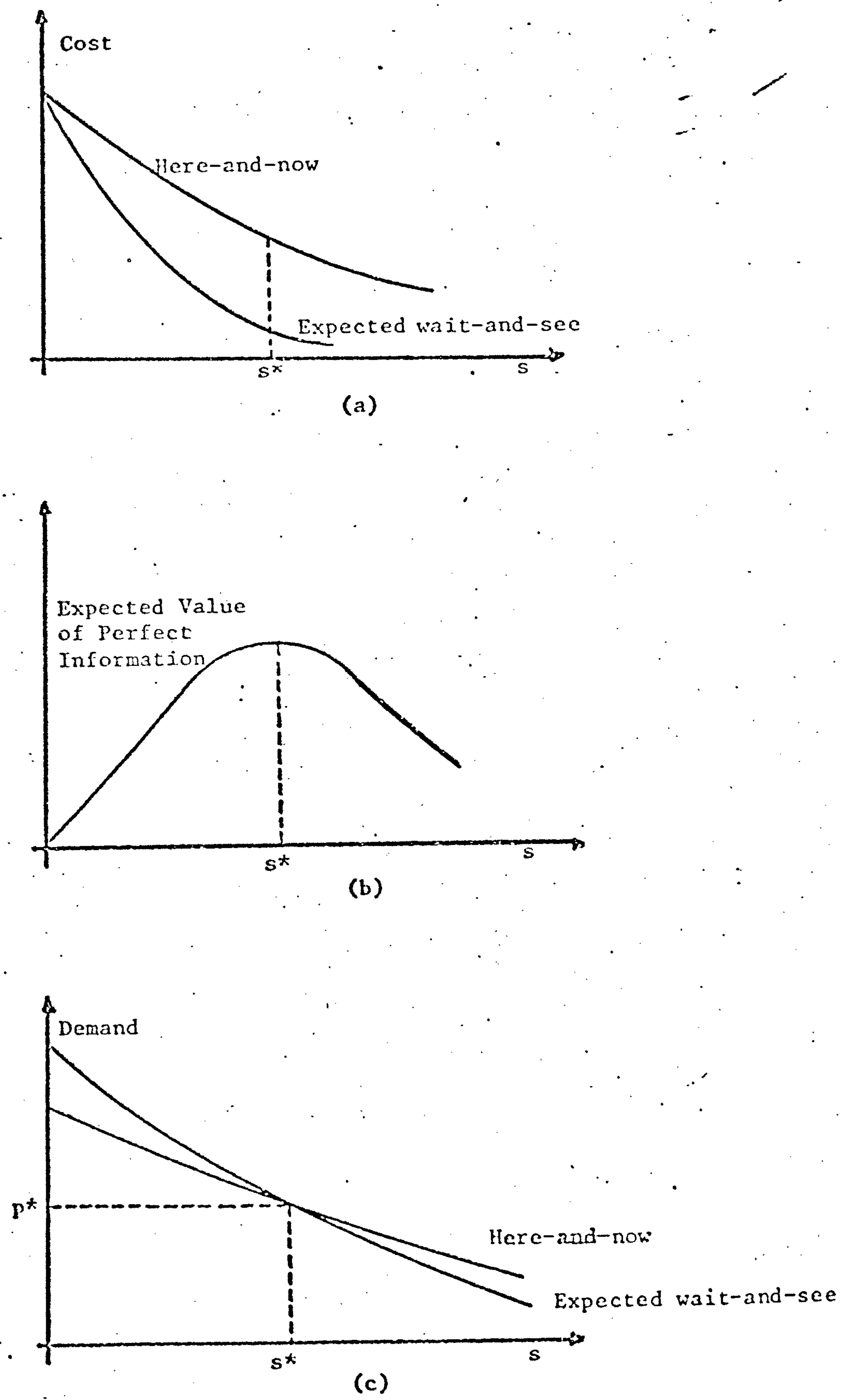

Cost
Here-and-now
Expected wait-and-see
s*
s
(a)
Expected Value
of Perfect
Information
s*
s
(b)
Demand
p*
Here-and-now
Expected wait-and-see
s*
s
(c)

Figure 1

When resource prices are high ($p > p^*$) the resource-using activities are expensive and may not always be operated given demand uncertainty. Opportunity losses are worth incurring. This implies that the here-and-now order is smaller than the expected wait-and-see order. Within this range, to secure consumption of an additional unit of the resource, the sector is willing to pay more for a perfect prediction of the end-use-demands and the EVPI curve rises. When the resource price is low ($p < p^*$), the resource-using activities become an interesting proposition, no matter what the demand outcome is. The expected opportunity cost of not having enough capacity to accomodate demand fluctuations may become larger than the resource acquisition costs; the here-and-now order becomes larger than the wait-and-see order. Perfect information becomes less valuable as the resource price futher declines and the EVPI curve decreases. A maximum of the expected value of perfect information is reached for $p^f = p^*$ and $\delta = 0$ when the here-and-now the wait-and-see orders coincide at s^*.

4. An Application to the U.S. Energy Sector: The Case of Coal

In this section, we discuss the application of the models described in sections two and three to the derivation of the U.S. energy sector demand for coal under uncertainty in the end-use demands. In determining the here-and-now cost function for coal, the first-stage decision is the optimal capacity of coal-fired electricity generation plants. In the second-stage, the optimal operational level of the sectoral activities are determined for each and every possible realization of the end-use demands. The sensitivity of the results is discussed under different scenarios for uncertainty, oil-import prices, nuclear power capacities and capital costs of coal-fired plants.

* LP Model of U.S. energy sector. For this study, we used a linear programing representation of the energy sector analogous to (3) derived in large part from the Brookhaven Energy System Optimization Model (BESOM). The model is essentially a network optimization problem consisting of source nodes corresponding to primary energy supplies and sink nodes corresponding to energy end-use demands. The arcs in the network connect source nodes to sink nodes and they correspond to activities converting and transmitting supply BTU's into satisfied demand. Losses in arc flows are experienced due to inefficiencies in conversion and transmission devices. Unit costs are associated with each arc. Demands are given exogenously, and supplies are variable with associated prices per BTU supplied up to fixed upper bound levels. In addition, there are side constraints on the BTU flows in the arcs due to capacity restrictions and policy constraints. The objective function is to minimize the supply and conversion costs of meeting exogenous energy end-use demands in a given year.

Table 1 gives the unit costs 9/ and exogenous demands and supplies for the reference year 2000. The costs are in 1974 dollars per 10^6 BTU's delivered to intermediate nodes in the energy network optimization problem; specifically, these are the nodes immediately preceding the conversion of BTU's to electric energy and end-use devices. No exogenous value is

9/ The unit costs in Table 1 include extraction and transportation cost for all primary fuels. With the introduction of a nonlinear oil and gas supply function it was necessary to deduct the costs of $9/bbl of oil and 82¢/1.000 ft^3 of natural gas adjusted by the efficiencies of the processes from the corresponding entries in the Table 1. In an analogous fashion, $16.40/ton and $14.50/ton for underground and stripmined coal respectively, have been deducted from Table 1.

Table 1

Cost Coefficients ($1974/106BTU) and Constraining Values (10^{15}BTU)
(Year 2000)

Supply \ Demand	Space Heat	Air-Conditioning	Base Load	Peak Load	Process Heat	Petrochemical	Air Transport	Public Transport	Private Transport	Intermediate Load	Water Heat Cooking	Exogenous Supply Limit
Coal Stripmined					.89	.73						
Coal Underground					.96	.80						
Oil	2.56				1.56	1.77	2.21	2.56	3.04		2.56	
Natural Gas	1.61	1.61			.96	.96					1.61	
Coal Eletric	12.08	14.22	8.31	39.45	8.31			10.70		10.70	8.14	
Oil Electric	12.22	13.88	9.07	33.43	9.31			11.16		11.16	9.17	
Turbine	15.90	17.04	13.72	30.53	13.87			15.17		15.17	13.79	
Light Water Reactor	12.51	14.95	7.85	43.92	8.20			10.94		10.94	8.00	27.78
Gas Electric	11.01	12.60	7.99	31.37	8.20			9.99		9.99	8.08	
Hydro Eletric			3.93	26.11	4.15			5.84		5.84		3.90
Geothermal			3.43	22.19	3.59			5.03		5.03		1.60
End-Use Demands	7.99	4.77	8.91		18.49	10.63	8.63	10.53	12.84	1.85	3.01	

Source: Brookhaven National Laboratory (1975) assigned to the demand for peak load electricity. This demand is assumed to be endogenous and formed by summing 10% of the demand for intermediate load electricity, 5% of the demand for base load electricity, 5% of the demand for electric process heat, 5% of the demand for electric water heat and cooking and 10% of electric public transport.

Side constraints, based on average and projected national figures, were included in the model to make it a more accurate description of the U.S. energy sector. These require: the oil-to-gas usage ratio in the petrochemical industry to be exactly 5.6; the amount of electricity used for ground transportation to be $.23 \times 10^{15}$ BTU; the amount of coal used in the petrochemical industry not to exced 2.60×10^{15} BTU; the amount of coal used for process heat not to exceed 9.39×10^{15} BTU; the amount of natural gas for air conditioning not to exceed 0.03×10^{15} BTU and turbine generators to handle at least half of the peak demand assigned endogenously. The cooling demand is required not to exceed 3/5 of the heating demand in proportion to the number of months in a year when cooling and heating are demanded by residential and commercial units. These constraints reflect the fuel mix and projected capacity of the end-users' present equipment and in most cases, extensive changes in equipment would be necessary to overcome them.

* <u>Nonlinear oil and natural gas supply model</u>. The nonlinear supply function we use is

$$f(s_1, s_2) = (\delta_1 s_1^c + \delta_2 s_2^c)^{w/c}$$ = cost of supplying s_1 BTU's of crude oil and s_2 BTU's of natural gas.10/

10/ Manne (1976) used a limiting case of this function, namely when c=0, to model endogenous demand for electric and non-electric energy.

Its analytic form is similar to the constant elasticity of substitution production function, but with the parametric restrictions $\delta_1 > 0$, $\delta_2 > 0$, $c > 1$ and $w > c$ which provide the appropriate convexity. The homogeneity of degree w (greater than 1) reflects the depletion of domestic crude oil and natural gas reserves. The non-zero cross-price elasticities encompass the cross-price effects found in the process of discovery of associated natural gas.11/ This function can be related to econometrically estimated supply functions by equating the long-run marginal costs of supply to supply prices.12/

The complete specification of the supply function requires the determination of the constant perameters δ_1, δ_2, w and c. For this purpose it suffices to make two observations from the supply curves. Two "reference prices" and"reference levels" of supply were set as depicted in Table 2. 13/

Table 2

Oil and Gas Supply Reference Points

	Oil Domestic Production (10^{15}BTU)	Gas Domestic Production (10^{15}BTU)
Oil C \$ 7/barrel Gas C \$ 8/10^3 ft.^{3}	23.1	23.9
Oil C \$11/barrel Gas C \$ 1/10^3 ft.^{3}	31.3	24.7

Source: Federal Energy Administration (1974), Tables F-8, F-9 and I-10.

11/ For example, see Erickson and Spann (1971), Khazoom(1971) and Pindyck (1972).

12/ See Modiano and Shapiro (1980).

13/ For further details on the specification of the supply function see Modiano and Shapiro (1980).

In order to retain the linear programming format of the sectoral model, the nonlinear oil and natural gas supply function was incorporated using a grid linearization of the convex supply function. 14/ The set of grid points used in our study ranged from [0,0] to [60,45] quads of crude oil and natural gas respectively.

* Nuclear Energy. Limits to the uranium supply to the U.S. energy sector were constructed from forecasted levels of nuclear generating capacity for year 2000, as depicted in Table 3. Assuming a load factor of 58% for nuclear plants, a conversion factor of 3413 BTU per KWh and a plant efficiency of .31, upper bounds on the uranium supply were derived for each of the three alternative scenarios for nuclear capacity.

Table 3

Scenarios	Nuclear Power* (GWe)	Uranium Supply Limits (10^{15}BTU
Low	380	21.11
Medium**	500	27.78
High	620	34.44

* Source: Workshop on Alternative Energy Strategies (1977)
** Constructed by averaging the low and high scenarios.

In this study the cost of the nuclear fuel input to the plant per million BTU in year 2000 is assumed to be 50c in 1974 prices. This value should be divided by the plant conversion efficiency (.31) to obtain the raw fuel cost share of the intermediate energy form costs in Table 1.

14/ The sectoral problem incorporating the linearization is further described in Modiano and Shapiro (1980).

* Oil Import Prices. Starting at the current 1979 import price of $15 per barrel three scenarios based on alternative growth rates of crude oil real prices were determined. The low scenario results from the assumption of no-growth in real prices between years 1979 and 2000. For the derivation of the medium-and high-price scenarios the growth rates of 2% and 5% per year respectively were assumed. Under an average conversion factor 5.6×10^6 BTU per barrel of oil and a price index of 1.37 the low-, medium- and high-growth oil import price scenarios are in 1974 prices, $1.96, $2.96 and $5.45 per million BTU respectively.

* Uncertainty in the End-use Demands. For this study, the end-use demands were assumed to be independently normally distributed with mean equal to the actual BESOM forecasts for year 2000 presented in Table 1. To study the sensitivity of the results, alternative values for the standard deviations were associated with three distinct levels of uncertainty in these predictions. The standard deviations for each category of end-use demand corresponding to the low-, medium- and high-uncertainty scenarios are depicted in Table 4.

The standard deviations for the high-uncertainty scenario were selected such as to yield infinitesimal probabilities of negative values for the normally distributed end-use demands. This was accomplished by equating the mean of the distribution with four times its standard deviation 15/. To facilitate the computations, the standard deviations were restricted to multiples of $.5 \times 10^{15}$ BTU. The entries for low- and medium-uncertainty are respectively 1/5 and 3/5 the standard deviations under the high-uncertainty scenario.

15/ The same standard deviations for the end-use demands would result from assuming for the high-uncertainty scenario, a multiplicative uncertainty form, $d = \theta d$, with θ normally distributed with mean 1 and standard deviation .25. These probabilities were concentrated on the five bracket-median points denoted; low, medium-low, medium, medium-high and high end-use demands.

Table 4

Standard Deviations for End-Use Demands (10^{15} BTU)

Demand Category	Low Uncertainty	Medium Uncertainty	High Uncertainty
Space Heat	.30	.90	1.50
Air Conditioning	.20	.60	1.00
Base and Intermediate Load	.50	1.50	2.50
Process Heat	.90	2.70	4.50
Petrochemicals	.50	1.50	2.50
Air Transport	.20	.60	1.00
Public Transport	.50	1.50	2.50
Private Transport	.60	1.80	3.00
Water Heat and Cooking	.10	.30	.50

The linear programming respresentation of the energy sector was retained by discretizing the continuous probability distributions into five brackets with probabilities .05, .20, .50, .20 and .05 respectively.

* Coal-fired electricity generation capacity. The construction and expansion of electricity generation plants requires a lead time. For this reason, electricity generation capacity built-up plans have to be undertaken, in general, several years prior to the date when the new capacity is expected to be operational. Once such capital investment decisions are made, adjustments in the sector's operations due to deviations from forecasted levels can only take place within the limits of available capacity.

We assume for the derivation of the demand for coal under uncertainty in the end-use demands, the optimal size of coal-fired electricity generation plants to be a first-stage decision. Contingent upon the choice of the optimal capacity in the first-stage all remaining decision variable (e.g., the energy flows in the network, oil imports, domestic crude oil and natural gas production) are determined in a second-stage for each and every possible realization of the end-use demands. Hance, the second-stage decision variables are further indexed in correspondence to the state of nature to which they refer. As previously described, discretizing the normally distributed random demands led to five states denoted by θ = 1, 2, 3, 4, and 5.

In our model, the first stage decision is implemented by the constraints

$$\sum_{\substack{j=\text{electric}\\ \text{demand}\\ \text{categories}}} (z_j^\theta/\gamma_j) - y \leq 0 \qquad \theta = 1,2,3,4, \text{ and } 5$$

$$\sum_{\substack{j=\text{electric}\\ \text{demand}\\ \text{categories}}} (z_j^\theta/e_j) + \sum_{\substack{q=\text{non-electric}\\ \text{demand}\\ \text{categories}}} (w_q^\theta/e_q) \leq s \qquad \theta = 1,2,3,4, \text{ and } 5$$

where y denotes the energy equivalent of the fully utilized capacity of the coal-fired power plants and z_j^θ denotes the energy flow from coal-fired plants to the electricity demand category j for each realization of the end-use demands. The utilization factor γ_j gives the load duration measured as a fraction of the year for each end-use demand category. Table 5 depicts the utilization factors used for the various end-use energy forms. The energy flow from coal to the non--electric demand category q for the state of nature θ is w_q^θ. The efficiencies in the path are e_j and e_q from coal to electric and non-electric end-use demands respectively. Coal availability is denoted by s. Notice that optimality conditions for linear programming require, for s>0, that the future price equals the sum of the shadow prices attached to the coal availability constraints.

Table 5

Utilization Factors for Power Plants

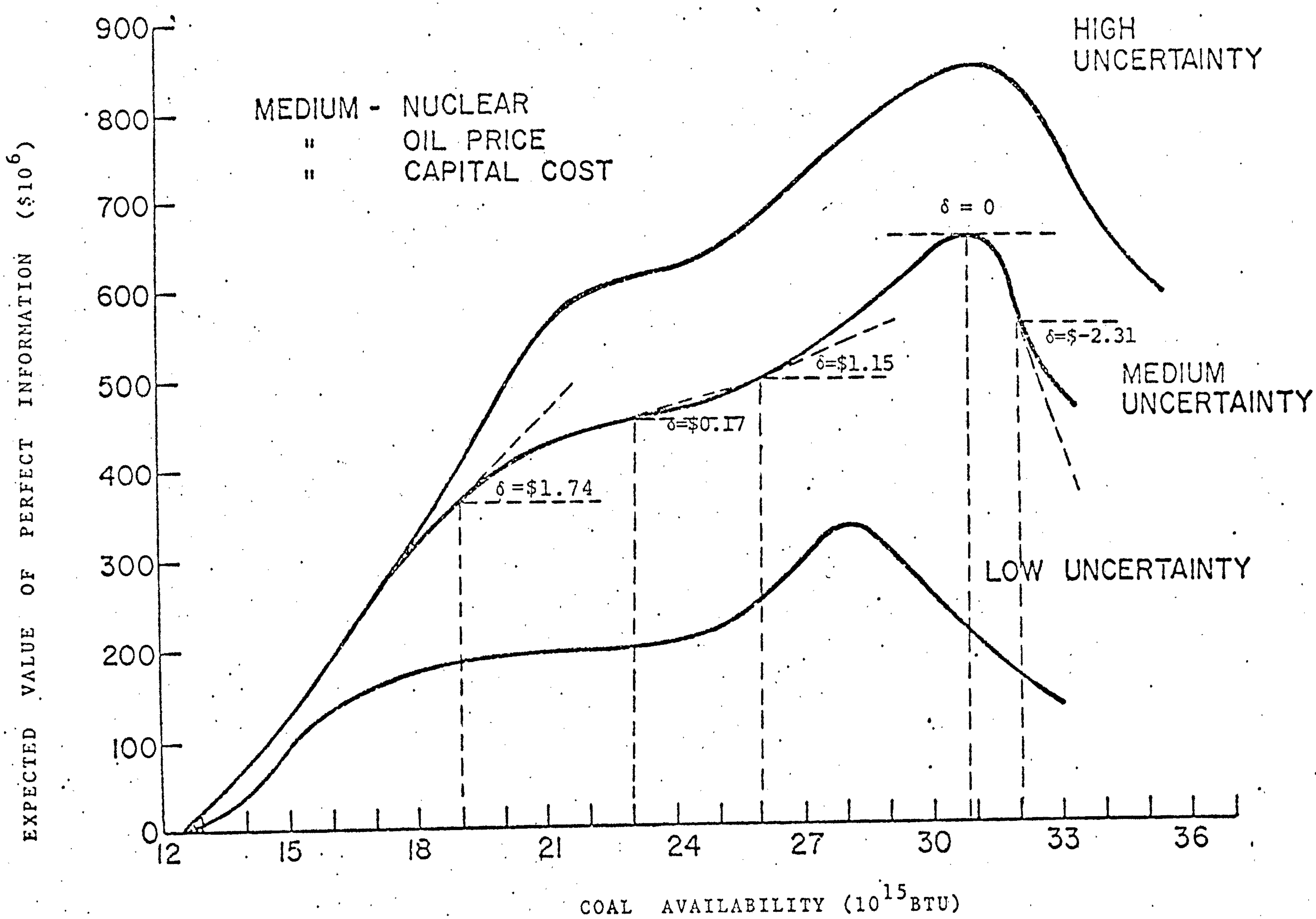
Demand Category	Utilization Factor
Space Heat	.42
Air Conditioning	.33
Base Load	.80
Peak Load	.10
Process Heat	.75
Intermediate Load	.50
Public Transport	.50
Water Heat & Cooking	.78

The capital recovery cost 16/ per BTU for power plants at the intermediate energy form is computed assuming a fixed charge rate of 0.15 at a 30-year life, a conversion factor of 3413 BTU per KWh and an electricity transmission and distribution efficiency of .91. The capital costs for coal-fired plants in this study are 350,425 and 500 dollars per kilowatt for the low, medium-and high-scenarios respectively 17/.

* Results. The expected value of perfect information derived from our model under the medium scenario for year 2000 is depicted in Figure 2. At a future price as high as $43.4 per ton of coal, the optimal coal consumption by the energy sector under the two-stage strategy is 19 x 10^{15}BTU or, at an average conversion factor of 23 x 10^{6}BTU per ton of coal, 826 million tons. At this level, the expected contingency price, could the sector wait-and-see the end-use demands prior to deciding on the capacity of coal-fired electricity

16/ The original formulation of BESOM includes the capital recovery cost for $425/KW as part of the cost coefficients of the energy flows. For this study it was necessary to further deduct $3.60/$10^{6}$BTU in Table 6 divided by the corresponding utilization factor in Table 5 from the cost coefficients for coal-fired electricity given in Table 1.

17/ The capital recovery costs include the capital costs of $75/KW and $145/KW for eletric transmission and distribution facilities respectively.



THE VALUE OF INFORMATION: UNCERTAINTY SCENARIOS

Figure 2

generation, is $45.08 per ton. The difference of $1.74 per ton (approximately 4.5% of the future price) represents the price of uncertainty, namely the amount coal producers are foregoing due to the fact that uncertainty in the end-use demands requires a two-stage strategy. At the price of $37.95 per ton which corresponds 23 x 10^{15}BTU (1,000 x 10^{6} tons) of coal the price of uncertainty represents less than 1% on the future price. When the coal future price reaches $32.43 per ton, the price of uncertainty increases to $1.15, approximately 4% of the future price, at 26 x 10^{15}BTU 91,343 x 10^{6}) as the expected of perfect information in Figure 2 reaches a maximum. The expected contingency price is $23.69 at the future price of $26.00 per ton of coal. The negative difference between those prices of $2.31 (approximately 10% of the future price) at 32 x 10^{15}BTU (1,391 x 10^{6} tons) of coal represents the amount the energy sector is paying coal producers over and above the expected contingency price due to the here-and-now strategy.

Under the medium uncertainty scenario perfect information is worthless up to 12.66 x 10^{15}BTU (550 x 10^{6} tons). This is due to the early substitution of coal for oil and natural gas for the two non-eletric end-use categories: process heat and petrochemicals 18/. Further increases in coal availability cause its introduction for electricity generation. Perfect information becomes valuable as capacity expansion decisions for coal-fired plants have to be made prior to the resolution of uncertainty. Substitution of coal for the competing electricity sources follows a well-defined hierarchy in the model: oil,natural gas, nuclear, hydroelectric and geothermal. When the substitution for oil and gas dominates, the value of information rises sharply and the price of uncertainty is high. For the medium uncertainty scenario, substitution for oil and natural gas for electricity generation for the low and medium-low states increases the

18/ See Table 1.

value of perfect information up to 21 x 10^{15}BTU (913 x 10^{6} tons). As substitution for nuclear energy in the hierarchy is reached, coal and nuclear being the two most competing sources for electricity generation, the price of uncertainty declines. Under medium-uncertainty coal starts to substitute for nuclear energy, specifically for the low and the medium--low states of the end-use demands, at 21 x 10^{15}BTU19/.

The discretization of the probability distribution of the end-use demands explains the shape of the expected value of perfect information in Figure 2. This is because the stages in the substitution hierarchy under the five different states occur at different coal levels. As coal availability reaches 25 x 10^{15}BTU (1,087 x 10^{6} tons) substitution for nuclear energy under the medium-low state of the end-use demands slows dows and the substitution for oil and natural gas for the medium, medium-high and high states takes over. Once again, the perfect information curve becomes steeper reaching the maximum level of $658 million at 30.8 x 10^{15}BTU (1,343 x 10^{6} tons) where the price of uncertainty vanishes. As coal starts to substitute for nuclear energy for the medium state of the end-use demands, which by construction corresponds to the mean of the probability distribution, the decline in the price of uncertainty reduces the value of perfect information away from its peak level.

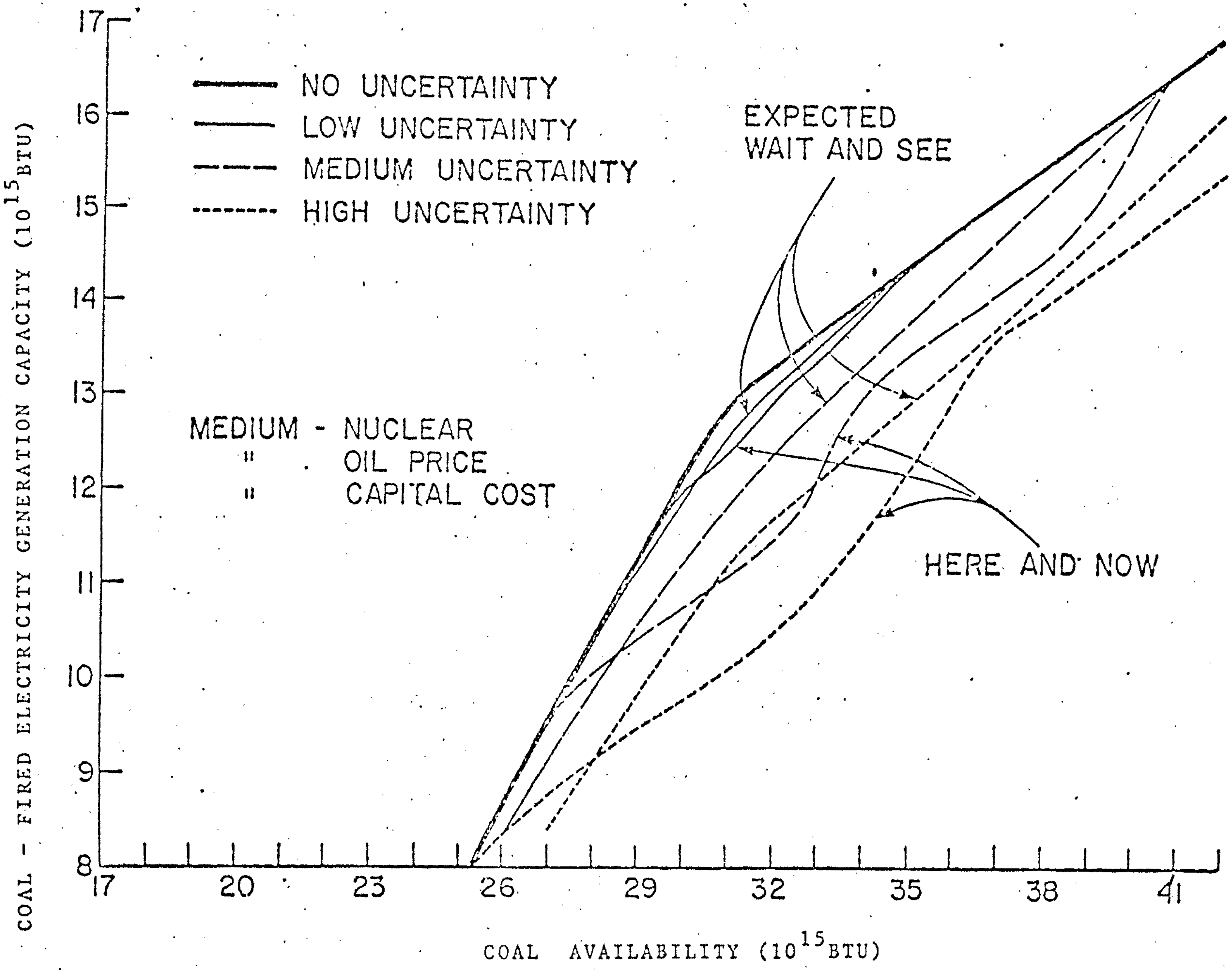
Figure 2 also displays the effect of the alternative uncertainty scenarios. Perfect information becomes more valuable as the uncertainty in the prediction of the end-use demands increases. Also, as uncertainty increases, the expected value of perfect information shifts rightwards. As one would expected, at each coal level the price of uncertainty is higher the larger are the standard deviations of the probability distribution of the end-use demands.

19/ Nuclear energy consumption for the medium-low state moves away from its upper bound of 27.78 x 10^{15}BTU.

Figure 3 depicts the effect uncertainty upon the optimal capacities of coal-fired plants. Uncertainty only seems to play a role in determining optimal capacities whithin certain ranges. This is because very low coal levels are associated with high prices which in turn make coal-fired electricity generation plants unattractive no matter the uncertainty in the end-use demands. Higher levels of coal availability correspond to lower prices which tend to make coal-fired electricity an increasingly attractive option. Depending upon the magnitude of the capital cost the role of uncertainty may eventually vanish again. Clearly the greater the standard deviations of the end-use demands, the wider is the range over uncertainty is a determinant factor of the optimal capacities.

The size of coal-fired plants is inversely related to uncertainty in the end-use demands at all levels of availability. This is because, at a fixed coal supply level, capacity cannot increase, if the non-electric coal using activities are already at their upper level, to accomodate the larger demands that may result from increasing uncertainty. On the other hand, the risk of excess capacity increases as lower demands have higher probabilities. Consequently both the optimal and the expected capacities decline. When the probability distribution collapses around the mean (the no-uncertainty scenario displayed in Figure 3) the size of coal-fired plants is severely overestimated. Under medium-uncertainty at 32 x 10^{15}BTU (1,391 x 10^{6} tons) the excess capacity that results from disregarding uncertainty or planning at the mean is approximately 62 GWe (1,8 x 10^{15}BTU equivalent energy units at full capacity). At the capital cost of $425 per KW, this represents an excessive investment of $26 billion.

The relative positions of the here-and-now and the expected wait-and-see curves suggest that at low levels of

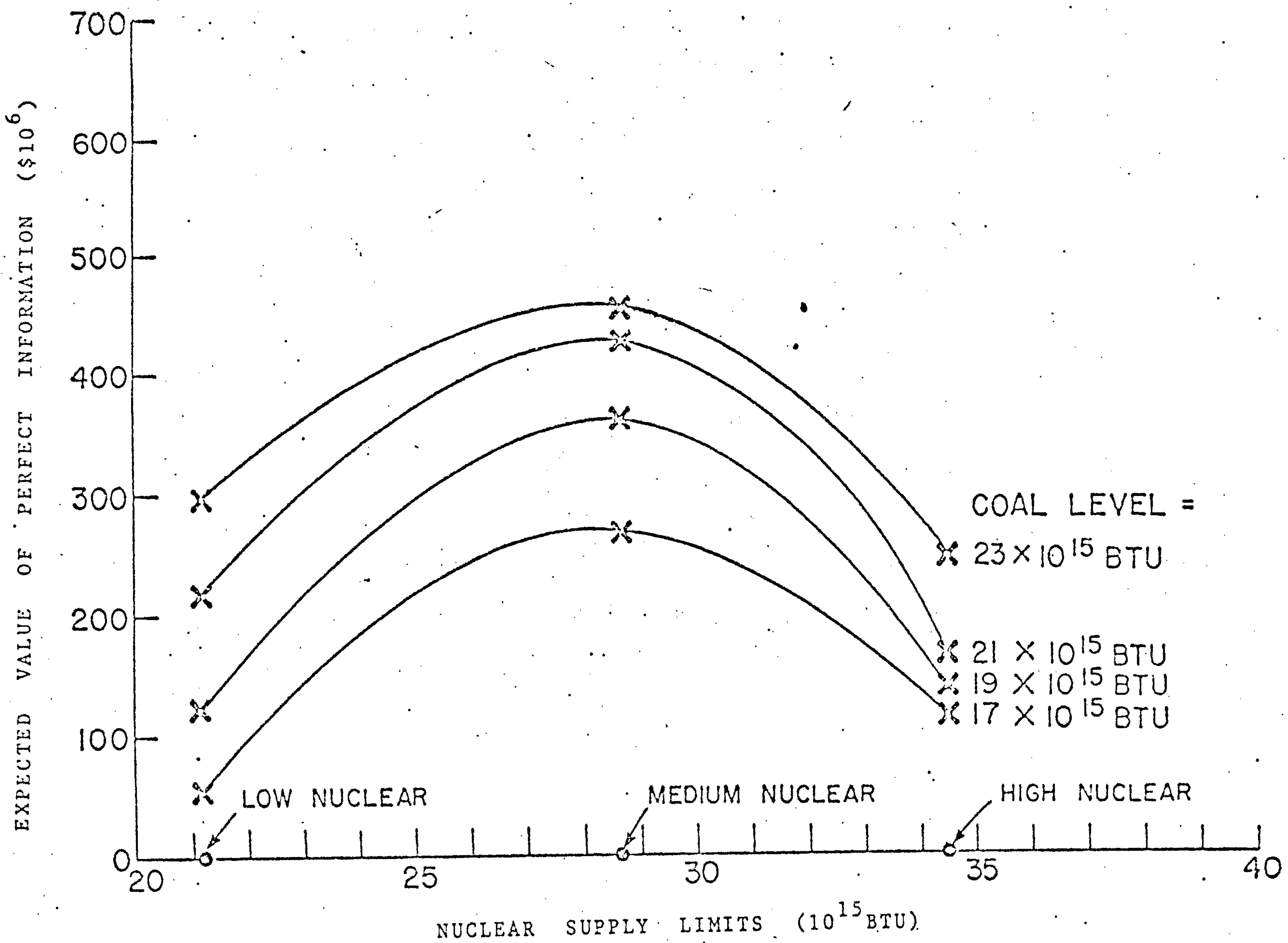


ELECTRICITY GENERATION CAPACITY: UNCERTAINTY SCENARIOS

Figure 3

coal availability the opportunity cost of a capacity shortage is larger than the capital cost of coal-fired plants. The risk of building up excess capacity is worth taking. The here-and-now order is larger than the expected optimal capacity could the sector wait and see the resolution of uncertainty. As coal availability increases, the resource price declines lowering opportunity costs which are eventually taken over by the capital cost. The here-and-now capacity becomes then smaller than under the wait-and-see strategy. The breakeven points occur at 1,217, 1,239 and 1,326 million tons of coal for the high -, medium - and low-uncertainty scenarios respectively.

The expected value of perfect information under the three alternative nuclear scenarios for different levels of coal availability is displayed in Figure 4. Given the substitution potential between coal-fired and nuclear energy for electricity generation coal prices will be higher the lower is the available nuclear capacity. Hence, at very low levels of nuclear supply, coal is more expensive than nuclear energy and a marginal increment in nuclear capacity may generate large savings in the coal bill. The value of perfect information might initially increase at the margin, as perfect information would allow cost reductions to be expected from the optimal allocation of the less expensive nuclear capacity increment. Eventually, increasing levels of nuclear capacity will drive coal prices downward enough to make perfect information decline in value at the margin. This the case for the range of nuclear supply limits depicted in Figure 4. At the extreme situation of unlimited nuclear capacity one would expect the value of perfect information to be low. As coal-fired plants become increasingly attractive, given the lower resource price and capital cost relative to nuclear energy, the role of uncertainty is reduced. In the other dimension, the expected value of perfect information increases with coal availability



THE VALUE OF INFORMATION: NUCLEAR SUPPLY LIMITS

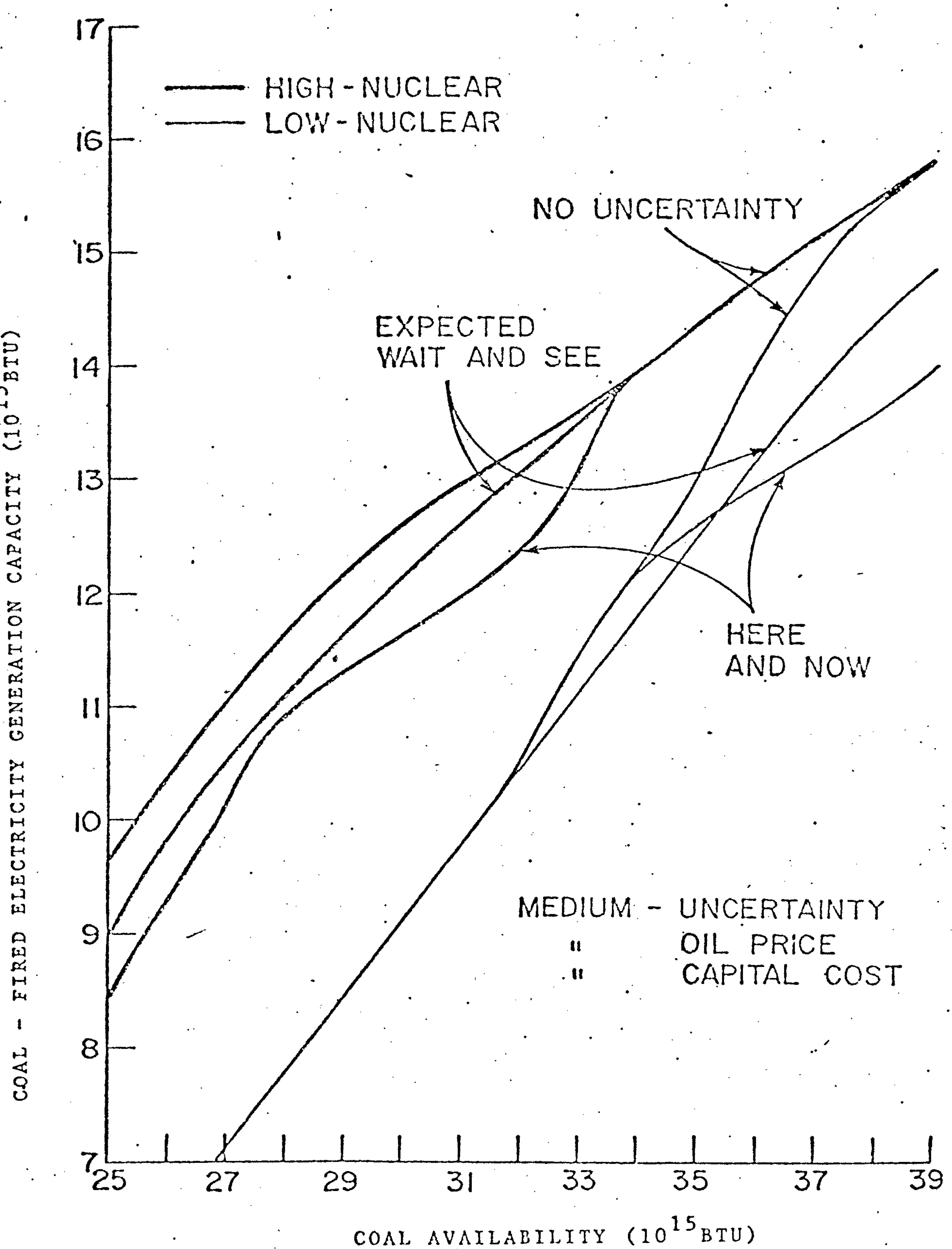
Figure: 4

at fixed levels of nuclear capacity from 17 to 23 x 10^{15}BTU of coal. This corresponds to the ascending range of the expected value of perfect information in Figure 2.

Figure 5 depicts the optimal capacities under the low-and high-nuclear acenarios as a function of coal availability. In accordance with the high degree of substitution between the two primary fuels, at any level of coal-fired electricity generation capacity, the lower is the nuclear limit the larger should be the demand for coal. Given the significance of capital costs in installing nuclear powered plants, the less expensive activities employing nuclear energy are characterized by high utilization factors and end-use demands [20/]. As the available nuclear capacity declines, the same coal-fired electricity capacity can be dislocated into servicing the highly demanding and utilized end-use activities, increasing the demand for coal.

Under the low-nuclear scenario, up to 35.5 x 10^{15}BTU of coal the expected opportunity cost of a capacity shortage for coal-fired plants exceeds the capital cost. This tends to make the here-and-now capacity no smaller than the expected wait-and-see order. As increases in coal availability shift coal prices downward, the marginal value of coal-fired plants diminishes. When the future price of coal reaches $30.79 por ton, which corresponds to a coal level of 35.5 x 10^{15}BTU and an optimal capacity of 12.57 x 10^{15}BTU, opportunity and capital costs are balanced. Further expansion of coal availability and consequently, still lower prices, will result in the capital cost overtaking opportunity costs. The here-and-now curve falls below the wait-and-see curve as the risk of excess capacity is no longer worth incurring. With coal prices depending upon both the availability of coal and its close substitute, nuclear energy, one would expect that the same future price could prevail at higher levels of nuclear capacity only if

20/ See Tables 1 and 4.



ELECTRICITY GENERATION CAPACITY: NUCLEAR SCENARIOS

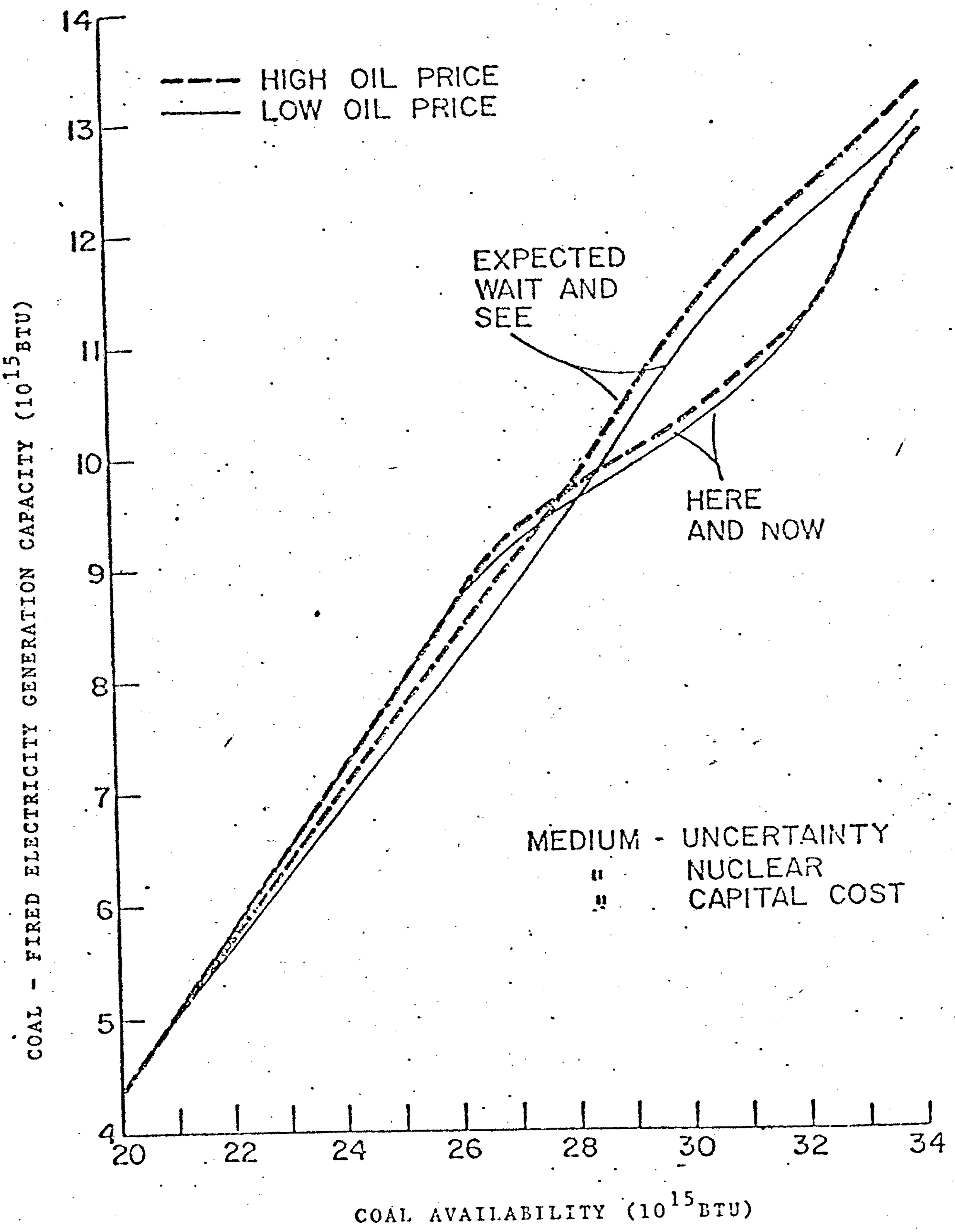
Figure 5

coal availability is reduced. At the high-nuclear scenario, opportunity and capital costs are balanced at the capacity of 6.56×10^{15} BTU for coal-fired electricity generation which corresponds to a coal level outside the range depicted in Figure 5. In this range, that encompasses most forecasts for coal availability in year 2000, the here-and-now order is always smaller than the expected wait-and-see capacity for high-nuclear.

As substitution between coal and oil is much weaker in this model 21/ than, between coal and nuclear, the low and high scenarios for oil import prices do not significantly affect the value of perfect information. However, as one would expect, the higher the oil import price the higher is the maximum amount the energy sector would be willing to pay for a perfect prediction of the end-use demands. For the same reason the impact of alternative oil prices upon the optimal capacities of coal fired plants is relatively small as illustrated by Figure 6. As intuition suggests the higher the oil price the larger is the coal-fired electricity generation capacity to be installed under both strategies.

In order to explain the relative positions of the here-and-now and the expected wait-and-see curves in Figure 6 we must resort, once again, to the competition between opportunity and capital costs for coal-fired plants. At low levels of coal availability capacity shortages for coal-fired plants are more likely. As the opportunity costs of not having enough plants on line overshadows the capital cost, the here-and-now capacity exceds the expected capacity under the alternative strategy. As coal availability increases, the opportunity cost declines and it is eventually overtaken by the capital cost. This drives the here-and-now curve below the expected wait-and-see at coal levels greater than 28×10^{15} BTU.

21/ See Modiano and Shapiro (1980).



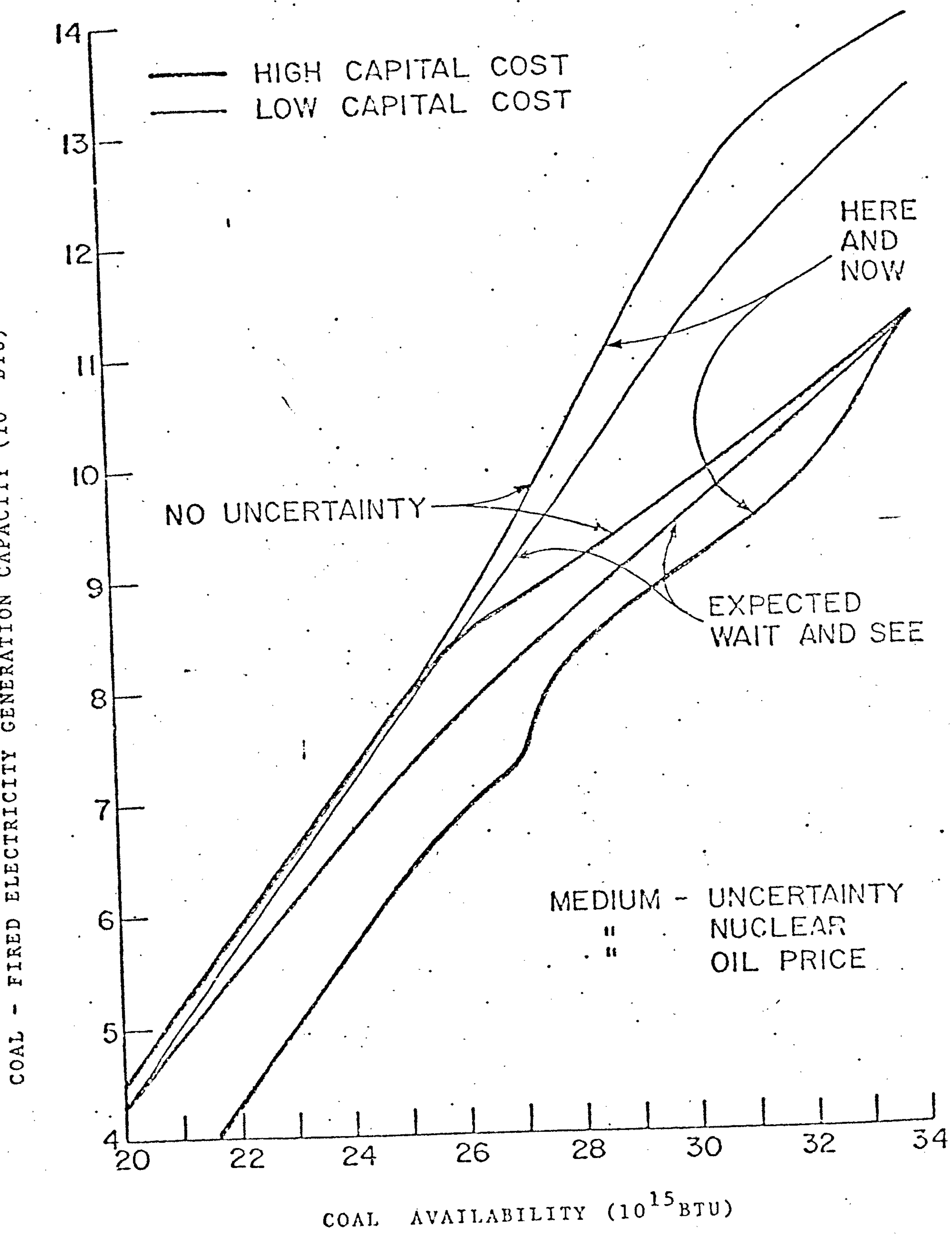
ELECTRICITY GENERATION CAPACITY: OIL PRICE SCENARIOS

Figure 6

Figure 7 depicts the optimal coal-fired electricity generation under the low-and high-capital costs scenarios. Clearly, the lower the capital cost the higher are the optimal capacities. At low capital costs and high levels of coal availability, the risk of building excess capacity is more attractive. As the opportunity cost of not having enough capacity outweighs the capital cost the here-and-now capacity tends to be larger than the wait-and-see order. This seems to be the case for the low cost scenario over the entire range of coal levels pictured in Figure 7. Conversely, one would expect at very high capital costs and very low coal supply levels the here-and-now capacity to be no larger than under wait-and-see strategy. For our high-capital cost scenario this is the situation over the entire range of coal availability levels considered. The breakeven points between opportunity and capital costs occur for the high-and low--capital cost scenario the below and above the limits of 20 to 34 x 10^{15}BTU of coal respectively. As coal availability increases, the higher the capital cost the sooner the capital cost overtakes the declining opportunity cost of a capacity shortage.

5. Policy Implications and Extensions

In Section 4 we applied the two-stage model described in section 2 to the U.S. energy sector demand for coal for the case where the first-stage decision is the size of coal--fired electricity generation plants. The subsequent analysis led to the evaluation of the price of uncertainty, built into coal future prices. The alternative scenarios considered permitted us to draw both qualitative and quantitative conclusions about the movement of such "prices" in response to different levels of uncertainty in the prediction of the end-use demands, nuclear capacity, oil import prices and capital costs of coal-fired plants.



ELECTRICITY GENERATION CAPACITY: CAPITAL COST SCENARIOS

Figure 7

The role played by uncertainty in determining the optimal capacity here and now varies sharply according to the scenario under study. Disregarding uncertainty in capacity planning leads in general, to an overestimate for the optimal size of the plants. Following the approach of computing expected capacities as if the sector could wait and see the end-use demands may lead to either over or under estimates of the optimal capacity. For both approaches, the error may reach significant levels in terms of the capital investment commited.

The implications of such two-stage models for policy making do not only refer to the capacity of the coal-fired electricity generation plants but also to the alternative generation sources. The smaller is the capacity of coal--fired plants the larger may be the capacity of others forms of electricity generation. Hence, the capacity of an alternative source may very well be seriously underestimated by disregarding uncertainty or acting as if the sector could wait and see. New technologies considered "infeasible" may turn out to be "feasible" as a recourse when coal-fired capacity plans have to be made prior to the resolution of uncertainty. At the other extreme, present available technologies may be overly dimensioned and even unoperative under the two-stage scheme.

While our analysis abstracts somehow from reality when we assume that only coal-fired capacity decisions are made here and now, our approach remains valid under more general conditions. For example, the decision on the introduction of new technologies and the dimensioning of their operating capacities can be analyzed basically in the same framework. However, the two-stage model may have to be extended to a mixed integer dynamic programing problem if we are to take into account intertemporal aspects of phasing technology introduction and the associated fixed charges. Our present results highlight the importance of dealing with uncertainty in energy planning models and suggest topics for further research in this area.

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