Monopoly Power in Dynamic Securities Markets

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Revised: August, 2000

1. Introduction

In a 1997 paper, Basak has analyzed the effect of monopoly power on asset pricing in a continuous time Lucas [1978] market setting. In his dynamic analysis, Basak restricts attention to the case in which the monopolist can make future commitments. In the present paper, we investigate the impact of monopoly power in a Lucas asset market when future commitments are impossible. Our analysis is motivated by the formal resemblance of assets in a dynamic Lucas portfolio model to durable goods. Because of this similarity, it is natural to conjecture that, when future commitments are impossible, the Coase [1972] analysis of a durable goods monopolist also applies to the dynamic asset pricing case. In his 1972 *Journal of Law and Economics* paper, Coase argued that a monopolist who can repeatedly sell a durable good will be unable to capture monopoly profits and will instead be forced to charge the competitive price. This happens because the monopolist's future sales effectively compete with his current sales and this eliminates his monopoly power. The Coase argument relies on the fact that the monopolist cannot commit himself to refrain from making future sales. If he could credibly make such commitments, buyers would have no alternative to present purchases and the monopolist would be able to earn monopoly profits by restricting supply and charging the monopoly price. It should be noted that the monopolist's inability to commit to refrain from making future sales would not cause a problem for him, if his incentives in the future were the same as in the present. That is not the case, however, since, when the future arrives and the monopolist decides whether to supply, he cannot raise past profits by restricting supply.

During the late 1970's and 1980's Coase's ideas were the subject of precise formal analyses in Stokey [1979] and [1981], Bulow [1982], Wilson [1985] and Gul, Sonnenschein and Wilson [1986]. The first chapter of Tirole's 1988 industrial organization textbook describes and exposits the analyses of these papers.

Here we begin an attempt to translate these ideas to the study of dynamic monopoly in a securities market. In a very special two-period, two asset Lucas portfolio model, we determine conditions under which Coase's argument applies and monopoly power cannot be completely exercised. In this model, a risk averse monopolist who begins as the initial owner of a risky firm sells shares in his firm to a representative competitive; i.e. price taking, trader. The second asset is a riskless technology. The risky firm yields dividends in two periods at which times consumption takes place. Trade in the assets occurs twice. The first trading date precedes the first consumption date and at this time the dividends to be paid by the risky firm are unknown to the traders. At this point the dividends are expected to be realizations of *iid* normally distributed random variables. After the close of the first trading period, the risky firm's first dividend is declared. At this time, the second period dividend remains unknown to the traders and the second trading period opens . The traders then decide how much of the wealth earned on their previous investments to consume and how much to save. At the same time they rebalance their portfolios.

In each trading period, the monopolist chooses the price of the shares in the risky firm. When, in the first trading period, the representative competitive trader chooses his demand for shares in the risky firm, he not only knows the price charged in that period by the monopolist, he also has correct expectations about the price the monopolist will charge in the second trading period.

We consider two scenarios. In the first scenario, the monopolist is able to make a commitment in the first trading period to the price he will charge in the second trading period. Thus, in this scenario, the monopolist can effectively chooses a price path in the first period. When he makes this choice, the monopolist knows how the competitor's demands in each of the trading periods are related to each price path he might choose. In this case, the monopolist can choose the second period price with a view to the effect that it's choice has on the competitive trader's first period demand. We show that, when commitment is possible, all of the shares sold by the monopolist are sold in the first period. He sells no additional shares in the second trading period. He accomplishes this by committing to a price path that induces the competitive trader to demand the same share in the risky firm in both trading periods. We also show that, in each trading period the price charged by the monopolist exceeds the competitive price and the monopolist is able to obtain this high price by effectively restricting the share of the firm that he offers for sale.

The second scenario considered is one in which, in the first trading period, the monopolist is unable to credibly commit to the price he will charge in the second trading period. In this setting, when the price charged in the second trading period is chosen, it is too late for the monopolist to take account of the effect that this choice will have on the competitive trader's first period expectations. Thus, the monopolist ignores the effect that his choice of a second period price has on the competitive trader's first period demand. However, we assume that, when the competitive trader makes his first period portfolio choice he not only knows the first period price charged by the monopolist, he also correctly anticipates the monopolist's second period price choice. Thus, the competitive trader's first period demand for shares in the risky firm does, in fact, depend on the second period price charged by the monopolist as well as on the first period price. In the first period, the monopolist not only knows the competitive trader's demand and how it relates to the expected price path, he also knows how he will react to the first period price when he chooses the second period price. He takes this reaction into account when making his first period choice.

Our major result is that the first period price chosen in this second scenario is indeed less than the first period price the monopolist would choose if he could commit to a price as assumed in the first scenario discussed.

The economic problem as posed here begins with a specification of the preferences for the assets. This asset preference specification appears to be analytically quite different that assumed for the durable goods in the durable goods literature referred to above. Nevertheless, as we demonstrate, the analytic reduced form of the model studied here is formally quite similar to that studied by Bulow as it is exposited by Dudey [1995]. In particular, like Bulow and Dudey, we have linear demand functions. The main analytic difference is that we, in effect, have increasing marginal costs while they assume that marginal cost is constant. In an asset pricing setting, when an investor supplies an asset he incurs a "cost" because he does not get the returns paid by the asset. For a risk averse investor this cost is reduced because of the riskiness of the asset's return. The value of this "cost reduction" becomes less important as the risk averse investor supplies more of the asset and bears less of the risk. Hence, the "increasing costs." As these remarks suggest the case of constant marginal cost corresponds to the case of a risk neutral monopoly supplier of an asset. In that case, however, the issues under study here become uninteresting because, in equilibrium, the risk neutral monopolist does not sell any shares in his the risky firm to the risk averse competitive trader. It should also be noted that, in an asset pricing setting, the "cost" of supplying the asset is borne not just once but is incurred repeatedly. When an asset is sold the returns are lost to the seller in every future period. This feature distinguishes our analysis of the increasing cost case from that in Kahn [1986].

A recent paper, Vayanos [1999], studies issues closely related to those investigated here. As in this paper, Vayanos assumes CARA preferences and normal dividend distributions. Since Vayanos uses a variation of the model of Kyle [1989], he, in effect, has many traders with market power and no competitive sector. Traders' asset endowments also fluctuate randomly over time. He also assumes that trading takes place many times and is able to investigate the convergence to the competitive case as trading frequency increases in his setting. Because of the fact that there are many traders with market power in the Vayanos model, he uses an equilibrium concept that differs from the monopolistic equilibrium of this paper and his results do not cover the case the investigated here.

2. The Model

In the model there are two trading periods, one and two, and two periods in which consumption takes place, periods two and three. Thus, there is no consumption in the first trading period, period one, and there is no trading in the last trading period, period three. Two assets are traded in each trading period.

- - One asset is a riskless.
    - In each period its return is r.
    - It is in infinitely elastic supply.
  - The second asset is a risky firm.
    - It pays random dividends $\tilde{D}_t$ at times $t = 2$ and $t = 3$
    - $\tilde{D}_2$ and $\tilde{D}_3$ are normal and *iid.*

$$E\tilde{D}_t = \mu$$

$$var\left(\tilde{D}_t\right) = \sigma^2$$

- There is a representative agent competitive trader, c,
  - He maximizes
    $$-e^{-\alpha_c \tilde{C}_2^c} - \delta e^{-\alpha_c \tilde{C}_3^c}$$
    where
    $$\tilde{C}_t^c$$
    is his period t consumption.
  - He begins period one with wealth W_1^c.

- There is a monopolist, m.
  - He begins period one holding wealth W_1^m
  - He also owns the risky firm at the beginning of period one.
  - He chooses the prices V_1 and V_2 at which shares in the firm trade in periods one and two
  - He maximizes
    $$-e^{-\alpha_m \tilde{C}_2^m} - \delta e^{-\alpha_m \tilde{C}_3^m}$$
    where
    $$\tilde{C}_t^m$$
    is his period t consumption.

3. THE COMPETITOR'S PROBLEM:

3.1. The Second Period Portfolio Problem. Having chosen his period two savings level, S_c, and

$\gamma_2 =$ the fractional share of the risky firm owned by c in period 2,

the competitor's period three consumption is

$$\tilde{C}_3^c = r\left(S_c - \gamma_2 V_2\right) + \gamma_2 \tilde{D}_3.$$

His optimal choice for γ_2 is $\hat{\gamma}_2\left(V_2\right)$ where

$$\begin{aligned}\hat{\gamma}_2\left(V_2\right) &= \arg\max_{\gamma_2} -e^{-\alpha_c r S_c - \alpha_c \gamma_2\left(\mu - rV_2\right) + \frac{\gamma_2^2}{2}\alpha_c^2\sigma^2} \\ &= \arg\max_{\gamma_c} \gamma_2\left(\mu - rV_2\right) - \frac{\gamma_2^2}{2}\alpha_c\sigma^2.\end{aligned}$$

The solution is

$$\hat{\gamma}_2\left(V_2\right) = \frac{\mu - rV_2}{\alpha_c\sigma^2}. \tag{1}$$

Substituting $\hat{\gamma}_2\left(V_2\right)$ in the maximand implies that

$$\begin{aligned}&\max_{\gamma_2} -e^{-\alpha_c r S_c - \alpha_c \gamma_2\left(\mu - rV_2\right) + \frac{\gamma_2^2}{2}\alpha_c^2\sigma^2} \\ = \; & -e^{-\alpha_c r S_c}\left(e^{-\frac{\left(\mu - rV_2\right)^2}{2\sigma^2}}\right).\end{aligned}$$

Remark 1. *Because the dividend distributions are independent and because CARA preferences imply that the optimal portfolio is independent of wealth, the second period portfolio choice described in (1) is independent of the dividend D_2 paid in period 2.*

3.2. The Savings Choice. In period 2, the competitor's wealth is

$$W_2^c = rW_1^c + \gamma_1\left(D_2 + V_2 - rV_1\right).$$

and he chooses S_c to maximize

$$-e^{-\alpha_c\left(W_2^c - S_c\right)} - \delta e^{-\alpha_c r S_c}\left(e^{-\frac{\left(\mu - rV_2\right)^2}{2\sigma^2}}\right).$$

The result is

$$\begin{aligned}&\max_{S_c}\left[-e^{-\alpha_c\left(W_2^c - S_c\right)} - \delta e^{-\alpha_c r S_c}\left(e^{-\frac{\left(\mu - rV_2\right)^2}{2\sigma^2}}\right)\right] \\ = \; & -\left(\frac{1+r}{r}\right)\left(e^{-\frac{r\alpha_c}{1+r}W_2^c}\right)\left(r\delta e^{-\frac{\left(\mu - rV_2\right)^2}{2\sigma^2}}\right)^{\frac{1}{(1+r)}}.\end{aligned}$$

While it is easily possible to compute the optimal savings level, it is unnecessary for our purposes so we do not bother to do so.

3.3. The First Period Portfolio Problem. In period one, the competitor chooses

$$\gamma_1 = \text{ the fractional share of the risky firm owned by } c \text{ in period } 1,$$

When the competitor makes this choice he knows V_1 and expects the period two value of the risky firm to be V_2. In general situations, it is to be expected that the second period price of the firm, V_2, will depend on the dividend, D_2, paid by the firm in period two. We noted earlier that, in the specific case considered here, independence of the dividend distributions and the fact that preferences are CARA imply that the second period portfolio choice is independent of D_2. For the same reasons, V_2, can be assumed to be independent of D_2.

The optimal choice for γ_1 is $\hat{\gamma}_1(V_1, V_2)$ where

$$\hat{\gamma}_1(V_1, V_2) = \underset{\gamma_1}{argmax} \; -E\left(e^{-\frac{r\alpha_c}{1+r}\tilde{W}_2^c}\right)$$

and where

$$\tilde{W}_2^c = rW_1^c + \gamma_1\left(\tilde{D}_2 + V_2 - rV_1\right).$$

The solution is

$$\hat{\gamma}_1(V_1, V_2) = \frac{\mu + V_2 - rV_1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}. \tag{2}$$

Remark 2. *The demand function faced by the monopolist in the first trading period is $\hat{\gamma}_1(V_1, V_2)$. The fact that second period sales at price V_2 compete with first period sales is captured by the fact that*

$$\frac{\partial\hat{\gamma}_1(V_1, V_2)}{\partial V_2} = -\left[\frac{1}{r}\right]\frac{\partial\hat{\gamma}_1(V_1, V_2)}{\partial V_1} = \frac{1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2} > 0. \tag{3}$$

This means that an expected decrease in the second period price V_2 of the risky shares will decrease the competitor's first period demand for the risky shares. Note also that, because r is assumed to exceed one,

$$\left|\frac{\partial\hat{\gamma}_1(V_1, V_2)}{\partial V_2}\right| = \left[\frac{1}{r}\right]\left|\frac{\partial\hat{\gamma}_1(V_1, V_2)}{\partial V_1}\right| < \left|\frac{\partial\hat{\gamma}_1(V_1, V_2)}{\partial V_1}\right|.$$

This means that a one dollar reduction in the second period price V_2 of the risky shares does not decrease the competitor's first period demand for the risky shares as much as a one dollar increase in the first period risky share price V_1. If the second *trading period occurred soon after the first, we might expect r to be close to one. In that case,*

$$\left|\frac{\partial\hat{\gamma}_1(V_1,V_2)}{\partial V_2}\right| = \left[\frac{1}{r}\right]\left|\frac{\partial\hat{\gamma}_1(V_1,V_2)}{\partial V_1}\right| \simeq \left|\frac{\partial\hat{\gamma}_1(V_1,V_2)}{\partial V_1}\right|$$

and the monopolist's second period sales of the risky shares would provide more competition for the first period sales.

Remark 3. *Note that the expression optimal first period portfolio $\hat{\gamma}_1(V_1,V_2)$ in (2) is independent of the discount rate δ.*

4. The Monopolist's Problem

4.1. The Monopolist's Savings Choice.

The monopolist will choose S_m knowing

$$W_2^m = rW_1^m + rV_1 + (1-\gamma_1)(D_2 + V_2 - rV_1)$$

and having already chosen V_1. In the commitment case, he will have also chosen V_2. In the noncommitment case, we can, for purposes of computation, also regard him as choosing V_2 before choosing S_m. His expected utility is

$$\begin{aligned} & -e^{-\alpha_m\left(W_2^m - S_m\right)} - \delta e^{-\alpha_m r S_m - \alpha_m(1-\hat{\gamma}_2(V_2))(\mu - rV_2) + \frac{(1-\hat{\gamma}_2(V_2))^2}{2}\alpha_m^2\sigma^2} \\ = \; & -e^{-\alpha_m\left(W_2^m - S_m\right)} - \delta e^{-\alpha_m r S_m}\left(e^{-\alpha_m(1-\hat{\gamma}_2(V_2))(\mu - rV_2) + \frac{(1-\hat{\gamma}_2(V_2))^2}{2}\alpha_m^2\sigma^2}\right) \end{aligned}$$

where $\hat{\gamma}_2(V_2)$ is given by (1). By an argument similar to that given for the competitor we observe that

$$\begin{aligned} & \max_{S_m} -e^{-\alpha_m\left(W_2^m - S_m\right)} - \delta e^{-\alpha_m r S_m}\left(e^{-\alpha_m(1-\hat{\gamma}_2(V_2))(\mu - rV_2) + \frac{(1-\hat{\gamma}_2(V_2))^2}{2}\alpha_m^2\sigma^2}\right) \quad (4) \\ = \; & -\left(\frac{1+r}{r}\right)\left(e^{-\frac{r\alpha_m}{1+r}W_2^m}\right)\left(r\delta e^{-\alpha_m(1-\hat{\gamma}_2(V_2))(\mu - rV_2) + \frac{(1-\hat{\gamma}_2(V_2))^2}{2}\alpha_m^2\sigma^2}\right)^{\frac{1}{1+r}}. \end{aligned}$$

For the monopolist as for the competitive trader, it is easily possible to compute the optimal savings level, but again we avoid doing so as it is unnecessary for our current purposes.

4.2. The Monopolist's (V_1, V_2) Choice with Commitment.

When commitment is possible, the monopolist chooses both V_1 and V_2, in period one, when

$$W_2^m = rW_1^m + r\gamma_1 V_1 + (1-\gamma_1)(D_2 + V_2)$$

is as yet unknown. Using (V_1^*, V_2^*) to denote the optimal price path we have

$$(V_1^*, V_2^*) = \underset{(V_1,V_2)}{argmax} - \left(\frac{1+r}{r}\right) E\left(e^{-\frac{r\alpha_m}{1+r}\tilde{W}_2^m}\right)\left(r\delta e^{-\alpha_m(1-\hat{\gamma}_2)(\mu - rV_2) + \frac{(1-\hat{\gamma}_2)^2}{2}\alpha_m^2\sigma^2}\right)^{\frac{1}{(1+r)}},$$

where

$$\begin{aligned} & E\left(e^{-\frac{r\alpha_m}{1+r}\tilde{W}_2^m}\right) \\ = \;& e^{-\frac{r^2\alpha_m}{1+r}W_1^m} E\left(e^{-\frac{r\alpha_m}{1+r}\left(r\hat{\gamma}_1(V_1,V_2)V_1+(1-\hat{\gamma}_1(V_1,V_2))\left(\tilde{D}_2+V_2\right)\right)}\right) \\ = \;& e^{-\frac{r^2\alpha_m}{1+r}W_1^m} e^{-\frac{r\alpha_m}{1+r}(r\hat{\gamma}_1(V_1,V_2)V_1+(1-\hat{\gamma}_1(V_1,V_2))(\mu+V_2))+\frac{(1-\hat{\gamma}_1(V_1,V_2))^2}{2}\left(\frac{r\alpha_m}{1+r}\right)^2\sigma^2} \end{aligned} \tag{5}$$

Using the expression for $E\left(e^{-\frac{r\alpha_m}{1+r}\tilde{W}_2^m}\right)$ in (5) we can rewrite the monopolist's maximand as

$$\begin{aligned} & -\left(\frac{1+r}{r}\right) E\left(e^{-\frac{r\alpha_m}{1+r}\tilde{W}_2^m}\right)\left(r\delta e^{-\alpha_m(1-\hat{\gamma}_2)(\mu-rV_2)+\frac{(1-\hat{\gamma}_2)^2}{2}\alpha_m^2\sigma^2}\right)^{\frac{1}{(1+r)}} \\ = \;& -\left(\frac{1+r}{r}\right)\left(e^{-\frac{r^2\alpha_m}{1+r}W_1^m}\right)\left(r\delta e^{-\frac{\alpha_m}{1+r}f(V_1,V_2)}\right). \end{aligned}$$

where

$$\begin{aligned} f(V_1,V_2) \;=\;& r^2V_1 + r\left[(1-\hat{\gamma}_1)(\mu+V_2-rV_1) - \frac{(1-\hat{\gamma}_1)^2}{2}\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right] \\ & +(1-\hat{\gamma}_2)(\mu-rV_2) - \frac{(1-\hat{\gamma}_2)^2}{2}\alpha_m\sigma^2. \end{aligned}$$

Clearly,

$$(V_1^*, V_2^*) = \underset{(V_1,V_2)}{argmax}\; f(V_1,V_2).$$

Proposition 1. *When the monopolist can commit to a second period price, he chooses*

$$V_2^* = \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r}. \tag{6}$$

and

$$\begin{aligned} V_1^* \;&=\; \frac{\mu + V_2^* - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c+\alpha_m}}{r} \\ &=\; \frac{\mu + \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r} - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c+\alpha_m}}{r}. \end{aligned} \tag{7}$$

The fractional share of the risky firm held by the competitive trader in each period is

$$\hat{\gamma}_1(V_1^*, V_2^*) = \hat{\gamma}_2(V_2^*) = \frac{\alpha_m}{2\alpha_c+\alpha_m}. \tag{8}$$

Remark 4. *We noted in the introduction that, when commitment is possible, all of the shares sold by the monopolist are sold in the first period. He sells no additional shares in the second trading period. Equation (8) demonstrates that the monopolist accomplishes this by committing to the price path* (V_1^*, V_2^*). *Specifically, equation (8) simply asserts that when the monopolist commits to this price path, the competitive trader is induced to demand the same share in the risky firm in both trading periods.*

Proof: In Remark 7 which follows the proof of this proposition we prove that f is a strictly concave function. Thus, we can derive the (V_1^*, V_2^*), by simply solving the first order conditions, which in this case, are linear functions of (V_1, V_2). The first order conditions are

$$\begin{aligned} 0 &= f_{V_1}(V_1, V_2) \\ &= r^2\hat{\gamma}_1 - r\left[(\mu + V_2 - rV_1) - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{\partial\hat{\gamma}_1}{\partial V_1} \end{aligned} \tag{9}$$

and

$$\begin{aligned} 0 &= f_{V_2}(V_1, V_2) \\ &= r(\hat{\gamma}_2 - \hat{\gamma}_1) - r\left[(\mu + V_2 - rV_1) - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{\partial\hat{\gamma}_1}{\partial V_2} \\ &\quad - \left[(\mu - rV_2) - (1-\hat{\gamma}_2)\alpha_m\sigma^2\right]\frac{\partial\hat{\gamma}_2}{\partial V_2}. \end{aligned} \tag{10}$$

Note first that the relationship between $\frac{\partial\hat{\gamma}_1(V_1,V_2)}{\partial V_2}$ and $\frac{\partial\hat{\gamma}_1(V_1,V_2)}{\partial V_1}$ obtained in (3) implies that

$$f_{V_2}(V_1, V_2) = r\hat{\gamma}_2 - \left[(\mu - rV_2) - (1-\hat{\gamma}_2)\alpha_m\sigma^2\right]\frac{\partial\hat{\gamma}_2}{\partial V_2} - \frac{f_{V_1}(V_1, V_2)}{r}.$$

Thus, when (9) holds, (10) reduces to

$$0 = r\hat{\gamma}_2 - \left[(\mu - rV_2) - (1-\hat{\gamma}_2)\alpha_m\sigma^2\right]\frac{\partial\hat{\gamma}_2}{\partial V_2}. \tag{11}$$

Using the expression for $\hat{\gamma}_2$ given in (1) and the fact that

$$\frac{\partial\hat{\gamma}_2}{\partial V_2} = -\frac{r}{\alpha_c\sigma^2}, \tag{12}$$

we can solve (11) to get (6). Similarly, substituting (2) and

$$\frac{\partial\hat{\gamma}_1}{\partial V_1} = -\frac{r}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2} \tag{13}$$

in (9) and solving we get (7).

Now let's derive $\hat{\gamma}_1(V_1^*, V_2^*)$ and $\hat{\gamma}_2(V_2^*)$. For this purpose, we substitute the expressions for $\hat{\gamma}_2(V_2^*)$ and $\frac{\partial \hat{\gamma}_2}{\partial V_2}$ given in (1) and (12) in (11) to get

$$0 = r\hat{\gamma}_2(V_2^*) + \left[\hat{\gamma}_2(V_2^*)\,\alpha_c\sigma^2 - (1-\hat{\gamma}_2(V_2^*))\,\alpha_m\sigma^2\right]\frac{r}{\alpha_c\sigma^2}.$$

The solution is

$$\hat{\gamma}_2(V_2^*) = \frac{\alpha_m}{2\alpha_c + \alpha_m}.$$

Similarly, if we substitute, the expressions for $\hat{\gamma}_1(V_1^*, V_2^*)$ and $\frac{\partial \hat{\gamma}_1}{\partial V_1}$ given in (2) and (13) in (9) we get

$$0 = r^2\hat{\gamma}_1 + r\left[\hat{\gamma}_1(V_1^*, V_2^*)\left(\frac{r\alpha_c}{(1+r)}\right)\sigma^2 - (1-\hat{\gamma}_1(V_1^*, V_2^*))\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{r}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}$$

The solution is

$$\hat{\gamma}_1(V_1^*, V_2^*) = \hat{\gamma}_2(V_2^*) = \frac{\alpha_m}{2\alpha_c + \alpha_m}. \ \|$$

Remark 5. *It is natural to interpret the term*

$$\mu + V_2 - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2$$

as the first period "marginal cost" borne by the monopolist when he increases $\hat{\gamma}_1$, the fractional share in the firm that he sells in the first period. Note that because

$$\frac{\partial\left[\mu + V_2 - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]}{\partial\hat{\gamma}_1} = \left(\frac{r\alpha_m}{1+r}\right)\sigma^2 > 0$$

and

$$\frac{\partial\left[\mu - (1-\hat{\gamma}_2)\,\alpha_m\sigma^2\right]}{\partial\hat{\gamma}_2} = \alpha_m\sigma^2 > 0$$

the first and second period "marginal costs" are increasing in $\hat{\gamma}_1$ and $\hat{\gamma}_2$ respectively.

Remark 6. *Motivated by the interpretation in the preceding remark, we can rewrite the first order condition (9) as*

$$r\hat{\gamma}_1\left(\frac{1}{\frac{\partial\hat{\gamma}_1}{\partial V_1}}\right) + rV_1 = \left(\mu + V_2 - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right). \tag{14}$$

Since the left side of (14) is the marginal revenue of an increase in $\hat{\gamma}_1$, this equation simply asserts that marginal revenue equals marginal cost. Alternatively we can rewrite (14) as

$$V_1 = \frac{\left(\mu + V_2 - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right)}{\left[1 - \frac{1}{\varepsilon_1}\right] r}$$

where

$$\begin{aligned}\varepsilon_1 \quad &\equiv \quad -\frac{V_1 \frac{\partial \hat{\gamma}_1}{\partial V_1}}{\hat{\gamma}_1} = \frac{V_1 \frac{r}{\left[\frac{r}{(1+r)}\right]\alpha_c \sigma^2}}{\frac{\mu + V_2 - rV_1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}} \\ &= \quad \frac{rV_1}{\mu + V_2 - rV_1}\end{aligned}$$

is the elasticity of demand. Thus, the "Lerner index" measuring the fractional markup of price over marginal cost is

$$\begin{aligned}\frac{V_1 - \frac{\left(\mu + V_2 - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right)}{r}}{V_1} \quad &= \quad \left[\frac{1}{\left[1 - \frac{1}{\varepsilon_1}\right]} - 1\right]\left[1 - \frac{1}{\varepsilon_1}\right] \\ &= \quad \frac{1}{\varepsilon_1}.\end{aligned}$$

In equilibrium,

$$\begin{aligned}\varepsilon_1 \quad &= \quad \frac{\mu + V_2^* - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c + \alpha_m}}{\frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c+\alpha_m}} \\ &= \quad \frac{\mu + \frac{\mu - \frac{\alpha_m\alpha_c\sigma^2}{2\alpha_c+\alpha_m}}{r}}{\left[\frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c+\alpha_m}\right]} - 1\end{aligned}$$

The Lerner index will be less than one in equilibrium only if the elasticity ε_1 exceeds one, i.e., only if

$$\frac{\mu}{\sigma^2} > \left[\frac{1}{1+r} + 2\left(\frac{r}{1+r}\right)^2\right]\left[\frac{1}{\frac{2}{\alpha_m} + \frac{1}{\alpha_c}}\right].$$

In that case, the marginal revenue and marginal cost in (15) will be positive. The parameter restriction required to get a positive first period price is less stringent only requiring that

$$V_2^* = \frac{\mu + \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r} - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c+\alpha_m}}{r} > 0$$

which is equivalent to

$$\frac{\mu}{\sigma^2} > \left[\frac{1}{1+r} + \left(\frac{r}{1+r}\right)^2\right]\left[\frac{1}{\frac{2}{\alpha_m} + \frac{1}{\alpha_c}}\right]$$

A parallel interpretation of (11) is obtained if we rewrite that equation as

$$r\hat{\gamma}_2\left(\frac{1}{\frac{\partial\hat{\gamma}_2}{\partial V_2}}\right) + rV_2 = \mu - (1-\hat{\gamma}_2)\,\alpha_m\sigma^2 \tag{15}$$

and interpret the term

$$\mu - (1-\hat{\gamma}_2)\,\alpha_m\sigma^2$$

as the second period "marginal cost" borne by the monopolist when he increases $\hat{\gamma}_2$*. Again we can rewrite (15) as*

$$V_2 = \frac{\left(\mu - (1-\hat{\gamma}_2)\,\alpha_m\sigma^2\right)}{\left[1 - \frac{1}{\varepsilon_2}\right] r}$$

where

$$\begin{aligned}\varepsilon_2 \quad &\equiv \quad -\frac{V_2\frac{\partial\hat{\gamma}_2}{\partial V_2}}{\hat{\gamma}_2} = \frac{V_2\frac{r}{\alpha_c\sigma^2}}{\frac{\mu - rV_2}{\alpha_c\sigma^2}} \\ &= \quad \frac{rV_2}{\mu - rV_2}\end{aligned}$$

is the elasticity of demand. In equilibrium,

$$\begin{aligned}\varepsilon_2 \quad &= \quad \frac{\mu - \frac{\alpha_m\alpha_c\sigma^2}{2\alpha_c+\alpha_m}}{\frac{\alpha_m\alpha_c\sigma^2}{2\alpha_c+\alpha_m}} \\ &= \quad \frac{\mu}{\left[\frac{\alpha_m\alpha_c\sigma^2}{2\alpha_c+\alpha_m}\right]} - 1.\end{aligned}$$

The Lerner index will be less than one in equilibrium only if the elasticity ε_2 *exceeds one, i.e., only if*

$$\frac{\mu}{\sigma^2} > 2\left[\frac{1}{\frac{2}{\alpha_m} + \frac{1}{\alpha_c}}\right] = \frac{1}{\frac{1}{\alpha_m} + \frac{1}{2\alpha_c}}.$$

In that case, the marginal revenue and marginal cost in (15) will be positive. Again the parameter restriction required to get a positive second period price is less stringent only requiring that

$$V_1^* = \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r} > 0$$

which is equivalent to

$$\frac{\mu}{\sigma^2} > \left[\frac{1}{\frac{2}{\alpha_m} + \frac{1}{\alpha_c}}\right].$$

Remark 7. *In order to prove that the function f is strictly concave we simply observe that*

$$\begin{aligned}
&f_{V_1V_1}(V_1, V_2) \\
&= 2r^2 \frac{\partial \hat{\gamma}_1}{\partial V_1} - \left(\frac{r^2 \alpha_m}{1+r}\right)\sigma^2 \left[\frac{\partial \hat{\gamma}_1}{\partial V_1}\right]^2 \\
&= -2r^2 \frac{r}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2} - \left(\frac{r^2\alpha_m}{1+r}\right)\sigma^2 \left[\frac{r}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right]^2 \\
&= -\frac{r^3}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\left(\frac{2\alpha_c + \alpha_m}{\alpha_c}\right) < 0,
\end{aligned}$$

$$\begin{aligned}
f_{V_2V_2}(V_1, V_2) &= r\left(\frac{\partial \hat{\gamma}_2}{\partial V_2} - \frac{\partial \hat{\gamma}_1}{\partial V_2}\right) - r\frac{\partial \hat{\gamma}_1}{\partial V_2} - r\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\left[\frac{\partial \hat{\gamma}_1}{\partial V_2}\right]^2 \\
&\quad + r\frac{\partial \hat{\gamma}_2}{\partial V_2} - \alpha_m\sigma^2\left[\frac{\partial \hat{\gamma}_2}{\partial V_2}\right]^2 \\
&= -2r\left[\frac{1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right] - r\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\left[\frac{1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right]^2 \\
&\quad -2r\left[\frac{r}{\alpha_c\sigma^2}\right] - \alpha_m\sigma^2\left[\frac{r}{\alpha_c\sigma^2}\right]^2 \\
&= -\left[\frac{r}{\alpha_c\sigma^2}\right]\left(\frac{2\alpha_c + \alpha_m}{\alpha_c}\right)\left[\frac{(1+r)}{r} + r\right] < 0
\end{aligned}$$

and

$$\begin{aligned}
&f_{V_1V_1}(V_1, V_2)\, f_{V_2V_2}(V_1, V_2) - \left[f_{V_1V_2}(V_1, V_2)\right]^2 \\
&= \left[\frac{r^3}{\left(\frac{r}{1+r}\right)\alpha_c\sigma^2}\left(\frac{2\alpha_c + \alpha_m}{\alpha_c}\right)\right]\left[\left(\frac{r}{\alpha_c\sigma^2}\right)\left(\frac{2\alpha_c + \alpha_m}{\alpha_c}\right)\left(\frac{(1+r)}{r} + r\right)\right] \\
&\quad - \left[\frac{r^2}{\left(\frac{r}{(1+r)}\right)\alpha_c\sigma^2}\left(\frac{2\alpha_c + \alpha_m}{\alpha_c}\right)\right]^2 \\
&= \left(\frac{2\alpha_c + \alpha_m}{\alpha_c}\right)^2\left(\frac{r^2}{\alpha_c\sigma^2}\right)^2\left(\left[\frac{(1+r)}{r}\right]\left[\frac{(1+r)}{r} + r\right] - \left[\frac{(1+r)}{r}\right]^2\right) > 0,
\end{aligned}$$

where

$$\begin{aligned}
& f_{V_1V_2}(V_1,V_2) \\
= \; & r^2\frac{\partial\hat{\gamma}_1}{\partial V_2} - r\frac{\partial\hat{\gamma}_1}{\partial V_1} - r^2\left(\frac{\alpha_m}{1+r}\right)\sigma^2\left[\frac{\partial\hat{\gamma}_1}{\partial V_2}\right]\left[\frac{\partial\hat{\gamma}_1}{\partial V_1}\right] \\
= \; & \frac{2r^2}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2} + r^2\left(\frac{\alpha_m}{1+r}\right)\sigma^2\left[\frac{1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right]\left[\frac{r}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right] \\
= \; & \frac{r^2}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\left(\frac{2\alpha_c+\alpha_m}{\alpha_c}\right).
\end{aligned}$$

Comparison with the Competitive Case. In the competitive case, price equals marginal cost in both periods so that

$$rV_1 = \left(\mu + V_2 - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right) \tag{16}$$

and

$$rV_2 = \mu - (1-\hat{\gamma}_2)\,\alpha_m\sigma^2 \tag{17}$$

Combining (17) with (1) and solving for the competitive price V_2^e we obtain

$$V_2^e = \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(\alpha_c+\alpha_m)}}{r} < \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r} = V_2^*.$$

Similarly, combining (16) with (2) and solving for the competitive price V_1^e we obtain

$$\begin{aligned}
V_1^e \; &= \; \frac{\mu + V_2^e - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{\alpha_c+\alpha_m}}{r} \\
&= \; \frac{\mu + \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(\alpha_c+\alpha_m)}}{r} - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{\alpha_c+\alpha_m}}{r} \\
&< \; \frac{\mu + \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r} - \frac{\left(\frac{r\alpha_m\alpha_c}{1+r}\right)\sigma^2}{2\alpha_c+\alpha_m}}{r} = V_1^*.
\end{aligned}$$

When we compute

$$\gamma_1^e = \hat{\gamma}_1(V_1^e, V_2^e) = \frac{\mu + V_2^e - rV_1^e}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}$$

and

$$\gamma_2^e = \hat{\gamma}_2(V_2^e) = \frac{\mu - rV_2^e}{\alpha_c\sigma^2},$$

the result is

$$\gamma_1^e = \gamma_2^e = \frac{\alpha_m}{\alpha_c + \alpha_m} = \frac{\frac{1}{\alpha_m}}{\frac{1}{\alpha_m} + \frac{1}{\alpha_m}} > \frac{\frac{1}{\alpha_m}}{\frac{2}{\alpha_m} + \frac{1}{\alpha_m}} = \frac{\alpha_m}{2\alpha_c + \alpha_m}.$$

Thus, as is typical with monopoly, supply is restricted so as to raise the price.

4.3. The Case Without Commitment.

The Monopolist's Second Period Problem:.

The Optimal V_2 and $\hat{\gamma}_2(V_2)$. In this case, V_2 is chosen in Period 2 when γ_1 and D_1 have been determined. In this case, the monopolist chooses V_2 to maximize (4) where

$$W_2^m = rW_1^m + r\gamma_1 V_1 + (1-\gamma_1)(D_1 + V_2). \tag{18}$$

Substituting (18) in (4) the monopolist's maximand becomes

$$\begin{aligned} & -\left(\frac{1+r}{r}\right)\left(e^{-\frac{r\alpha_m}{1+r}W_2^m}\right)\left(r\delta e^{-\frac{\alpha_m}{(1+r)}(1-\hat{\gamma}_2(V_2))(\mu - rV_2) + \frac{(1-\hat{\gamma}_2(V_2))^2}{2(1+r)}\alpha_m^2\sigma^2}\right) \\ = \; & -\left(\frac{1+r}{r}\right)\left(e^{-\frac{r^2\alpha_m}{1+r}W_1^m}\right)\left(r\delta e^{-\frac{r\alpha_m}{1+r}(r\gamma_1 V_1 + (1-\gamma_1)(D_1+V_2)) - \frac{\alpha_m}{(1+r)}(1-\hat{\gamma}_2(V_2))(\mu - rV_2) + \frac{(1-\hat{\gamma}_2(V_2))^2}{2(1+r)}\alpha_m^2\sigma^2}\right). \end{aligned}$$

The maximizing choice is easily seen to equal

$$\bar{V}_2(\gamma_1) \equiv \underset{V_2}{argmax}\, \pi(\gamma_1, V_2),$$

where

$$\pi(\gamma_1, V_2) \equiv r(\hat{\gamma}_2(V_2) - \gamma_1)V_2 + (1 - \hat{\gamma}_2(V_2))\mu - \frac{(1-\hat{\gamma}_2(V_2))^2}{2}\alpha_m\sigma^2.$$

Proposition 2. *When the monopolist cannot commit to a second period price in the first trading period the second period price he charges is*

$$\bar{V}_2(\gamma_1) = \frac{\mu - \frac{(\alpha_c\gamma_1 + \alpha_m)\alpha_c\sigma^2}{(2\alpha_c + \alpha_m)}}{r}. \tag{19}$$

When the monopolist charges $\bar{V}_2(\gamma_1)$ in period 2, the fractional share of the risky firm held by the competitor after trading in period 2 is

$$\hat{\gamma}_2\left(\bar{V}_2(\gamma_1)\right) = \frac{\alpha_c\gamma_1 + \alpha_m}{(2\alpha_c + \alpha_m)}.$$

Note that, when $\gamma_1 > 0$*,*

$$\hat{\gamma}_2\left(\bar{V}_2(\gamma_1)\right) > \frac{\alpha_m}{(2\alpha_c+\alpha_m)} = \hat{\gamma}_2(V_2^*)$$

and

$$V_2^* = \frac{\mu - \frac{\alpha_c\alpha_m\sigma^2}{(2\alpha_c+\alpha_m)}}{r} > \frac{\mu - \frac{(\alpha_c\gamma_1+\alpha_m)\alpha_c\sigma^2}{(2\alpha_c+\alpha_m)}}{r} = \bar{V}_2(\gamma_1).$$

Thus, if the monopolist makes any sales in the first period, the fractional share of the risky firm held by the competitor after trading in period 2 is larger when the monopolist cannot commit to V_2 *in period* 1 *than when he can commit. Also if the monopolist makes sales in the first period, then the price he charges in the second period when he can't commit in the first period is lower than the second period price he would charge if he could commit.*

Proof: The proof that π is a strictly concave function of V_2 is given in Remark 8 that follows the proof of this proposition. The first order condition satisfied at $\bar{V}_2(\gamma_1)$ is

$$0 = \pi_{V_2}(\gamma_1, V_2) = r(\hat{\gamma}_2 - \gamma_1) - \left[\mu - rV_2 - (1-\hat{\gamma}_2)\alpha_m\sigma^2\right]\frac{\partial\hat{\gamma}_2}{\partial V_2}. \tag{20}$$

Substituting the expression for $\hat{\gamma}_2(V_2)$ given in (1) and the expression for $\frac{\partial\hat{\gamma}_2}{\partial V_2}$ given in (12) into (20), we can solve for V_2 to get

$$\bar{V}_2(\gamma_1) = \frac{\mu - \frac{(\alpha_c\gamma_1+\alpha_m)\alpha_c\sigma^2}{(2\alpha_c+\alpha_m)}}{r}.$$

In addition, we can substitute the expression for $\frac{\partial\hat{\gamma}_2}{\partial V_2}$ given in (12) into (20) and then use (1) to replace

$$\mu - rV_2$$

by

$$\hat{\gamma}_2(V_2)\,\alpha_c\sigma^2.$$

The result is an equation involving $\hat{\gamma}_2(V_2)$ that we can solve to get

$$\hat{\gamma}_2\left(\bar{V}_2(\gamma_1)\right) = \frac{\alpha_c\gamma_1+\alpha_m}{(2\alpha_c+\alpha_m)} > \frac{\alpha_m}{(2\alpha_c+\alpha_m)}. \,\| \tag{21}$$

Remark 8. *In order to demonstrate that* π *is a strictly concave function of* V_2 *we simply note that*

$$\begin{aligned}\pi_{V_2,V_2}(\gamma_1, V_2) &= 2r\frac{\partial\hat{\gamma}_2}{\partial V_2} - \alpha_m\sigma^2\left[\frac{\partial\hat{\gamma}_2}{\partial V_2}\right]^2 \\ &= -\frac{2r^2}{\alpha_c\sigma^2} - \alpha_m\sigma^2\left[\frac{r}{\alpha_c\sigma^2}\right]^2 < 0.\end{aligned}$$

Remark 9. *When commitment is possible in period one, and the monopolist chooses V_1 and V_2 so that*

$$\begin{aligned} 0 &= f_{V_2}(V_1, V_2) \\ &= r(\hat{\gamma}_2 - \hat{\gamma}_1) - r\left[(\mu + V_2 - rV_1) - (1 - \hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{\partial \hat{\gamma}_1}{\partial V_2} \\ &\quad - \left[(\mu - rV_2) - (1 - \hat{\gamma}_2)\alpha_m \sigma^2\right]\frac{\partial \hat{\gamma}_2}{\partial V_2}, \end{aligned}$$

he accounts for the effect of V_2 on his first period demand and profits. When, however, the monopolist chooses V_2 in the second period, he sets

$$0 = \pi_{V_2}(\gamma_1, V_2) = r(\hat{\gamma}_2 - \gamma_1) - \left[\mu - rV_2 - (1 - \hat{\gamma}_2)\alpha_m \sigma^2\right]\frac{\partial \hat{\gamma}_2}{\partial V_2}.$$

He ignores the term

$$-r\left[(\mu + V_2 - rV_1) - (1 - \hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{\partial \hat{\gamma}_1}{\partial V_2}$$

that measures the effect of V_2 on the monopolist's first period demand and profits. As a consequence, when the monopolist chooses V_2 in the second period, the condition

$$r\hat{\gamma}_2 \frac{1}{\frac{\partial \hat{\gamma}_2}{\partial V_2}} + rV_2 = \left[\mu - (1 - \hat{\gamma}_2)\alpha_m \sigma^2\right] \tag{22}$$

that we observed to hold in the commitment case (see Remark 6) is replaced by the condition

$$r(\hat{\gamma}_2 - \gamma_1)\frac{1}{\frac{\partial \hat{\gamma}_2}{\partial V_2}} + rV_2 = \left[\mu - (1 - \hat{\gamma}_2)\alpha_m \sigma^2\right].$$

Thus, in the noncommitment case, the difference between the price, V_2, and the marginal revenue is only $(\hat{\gamma}_2 - \gamma_1)\left|\frac{1}{\frac{\partial \hat{\gamma}_2}{\partial V_2}}\right|$ rather than $\hat{\gamma}_2\left|\frac{1}{\frac{\partial \hat{\gamma}_2}{\partial V_2}}\right|$. When the monopolist chooses V_2 in the second period he overstates the marginal revenue by the amount $-\gamma_1 \frac{1}{\frac{\partial \hat{\gamma}_2}{\partial V_2}}$ because he ignores the effect of V_2 on the demand in period one.

Comparing $\hat{\gamma}_2\left(\bar{V}_2(\gamma_1)\right)$ and γ_1.

Proposition 3. *If γ_1, the fractional share in the firm sold by the monopolist in the first period, is less than the competitive level*

$$\hat{\gamma}_1^e = \frac{\alpha_m}{(\alpha_m + \alpha_c)},$$

then the monopolist will make additional sales in the second trading period; i.e.,

$$\hat{\gamma}_2\left(\bar{V}_2\left(\gamma_1\right)\right) > \gamma_1.$$

In this case, we will still have

$$\hat{\gamma}_2\left(\bar{V}_2\left(\gamma_1\right)\right) < \hat{\gamma}_1^e$$

and we will also have

$$\bar{V}_2\left(\gamma_1\right) = \frac{\mu - \frac{(\alpha_c\gamma_1+\alpha_m)\alpha_c\sigma^2}{(2\alpha_c+\alpha_m)}}{r} > \frac{\mu - \frac{\alpha_m\alpha_c\sigma^2}{(\alpha_c+\alpha_m)}}{r} = V_2^e.$$

Proof: Note that, in general,

$$\begin{aligned}
\hat{\gamma}_2\left(\bar{V}_2\left(\gamma_1\right)\right) - \gamma_1 &= \frac{\alpha_c\gamma_1 + \alpha_m}{(2\alpha_c + \alpha_m)} - \gamma_1\frac{(2\alpha_c + \alpha_m)}{(2\alpha_c + \alpha_m)} \\
&= \frac{\alpha_m - \left(\alpha_m + \alpha_c\right)\gamma_1}{(2\alpha_c + \alpha_m)}.
\end{aligned}$$

so $\hat{\gamma}_2\left(\bar{V}_2\left(\gamma_1\right)\right)$ will exceed γ_1 whenever

$$\alpha_m > \left(\alpha_m + \alpha_c\right)\gamma_1$$

which is equivalent to

$$\gamma_1 < \frac{\alpha_m}{(\alpha_m + \alpha_c)} = \hat{\gamma}_1^e.$$

Note also that

$$\begin{aligned}
\hat{\gamma}_2\left(\bar{V}_2\left(\gamma_1\right)\right) &= \frac{\alpha_c\gamma_1 + \alpha_m}{(2\alpha_c + \alpha_m)} \\
&< \frac{\alpha_c\frac{\alpha_m}{(\alpha_m+\alpha_c)} + \alpha_m}{(2\alpha_c + \alpha_m)} \\
&= \frac{\left(\frac{\alpha_c}{(\alpha_m+\alpha_c)} + 1\right)\alpha_m}{(2\alpha_c + \alpha_m)} \\
&= \frac{\left(\frac{(2\alpha_c+\alpha_m)}{(\alpha_m+\alpha_c)}\right)\alpha_m}{(2\alpha_c + \alpha_m)} \\
&= \frac{\alpha_m}{(\alpha_m + \alpha_c)} = \hat{\gamma}_1^e
\end{aligned}$$

when

$$\gamma_1 < \frac{\alpha_m}{(\alpha_m + \alpha_c)} = \hat{\gamma}_1^e.$$

Finally, since

$$\hat{\gamma}_2\left(\bar{V}_2(\gamma_1)\right) < \hat{\gamma}_1^e$$

when

$$\hat{\gamma}_1 < \hat{\gamma}_1^e$$

(1) implies that we also have

$$\bar{V}_2(\gamma_1) = \frac{\mu - \frac{(\alpha_c\gamma_1+\alpha_m)\alpha_c\sigma^2}{(2\alpha_c+\alpha_m)}}{r} > \frac{\mu - \frac{\alpha_m\alpha_c\sigma^2}{(\alpha_c+\alpha_m)}}{r} = V_2^e.$$

The Monopolist's Second Period Reaction Function $V_2(V_1)$. Let's define $V_2(V_1)$ as the solution to the equation

$$V_2(V_1) = \bar{V}_2(\hat{\gamma}_1(V_1, V_2(V_1))). \tag{23}$$

Remark 10. *Note that $V_2(V_1)$ has been constructed so that when the monopolist charges V_1 in the first period and the competitive trader expects him to charge $V_2(V_1)$ in the second period the competitive trader's first period demand for a fractional share in the risky firm is indeed given by $\hat{\gamma}_1(V_1, V_2(V_1))$ and the monopolist responds by charging $\bar{V}_2(\hat{\gamma}_1(V_1, V_2(V_1)))$ in the second period. Since $V_2(V_1)$ is a solution to (23), the second period price charged by the monopolist is the one expected by the competitive trader.*

Proposition 4. *When the monopolist charges V_1 in the first period he reacts by charging*

$$V_2(V_1) = a + bV_1. \tag{24}$$

in the second, where

$$a = \frac{\mu\left[1 - \frac{(1+r)\alpha_c}{r(2\alpha_c+\alpha_m)}\right] - \frac{\alpha_m\alpha_c\sigma^2}{(2\alpha_c+\alpha_m)}}{r\left[1 + \frac{(1+r)\alpha_c}{r^2(2\alpha_c+\alpha_m)}\right]}$$

and

$$b = \frac{r}{\left[\frac{r^2(2\alpha_c+\alpha_m)}{(1+r)\alpha_c} + 1\right]} < r.$$

.

Proof: When we substitute

$$\hat{\gamma}_1(V_1, V_2(V_1)) = \left[\frac{\mu + V_2(V_1) - rV_1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right] \tag{25}$$

in the expression for $\bar{V}_2(\gamma_1)$ given in (19) we get

$$\begin{aligned} V_2(V_1) &= \bar{V}_2(\hat{\gamma}_1(V_1, V_2(V_1))) \\ &= \frac{\mu - \frac{\alpha_m \alpha_c \sigma^2}{(2\alpha_c+\alpha_m)} - \frac{\alpha_c^2\sigma^2}{(2\alpha_c+\alpha_m)}\left(\frac{\mu+V_2(V_1)-rV_1}{\left[\frac{r}{(1+r)}\right]\alpha_c\sigma^2}\right)}{r} \\ &= \frac{\mu - \frac{\alpha_m \alpha_c \sigma^2}{(2\alpha_c+\alpha_m)} - \frac{(1+r)\alpha_c}{r(2\alpha_c+\alpha_m)}(\mu - rV_1)}{r} - \frac{(1+r)\alpha_c}{r^2(2\alpha_c+\alpha_m)}V_2(V_1) \end{aligned}$$

which we can solve to get

$$V_2(V_1) = \frac{\mu - \frac{\alpha_m \alpha_c \sigma^2}{(2\alpha_c+\alpha_m)} - \frac{(1+r)\alpha_c}{r(2\alpha_c+\alpha_m)}(\mu - rV_1)}{r\left[1 + \frac{(1+r)\alpha_c}{r^2(2\alpha_c+\alpha_m)}\right]}$$

from which (24) follows immediately.‖

The Monopolist's First Period Problem.

Choosing V_1. Now the monopolist chooses V_1 to maximize

$$g(V_1) \equiv f(V_1, V_2(V_1))$$

where $V_2(V_1)$ is given by (24). We let

$$V_1^{nc} \equiv \arg\max_{V_1} g(V_1)$$

and

$$\gamma_1^{nc} \equiv \hat{\gamma}_1(V_1^{nc}, V_2(V_1^{nc}))$$

Proposition 5. *In the first period, the monopolist sells the fractional share*

$$\gamma_1^{nc} = \frac{\alpha_m}{\left(\left[1 + \frac{1}{1-\frac{b}{r}}\right]\alpha_c + \alpha_m\right)}, \tag{26}$$

where, it will be recalled that

$$\frac{b}{r} = \frac{V_2'(V_1)}{r} = \frac{1}{\left[\frac{r^2(2\alpha_c+\alpha_m)}{(1+r)\alpha_c} + 1\right]} < 1.$$

His first period price is

$$rV_1^{nc} = \mu + V_2(V_1^{nc}) - \frac{\left(\frac{r}{1+r}\right)\alpha_m\alpha_c\sigma^2}{\left(\left[1 + \frac{1}{1-\frac{b}{r}}\right]\alpha_c + \alpha_m\right)} \tag{27}$$

where

$$V_2\left(V_1^{nc}\right) = \bar{V}_2\left(\gamma_1^{nc}\right) = \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c+\alpha_m)} - \gamma_1^{nc} \frac{\alpha_c^2 \sigma^2}{(2\alpha_c+\alpha_m)}}{r} \tag{28}$$

$$= \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c+\alpha_m)} - \frac{\alpha_m \alpha_c^2 \sigma^2}{\left(\left[1+\frac{1}{1-\frac{b}{r}}\right]\alpha_c+\alpha_m\right)(2\alpha_c+\alpha_m)}}{r}.$$

The monopolist's first period price can also be written as

$$rV_1^{nc} = \mu + \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c+\alpha_m)}}{r} - \left(\frac{r}{1+r}\right) \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)} \xi \tag{29}$$

where

$$\xi \equiv \left[\frac{r\left(2\alpha_c + \alpha_m\right) + \frac{(1+r)}{r}\alpha_c}{r\left(2\alpha_c + \alpha_m\right) + \left(\frac{\alpha_c}{2\alpha_c+\alpha_m}\right)\left(\frac{1+r}{r}\right)\alpha_c}\right] > 1.$$

Since

$$\gamma_1^{nc} < \frac{\alpha_m}{(2\alpha_c + \alpha_m)} = \hat{\gamma}_1\left(V_1^*, V_2^*\right),$$

the monopolist restricts his sales even more when he can't commit than when he can. In spite of the additional restriction in sales, the monopolist still charges a lower price in the first period than he would if he could commit since

$$rV_1^{nc} < rV_1^* = \mu + \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c+\alpha_m)}}{r} - \left(\frac{r}{1+r}\right) \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)}.$$

Remark 11. *The fact that*

$$\gamma_1^{nc} < \hat{\gamma}_1\left(V_1^*, V_2^*\right)$$

implies that when he cannot commit to a second period price, the monopolist sells less than he would if the market were competitive since

$$\gamma_1^e > \hat{\gamma}_1\left(V_1^*, V_2^*\right).$$

This means that the results of Propositions 3 apply. The monopolist does make additional sales in the second period but he still sells less than the competitive amount; i.e.,

$$\gamma_1^e > \hat{\gamma}_2\left(\bar{V}_2\left(\hat{\gamma}_1^{nc}\right)\right) > \hat{\gamma}_1^{nc}.$$

Since $\hat{\gamma}_1^{nc} > 0$, the Propositions 3 also implies that the monopolist sells more in the second period than he would if he could commit to a second period price in the first period; i.e.,

$$\hat{\gamma}_2\left(\bar{V}_2\left(\gamma_1^{nc}\right)\right) = \frac{\alpha_c \gamma_1^{nc} + \alpha_m}{(2\alpha_c + \alpha_m)} > \frac{\alpha_m}{(2\alpha_c + \alpha_m)} = \hat{\gamma}_2\left(V_2^*\right).$$

In addition, Proposition 3 implies that the second price charged by the monopolist when he cannot commit is less than the one he would commit to if could credibly do so; i.e.,

$$V_2^* = \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)}}{r} > \frac{\mu - \frac{\left(\alpha_c \gamma_1^{nc} + \alpha_m\right)\alpha_c \sigma^2}{(2\alpha_c + \alpha_m)}}{r} = \bar{V}_2\left(\gamma_1^{nc}\right).$$

Finally, Proposition 3 implies that we will also have a second period price above the competitive second period price; i.e.,

$$\bar{V}_2\left(\gamma_1^{nc}\right) = \frac{\mu - \frac{\left(\alpha_c \gamma_1^{nc} + \alpha_m\right)\alpha_c \sigma^2}{(2\alpha_c + \alpha_m)}}{r} > \frac{\mu - \frac{\alpha_m \alpha_c \sigma^2}{(\alpha_c + \alpha_m)}}{r} = V_2^e.$$

Proof: First note that since f is strictly concave and $V_2(V_1)$ is linear, $g(V_1) = f(V_1, V_2(V_1))$ is strictly concave.

The first order condition is

$$g'(V_1) = 0. \tag{30}$$

Since

$$\begin{aligned} g'(V_1) &= f_{V_1}(V_1, V_2(V_1)) + f_{V_2}(V_1, V_2(V_1))\, V_2'(V_1) \\ &= f_{V_1}(V_1, V_2(V_1)) \\ &\quad + \left(-r\left[(\mu + V_2 - rV_1) - (1-\hat{\gamma}_1)\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{\partial \hat{\gamma}_1}{\partial V_2} + \pi_{V_2}(\hat{\gamma}_1, V_2(V_1))\right) V_2'(V_1), \end{aligned}$$

and

$$\pi_{V_2}(\hat{\gamma}_1, V_2(V_1)) = 0,$$

$$\begin{aligned} g'(V_1) &= r^2 \hat{\gamma}_1(V_1, V_2(V_1)) \\ &\quad - r\left[(\mu + V_2(V_1) - rV_1) - (1 - \hat{\gamma}_1(V_1, V_2(V_1)))\left(\frac{r\alpha_m}{1+r}\right)\sigma^2\right]\frac{d\hat{\gamma}_1(V_1, V_2(V_1))}{dV_1}, \end{aligned} \tag{31}$$

where

$$\frac{d\hat{\gamma}_1(V_1, V_2(V_1))}{dV_1} = \frac{\partial \hat{\gamma}_1(V_1, V_2(V_1))}{\partial V_1} + \frac{\partial \hat{\gamma}_1(V_1, V_2(V_1))}{\partial V_2} V_2'(V_1) = \frac{b - r}{\left[\frac{r}{(1+r)}\right]\alpha_c \sigma^2}. \tag{32}$$

Combining the expression for $\hat{\gamma}_1(V_1, V_2(V_1))$ in (2) with (31) and (32) we observe that (30) reduces to

$$0 = (\mu + V_2(V_1) - rV_1)\left(\left[1 + \frac{1}{1 - \frac{b}{r}}\right]\alpha_c + \alpha_m\right) - \left(\frac{r}{1+r}\right)\alpha_m \alpha_c \sigma^2. \tag{33}$$

First note that we can use the expression for $\hat{\gamma}_1 (V_1, V_2 (V_1))$ in (2) to replace

$$(\mu + V_2 (V_1) - rV_1)$$

by

$$\hat{\gamma}_1 (V_1, V_2 (V_1)) \left[\frac{r}{(1+r)} \right] \alpha_c \sigma^2$$

in (30) to get the expression

$$0 = \hat{\gamma}_1 (V_1, V_2 (V_1)) \alpha_c \sigma^2 \left(\left[1 + \frac{1}{1 - \frac{b}{r}} \right] \alpha_c + \alpha_m \right) - \left(\frac{r}{1+r} \right) \alpha_m \alpha_c \sigma^2 .$$

This expression can easily be solved for γ_1^{nc}. The solution is (26). Now we can solve (33) to get (27) and recall that

$$V_2 (V_1^{nc}) = \bar{V}_2 (\hat{\gamma}_1 (V_1^{nc}, V_2 (V_1^{nc}))) = \bar{V}_2 (\gamma_1^{nc})$$

implies (28). When we substitute (28) in (27) and simplify the resulting expression we get

$$\begin{aligned} rV_1^{nc} &= \mu + \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)}}{r} - \left(\frac{r}{1+r} \right) \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)} \left[\frac{r (2\alpha_c + \alpha_m) + \frac{\alpha_c}{\left(\frac{r}{(1+r)} \right)}}{r \left(\alpha_c \left[1 + \frac{1}{1 - \frac{b}{r}} \right] + \alpha_m \right)} \right] \\ &= \mu + \frac{\mu - \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)} - \left(\frac{r}{1+r} \right) \frac{\alpha_c \alpha_m \sigma^2}{(2\alpha_c + \alpha_m)} \left[\frac{r(2\alpha_c + \alpha_m) + \frac{\alpha_c}{\left(\frac{r}{(1+r)} \right)}}{r(2\alpha_c + \alpha_m) + \frac{\alpha_c}{\frac{1}{b} - \frac{1}{r}}} \right]}{r} . \end{aligned}$$

Now (29) follows from this expression and the fact that

$$\frac{1}{b} - \frac{1}{r} = \frac{r (2\alpha_c + \alpha_m)}{(1+r) \alpha_c} . \, \|$$

5. SUMMARY:

The example described here simply illustrates that the forces at work in a durable goods monopoly setting also operate to some extent in dynamic security markets of the type studied by Lucas. We began by considering the case in which a monopoly seller of firm shares can commit to a second period price when he sets his price in the first round of trading. The first result, stated as Proposition 1, demonstrates that, in that case, the monopolist will set the price in each period so that he effectively makes all his sales in the first round of trading. When he does this he sells a smaller fraction of the firm than he would if the market were competitive. by restricting supply he is able to charge first and second period prices that exceed the corresponding first and second period competitive prices.

When commitment is not possible, the monopolist does make additional sales in the second period. That is, he sells a larger fraction of the firm in the second period than he does in the first trading period. In fact, the amount the monopolist sells in the second period also exceeds the amount he would sell in the second period in the commitment case. He also charges a lower second period price than he would commit to charge if he could credibly make such a commitment. These results are proved in Propositions 2, 3 and 5. Because the competitive trader recognizes that he will be able to get a better price in the second trading period, the monopolist's first period monopoly power is eroded. The major result of the paper is Proposition 5 which demonstrates that this erosion in his first period monopoly power causes the monopolist to sell at a lower price than he would if he could commit to a second period price.

The model investigated here is obviously quite special and the analysis of this special case represents only a first step in the analysis of dynamic monopoly in securities markets. The natural extension requires an investigation of the case in which trading takes place frequently. If trading takes place infinitely often, then, in every trading period, the monopolist must anticipate intense competition from his own future sales. Also, with very frequent trading, future sales do become very good substitutes for present sales. For both of these reasons repeated and frequent trading should be expected, as Coase argued, to eliminate monopoly power effectively and force the monopolist to sell at the competitive price.

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