General Ambiguity Index for Bewley Preferences

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This paper characterize preference relations $\succsim$ over the set $\mathcal{F}$ of Anscombe and Aumann acts (*i.e.*, $\mathcal{F}$ is the family of all random variables on a measurable state space (S, Σ) with values on a consequence space X) and give necessary and sufficient conditions that guarantee the existence of a utility function u on consequences and an *ambiguity index* η on the set of probabilities on the the states of the nature such that, for any acts f and g,

$$f \succsim g \Leftrightarrow \int u(f)dp + \eta(p) \geq \int u(g)dp, \; \forall p \in \Delta.$$

The function u represents the decision maker´s risk attitudes, while the ambiguity index η describes his confidence among the universe of probabilities laws. In this model, the decision maker´s subjective plausibility level among the universe of *priors that matthers*[2] is not uniform in many cases. Here, the ambiguity index η presents a similar role as proposed by Maccheroni, Marinacci and Rustichini (2006)[3][4].

The axiomatic foundations of this class of preferences rests on a simple set of axioms that generalizes the subjective expected utility axiomatization of Anscombe and Aumann (1963) and the Knightian decision theory axiomatization of Bewley (2002): $\succsim$ satisfies reflexibility, continuity, monotonicity, independence, and its restriction $\succsim|_{X \times X}$ over the set of consequences is a nontrivial weak order.

An important fact is that the von Neumann-Morgenstern utility index u is unique up to positive linear transformation, and for each u the minimal ambiguity index η^* consistente with the decision rule above satisfies,

$$\eta^*(p) = \underset{(f,g)\in\succsim}{sup} \left(\int \left(u(g) - u(f)\right) dp \right), \; \forall p \in \Delta.$$

As special case we have:

1. If $\eta = \delta_C : \Delta \rightarrow \{0, \infty\}$, where $\delta_C\left(p\right) = 0$ iff $p \in C$, we obtain the Bewley model and it is true iff $\succsim$ above is transitive. Also, if $C = \{q\}$ for some $q \in \Delta$, we obtain the expected utility model and it is true iff $\succsim$ above is a complete order.

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[2] *i.e.* the set of priors p such that $\eta\left(p\right) < \infty$.

[3] Formally, ambiguity index are convex, lower semicontinous functions η on Δ with values on $[0, \infty]$ such that $\{\eta = 0\}$ is nonempty.

[4] Another interesting interpretation for η says that $\eta\left(p\right)$ measure the *maximal expected loss* acepted by the decision maker if p is the true probability and his change f by g: in fact, note that f is (*weakly*) better than g iff $\int \left(u(g) - u\left(f\right)\right)dp \geq -\eta\left(p\right)$ for any $p \in \Delta$.

2. If $\eta = R\left(\cdot \parallel q\right) : \Delta \to [0,\infty]$, where

$$R\left(p \parallel q\right) = \begin{cases} \int \log\left(\frac{dp}{dq}\right) dp \text{ if } p \ll q \\ \infty, \text{ otherwise} \end{cases}$$

   is the relative entropy index (w.r.t q), we obtain a preference relation in a similar spirit of Hansen and Sargent (2001) robutness model, but with a decision rule *a la* Bewley.

3. Another possibility is when $\eta = G\left(\cdot \parallel q\right) : \Delta \to [0,\infty]$, where

$$G\left(p \parallel q\right) = \begin{cases} \frac{1}{2} \int \left(\frac{dp}{dq} - 1\right)^2 dq \text{ if } p \ll q \\ \infty, \text{ otherwise} \end{cases}$$

   is the classic concentration Gine index.

References

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[4] Maccheroni, F., M. Marinacci and A. Rustichini.(2006): *Ambiguity aversion, Robustness and the variational representation of preferences.* **Econometrica**, 74, 1447-1498.