Corruption and Auctions*

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Abstract

We investigate the outcome of an auction where the auctioneer approaches one of the existing bidders and offers an opportunity for him to match his opponent's bid. In particular, the auctioneer approaches the winner to offer the possibility of a reduction in his bid to match the loser's bid in exchange for a bribe. The bribe may take any format including a fixed amount, a percentage of the gain (difference between the original bid and the new revised bid) or a combination of the two. We investigate how corruption affects bidding behavior, efficiency and the seller's expected revenue in a first-price auction.
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1 Introduction

Standard auction theory does not distinguish between the seller of the item being auctioned off and the auctioneer. However, it is often the case that

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these two agents are indeed two distinct individuals. Government procurement is typically organized by a bureaucrat on behalf of the government. Private firms' procurement decisions are made by firm officials belonging to the purchasing department and not by general managers. When selling a particular good, such as a house, car, painting, or bottle of wine, individual sellers usually do not conduct the auction themselves but instead turn to an auction house.

This distinction between the seller and the auctioneer creates the possibility of illicit behavior by the auctioneer as he might have incentives that differ from those of the seller. Whereas the seller wants the highest price for the object being sold, the auctioneer might want to illicitly solicit a bribe to somehow change the auction result.

Previous studies of illicit behavior in auctions have focused on collusion between bidders (bidding rings).[1] Laffont and Tirole (1991) examine the design of auctions to favor specific bidders, such as the case of government procurement favoring domestic suppliers. Graham and Marshall (1987) and Mailath and Zemsky(1991) show that in second-price sealed-bid and English oral auctions, when collusion among players is allowed, cooperative strategies are dominant. The optimal response of the auctioneer is to establish a reserve price that is a function of the coalition's size. Furthermore, they show that the revenue equivalence between second-price and English auctions holds.

McAfee and McMillan (1992) examine collusion among players in first-price auctions where they show that the price paid is the minimum price. Again, sellers can react by increasing the reserve price. While McAfee and McMillan cite evidence for collusion between bidders for government contracts where "it has been often the case that all bids are identical to the last cent," this could also be evidence of corruption involving the auctioneer. We claim that any evidence based on little dispersion amongst observed bids can also be consistent with the existence of the type of corruption we investigate.

In this paper we investigate the outcome of an auction where the auctioneer approaches the winner of the auction to offer a reduction in his bid to match the second highest bid in exchange for a bribe. Therefore, if corruption does occur, the two highest published bids will be identical. (In practice, the

[1]One exception is Beck and Maher (1986), who compare competitive bidding (in the form of a first price sealed bid auction) and a bribery competition, and find that the expected payoffs are the same under the two allocation mechanisms. In contrast, by allowing corruption between the auctioneer and one of the players, we show that seller's expected revenue and the expected bribe may depend on how the bribery market is organized.

observed winner's bid might be a few cents above the observed loser's bid).

Other potential applications include takeover and privatization auctions. For example, Wall street lore has it that a target of a takeover will sometimes run a sealed-bid auction, release the results to a favored bidder and then permit rebidding. Although there is no documented evidence of this phenomenon, target officer might do that in order to keep their jobs under new ownership or to receive favorable treatment if the takeover does occur.

Similarly, there are newspapers reports of a taped phone conversation between the government official in charge of selling the former state-owned Brazilian Telecomm company and one of the bidders. The government official, it is allegedly, was concerned that the highest standing bid was made by a consortium that did not have the appropriate technological expertise and urged this bidder to resubmit a higher bid. Note that even if there is no bribe payment to the auctioneer, the fact that a bidder might be given a chance to resubmit his bid will affect behavior.

Thus, in this paper we investigate how this type of corruption affects both bidding behavior and the seller's expected revenue in a first-price auction. We show that there exists a symmetric bidding strategy equilibrium that is monotone (higher valuation buyers bid higher). Incentive compatibility is then used to obtain first-order conditions for the candidate equilibrium. These conditions are presented for the different types of bribes. Standard economics arguments are then used to explain these conditions and to help us understand the predicted effects of corruption.

2 An Example

In this section we provide a simple example to illustrate some of the implications of having a corrupt auctioneer. The next sections contain a more general and detailed analysis.

Consider for the moment two risk-neutral bidders, 1 and 2, whose valuations are independent draws from the uniform [0,1] distribution. Each individual's valuation is private information. The auctioneer approaches the individual with the highest bid and allows him to reduce his bid to match the second highest bid (i.e., the lowest of the two bids) in exchange for a bribe. In the next section we will analyze different types of bribe payments, but here we assume that the bribe is a proportion (ρ) of the difference between the highest and the lowest bid. We can write Bidder 1's expected profit as a function of his private value v, his choice of bid x, assuming that Bidder 2 follows some bidding strategy $b(y)$, as follows:

$$\pi(v, x, b(y)) = E\left[(v - b(y) - \rho(x - b(y))\, I_{x>b(y)}\right]. \tag{1}$$

Where $I_{x>b(y)}$ denotes the indicator function of the set $\{y; b(y) < x\}$, i.e. it is one if $b\,(y) < x$ and zero otherwise. Bidder 1 can win only if he is the highest bidder and if he does win he will pay Bidder 2's bid plus the bribe, as $b(y)+\rho(x-b(y)) \leq x$. We can take the expected value of (1) by integrating over the appropriate interval, which yields:

$$\pi(v, x, b(y)) = \int_{0}^{b^{-1}(x)} (v - b(y) - \rho(x - b(y))\, dy.$$

Bidder 1 will then choose $x \geq 0$ to maximize his expected profits. Differentiating the above expression with respect to x and using that at the optimum $x = b(v)$, as we are looking for a symmetric equilibrium, yields the following differential equation:

$$\frac{v - b(v)}{b'(v)} - \rho v = 0$$

One can check directly that the solution to this differential equation – our symmetric equilibrium bidding strategy, which is increasing in v, is given by

$$b(v) = \frac{v}{1 + \rho}.$$

To help develop the intuition, consider some polar cases. For $\rho = 1$, the winner's total payment is equal to his bid as

$$b(y) + (b(v) - b(y)) = b(v)$$

where y denotes the loser's valuation. This is identical to the winner's total payment in a standard first-price sealed-bid auction. Accordingly, $b(v) = v/2$ in this case, where $v/2$ is the expected valuation of the other bidder conditional on v being the highest value. Note that the winner's total payment is split into a payment to the seller that is equal to the loser's bid and a payment to the auctioneer that is equal to the difference between the two bids.

For $\rho = 0$, the winner's total payment is equal to the second-highest bid, as in a standard second-price sealed-bid auction. Accordingly, $b(v) = v$ in this case, the winner pays the seller the lowest value and the auctioneer receives no bribe.

For $0 < \rho < 1$, the winner's total payment is equal to

$$b(y) + \rho(b(v) - b(y)) < b(v).$$

The intuition is that conditional on having the highest value, a bidder bids in such a way to outbid his opponent, just as he does in a standard first-price auction, but now winning is more valuable as it results in a lower total payment to the auctioneer. Therefore, in equilibrium, a bidder bids more than what would in the absence of corruption. In the next section we will show how to use the revenue equivalence theorem in some instances to argue that the winner's expected payment is actually fixed across a variety of bribery arrangements, as long as there is an increasing equilibrium bidding strategy.

For this example, it is also possible to compute the seller's expected revenue and the expected bribe as a function of ρ. The seller's expected revenue (ER) is simply the expected value of the second highest bid:

$$ER = \frac{1}{1+\rho} \cdot \frac{1}{3}$$

where $\frac{1}{3}$ is the expected value of the lowest valuation. The expected bribe (EB) is simply ρ times the difference between the highest and second-highest bids:

$$EB = \frac{\rho}{1+\rho} \cdot \frac{1}{3}$$

where $\frac{1}{3}$ is the difference between the highest and the second-highest expected values. Notice that ER and EB add up to $1/3$, which is the expected total payment of the winner in any standard auction under no corruption with two bidders with values drawn independently from the uniform [0,1] distribution.

In this environment, the first-price sealed-bid auction is still efficient as the object is allocated to the individual with the highest valuation. The winner's total payment, however, is now split between the seller and the auctioneer.

3 The Model

There are n risk-neutral bidders who are competing for an object to be auctioned off. According to the independent private values assumption, Bidder $i \in I$ knows his own value (v_i) for the object but only knows the distribution $F(v_j)$, $\forall j \neq i$, of the other bidder's value. It is assumed that values are independently drawn from the distribution F. We assume that F has a continuous density $f(\cdot)$ strictly positive on its support $[0, 1]$.

The auctioneer approaches the winner and offers him the possibility to reduce his bid to the second highest bid in exchange for a bribe. If this bidder agrees to pay the bribe, then he wins the object paying what is effectively the second highest bid (plus the bribe to the auctioneer). If this player does not agree to pay the bribe, then he still wins the object and pays accordingly to the auction rules — that is, his bid in a first price auction. The bribery arrangement is common knowledge.

Our formulation in the next section considers different possible types of bribe payments, such as fixed bribe B or a function of the difference between the highest bid and the second highest bid. A fixed bribe B, for example, might have been determined by a social convention or norm. Similarly, it might be that the convention is to pay the auctioneer a percentage of the difference between the highest and second highest bids. Although a more complete model would have the size of the bribe as endogenous, the analysis presented yields several interesting results.

We also ignore the principal-agent relationship between the seller and the auctioneer. As noted earlier, our motivating example is that of a government official who has been delegated the authority to purchase a given good or service on behalf of the government or an auctioneer who has been given an object to sell on behalf of an individual seller.

Before we proceed to the next section it is convenient to explain why we do not analyze second-price auctions: they are corruption free. The reason is that for the type of corruption we investigate the winner in a second-price sealed bid auction always pays the second highest bid without the need to pay a bribe.

4 Corruption in First-Price Auctions

In this section we investigate the effects of corruption on bidding behavior in a first-price auction. Denote the maximum of bidders $2, \ldots, n$ values by $Y = \max_{j \geq 2} v_j$. This random variable has distribution $F_Y(x) = F^{n-1}(x)$ and density $f_Y(x) = (n-1) F^{n-2}(x) f(x)$. We denote by $b_1(v) = \frac{\int_0^v y f_Y(y) dy}{F_Y(v)}$ the equilibrium bidding function of the first price sealed bid auction. Note that $b_1(1) = E[Y]$. To find an equilibrium we could follow the approach used in Section 2 of determining the optimal bid $x \geq 0$ of Bidder 1, when his opponents are bidding according to a function $b(v_j)$, $j = 2, \ldots, n$, by means of the first-order condition of Bidder 1's expected utility maximization problem, and then using that in a symmetric equilibrium $x = b(v)$. Instead, we will follow a more general and neater approach. First, note that if there is a symmetric equilibrium strictly increasing strategy $b(\cdot)$, the object will be allocated to the bidder with the highest value. As the bidder with the lowest possible value obtains a zero payoff (see below for a formal justification), the winner's total payments is actually fixed across any efficient auction format. This is a consequence of the celebrated Revenue Equivalence Theorem.[2] The distinction here is that part of the bidders' payment now goes to the auctioneer as a bribe rather than to the seller. In the next proposition we characterize a symmetric equilibrium bidding strategy.

Proposition 1 *Suppose the strictly increasing function $b : [0,1] \to \mathbb{R}_+$ is a symmetric equilibrium. Suppose the auctioneer approaches the winner and for a bribe $B(b(v), b(Y))$, where $b(v)$ is the winner´s bid and $b(Y)$ is the highest losing bid, offers the winner to change his bid to match the second highest bid. Then for every $v \in [0,1]$,*

$$\int_0^v \min\{b(v), b(y) + B(v,y)\} f_Y(y) dy = \int_0^v y f_Y(y) dy. \tag{2}$$

Proof: The winning bidder's payment is $b(v)$ if he does not accept the bribe and is $b(Y) + B(v,Y)$ if he accepts the bribe. Thus his expected payment $E[\min\{b(v), b(Y) + B(v,Y)\}| v > Y]$. From the Revenue equivalence theorem, the winner's total payment in a first-price auction has to be identical to his expected payment in any efficient auction and, in particular, in a second-price auction. This is equivalent to (2). QED

[2] See for example a general argument in theorem 2 in Milgrom and Segal (2000).

Note that we have not specified the type of bribe payments. The idea is that for a given type of bribe payment – for example, a fixed bribe $B(\cdot,\cdot) = B$ – one should try to find a strictly increasing strategy $b(\cdot)$ that satisfies (2). This yields a differential equation that $b(\cdot)$ has to satisfy together with the boundary condition $b(0) = 0$.[3]

Note that the type of reasoning that leads to (2) does not ensure either uniqueness or existence. It is a necessary condition. However, the equation (2), as we will see from the reasoning below for various particular bribe payments, will imply a unique solution. Moreover, if such a strictly increasing equilibrium strategy exists, then the object will still be allocated to the individual with the highest valuation so that the first-price auction is still efficient under the type of corruption we investigate.

4.1 Fixed bribe

In this subsection we consider the fixed bribe case, that is, when $B(\cdot,\cdot)$ is equal to a constant B. The equilibrium is a solution to a complicated differential equation. This differential equation can be transformed into a collection of differential equations. See (4) below. The smaller B the more complicated the solution is. The following lemma characterizes the symmetric equilibrium bidding function in the fixed bribe case. Define v_1 implicitly by $b_1(v_1) = \frac{\int_0^{v_1} y f_Y(y)dy}{F_Y(v_1)} = \min\{E[Y], B\}$. Thus $v_1 < 1$ if and only if $B < E[Y]$.

Lemma 1 *A differentiable, increasing symmetric equilibrium of the fixed bribe auction with corruption, $b_B(\cdot) = b(\cdot)$ coincides with $b_1(v)$ if $v \leq v_1$ and if $v \geq v_1$ satisfies the differential equation*

$$b'(v) = \frac{(v - b(v)) f_Y(v)}{F_Y(v) - F_Y(b^{-1}(b(v) - B))}. \tag{3}$$

with initial condition, $b(v_1) = B$.Reciprocally any solution of the differential equation above for $v \geq v_1$ and such that it is equal to $b_1(v)$ if $v \leq v_1$ is an equilibrium bidding function.

[3] Given that bidders decide whether or not to pay the bribe, the individual rationality constraint is the same as under no corruption, that is, the individual with the lowest possible valuation bids zero so that he is indifferent between participating or not in the auction.

The proof is in the appendix. The equation (3) is non-standard due to the presence of b^{-1}. In general we have the following:

Theorem 1 *The function $b_B(\cdot) = b(\cdot)$ defined below is a symmetric strictly increasing equilibrium of the first price auction with corruption.*

$$b(v) = \begin{cases} b_1(v) & \text{if} \quad 0 \le v \le v_1 \\ b_2(v) & \text{if} \quad v_1 \le v \le v_2 \\ \dots & \dots \quad \dots \\ b_m(v) & \text{if} \quad v_{m-1} \le v \le 1 \end{cases} \tag{4}$$

The function $b_k(\cdot)$, $2 \le k \le m$ is defined by the differential equation:

$$b_k'(v) = \frac{(v - b_k(v)) f_Y(v)}{F_Y(v) - F_Y\left(b_{k-1}^{-1}(b_k(v) - B)\right)}, v_{k-1} \le v \le v_k \tag{5}$$

with initial condition $b_k(v_{k-1}) = b_{k-1}(v_{k-1})$ and v_k is defined by $b_k(v_k) = kB$. Here $v_0 = 0$ and $b_m(1) \le mB$.

Remark 1 *There are at most $1/B$ functions b_k since $b_m(1) \le 1$.*

The existence of the function $b_B(\cdot)$ defined above is proved in the appendix. Instead here we develop some of the intuition behind Theorem 1. Consider a candidate equilibrium $b(v)$, and gains and losses for a buyer that considers a deviation by announcing that his valuation is $v - \varepsilon$ instead of v for an arbitrarily small $\varepsilon > 0$. As a result, assume that this bidder reduces his bid by $b'(v)\epsilon$.

If the highest valuation of the other bidders, Y, belonged to the interval $(b^{-1}(b(v) - B), v - \varepsilon)$ then this bidder would still win the auction and pay $b'(v)\epsilon$ less. In this event, which happens with probability $F_Y(v) - F_Y\left(b_{n-1}^{-1}(b(v) - B)\right)$, the winner would not accept the offer to revise the bid down and pay the bribe because the cost of doing so, B, would exceed the benefit, $b(v) - b(Y)$.

A second possibility is the highest of the other bidders' valuation, Y, was less than $b^{-1}(b(v) - B)$. In this event our bidder would still win the auction and pay the price plus the bribe, $b(Y) + B$, so that he would gain nothing by deviating. In summary, the possible expected benefit from a deviation is equal to:

$$\epsilon b'(v)\left(F_Y(v) - F_Y\left(b^{-1}(b(v) - B)\right)\right).$$

It may be the case, however, that Y belonged to the interval $(v-\varepsilon, v)$. In this event, which happens with probability $f_Y(v)\,\epsilon$, our bidder would now lose the auction because of the reduction in his bid. The loss would be equal to $v-b(v)$. That is, the expected loss from reducing the bid is equal to

$$(v-b(v))\,f_Y(v)\,\epsilon.$$

In equilibrium we need $b(v)$ to be optimal for our bidder, so that the losses should exactly compensate the gains, which gives us the first-order condition in Theorem 1.

We now explain why the solution changes at intervals of length B in the bid. It turns out that there is a simple reason behind it. Start with the case where one's bid is below B. This bidder will not accept an offer to revise his bid and pay a bribe B, since the gain would be smaller than the loss. In this case, the bid is the same as in a first-price auction under no corruption. If one bids B (or above), this bidder may accept the bribe. This would happen when the rival highest bidder had bid very close to 0. In this sense the gain from being the winner is now higher than in the absence of corruption. Thus one would bid more aggressively. By the same reasoning, one expects that rivals with similar valuations would bid more aggressively as well. The result is that the slope of the bid function jumps up at $b(v)=B$.

Why does the bid function jumps again at $2B$? At the margin what matters for a bidder that bids $b(v)$ is not only the behavior of a direct competitor, i.e., bidders with valuation v, but also the behavior of bidders whose bid may determine whether the auctioneer's proposal for a bribe is accepted or not. That is, the behavior of bidders with valuation $\left(b^{-1}(b(v)-B)\right)$. If the behavior of these bidders becomes more aggressive, so does the behavior of our bidder with valuation v. This is why the slop of the bidding function jumps in equilibrium at $B, 2B, 3B$, and so on.

In addition, it is worth noting that if the fixed bribe goes to zero the equilibrium bidding function must approach $b(v)=v$ since the highest bidder will pay the bribe and pay the second highest bid. We next provide a numerical example to help illustrate the potential magnitude of the effect of corruption on the seller's revenue.

Example 1 *Consider the particular case of the uniform distribution with two bidders. We know already that if* $B \geq E[Y]=1/2$ *the equilibrium is*

$b_1(v) = \frac{\int_0^v y dy}{v} = \frac{v}{2}$. *Suppose* $\frac{1}{2} \geq B \geq \frac{1}{2+\sqrt{2}} = 0.29289$. *Then*

$$b(v) = \begin{cases} \frac{v}{2} & \text{if } 0 \leq v \leq 2B \\ \frac{v}{2} + B - \frac{\sqrt{4Bv - v^2}}{2} & \text{if } 2B \leq v \leq 1 \end{cases}$$

is an equilibrium when the auctioneer approaches the winner. The seller's expected revenue is $\frac{1}{3} - R_B$ *and the expected bribe* $R_B = B - 4B^3 - 8B^3 \int_0^{\frac{1-2B}{2B}} \sqrt{1 - w^2} dw$. *This function is decreasing in* B *and the maximum expected bribe is* $R_{0.29289} = 0.063$. *The calculations are shown in the appendix.*

In the example above the presence of a corrupt auctioneer caused a reduction of 18 per cent in the seller's expected revenue.

4.2 Proportional bribe

We now generalize the example of Section 2 where the auctioneer approaches the winner and asks for a bribe that is proportional to the difference between the highest and the second highest bids. Suppose $0 < \rho \leq 1$ so that if b_1 is the highest bid and b_2 is the second highest bid then the auctioneer asks for a bribe $B(b_1, b_2) = \rho(b_1 - b_2)$. To find the equilibrium bidding strategy, we assume that our candidate for a symmetric equilibrium $b(\cdot)$ is strictly increasing. Replacing this particular type of bribe payment into (2) yields:

$$\int_0^v [b(y) + \rho(b(v) - b(y))] f_Y(y) dy = \int_0^v y f_Y(y) dy. \tag{6}$$

This allows us to obtain a first-order condition.

Proposition 2 *The following condition has to be satisfied by a symmetric, monotone equilibrium of a first-price auction with proportional bribes:*

$$\rho b'(v) = (v - b(v)) \frac{f_Y(v)}{F_Y(v)} = (v - b(v)) \frac{nf(v)}{F(v)}. \tag{7}$$

The proof is omitted as it is obtained by differentiating (6) with respect to v. For example, when there are n bidders with values distributed uniformly on $[0, 1]$, our strictly increasing symmetric equilibrium is $b(v) = \frac{n-1}{n-1+\rho} v$. One can check that this approach yields the same equilibrium as obtained in the example of Section 2 when $n = 2$.

Let us now develop the intuition behind this proposition. Consider again a candidate equilibrium $b(v)$, and gains and losses for a buyer that considers a deviation by announcing that his valuation is $v-\varepsilon$ instead of v for an arbitrarily small positive ε so that this bidder reduces his bid by $b'(v)\varepsilon$.

If this player still wins with this bid, an event with probability $F_Y(v)$, then this bidder reduces his expected total payment by $\rho b'(v)\varepsilon$ as this bidder's total payment is given by (6). Thus, the potential gains from change his bid are

$$\rho b'(v) F_Y(v)\varepsilon$$

It may be the case, however, that Y belonged to the interval ,$(v-\varepsilon, v)$ In this event, which happens with probability $f_Y(v)\varepsilon$, our bidder would now lose the auction because of the reduction in his bid. The loss would be equal to $v-b(v)$. That is, the expected loss from reducing the bid is equal to

$$(v-b(v)) f_Y(v)\varepsilon.$$

In equilibrium we need $b(v)$ to maximize our bidder's expected profits, so that the losses should exactly offset the gains, which gives us the first-order condition:

$$(v-b(v)) f_Y(v)\varepsilon = \rho b'(v) F_Y(v)\varepsilon. \tag{8}$$

Rearranging (8) we obtain the first-order condition in the above proposition.

Note that it is possible to compare bidding behavior in a first-price auction with proportional bribe with behavior in a first-price auction under no corruption. For this latter case, the first-order condition is given by[4]:

$$(v-b_1(v)) f_Y(v) = b_1'(v) F_Y(v) \tag{9}$$

where $b_1(\cdot)$ is the symmetric, monotone equilibrium in a first-price auction under no corruption. Note that both $b(0)=0$ (see justification above) and $b_1(0)=0$. Thus, we can compare the two first-order conditions (see below) to conclude that bidders bid uniformly more aggressively in the presence of a corrupt auctioneer. The reason was outlined in Section 2; the value of winning the auction is higher under corruption in the sense that the winner will now pay at most his bid and often less than his bid. Although bidders are more aggressive under corruption, their total expected payments is the

[4]For a derivation of this first-order condition see, for example, the survey article by McAfee and McMillan (1987). For a derivation of $b^*(\cdot)$ using the Revenue Equivalence Theorem see the recent survey by Klemperer (1999).

same as under no corruption. This is precisely because they will often pay less than their bids. As a result, we can conclude that the seller's revenue is lower under corruption as bidders' total expected payments are now split between the seller and the auctioneer.

4.3 General linear bribe

Suppose the bribe now takes the form $B+\rho\left(b\left(v\right)-b\left(y\right)\right)$ where $\rho\in(0,1)$. We now obtain the bidding function differential equation. The procedure is analogous to the fixed bribe case proved in the appendix. First note that equation (7) is now of the form

$$\int_{0}^{v}\min\left\{b\left(v\right),b\left(y\right)+B+\rho\left(b\left(v\right)-b\left(y\right)\right)\right\}f_{Y}\left(y\right)dy=\int_{0}^{v}yf_{Y}\left(y\right)dy.$$

We have that $b\left(v\right)\leq b\left(y\right)+B+\rho\left(b\left(v\right)-b\left(y\right)\right)$ if and only if $b\left(v\right)\leq\frac{B}{1-\rho}+b\left(y\right)$. Thus if $b\left(v\right)\leq\frac{B}{1-\rho}$ we obtain that $b\left(v\right)=b_{1}\left(v\right)$ where $b_{1}\left(v\right)=\frac{\int_{0}^{v}yf_{Y}(y)dy}{F_{Y}(v)}$. Define v_{1} as the solution of $b_{1}\left(v_{1}\right)=\frac{B}{1-\rho}$ or as $v_{1}=1$ if $b_{1}\left(1\right)\leq\frac{B}{1-\rho}$. If we proceed analogously as in the fixed bribe case, Equation (**??**) changes to

$$b'\left(v\right)=\frac{\left(v-b\left(v\right)\right)f_{Y}\left(v\right)}{F_{Y}\left(v\right)-\left(1-\rho\right)F_{Y}\left(b^{-1}\left(b\left(v\right)-\frac{B}{1-\rho}\right)\right)}.$$

If ρ is high enough that $\frac{B}{1-\rho}\geq1$ there will be no bribe payment. For smaller ρ, the general linear bribe case is a combination of the fixed and proportional bribe cases.

4.4 Comparing bribe payments

As different types of bribe payments yield different results for the corrupt auctioneer, one could ask the question of which type the auctioneer would choose. We cannot provide a general result as to be able to make such comparison we have to obtain an explicit equilibrium bidding function for each type of bribe payment. Instead we indicate below how to compute the expected bribe under the different bribe payments as a function of the equilibrium bidding strategies. We also provide an example in which we can explicitly compute the expected bribe.

4.4.1 Proportional bribe

Let us consider the proportional bribe payment. If $b_\rho(\cdot)$ is the symmetric equilibrium, from equation (7) we have that

$$\rho b'_\rho(v) = (v - b_\rho(v)) \frac{f_Y(v)}{F_Y(v)} = (v - b_\rho(v)) \frac{nf(v)}{F(v)}. \tag{10}$$

The proposition below, proved in an appendix, formalizes our intuition expressed in subsection 4.2 that bidders bid uniformly more aggressively in the presence of a corrupt auctioneer.

Proposition 3 *The bidding function $b_\rho(v)$ is decreasing with ρ.*

Now define Z as the highest valuation amongst the n bidders. Its density is given by

$$f_Z(y) = nF^{n-1}(y) f(y).$$

Define Z_2 as the second highest valuation amongst the n bidders. Its density is given by

$$f_{Z_2}(y) = n(n-1)(1-F(y)) F^{n-2}(y).$$

Thus we can express the auctioneer's expected bribe under a proportional bribe as:

$$\begin{aligned}
R_\rho &= \rho E[b_\rho(Z) - b_\rho(Z_2)] = \rho E[b_\rho(Z)] - \rho E[b_\rho(Z_2)] = \\
&= \rho \int b_\rho(y) nF^{n-1}(y) f(y)\, dy - \rho \int b_\rho(y) n(n-1)(1-F(y)) F^{n-2}(y)\, dy = \\
&= \rho \int b_\rho(y) \left[nF^{n-1}(y) - n(n-1)\left(F^{n-2}(y) - F^{n-1}(y)\right)\right] f(y)\, dy = \\
&= \rho \int b_\rho(y) \left[n^2 F^{n-1}(y) - n(n-1) F^{n-2}(y)\right] f(y)\, dy.
\end{aligned}$$

Integrating by parts and using (10) we obtain

$$\begin{aligned}
R_\rho &= \rho b_\rho(y) \left[nF^n(y) - nF^{n-1}(y)\right] |_0^1 - \int n\rho b'_\rho(y) \left[F^n(y) - F^{n-1}(y)\right] dy = \\
&= n \int (y - b_\rho(y)) \frac{nf(y)}{F(y)} \left[F^n(y) - F^{n-1}(y)\right] dy. \qquad (11)
\end{aligned}$$

From Proposition 3 and inspection of (11) we can conclude that the auctioneer expected bribe is maximized at $\rho = 1$ as we suggested earlier on. This is summarized in the next result.

Theorem 2 *The optimal proportional bribe is* $B(b_1, b_2) = b_1 - b_2$. *The auctioneer expected bribe is given by*

$$R_1 = \int b_1'(y)\left[F^n(y) - F^{n-1}(y)\right]dy = \int \frac{\int_0^y z f_Y(z)\,dz}{F(y)}\left[-nF(y) + (n-1)\right]f(y)\,dy. \tag{12}$$

4.4.2 Fixed bribe

The bidding function in the fixed bribe case is not smaller than $b_1(\cdot)$. Denote by b_B the bidding function defined in theorem 5. If $v \leq v_1, b_B(v) = b_1(v)$. Note that

$$b_B'(v_1) = \frac{(v_1 - b_1(v_1))\,f_Y(v_1)}{F_Y(v_1) - F_Y\left(b_1^{-1}(b_B(v_1) - B)\right)} > \frac{(v_1 - b_1(v_1))\,f_Y(v_1)}{F_Y(v_1)} = b_1'(v_1). \tag{13}$$

Thus if $b_B(v) - b_1(v) \leq 0$ for some $v > v_1$ a reasoning similar to (13) shows that the minimum of $b_B - b_1$ is not interior. But it can be neither v nor v_1, a contradiction.

The auctioneer's expected revenue is given by

$$R_B = B\Pr(b_B(Z) - b_B(Z_2) > B).$$

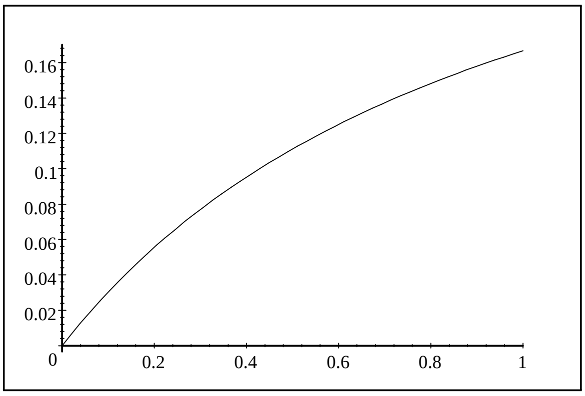
To be able to compute this expression explicitly we need the equilibrium bidding functions that are defined by the size of the fixed bribe B. For B sufficiently large, there is no corruption and therefore a corrupt auctioneer would prefer a proportional bribe scheme with $\rho = 1$. For B sufficiently close to zero, there is always corruption but the expected bribe is small and again a corrupt auctioneer would prefer a proportional bribe scheme with $\rho = 1$. For intermediate cases the comparison is more complex. Next we provide an example where we can explicitly compare the expected bribe under the two bribe payment schemes.

4.4.3 Revisiting example 1

Consider the case of two players with values drawn from a uniform distribution on [0,1]. From the example in Section 2, the expected bribe under a proportional bribe is equal to:

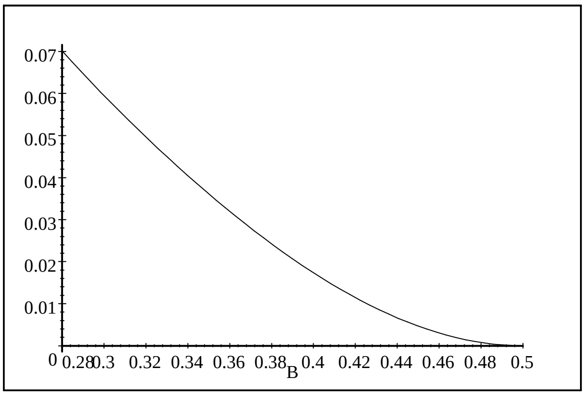
$$R_\rho = \frac{\rho}{1+\rho}\frac{1}{3}$$

The diagram below plots this function for $0 \leq \rho \leq 1$:



Expected bribe as a function of ρ

As established in Proposition 3, the auctioneer's expected bribe under a proportional bribe is maximized at $\rho = 1$.From example 1 above, the expected bribe under a fixed bribe is given by



Expected bribe as a function of B

It is clear from the above diagrams that the fixed bribe would be preferred if $\rho < .265\,82$.

4.4.4 Discussion

Mote that changing the way the auctioneer is compensated, say for example from a fixed salary to a fixed percentage of the sales revenue as commission, will not eliminate or even reduce corruption. This finding contrasts with that of the classical economic theory of corruption — e.g., Becker and Stigler

(1974) – and with more recent findings (Mookherjee and Png, 1995) that to wipe out corruption a large discrete jump in the official's compensation is required.

4.5 Other corruption arrangements

Other possible corruption arrangements exist. For instance, the auctioneer might approach any of the losers and offer the possibility of revising the losing bid to match the highest bid and win the auction in exchange for a bribe. For example, the auctioneer might approach the loser with the highest bid. It turns out that such arrangement might result in the failure of existence of a symmetric, monotone equilibrium as it is illustrated in the next proposition.

Proposition 4 *In general there is no increasing symmetric bidding strategy equilibrium if the auctioneer approaches the highest losing bidder.*

The formal proof is in the appendix but the intuition is quite straightforward. Suppose bidders $2, ..., n$ follow a symmetric increasing bidding strategy. Consider Bidder 1's best response. If he has the highest value and chooses the same symmetric increasing bidding strategy he will win the auction but might lose the object as the auctioneer will approach the highest losing bidder. In such scenario, the highest valued bidder will actually wants to have the second highest bid and an increasing symmetric equilibrium bidding strategy will then not exist in general.

Of course, the inexistence of an increasing symmetric equilibrium bidding strategy does not mean that the object will never be allocated to the individual with the highest value. This is still possible, but not guaranteed, if this individual has the second highest bid.

5 Conclusion

In this paper we analyzed the effects of corruption on auctions under the independent private values setting with risk neutral bidders. We examined how bidding behavior in a first price auction is affected by corruption. We show how to compute an increasing symmetric equilibrium bidding strategy for any type of bribe payment when the auctioneer approaches the winner of the auction to offer him the possibility of reducing his bid to match his opponent's bid in exchange for a bribe. If such equilibrium exists, then the auction is still efficient. For example, we show that in the case of a large fixed bribe, the probability of corruption is high and, consequently, the seller's expected revenue is low (as the winning bidder reduces his bid).

These results are only suggestive of how the existence of corruption might affect both the seller's expected revenue and the efficiency of an auction. This seems to be a research line worth pursuing as developing countries, naturally more prone to corruption, are increasingly using auctions to allocate goods and services and to privatize government owned assets.

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A Appendix

Proof of Lemma 1: To obtain a differential equation for $b_B(\cdot) = b(\cdot)$ we use (2). Let us rewrite (2) for a fixed bribe:

$$\int_0^v \min\{b(v), b(y) + B\} f_Y(y)\, dy = \int_0^v y f_Y(y)\, dy. \tag{14}$$

First assume that $b(v) \leq B$. Then since $\min\{b(v), b(y) + B\} = b(v)$:

$$b(v) F_Y(v) = \int_0^v y f_Y(y)\, dy.$$

Thus necessarily

$$b(v) = b_1(v) \text{ if } b(v) \leq B, \tag{15}$$

where $b_1(v) = \frac{\int_0^v y f_Y(y) dy}{F_Y(v)}$. Therefore if $B \geq E[Y] = b_1(1)$ then $b(v) = b_1(v), 0 \leq v \leq 1$. Suppose now that $0 < B < E[Y]$. Then define v_1 as the solution of $b_1(v_1) = B$. Now define $\alpha(v) < v$ implicitly by $b(v) = B + b(\alpha(v)), v \geq v_1$. Then we may rewrite (14) as

$$\int_0^{\alpha(v)} (B + b(y)) f_Y(y)\, dy + b(v) \int_{\alpha(v)}^v f_Y(y)\, dy = \int_0^v y f_Y(y)\, dy.$$

Differentiating we have

$$\alpha'(v) b(v) f_Y(\alpha(v)) + b'(v) \int_{\alpha(v)}^v f_Y(y)\, dy + b(v)\left(f_Y(v) - \alpha'(v) f_Y(\alpha(v))\right) = v f_Y(v).$$

Above we used that $B + b(\alpha(v)) = b(v)$ in the first summand. Cancelling $\alpha'(v) b(v) f_Y(\alpha(v))$ on the left hand side and using that $\int_{\alpha(v)}^v f_Y(y)\, dy = F_Y(v) - F_Y(\alpha(v))$ we obtain

$$b'(v)\left(F_Y(v) - F_Y(\alpha(v))\right) + b(v) f_Y(v) = v f_Y(v).$$

Finally since $\alpha(v) = b^{-1}(b(v) - B)$ we obtain the non-standard differential equation:

$$b'(v) = \frac{(v - b(v)) f_Y(v)}{F_Y(v) - F_Y(b^{-1}(b(v) - B))}, v \geq v_1. \tag{16}$$

We now show that a solution of this differential equation is necessarily an equilibrium. Suppose bidder 1 bids $x \geq 0$ and the other bidders bid $b(v_j), j \geq 2$. Without loss of generality we may suppose that $x = b(a), a \in [0,1]$. His expected utility is

$$\phi(a) = \int_0^a (v - \min\{b(a), b(y) + B\}) f_Y(y)\, dy.$$

If $a \leq v_1$, $\phi(a) = (v - b(a)) F_Y(a)$.In this case $\phi'(a) = (v - a) f_Y(a)$ If $a > v_1$, there exists an $\alpha(a) < a$ defined implicitly by $b(a) = b(\alpha(a)) + B$. Namely $\alpha(a) = b^{-1}(b(a) - B)$. Thus

$$\phi(a) = \int_0^{\alpha(a)} (v - b(y) - B) f_Y(y)\, dy + (v - b(a)) \int_{\alpha(a)}^a f_Y(y)\, dy.$$

Differentiating and using that $b(\alpha(a)) + B = b(a)$, in the first summand on the first line we obtain

$$\begin{aligned} &\phi'(a) = \\ &(v - b(a)) f_Y(\alpha(a)) \alpha'(a) - b'(a) \int_{\alpha(a)}^a f_Y(y)\, dy + (v - b(a)) (f_Y(a) - f_Y(\alpha(a)) \alpha'(a)) = \\ &= -b'(a) (F_Y(a) - F_Y(\alpha(a))) + (v - b(a)) f_Y(a) = \\ &= -(a - b(a)) f_Y(a) + (v - b(a)) f_Y(a) = \\ &= (v - a) f_Y(a). \end{aligned}$$

Thus $\phi(a)$ is maximized at $a = v$.

Proof of Theorem 1: It suffices to show that $b_B(\cdot)$ defined in (4) satisfy the differential equation (16). The sequence $(v_n)_n$ terminates since each $b_n(\cdot)$ is defined for $v \geq v_{n-1}$. Thus either $b_n(1) \leq nB$ or there exists $v_n > v_{n-1}$ such that $b_n(v_n) = nB$. Eventually $nB > 1$ and $b_n(1) \leq nB$. Consider now $v \in [v_{n-1}, v_n]$. Then $b(v) = b_n(v)$. Moreover since $(n-2)B \leq b_n(v) - B \leq nB - B = (n-1)B$ we have that $b_{n-1}^{-1}(b_n(v) - B) = b^{-1}(b(v) - B)$. The differential equation defining $b_n(\cdot)$ thus imply that $b(\cdot)$ satisfy (16), ending the proof.

Proof of Proposition 3:

Suppose $\mu > \rho$ and define $k(v) = b_\rho(v) - b_\mu(v)$. If $v > 0$ we have that

$$\begin{aligned} k'(v) &= b'_\rho(v) - b'_\mu(v) = \frac{nf(v)}{F(v)}\left(\frac{v - b_\rho(v)}{\rho} - \frac{v - b_\mu(v)}{\mu}\right) = \\ &= \frac{nf(v)}{F(v)}\left((v - b_\mu(v))\left(\frac{1}{\rho} - \frac{1}{\mu}\right) - \frac{k(v)}{\rho}\right). \end{aligned}$$

Thus if $v > 0$,

$$k'(v) = \frac{nf(v)}{F(v)}\left((v - b_\mu(v))\left(\frac{1}{\rho} - \frac{1}{\mu}\right) - \frac{k(v)}{\rho}\right)$$

If $v > 0$ and $k(v) \leq 0$ then $k'(v) > 0$. Thus in this case there is a $v' \in (0, v)$ such that $k(v') < k(v)$. Consider $\omega \in [0, v]$ that minimizes $k(\omega)$ in $[0, v]$. The reasoning above shows that the minimum cannot be interior and cannot be v. Thus $\omega = 0$. However $k(0) = 0$ and k is continuous we have a contradiction. Thus $k(v) > 0$ if $v > 0$.

Calculations of Example 1.

If $Z = \max\{v_i; i = 1, \ldots, n\}$ and Z_2 is the second highest value of $\{v_i; i = 1, \ldots, n\}$ then $R_B = B \Pr(b_B(Z) - b_B(Z_2) > B)$ is the auctioneer expected bribe. The bidding function is given by

$$b_B(v) = \begin{cases} \frac{v}{2} & \text{if } 0 \leq v \leq 2B; \\ \frac{v}{2} + B - \frac{\sqrt{4Bv - v^2}}{2} & \text{if } 2B \leq v \leq 1. \end{cases}$$

Thus

$$\begin{aligned} R_B &= 2B \Pr(b_B(v_1) - b_B(v_2) > B) = 2B \Pr(b_B(v_1) - b_B(v_2) > B, v_1 > 2B > v_2) = \\ &= 2B \Pr\left(\frac{v_1}{2} + B - \frac{\sqrt{4Bv_1 - v_1^2}}{2} - \frac{v_2}{2} > B, v_1 > 2B > v_2\right) = \\ &= 2B \Pr\left(v_1 - \sqrt{4Bv_1 - v_1^2} > v_2, v_1 > 2B > v_2\right) = \\ &= 2B \Pr\left(v_1 - \sqrt{4Bv_1 - v_1^2} > v_2, v_1 > 2B > v_2\right) = 2B \int_{2B}^{1} \left(v_1 - \sqrt{4Bv_1 - v_1^2}\right) dv_1 = \\ &= B - 4B^3 - 2B \int_{2B}^{1} \sqrt{4Bv_1 - v_1^2} dv_1 = \\ &= B - 4B^3 - 8B^3 \int_{0}^{\frac{1-2B}{2B}} \sqrt{1 - w^2} dw. \end{aligned}$$

Proof of Proposition 4: Suppose $b(\cdot)$ is an increasing symmetric equilibrium strategy. Bidder 1 expected payment is

$$(v - b(v)) \Pr(b(v) > b(Y), b(v) + B > Y) + E[(v - b(Y) - B)^+ \chi_{Y>v}] = \int_0^v y f_Y(y)\, dy.$$

Since $b(\cdot)$ is increasing this is equivalent to

$$(v - b(v)) F_Y(\min\{v, b(v) + B\}) + \int_v^1 (v - b(Y) - B)^+ f_Y(y)\, dy = \int_0^v y f_Y(y)\, dy.$$

Efficiency requires that if $Y > v$ then $b(Y) + B > v$. Therefore

$$\int_v^1 (v - b(Y) - B)^+ f_Y(y)\, dy = 0$$

and

$$(v - b(v)) F_Y(\min\{v, b(v) + B\}) = \int_0^v y f_Y(y)\, dy.$$

Consider now $v = 1$.

$$(1 - b(1)) F_Y(b(1) + B) = \int_0^1 y f_Y(y)\, dy = E[Y].$$

If B is small enough this will not have a solution. For the uniform distribution and $n = 2$, there is no solution if $B < 1/4$:

$$\frac{1}{4} + (1 - b(1)) B \geq (1 - b(1))(b(1) + B) = \frac{1}{2}$$

and therefore $B \geq (1 - b(1)) B \geq 1/4$.