Spillover Effects in Forecasting Volatility and VaR[*]

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Abstract: Accurate modelling of volatility (or risk) is important in finance, particularly as it relates to the modelling and forecasting of Value-at-Risk (VaR) thresholds. As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past as well as the past of other assets. The need to create volatility models that can be used to estimate large covariance matrices has become especially relevant following the 1995 amendment to the Basel Accord, whereby banks are permitted to use internal models to calculate their VaR thresholds. While the amendment was designed to reward institutions with superior risk management systems, a backtesting procedure, whereby the realized returns are compared with the VaR forecasts, was introduced to asses the quality of the internal models. Banks using models that lead to a greater number of violations than can reasonably be expected, given the confidence level, are penalized by having to hold higher levels of capital. The imposition of penalties is severe as it has an impact on the profitability of the bank directly through higher capital charges, may damage the banks reputation, and may also lead to the imposition of a more stringent external model to forecast the VaR thresholds. In this paper we analyse the importance of considering spillover effects when forecasting financial volatility. The forecasting performance of the VARMA-GARCH model of Ling and McAleer (2002), which includes spillover effects from all assets, the CCC model of Bollerslev (1990), which includes no spillovers, and the new PS-GARCH model, which accommodates aggregate spillovers parsimoniously, are compared using a VaR example. The empirical results suggest that the inclusion of spillover effects is not particularly important in forecasting VaR thresholds, even when these spillovers are statistically significant.

JEL classifications: C32, C51, C53, F37, G15, G21

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1. Introduction

Accurate modelling of volatility (or risk) is of paramount importance in finance. As risk is unobservable, several modelling procedures have been developed to measure and forecast risk. The Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986) have subsequently led to a family of autoregressive conditional volatility models. The success of GARCH models can be attributed largely to their ability to capture several stylised facts of financial returns, such as time-varying volatility, persistence and clustering of volatility, and asymmetric reactions to positive and negative shocks of equal magnitude. This has also contributed to the modelling and forecasting of Value-at-Risk (VaR) thresholds.

As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past risk as well as on the past risk of other assets (see McAleer (2005) for a discussion of a variety of univariate and multivariate conditional and stochastic volatility models). da Veiga and McAleer (2005) showed that the multivariate VARMA-GARCH model of Ling and McAleer (2003) and VARMA-AGARCH model of Hoti et al. (2003) provided far superior volatility and VaR threshold forecasts than their nested univariate counterparts, namely the GARCH model of Bollerslev (1986) and the GJR model of Glosten, Jagannathan and Runkle (1992), respectively.

Multivariate extensions have great intuitive and empirical appeal as they enable modelling of the relationship between subsets of the portfolio and allow for scenario and sensitivity analyses. Moreover, their structural and asymptotic properties have been well established, especially for multivariate GARCH models (for further details, see Ling and McAleer (2003) and Hoti et al. (2002), which extend the results for a range of univariate GARCH models in Ling and McAleer (2002a,b)). However, the practical usefulness of this result can be affected by the computational difficulties in estimating the VARMA-GARCH and VARMA-AGARCH models for a large number of assets, as the number of parameters to be estimated can increase dramatically with the number of assets, and hence spillover effects.

Several parsimonious multivariate models have been proposed to deal with the over-parameterization problem. The CCC model of Bollerslev (1990), the Dynamic Conditional Correlation (DCC) model of Engle (2002), and the Varying Conditional Correlation (VCC) model of Tse and Tsui (2002) use a two-step estimation procedure to facilitate estimation. Chan et al. (2003) extended these conditional correlation models by specifying the shocks to returns as being time dependent, and established the structural and asymptotic properties of the more general model. The Orthogonal GARCH (O-GARCH) model of Alexander (2001) uses principal component analysis to reduce the number of parameters to be estimated.

The need to develop volatility models to estimate accurately large covariance matrices has become especially relevant following the 1995 amendment to the Basel Accord, whereby banks were permitted to use internal models to calculate their VaR thresholds. This amendment was a reaction to widespread criticism that the ‘Standardized’ approach, which banks were originally required to use in calculating their VaR thresholds, led to excessively conservative forecasts. Excessive conservatism has a negative impact on the profitability of banks as higher capital charges are subsequently required. While the amendment was designed to reward institutions with superior risk management systems, a backtesting procedure, whereby the realized returns are compared with the VaR forecasts, was introduced to assess the quality of the internal models. Banks using models that lead to a greater number of violations than can reasonably be expected, given the confidence level, are required to hold higher levels of capital (see the discussion in Section 5 and Table 13 for the penalties imposed under the Basel Accord). If a bank’s VaR forecasts are violated more than 9 times in a financial year, the bank is required to adopt the ‘Standardized’ approach. The imposition of such a penalty is severe as it has an impact on the profitability of the bank directly through higher capital charges, may damage the bank’s reputation, and may also lead to the imposition of a more stringent external model to forecast the VaR thresholds.

In this paper we investigate the importance of including spillover effects when modelling and forecasting financial volatility. We compare the forecasted conditional variances produced by the VARMA-GARCH model of Ling and McAleer (2002), in which the conditional variance of asset *i* is specified to depend dynamically on past squared unconditional shocks and past conditional variances of each asset in the portfolio, with the forecasted conditional variances produced by the CCC model of Bollerslev (1990), where the conditional variance of asset *i* is specified to depend only on squared unconditional shocks and past conditional variances of asset *i*. We also develop a new Portfolio Spillover GARCH (PS-GARCH) model, which allows spillover effects to be included in a more parsimonious manner. This parsimonious model is found to yield volatility and VaR threshold forecasts that are very similar to those of the VARMA-GARCH and VARMA-AGARCH models.

The plan of the paper is as follows. Section 2 presents the new portfolio spillover GARCH (PS-GARCH) model, discusses alternative multivariate GARCH models with and without spillover effects, and presents a simple two-step estimation method for PS-GARCH. The data is discussed in Section 3, forecasting is examined in Section 4, the economic significance of the results is analysed in Section 5, and some concluding remarks are given in Section 6.

Equation Section 1

2. Models and Estimation

This section proposes a parsimonious PS-GARCH model which captures aggregate portfolio spillover effects, and compares the new model with two constant conditional correlation models, one of which models spillover effects from each of the other assets in the portfolio and another which has no spillover effects.

2.1 PS-GARCH

Let the vector of returns on *m* financial assets be given by

$$Y_t = E(Y_t / F_{t-1}) + \varepsilon_t \tag{1.1}$$

where the conditional mean of the returns follows a VARMA process:

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \tag{1.2}$$

The return on the portfolio consisting of the *m* assets is denoted as:

$$Y_{p,t} = E(\sum_{i=1}^{m} x_{i,t} y_{i,t} / F_{t-1}) + \varepsilon_{p,t} \tag{1.3}$$

where $y_{i,t}$ denotes the return on asset *i* at time *t* and x_{it} denotes the portfolio weight of asset *i* at time *t*, such that:

$$\sum_{i=1}^{m} x_{i,t} = 1 \quad \forall\, t\,. \tag{1.4}$$

The portfolio spillover GARCH (PS-GARCH) model assumes that the returns on the portfolio follow an ARMA process, as follows:

$$\Phi(L)(Y_{p,t} - \mu_p) = \Psi(L)\varepsilon_{p,t} \tag{1.5}$$

$$\varepsilon_t = D_t \eta_t \tag{1.6}$$

$$\varepsilon_{p,t} = h_{p,t}^{1/2} \eta_{p,t} \tag{1.7}$$

$$h_{p,t} = \omega_p + \sum_{l=1}^{s} \alpha_{p,l} \varepsilon_{p,t-l}^2 + \sum_{l=1}^{s} \beta_{p,l} h_{p,t-l} \tag{1.8}$$

$$H_t = W + \sum_{l=1}^{r} A_l \, \varepsilon'_{t-l} + \sum_{l=1}^{r} C_l I(\eta_{t-l}) \, \varepsilon'_{t-l} + \sum_{l=1}^{s} B_l H_{t-l} + \sum_{l=1}^{s} G_l \hat{\varepsilon}_{p,t-l} + \sum_{l=1}^{s} K_l \hat{h}_{p,t-l} \tag{1.9}$$

where $H_t = (h_{1t},...,h_{mt})'$, $W = (\omega_1,...,\omega_m)'$, $D_t = diag(h_{it}^{1/2})$, $\eta'_t = (\eta_{1t},...,\eta_{mt})$, $\varepsilon'_t = (\varepsilon_{1t}^2,...,\varepsilon_{mt}^2)$, and $\hat{\varepsilon}_{p,t-l}^2$ and $\hat{h}_{p,t-l}$ are the fitted values from and (1.5) and (1.8), respectively. The matrices A_l, B_l, C_l, G_l and K_l are diagonal, with typical elements α_{ii}, $\beta_{ii}, \gamma_{ii}, \lambda_{ii}$ and δ_{ii}, respectively, for $i = 1,...,m$, $I(\eta_t) = diag(I(\eta_{it}))$ is an $m \times m$ diagonal matrix, $\Phi(L) = I_m - \Phi_1 L - ... - \Phi_p L^p$ and $\Psi(L) = I_m - \Psi_1 L - ... - \Psi_q L^q$ are polynomials in *L*, the lag operator, F_t is the past information available to time *t*, I_m is the $m \times m$ identity matrix, and $I(\eta_{it})$ is an indicator function, given as:

$$I(\eta_{it}) = \begin{cases} 1, \ \varepsilon_{it} \le 0 \\ 0, \ \varepsilon_{it} > 0. \end{cases} \tag{1.10}$$

The indicator function distinguishes between the effects of positive and negative shocks of equal magnitude on conditional volatility.

Using (1.6), the conditional covariance matrix for the PS-GARCH model is given by $Q_t = D_t \, \Gamma \, D_t$, for which the matrix of conditional correlations is given by $E(\eta_t \eta'_t) = \Gamma$. The matrix Γ is the constant conditional correlation matrix of the unconditional shocks which is, by definition, equivalent to the constant conditional correlation matrix of the conditional shocks.

2.2 VARMA-GARCH

The VARMA-GARCH model of Ling and McAleer (2003), which assumes symmetry in the effects of positive and negative shocks on conditional volatility, is given by:

$$Y_t = E(Y_t / F_{t-1}) + \varepsilon_t \tag{1.11}$$

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \tag{1.12}$$

$$\varepsilon_t = D_t \eta_t \tag{1.13}$$

$$H_t = W + \sum_{l=1}^{r} A_l \vec{\varepsilon}_{t-l} + \sum_{l=1}^{s} B_l H_{t-l} \tag{1.14}$$

where $H_t = (h_{it},...,h_{mt})'$, $W = (\omega_1,...,\omega_m)'$, $D_t = diag(h_{i,t}^{1/2})$, $\eta_t = (\eta_{i,t},...,\eta_{mt})'$, $\vec{\varepsilon}_t = (\varepsilon_{it}^2,...,\varepsilon_{mt}^2)$, A_l and B_l are $m \times m$ matrices with typical elements α_{ij} and β_{ij}, respectively, for $i, j = 1,...,m$, $I(\eta_t) = diag(I(\eta_{it}))$ is an $m \times m$ matrix, $\Phi(L) = I_m - \Phi_1 L - ... - \Phi_p L^p$ and $\Psi(L) = I_m - \Psi_1 L - ... - \Psi_q L^q$ are polynomials in *L*, the lag operator, and F_t is the past information available to time t. Spillover effects are given in conditional volatility for each asset in the portfolio. Based on equation (1.13), the VARMA-GARCH model also assumes that the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$.

An extension of the VARMA-GARCH model is the VARMA-AGARCH model of Hoti et al. (2002), which captures the asymmetric spillover effects from each of the other assets in the portfolio. The VARMA-AGARCH model is also a multivariate extension of the univariate GJR model.

2.3 CCC

The VARMA-GARCH and VARMA-AGARCH models have several popular constant conditional correlation univariate and multivariate models as special cases. If the model given by equation (1.14) is restricted so that A_l and B_l are diagonal matrices, the VARMA-GARCH model reduces to:

$$h_{it} = \omega_i + \sum_{l=1}^{r} \alpha_i \varepsilon_{i,t-l} + \sum_{l=1}^{s} \beta_i h_{i,t-l} \tag{1.15}$$

which is the constant conditional correlation (CCC) model of Bollerslev (1990). The CCC model also assumes that the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$. As given in equation (1.15), the CCC model does not have volatility spillover effects across different financial assets, and hence is intrinsically univariate in nature. Moreover, CCC also does not capture the asymmetric effects of positive and negative shocks on conditional volatility.

2.4 Estimation

The parameters in models (1.11), (1.14), (1.15) can be obtained by maximum likelihood estimation (MLE) using a joint normal density, namely:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{2} \sum_{t=1}^{n} \left(\log \left| Q_t \right| + \varepsilon_t^{'} Q_t^{-1} \varepsilon_t \right) \tag{1.16}$$

where θ denotes the vector of parameters to be estimated in the conditional log-likelihood function, and $\left| Q_t \right|$ denotes the determinant of Q_t, the conditional covariance matrix. When η_t does not follow a joint multivariate normal distribution, equation (1.16) is defined as the Quasi-MLE (QMLE).

The models described above can also be estimated using the following simple two-step estimation procedure:

(1) For each financial index return series, the univariate GARCH (1,1) model with an AR(1) conditional mean specification is estimated, and the unconditional shocks and standardized residuals of all *m* returns are saved.

(2) For the portfolio returns, as defined by equation (1.3), the univariate GARCH (1,1) model with VARMA(1,1) conditional mean specification is estimated, and the unconditional shocks and standardized residuals are saved.

(3) For each financial returns series, the univariate VARMA(1,1)-GARCH(1,1) model is estimated, including the lagged squared unconditional shocks and the lagged conditional variances of the remaining *m*-1 assets. The standardized residuals of the *m*-1 financial returns are saved.

(4) For each financial returns series, the VARMA(1,1)-PS-GARCH(1,1) model is estimated, including the lagged squared unconditional shocks and the lagged conditional variances from step (2). The standardized residuals of all *m* financial returns are saved.

(5) For each returns series, the constant conditional correlation matrices of the VARMA(1,1)-GARCH(1,1) model are estimated by direct computation using the standardized residuals from step (3). Bollerslev's (1990) CCC matrix is estimated directly using the standardized residuals from step (1). Finally, the constant conditional correlation matrix of the PS-GARCH model is estimated using the standardized residuals from step (4).

The tests of spillover and asymmetric effects are valid under the null hypothesis of independent (that is, no spillovers) and symmetric effects, so that steps (3) and (4) are valid under the joint null hypothesis. The primary purpose of the structural and asymptotic theory derived in Ling and McAleer (2003) is to demonstrate that such testing is statistically valid. This is in contrast to, for example, Nelson's (1991) univariate and multivariate EGARCH models, for which the asymptotic theory has yet to be established.

Using extensions of the results in Ling and McAleer (2003), Hoti et al. (2002) and Chan et al. (2003), it can be shown that the QMLE of the parameters in the PS-GARCH model are consistent and asymptotically normal in the absence of normality in the standardized shocks $\eta_{p,t}$ in (1.7) (the proof is available on request).

The VARMA-GARCH and VARMA-AGARCH models are available in, for example, the RATS 5 econometric software package.

3. Data

The data used in the empirical application are daily prices measured at 16:00 Greenwich Mean Time (GMT) for four international stock market indices (henceforth referred to as synchronous data), namely S&P500 (USA), FTSE100 (UK), CAC40 (France), and SMI (Switzerland). All prices are expressed in US dollars. The data were obtained from DataStream for the period 3 August 1990 to 5 November 2004. At the time the data were collected, this period was the longest for which data on all four variables were available. The rationale for employing daily synchronous data in modelling stock returns and volatility transmission is four-fold.

First, the Efficient Markets Hypothesis would suggest that information is quickly and efficiently incorporated into stock prices. While information generated yesterday may be significant in explaining stock price changes today, it is less likely that news generated last month would have any explanatory power today.

Second, it has been argued by Engle et al. (1990) that volatility is caused by the arrival of unexpected news and that volatility clustering is the result of investors reacting differently to news. The use of daily data may help in modelling the interaction between the heterogeneity of investor responses in different markets.

Third, studies that use close-to-close non-synchronous returns suffer from the non-synchronicity problem, as highlighted in Scholes and Williams (1977). In particular, these studies cannot distinguish a spillover from a contemporaneous correlation when markets with common trading hours are analysed. Kahya (1997) and Burns et al. (1998) also observe that, if cross market correlations are positive, the use of close-to-close returns for non-synchronous markets will underestimate the true correlations, and hence underestimate the true risk associated with a portfolio of such assets.

Finally, the use of synchronous data allows the system to be written in a simultaneous equations form, which can be estimated jointly. Such joint estimation of the parameters eliminates potential econometric problems associated with generated regressors (see, for example, Pagan(1984) and Oxley and McAleer (1993, 1994)), improves efficiency in estimation, increases the power of the test for cross-market spillovers, and analyses market interactions simultaneously. This allows all the relationships to be tested jointly. Joint estimation is also consistent with the notion that spillovers are the impact of global news on each market.

The synchronous returns for each market i at time t $(R_{i,t})$ are defined as:

$$R_{i,t} = \log(P_{i,t} / P_{i,t-1}),$$

where $P_{i,t}$ is the price in market i at time t, as recorded at 16:00 GMT.

The descriptive statistics for the synchronous returns of the four indexes are given in Table 1. All series have similar means and medians at close to zero, minima which vary between -5.533 and -10.251, and maxima that range between 5.771 and 10.356. Although the four standard deviations vary slightly, the coefficients of variation (CoV) are quite different, ranging from 31.227 for S&P500 to 66.002 for CAC40. The skewness differs among all four series, but the kurtosis is similar for all series. The Jarque-Bera test strongly rejects the null hypothesis of normally distributed returns, which may be due to the presence of extreme observations. As each of the series displays a high degree of kurtosis, this would seem to indicate the existence of extreme observations.

[Insert Figures 1a-d here]

[Insert Table 1 here]

Several definitions of volatility are available in the literature. This paper adopts the measure of volatility proposed in Franses and van Dijk (1999), where the *true* volatility of returns is defined as:

$$V_{i,t} = (R_{i,t} - E(R_{i,t} / F_{t-1}))^2$$

where F_{t-1} is the information set at time *t*-1.

The descriptive statistics for the volatility of the synchronous returns of the four indexes are given in Table 2. The CAC40 displays the highest mean (median) volatility at 1.812 (0.543), while the S&P500 has the lowest mean (median) volatility at 1.045 (0.264). Both FTSE100 and SMI have similar mean (median) volatilities at 1.139 (0.352) and 1.355 (0.419). The maxima of all volatility series differ substantially, with CAC40 displaying the highest maxima while S&P500 displays the lowest. Although the four standard deviations vary, the coefficients of variation (CoV) are similar. All series are highly skewed. As each of the series displays a high degree of kurtosis, this would seem to indicate the existence of extreme observations.

[Insert Table 2 here]

The plots of the volatilities of the synchronous returns are given in Figures 2a-d. Each of the series exhibits clustering, which needs to be captured by an appropriate model. The volatility of all series appears to be high during the early 1990’s, followed by a quiet period from the end of 1992 to the beginning of 1997. Finally, the volatility of all series appears to increase dramatically around 1997, due in large part to the Asian economic and financial crises. This increase in volatility persists until the end of the period, and is likely to have been affected by the September 11, 2001 terrorist attacks and the conflicts in Afghanistan and Iraq.

[Insert Figures 2a-d here]

4. Forecasts

The aim of this section is to compare the volatility and conditional correlation forecasts produced by the CCC model of Bollerslev (1990), the VARMA-GARCH model of Ling and McAleer (2003), and the PS-GARCH model proposed in this paper. We use a rolling window approach to forecast the 1-day ahead conditional correlations and conditional variances. The sample ranges from 3 August 1990 to 5 November 2004. In order to strike a balance between efficiency in estimation and a viable number of rolling regressions, the rolling window size is set at 2000 for all four data sets, which leads to a forecasting period from 6 April 1998 to 5 November 2004.

[Insert Figures 3-6 here]

Figures 3-6 plot the forecasted volatilities for each returns series using the 3 models, and Tables 3-6 show the correlations between the 3 forecasts for each returns series. The volatility forecasts produced by all models are remarkably similar, with correlation coefficients of the volatility forecasts ranging from 0.955 to 0.990, suggesting that the PS-GARCH model provides a good parsimonious approximation to the VARMA-GARCH model.

[Insert Tables 3-6 here]

The forecasted conditional correlations and the correlation of the conditional correlation forecasts are given in Figures 7-12 and Tables 7-12, respectively. The conditional correlation forecasts are virtually identical for all three models, with correlation coefficients ranging from 0.996 to 0.999. This result suggests that for applications where the required input are forecasts of the conditional variances and/or the conditional correlation matrix, all three models considered above yield very similar results.

[Insert Figures 7-12 here]

[Insert Tables 7-12 here]

5. Economic Significance

The 1988 Basel Capital Accord, which was originally concluded between the central banks from the Group of Ten (G10) countries, and has since been adopted by over 100 countries, sets minimum capital requirements which must be met by banks to guard against credit and market risks. These capital requirements are a function of the forecasted VaR thresholds, in which VaR summarizes the maximum expected loss over a target horizon for a given level of confidence. The Basel Accord stipulates that the daily capital charge must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days, multiplied by a factor k. The multiplicative factor k is set by the local regulators, but must not be lower than 3. In 1995, the 1988 Basel Accord was amended to allow banks to use internal models to determine their VaR. However, banks wishing to use internal models must demonstrate that the models are sound. Furthermore, the Basel Accord imposes penalties in the form of a higher multiplicative factor k on banks which use models that lead to a greater number of violations than would reasonably be expected given the specified confidence level of 1%. Table 13 shows the penalties imposed for a given number of violations for 250 business days.

In certain cases, where the number of violations is deemed to be excessively large, regulators may penalize banks even further by requiring that their internal models be reviewed. In circumstances where the internal models are found to be inadequate, banks can be required to adopt the standardized method originally proposed in 1993 by the Basel Accord. The standardized method suffers from several drawbacks, the most noticeable of which is its systematic overestimation of risk, which stems from the assumption of perfect correlation across different risk factors. Overestimating risk leads to higher capital charges which negatively impact both the profitability and reputation of the bank.

[Insert Table 13 here]

The economic significance of the PS-GARCH model proposed above is highlighted by forecasting VaR thresholds using the PS-GARCH, VARMA-GARCH and CCC models (see Jorion (2000) for a detailed coverage of VaR). In order to simplify the analysis, it is assumed that the portfolio returns are normally distributed, with equal and constant weights. We control for exchange rate risk by converting all prices to a common currency, namely the US Dollar. We use the forecasted variances and correlations from Section 4 to produce VaR forecasts for the period 6 May 1998 to 5 November 2004. The backtesting procedure is used to test the soundness of the models by comparing the realised and forecasted losses (see Basel Committee (1988, 1995, 1996) for further details).

Figures 13-15 show the VaR forecasts and realized returns for each model considered. Both the CCC and PS-GARCH VaR forecasts violate the thresholds 7 times from 1720 forecasts, while the VARMA-GARCH model leads to 6 violations from 1720 forecasts.

Table 14 shows that the mean daily capital charge, which is a function of both the penalty and the forecasted VaR, implied by PS-GARCH is the largest at 10.70%, followed by VARMA-GARCH at 9.76% and CCC at 9.67%. A high capital charge is undesirable, other things equal, as it reduces profitability. Table 14 also shows that CCC leads to violations that are greater in terms of mean absolute deviations than the VARMA-GARCH and PS-GARCH models. This is particularly important because large violations may lead to bank failures, as the capital requirements implied by the VaR threshold forecasts may be insufficient to cover the realized losses. Finally, CCC also leads to the largest maximum violation.

[Insert Figures 13-15 here]

[Insert Table 14 here]

6. Conclusion

Accurate modelling of volatility (or risk) is important in finance, particularly as it relates to the modelling and forecasting of Value-at-Risk (VaR) thresholds. As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past, as well as the past of other assets. The inclusion of multivariate spillover effects was found not to improve the accuracy of the forecasts significantly relative to models without spillover effects.

The need to create volatility models that can be used to estimate large covariance matrices has become especially relevant following the 1995 amendment to the Basel Accord, whereby banks are permitted to use internal models to calculate their VaR thresholds. While the amendment was designed to reward institutions with superior risk management systems, a backtesting procedure, whereby the realized returns are compared with the VaR forecasts, was introduced to assess the quality of the internal models. Banks using models that lead to a greater number of violations than can reasonably be expected, given the confidence level, are penalized by having to hold higher levels of capital. The imposition of penalties is severe as it has an impact on the profitability of the bank directly through higher capital charges, may damage the banks reputation, and may also lead to the imposition of a more stringent external model to forecast the VaR thresholds.

This paper examined various conditional volatility models for purposes of forecasting financial volatility and VaR thresholds. Two constant conditional correlation models for estimating the conditional variances and covariances are the CCC model of Bollerslev (1990) and the VARMA-GARCH model of Ling and McAleer (2003). Although the VARMA-GARCH model accommodates spillover effects from the returns shocks of all assets in the portfolio, which are typically estimated to be significantly different from zero, the forecasts of the conditional volatility and VaR thresholds produced by the VARMA-GARCH model are very similar to those produced by the CCC model. Furthermore, the models with spillover effects can be computationally difficult as the number of assets becomes large. The paper also developed a new Portfolio Spillover GARCH (PS-GARCH) model, which allowed spillover effects to be included in a more parsimonious manner. This parsimonious model was found to yield volatility and VaR threshold forecasts that were very similar to those of the CCC and VARMA-GARCH models.

References

Alexander, C.O. (2001), "Orthogonal GARCH", in C.O. Alexander (ed.), *Mastering Risk, Volume II*, FT-Prentice Hall**,** pp. 21-38.

Basel Committee on Banking Supervision, (1988), "International Convergence of Capital Measurement and Capital Standards", BIS, Basel, Switzerland.

Basel Committee on Banking Supervision, (1995), "An Internal Model-Based Approach to Market Risk Capital Requirements", BIS, Basel, Switzerland.

Basel Committee on Banking Supervision, (1996), "Supervisory Framework for the Use of "Backtesting" in Conjunction with the Internal Model-Based Approach to Market Risk Capital Requirements", BIS, Basel, Switzerland.

Bollerslev, T. (1986), "Generalised Autoregressive Conditional Heteroscedasticity", *Journal of Econometrics*, 31, 307-327.

Bollerslev, T. (1990), "Modelling the Coherence in Short-Run Nominal Exchange Rate: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistic*, 72, 498-505.

Burns, P, R. Engle and J. Mezrich (1998), "Correlations and Volatilities of Asynchronous Data", *Journal of Derivatives*, 5, 7-18.

Chan, F., S. Hoti and M. McAleer (2003), "Generalized Autoregressive Conditional Correlation", unpublished paper, School of Economics and Commerce, University of Western Australia.

Engle, R.F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica*, 50, 987-1007.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business and Economic Statistics*, 20, 339-350.

Engle, R.F., T. Ito and W. Lin (1990), "Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market", *Econometrica*, 58, 525-542.

Franses, P.H. and D. van Dijk (1999), *Nonlinear Time Series Models in Empirical Finance*, Cambridge, Cambridge University Press.

Glosten, L.R., R. Jagannathan, and D.E. Runkle (1992), “On the Relation between the Expected Value and Volatility of the Nominal Excess Return on Stocks”, *Journal of Finance*, 46, 1779-1801.

Hoti, S., F. Chan and M. McAleer (2002), “Structure and Asymptotic Theory for Multivariate Asymmetric Volatility: Empirical Evidence for Country Risk Ratings”, Invited paper presented to the Australasian Meeting of the Econometric Society, Brisbane, Australia, July 2002.

Jorion, P. (2000), “Value at Risk: The New Benchmark for Managing Financial Risk”, McGraw-Hill, New York.

Kahya, E. (1997), “Correlation of Returns in Non-Contemporaneous Markets”, *Multinational Finance Journal*, 1, 123-135.

Ling, S. and M. McAleer (2002a), “Stationarity and the Existence of Moments of a Family of GARCH Processes”, *Journal of Econometrics*, 106, 109-117.

Ling, S. and M. McAleer (2002b), “Necessary and Sufficient Moment Conditions for the GARCH(r,s) and Asymmetric Power GARCH(r,s) Models”, *Econometric Theory*, 18, 722-729.

Ling, S. and M. McAleer (2003), “Asymptotic Theory for a Vector ARMA-GARCH Model”, *Econometric Theory*, 19, 278-308.

McAleer, M. (2005), “Automated Inference and Learning in Modeling Financial Volatility”, *Econometric Theory*, 21, 232-261.

Nelson, D. (1991), “Conditional Heteroskedasticity in Asset Returns: A New Approach”, *Econometrica*, 59, 347-370.

Oxley, L. and M. McAleer (1993), “Econometric Issues in Macroeconomic Models with Generated Regressors”, *Journal of Economic Surveys*, 7, 1-40.

Oxley, L. and M. McAleer (1994), “Testing the Rational Expectations Hypothesis in Macroeconometric Models with Unobserved Variables”, in L. Oxley et al. (eds.), *Surveys in Econometrics*, Basil Blackwell, Oxford, 1994, pp. 299-349.

Pagan, A.R. (1984), “Econometric Issues in the Analysis of Regressions with Generated Regressors”, *International Economic Review*, 2, 221-247.

Scholes, M., and J. Williams (1977), “Estimating Betas from Nonsynchronous Data”, *Journal of Financial Economics*, 5, 309-327.

Tse, Y.K. and A.K.C. Tsui (2002), "A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations", *Journal of Business and Economic Statistics*, 20, 351-362.

da Veiga, B. and M. McAleer (2005), "Univariate and Multivariate Forecasts of Volatility and Value-at-Risk Thresholds", unpublished paper, School of Economics and Commerce, University of Western Australia.

Table 1: Descriptive Statistics for Returns

Statistics	S&P500	FTSE100	CAC40	SMI
Mean	0.033	0.020	0.020	0.036
Median	0.029	0.013	0.043	0.037
Maximum	5.771	8.336	10.356	7.049
Minimum	-5.533	-5.681	-10.251	-9.134
Std. Dev.	1.022	1.067	1.346	1.164
Skewness	-0.018	0.118	0.015	-0.120
Kurtosis	6.160	6.254	7.391	7.044
CoV	31.227	54.520	66.002	32.558
Jarque-Bera	1548.350	1649.464	2988.976	2543.419

Table 2: Descriptive Statistics for Volatilities

Statistics	S&P500	FTSE100	CAC40	SMI
Mean	1.463	1.357	2.029	1.539
Median	0.443	0.425	0.665	0.486
Maximum	33.088	47.700	42.249	61.433
Minimum	0.000	0.000	0.000	0.000
Std. Dev.	2.929	2.683	3.876	3.434
Skewness	5.014	6.076	4.668	7.477
Kurtosis	38.493	67.192	33.457	90.359
CoV	2.00245	1.97735	1.91041	2.23063

Table 3: Correlations Between Conditional Volatility Forecasts for S&P500

CCC	VARMA-GARCH	PS-GARCH
1	0.966	0.981
	1	0.989
		1

Table 4: Correlations Between Conditional Volatility Forecasts for FTSE100

CCC	VARMA-GARCH	PS-GARCH
1	0.955	0.97
	1	0.972
		1

Table 5: Correlations Between Conditional Volatility Forecasts for CAC40

CCC	VARMA-GARCH	PS-GARCH
1	0.956	0.990
	1	0.970
		1

Table 6: Correlations Between Conditional Volatility Forecasts for SMI

CCC	VARMA-GARCH	PS-GARCH
1	0.961	0.986
	1	0.983
		1

Table 7: Correlations of Rolling Conditional Correlation Forecasts Between S&P500 and FTSE100

CCC	VARMA-GARCH	PS-GARCH
1	0.996	0.999
	1	0.997
		1

Table 8: Correlations of Rolling Conditional Correlation Forecasts Between S&P500 and CAC40

CCC	VARMA-GARCH	PS-GARCH
1	0.996	0.998
	1	0.997
		1

Table 9: Correlations of Rolling Conditional Correlation Forecasts Between S&P500 and SMI

CCC	VARMA-GARCH	PS-GARCH
1	0.995	0.999
	1	0.996
		1

Table 10: Correlations of Rolling Conditional Correlation Forecasts Between FTSE100 and CAC40

CCC	VARMA-GARCH	PS-GARCH
1	0.992	0.996
	1	0.996
		1

Table 11: Correlations of Rolling Conditional Correlation Forecasts Between FTSE100 and SMI

CCC	VARMA-GARCH	PS-GARCH
1	0.984	0.995
	1	0.992
		1

Table 12: Correlations of Rolling Conditional Correlation Forecasts Between CAC40 and SMI

CCC	VARMA-GARCH	PS-GARCH
1	0.998	0.996
	1	0.996
		1

Table 13: Basel Accord Penalty Zones

Zone	Number of Violations	Increase in *k*
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days.

Table 14: Mean Daily Capital Charge and AD of Violations

Model	Mean Daily Capital Charge	AD of Violations		
		Minimum	Maximum	Mean
CCC	9.685317	0.0005	2.125144	0.497947
VARMA-GARCH	9.760421	0.0001	1.973546	0.453573
PS-GARCH	10.69713	0.0001	1.90225	0.441717

Note: The daily capital charge is given as the negative of (3+*k*)VaR, where $0 \leq k \leq 1$ is the penalty. AD is the absolute deviation of violations.

Figure 1

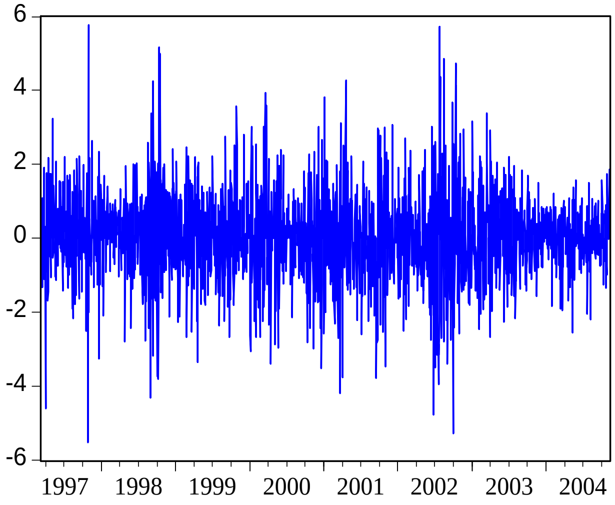

Figure 1a: S&P500 Returns

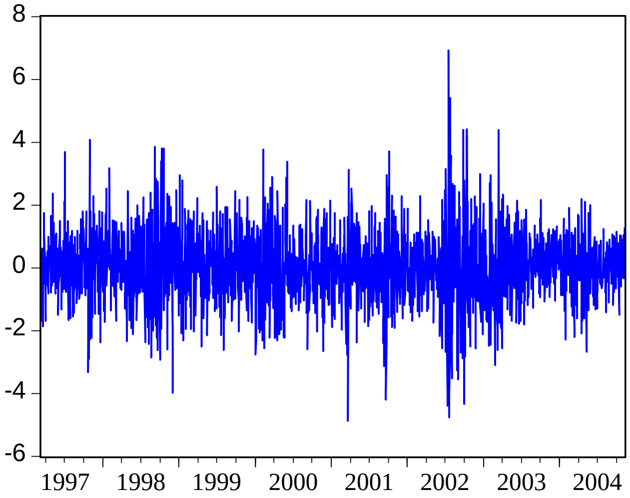

Figure 1b: FTSE100 Returns

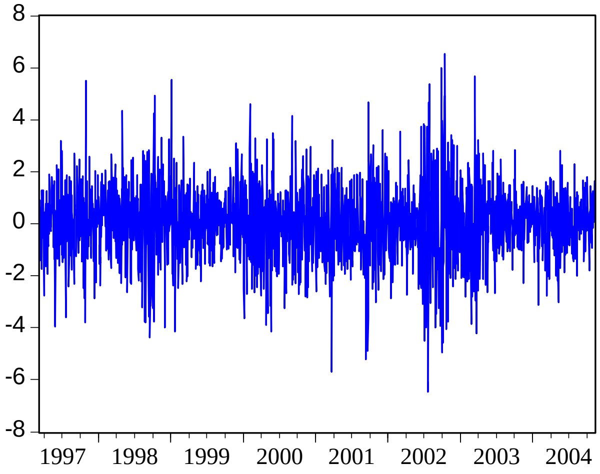

Figure 1c: CAC40 Returns

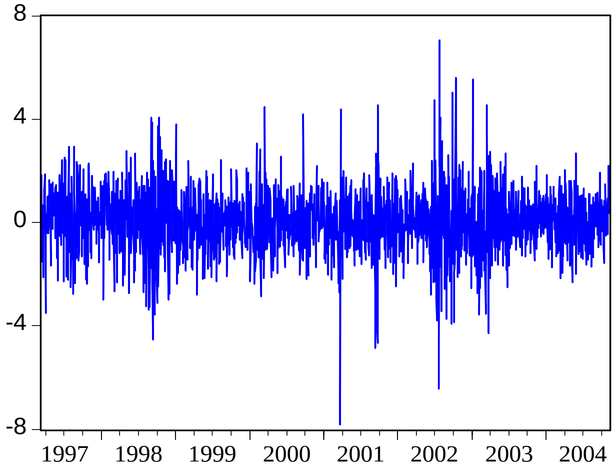

Figure 1d: SMI Returns

Figure 2

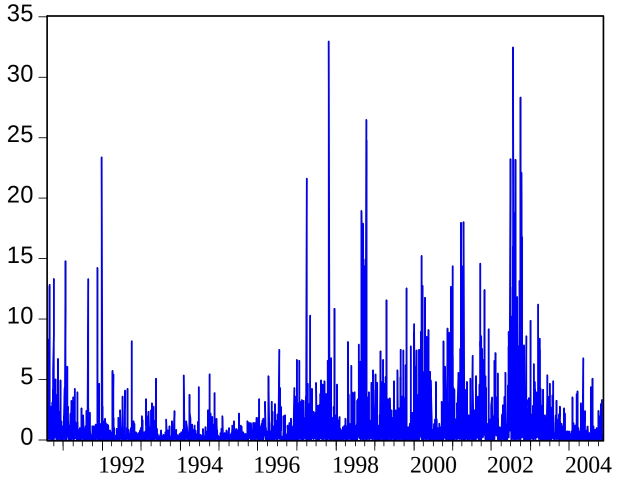

Figure 2a: S&P500 Volatility

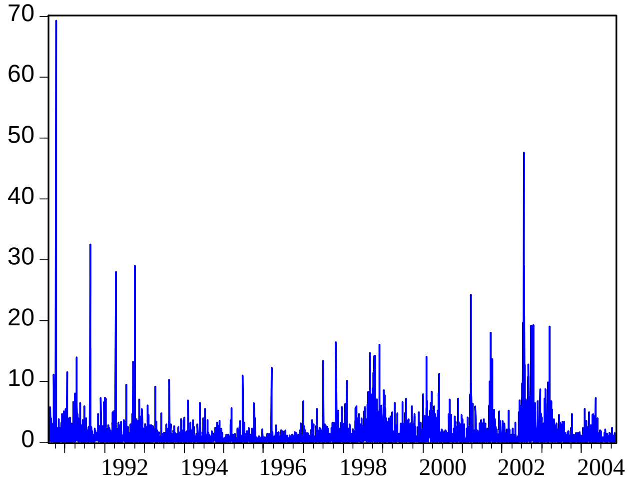

Figure 2b: FTSE100 Volatility

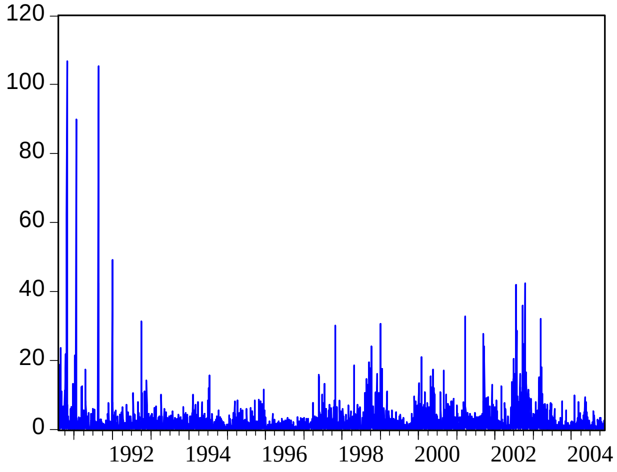

Figure 2c: CAC40 Volatility

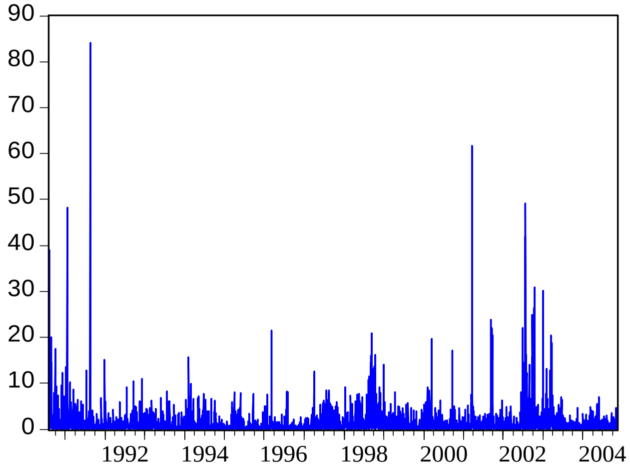

Figure 2d: SMI Volatility

Figure 3: S&P500 Volatility Forecasts

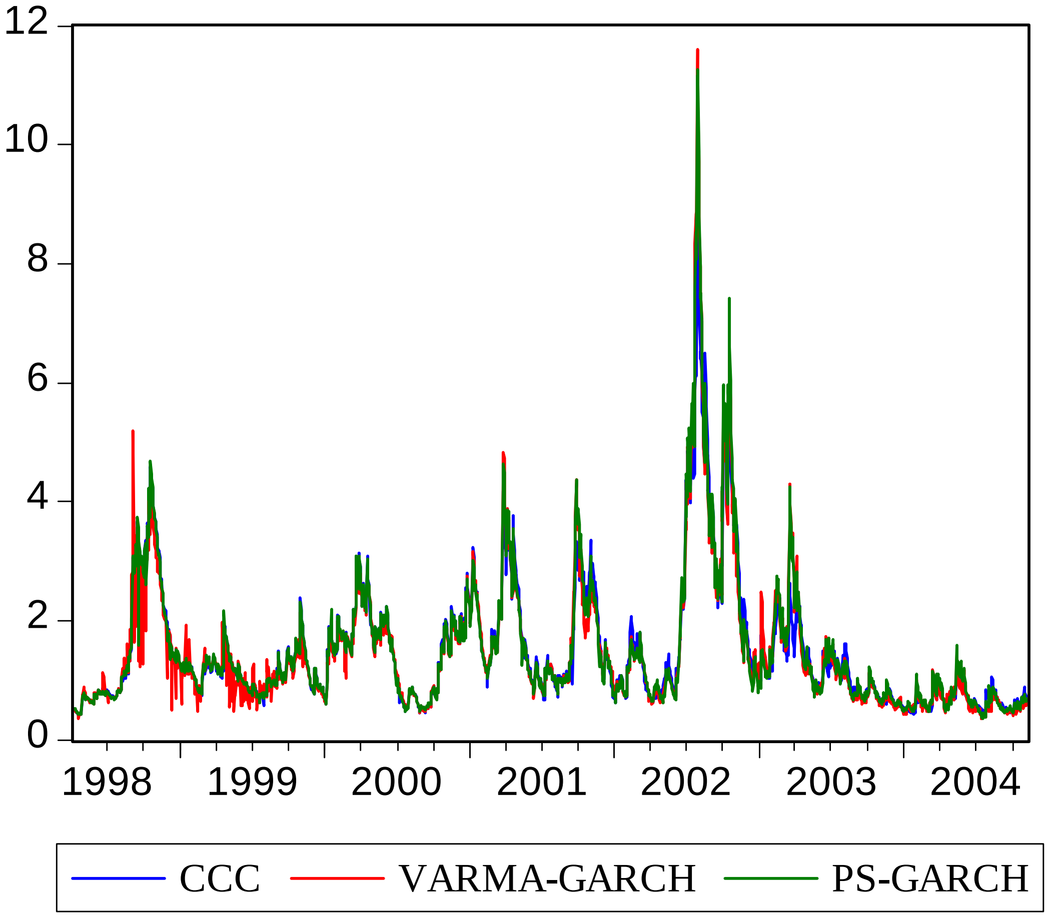

Figure 4: FTSE100 Volatility Forecasts

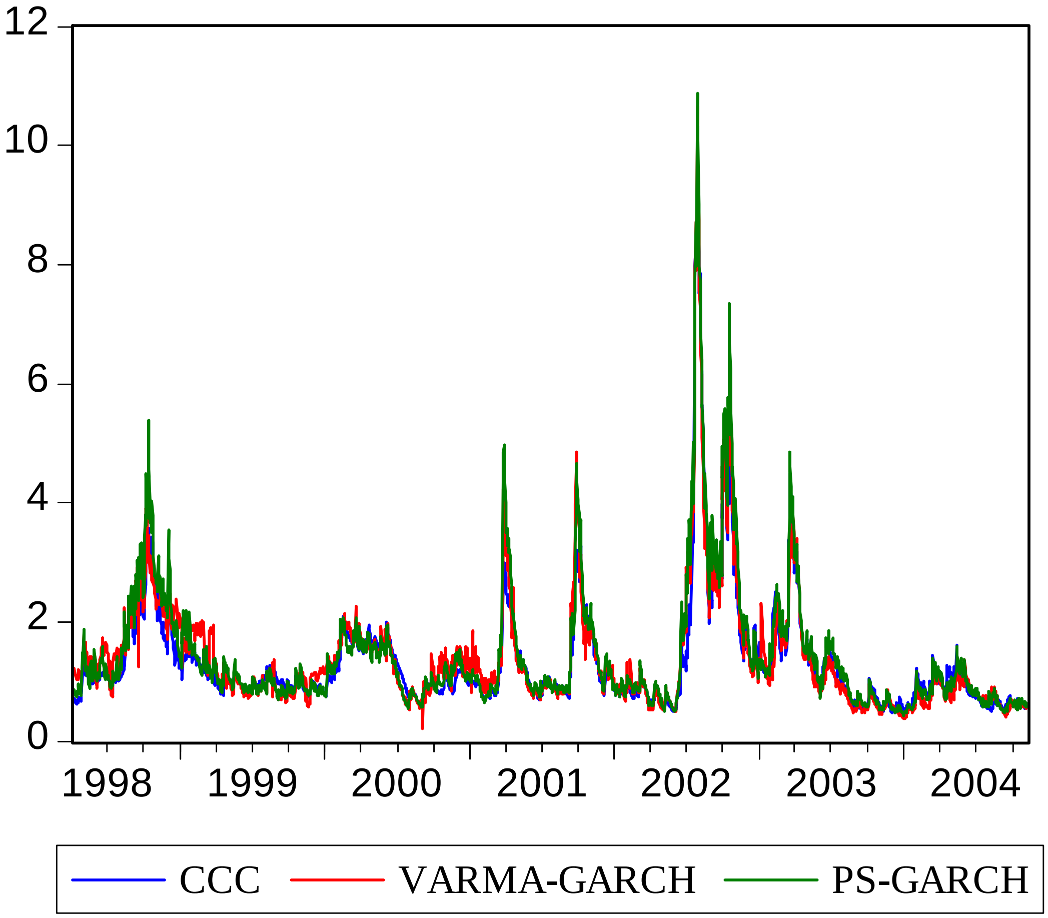

Figure 5: CAC40 Volatility Forecasts

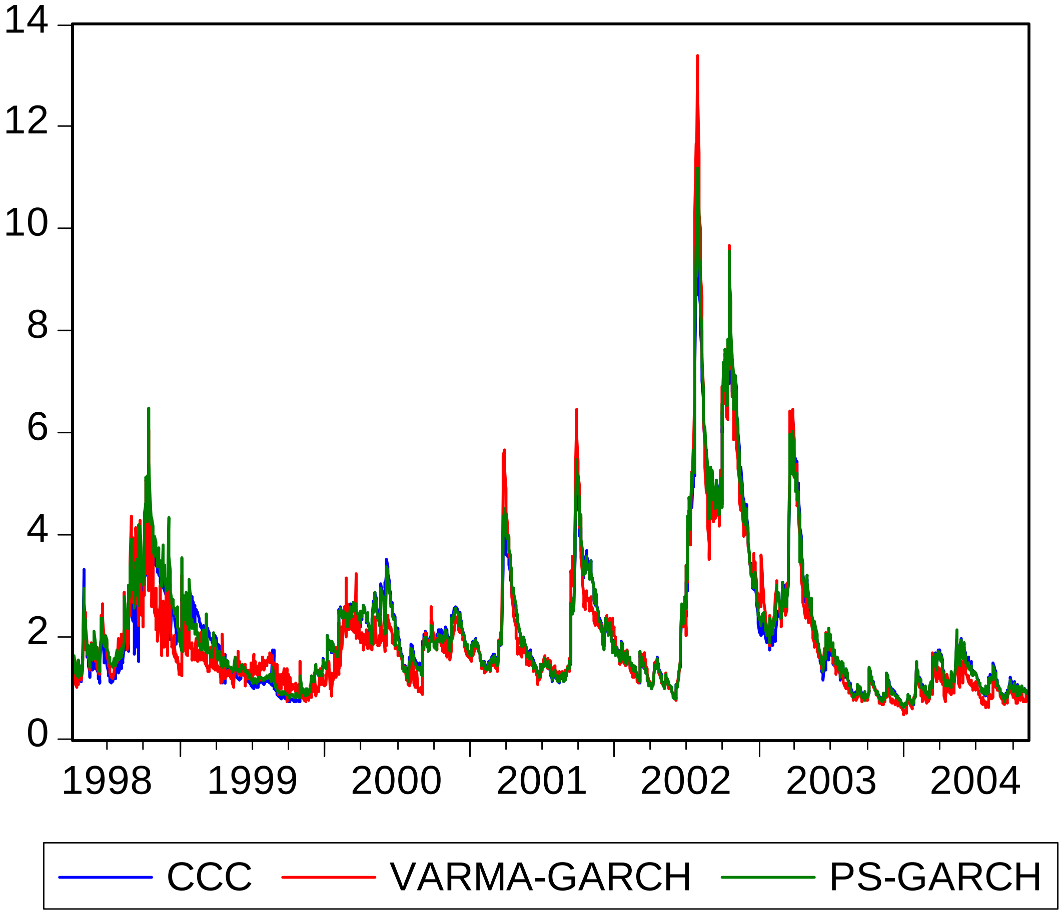

Figure 6: SMI Volatility Forecasts

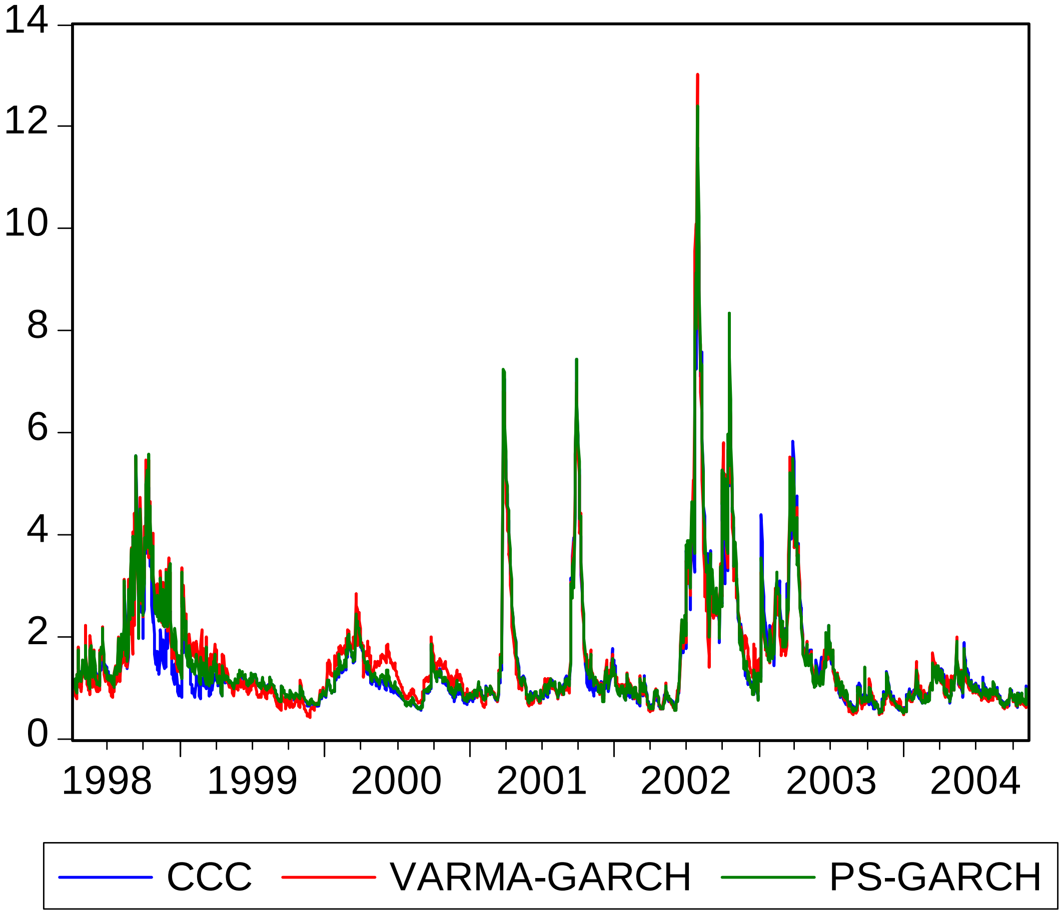

Figure 7: Rolling Conditional Correlation Forecasts Between S&P500 and FTSE100

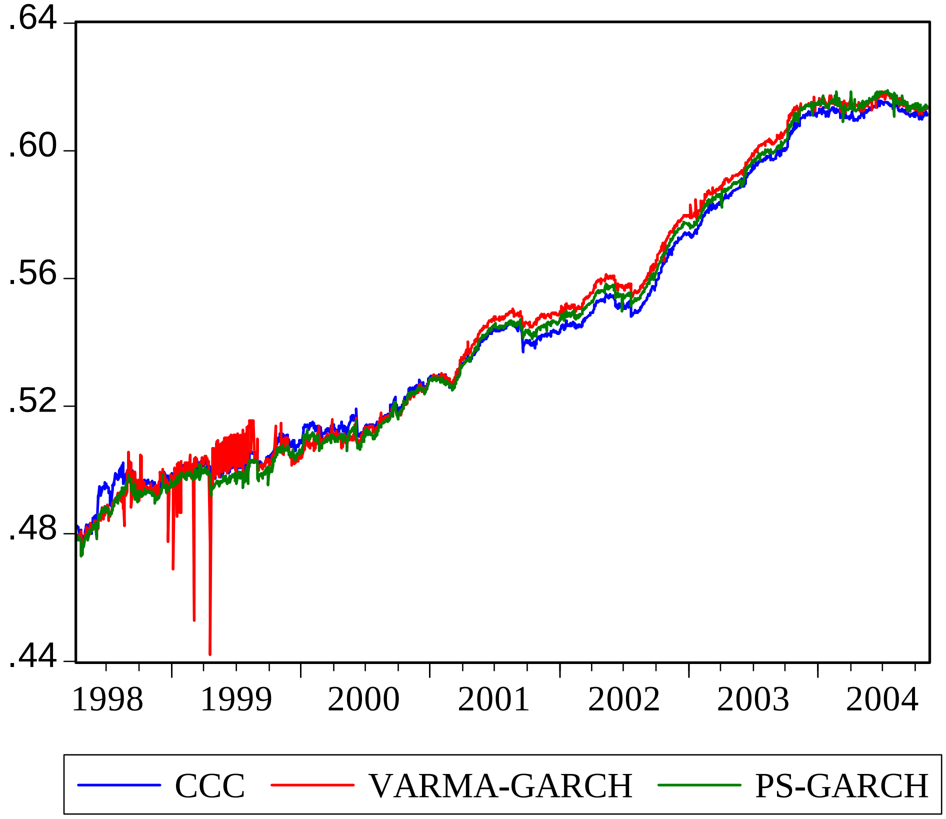

Figure 8: Rolling Conditional Correlation Forecasts Between S&P500 and CAC40

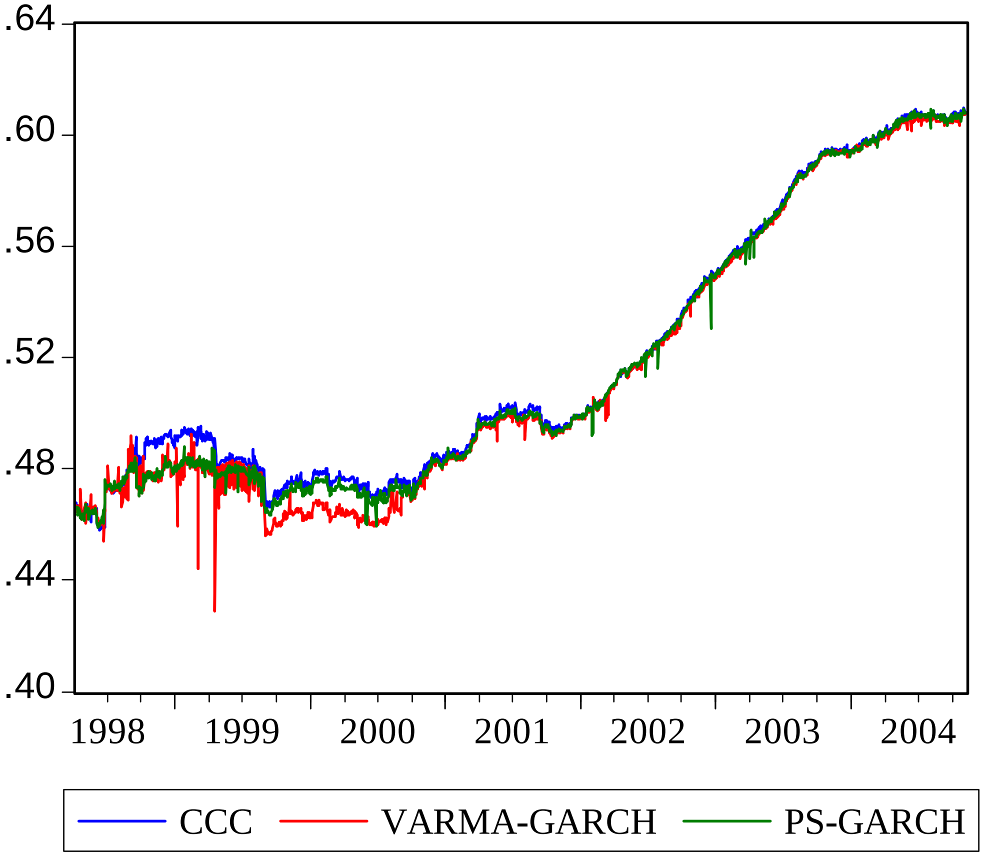

Figure 9: Rolling Conditional Correlation Forecasts Between S&P500 and SMI

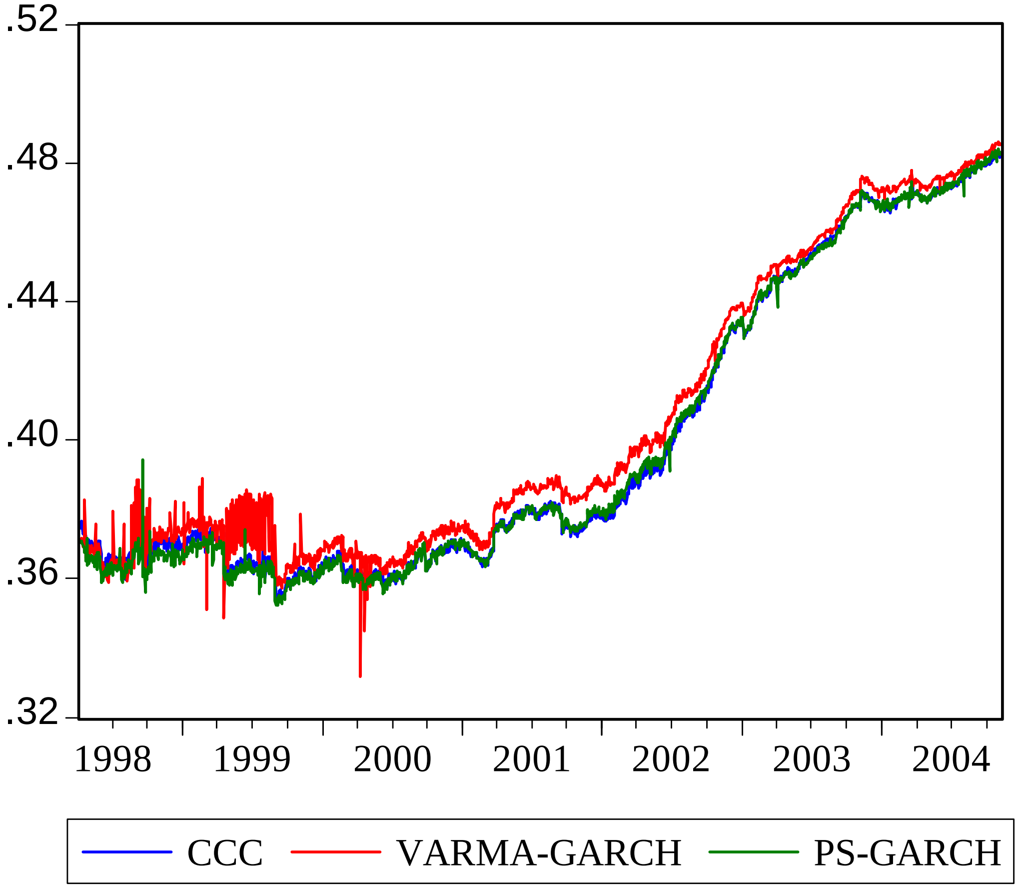

Figure 10: Rolling Conditional Correlation Forecasts Between FTSE100 and CAC40

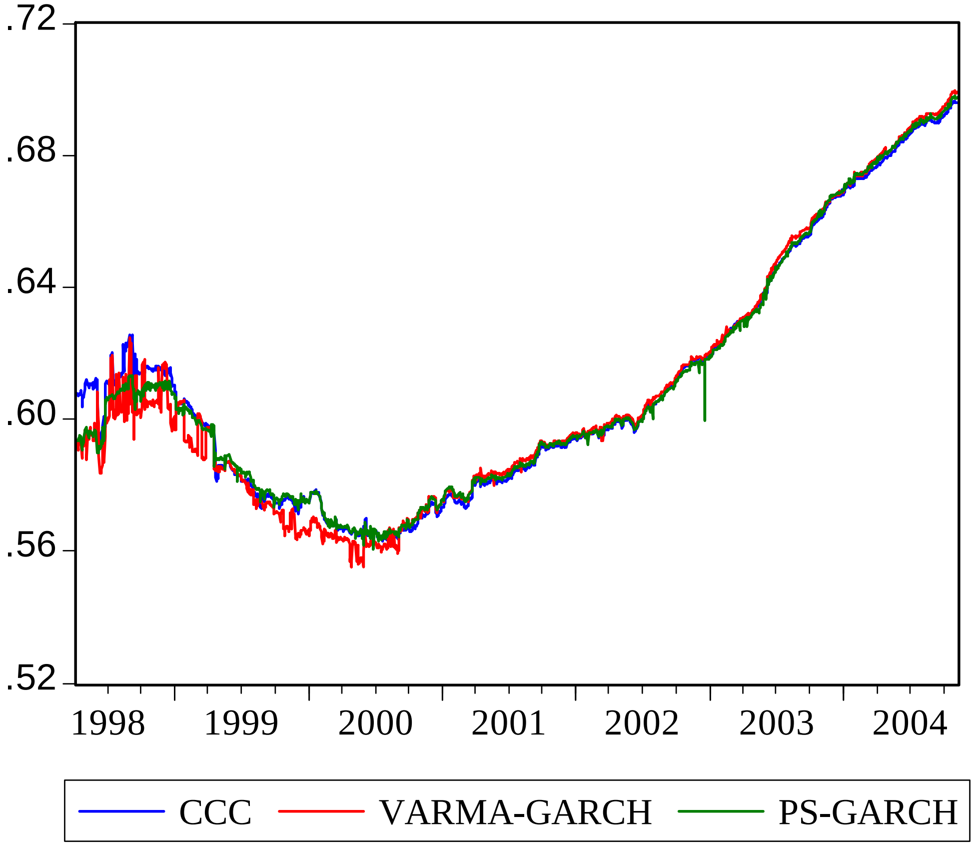

Figure 11: Rolling Conditional Correlation Forecasts Between FTSE100 and SMI

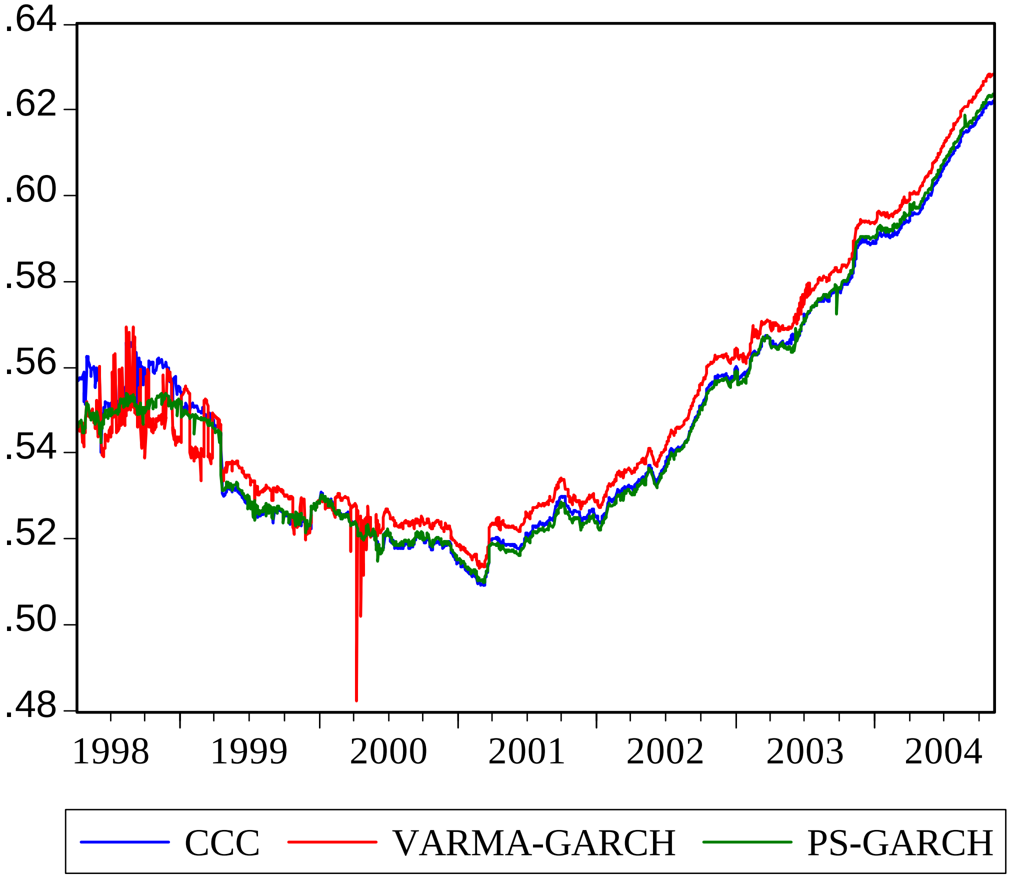

Figure 12: Rolling Conditional Correlation CAC40 and SMI

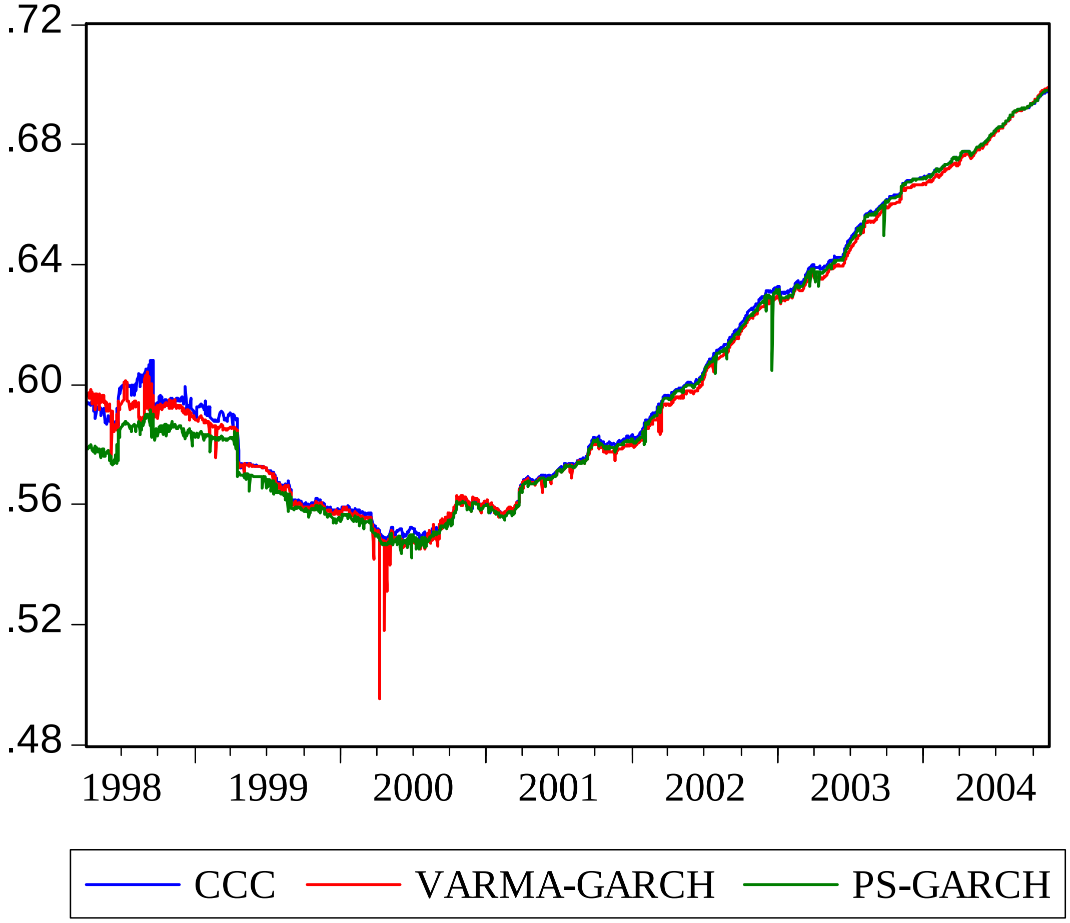

Figure 13: Realized Returns and CCC VaR Forecasts

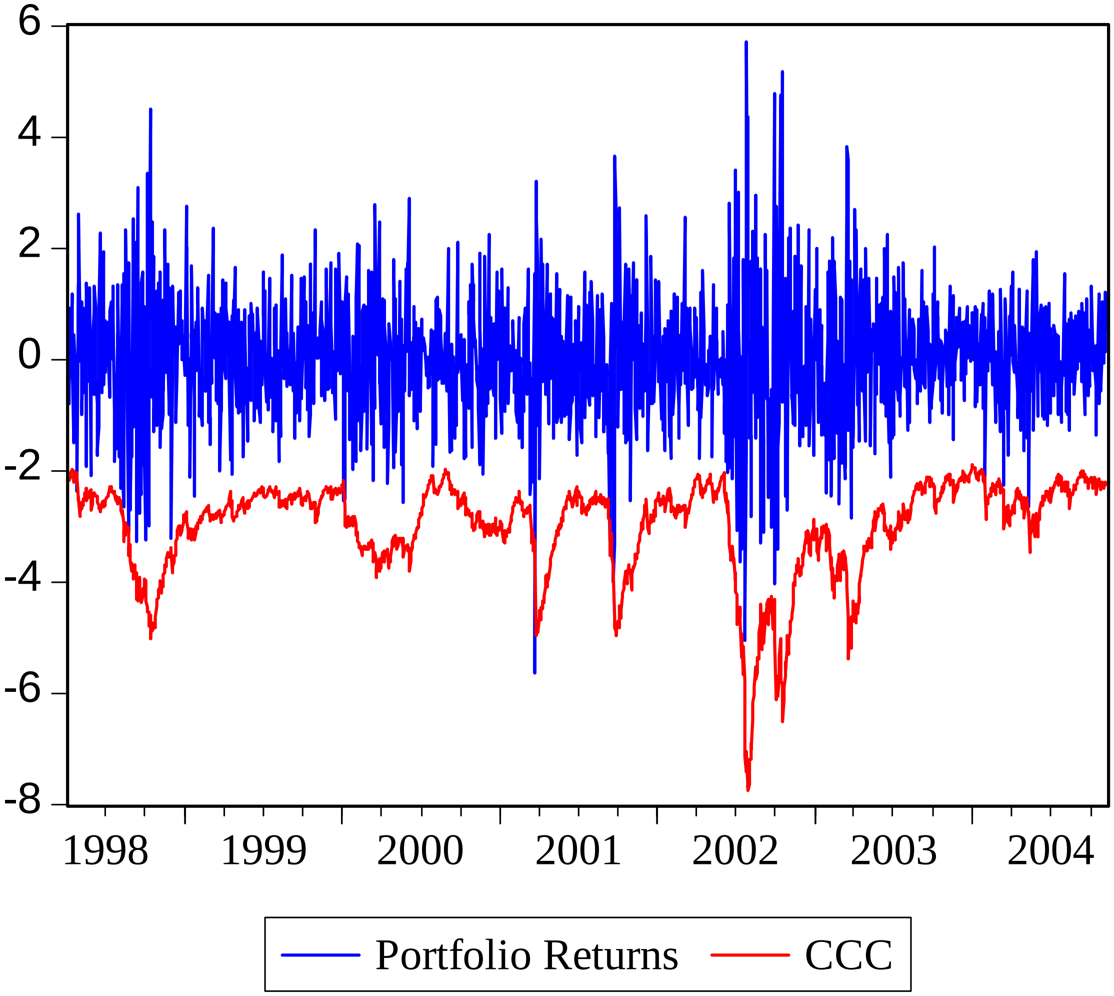

Figure 14: Realized Returns and VARMA-GARCH VaR Forecasts

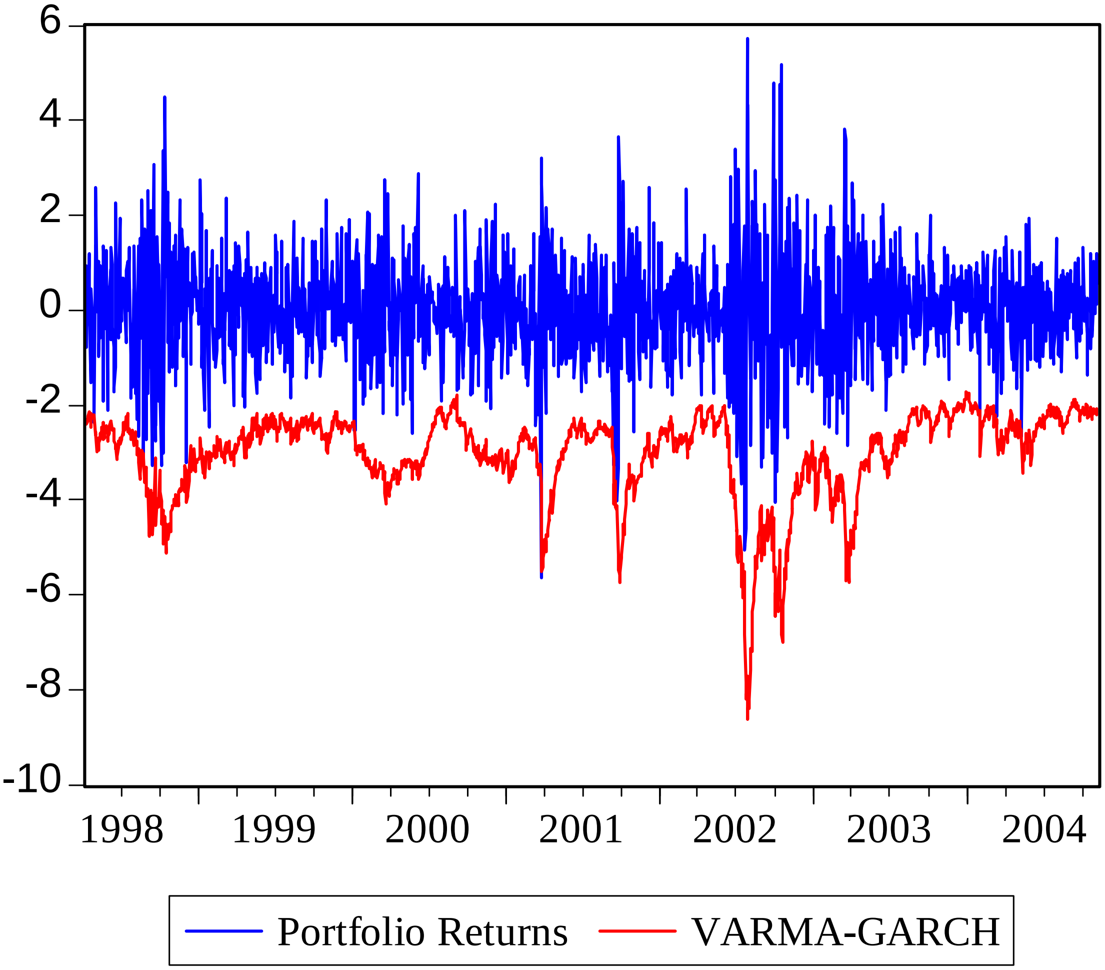

Figure 15: Realized Returns and PS-GARCH VaR Forecasts