The New Empirics of Economic Growth: Quantile Regression Estimation of Growth Equations

Marcelo Mello[1]

Álvaro Novo

Preliminary draft (please do not quote) – June 05, 2002

Department of Economics, University of Illinois at Urbana-Champaign
1206 S. 6th St., 484 Wohlers Hall, Champaign, IL 61820

Abstract

We propose a novel approach to estimate and make inference from growth equations. We use quantile regression to assess income convergence and the effects of policy variables on the conditional distribution of the GDP growth rates. Estimation of the regression quantiles allows us to characterize the entire conditional distribution of the GDP growth rate for a given set of regressors. This allows us to identify different responses of the GDP growth rate to policy variables associated with different points on its conditional distribution. Moreover, recent inferential procedures proposed by Koenker and Xiao (2001), permit us to test if policy variables affect the location and/or the scale of the conditional distribution of GDP growth rates. For a sample of 98 countries in the Barro-Lee data set we find that conditional on being in the top 35% of the conditional distribution of GDP growth rates countries exhibit a negative relationship between average rate of growth and initial income. Changes in human capital have a stronger impact on the GDP growth rate for fast-growing countries than for slow-growing countries. Tests of the location and location-scale hypothesis suggest that the initial income per capita and other policy variables affect the conditional distribution of GDP growth rate in more complex ways than a location and/or scale shift prescribed in the test. For example, they may affect other aspects of the distribution such as the skewness or the kurtosis. Our findings suggest that previous empirical growth studies relying on conditional mean estimation methods such as least squares give a misleading picture of the growth dynamics. Furthermore, it is suggested that the underlying growth model generating the growth experience of fast-growing countries is of neoclassical type while for slow-growing counties is of endogenous growth type.

Keywords: Solow Growth model, Income Convergence, Quantile Regression, Location shift, Location-scale shift

JEL Categories: C32, O41

[1] Mello gratefully acknowledges financial support from CNPQ-Brazil. Novo thanks the financial support provided by the Subprograma Ciência e Tecnologia do 2º Quadro de Apoio, Portugal. Corresponding author, email: mello@uiuc.edu, (phone: (217)-333-0120; fax: (217)-244-6678). We would like to thank for comments and suggestions on earlier versions of this work Roger Koenker, Anne Villamil, Dan Bernhardt, Stephen Parente, Zhijie Xiao, seminars participants at Bowling Green State University, Virginia Tech and the 66th Midwest Economics Association meeting in Chicago. The usual disclaimer applies.

1. Introduction

Predictions of the neoclassical growth model have generated a vast amount of research in macroeconomics. A central implication of the Solow (1956) growth model is that countries with the same technological parameters and preferences, differing only in the initial level of wealth, should converge over time to the same steady-state level of income per capita. This result became known in the literature as unconditional β-convergence. When countries differ in their microeconomic specifications, and consequently have different steady-state levels of income per capita, the Solow model predicts that, after controlling for steady-state differences, poor countries should grow faster then rich countries. This prediction of the model is known as conditional β-convergence.

The seminal article testing unconditional β-convergence is Baumol (1986), followed by many others including Barro (1991), Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992), henceforth MRW. These articles test income per capita convergence using mean regression methods such as ordinary least squares on a cross-section[2] of economies, where the dependent variable is the average rate of growth of income per capita and the explanatory variable is the initial income per capita. A negative sign on the coefficient of the initial income per capita is interpreted as evidence of income convergence.

The literature finds little evidence of unconditional convergence for broadly constituted samples. However, there seems to be enough evidence supporting the convergence hypothesis for selected sub-samples. For instance, Baumol (1986) suggests

[2] Income per capita convergence can also be tested in a time series or panel data setting. For a more detailed discussion of these alternative tests the reader is invited to check the articles by Bernard and Durlauf (1995), Durlauf and Quah (2000), Mello and Guimarães (2001), and Islam (1995).

that there are “convergence clubs” where rich OECD countries exhibit a faster speed of convergence while centrally planned economies[3] have a slower speed of convergence, and less developed countries exhibit no convergence at all.

Barro (1991) and MRW suggest that when appropriate policy variables are used to control for differences in steady-state levels of income, one can find a negative relationship between rate of growth and initial income. Thus, providing support for the conditional convergence hypothesis. Several policy variables can be used as control variables for conditional convergence equations, such as investment rate, proxies for human capital, measures of openness, terms of trade, among many others[4].

Convergence tests are important not only to assess income convergence but they can also be used to discriminate among growth theories. A finding of income convergence suggests that the underlying growth model is of neoclassical type. On the other hand, as pointed out in Durlauf (2000), some endogenous growth theories predict a positive sign on the initial income coefficient. In this case, we have a situation of divergence of income per capita, that is, the higher the initial income the higher the rate of growth. Therefore, if the estimated sign on the initial income coefficient is positive we have evidence that the underlying growth model is of endogenous growth type. Thus, determining empirically the sign of the initial income coefficient can be important for economic policy making.

The empirical growth literature, however, suffers from serious statistical pitfalls. First, the literature seems to ignore the problem of the “regression fallacy”. In an important article, Friedman (1992) points out that a negative relationship between the

[3] The centrally planned economies at the time of Baumol’s (1986) article were: Bulgaria, China, the former Czechoslovakia, the former East Germany, Hungary, Poland, Romania, the countries that formed the old USSR and Yugoslavia.

average rate of growth and initial income may reflect regression to the mean and not convergence. Friedman (1992) gives an example where one can find a negative relationship between average growth rate and initial period income and, when the latter is replaced by final period income in the regression equation, one finds no relationship between the dependent variable and the average growth rate[5]. Hence, one should be careful when interpreting a negative relationship between average growth rate and initial income as evidence of convergence. Quah (1993), and Bernard and Durlauf (1996) also show that β-convergence is consistent with a stable cross-section distribution of income per capita over time; a situation where there is no convergence.

Second, studies assessing conditional convergence typically use mean regression estimation methods what constrains the coefficient on the policy variables to be the same for all countries. This implies that the impact of a change in a policy variable say, human capital, on a rich country's GDP growth rate should be the same as the impact on a poor country's GDP growth rate. This is unlikely to be the case. This effect is neither a theoretical result nor an empirical regularity. One would expect the interaction between policy variables and growth rates to be more complex than the description given by an average correlation. Durlauf (2001) suggests that modeling parameter heterogeneity is one of the crucial topics in the agenda for empirical growth. More specifically, he suggests that the coefficient on the policy variables should depend on some measure of degree of the development of a country. In an unpublished article, Durlauf *et at*. (2000) assume that the Solow model is valid for each individual country however the model's parameters can vary from country to country. In particular, the country's individual

[4] See the recent survey by Durlauf and Quah (2000) for a long list of conditioning policy variables used in the growth literature.

parameters depend on their initial income. Their findings suggest that there is ample evidence of parameter heterogeneity. And that parameter heterogeneity is especially strong among poorer countries.

Third, several growth studies, Baumol (1986), and De Long and Dowrick (2001), to name a few, work with the idea of "convergence clubs". This typically implies sample segmentation. It is well known that segmenting the sample by the explanatory variable is in general not a problem. While sample segmentation based on the dependent variable can bias coefficient estimates as explained in Heckman (1979). In growth regressions the initial income variable appears on both sides of the equations[6]. Therefore, in this case is not clear, a priori, if sample segmentation bias coefficient estimates or not[7]. This remark is particularly important if the researcher is using the sign of the estimated coefficient on the initial income in order to discriminate growth models.

These concerns make clear that it would be useful to have a technique that could address the issue of income convergence in a more meaningful way providing a more complete picture of the association between policy variables and growth performance. This paper adopts a quantile regression approach to estimating and testing for income convergence. The quantile regression estimation procedure allows us to address the problems of parameter heterogeneity and sample selection, while the inferential procedure on the quantile regression process yield a new test of income convergence[8] and

---

[5] This statistical fallacy comes from the famous study of Sir Francis Galton on the height of fathers and sons. Sir Galton showed that sons of tall fathers tended to be shorter than their parents and fathers of tall sons tended to be shorter than their sons. That is the idea of regression to the mean and not convergence.

[6] Recall that the rate of growth is simply the final period income divided by the initial period income.

[7] Mello (2002) argues that apparently conflicting results on convergence tests, for instance, evidence of convergence for OECD countries but not for broadly constituted samples, can be reconciled once one takes into account the problem of sample segmentation.

[8] However, the tests of income convergence based on these inferential procedures are discussed in Mello (2002). Here we focus on the relationship between the policy variables and the growth rates.

allow us to analyze the effects of policy variables on the entire conditional distribution of GDP growth rates.

The quantile regression estimation procedure potentially yields a family of quantile coefficients; one for each sample quantile. Each slope coefficient can be interpreted as a different response of the GDP growth rate to a change in a policy variable corresponding to a different position on the conditional distribution of growth rates. Thus, in the quantile framework we may find that GDP growth rate responds differently to changes in human capital depending if the country is in the upper tail of the distribution (fast-growing country) or in the lower tail of the distribution (slow-growing country). This is one way to generate parameter heterogeneity in the relationship between policy variables and GDP growth rates. Furthermore, since we can generate the entire conditional distribution of GDP growth rate we can have a more complete picture of the relationship between rate of growth and policy variables without segmenting the sample.

Recent inferential procedures developed by Koenker and Xiao (2001) allow us to test hypothesis on the entire conditional distribution of GDP growth rates. In particular, we can test if policy variables affect the mean and the dispersion of the conditional distribution of GDP growth rates. The pure location shift hypothesis corresponds to a statistical model where the policy variables affect only the mean, and not any other aspect, of the conditional distribution of GDP growth rate. Thus, the pure location shift hypothesis requires the distribution of the error term to be independent of the policy variables. Note that, the classical least squares model corresponds to the pure location shift hypothesis. The location-scale hypothesis corresponds to a statistical model where policy variables affect the mean and the dispersion of the conditional distribution of GDP growth rate. The location-scale hypothesis can be thought to arise, for instance, from a regression model where the distribution of the error term is a function of the policy variables. Koenker and Xiao's (2001) test allows us to test the adequacy of least squares procedures in estimating growth equations; we can also use it as a new test of income convergence, as shown in Mello (2002).

Other attractive properties of quantile regression estimation are discussed in Koenker and Hallock (2000). One that is very important is the robustness of the quantile regression estimator. While the least squares estimator magnifies the effect of outlying observations, the quantile regression estimator is robust with respect to outliers in the dependent variable. This property is particularly convenient in our exercise given the fact that the data suggests that the distribution of GDP growth rates is fat-tailed[9].

In the next section, we briefly discuss the testable implications of growth models and concepts of income convergence. In section 3, we discuss the quantile regression estimation procedure. Section 4 is devoted to inferential procedures in the quantile regression framework. In section 5, we present our estimates of growth equations and the results of the aforementioned tests. Finally, section 6 concludes.

2. Exogenous Growth, Endogenous Growth and Concepts of Income Convergence

We follow Durlauf and Quah (2000) by defining a growth model to be of endogenous growth if an increase in the savings rate can affect the long-run equilibrium growth rate of income per capita; and of exogenous growth model if the savings rate is independent of the rate of growth. Recall that, in the traditional neoclassical exogenous growth model an increase in the savings rate affect the level of income but not the long-run equilibrium growth rate[10]. Examples of endogenous growth model include the articles by Romer (1986), Lucas (1988) and Rebelo (1991) among many others. Examples of exogenous growth models include, of course, Solow (1956) and MRW.

It is important to emphasize that endogenous growth model do not imply any specific sign on the coefficient on the initial income. When the savings rate can influence the long-run equilibrium growth rate, the regression equation consists of the GDP growth rate regressed on policy variables similar to the ones used in conditional convergence test. In this case, there is no prior on the sign of the initial income coefficient. This result can be used as a criterion to distinguish between exogenous and endogenous growth models.

Barro and Sala-i-Martin (1992) estimate the following equation derived from the Ramsey exogenous growth model to assess income per capita convergence:

$$(1/T)\log(y_{i,T}/y_{i,0}) = a - [(1-e^{-bT})/T]\log(y_{i,0}) + u_{i,0,T} \qquad (1)$$

where $y_{i,T}$ is the income per capita at the final period T, $y_{i,0}$ is income per capita in the initial period and disturbance term $u_{i,0,T}$ represents an average of the original errors term and β is the parameter that governs the speed of income convergence to the steady-state equilibrium[11]. The constant term is given by $a = g + [(1-e^{-bT})/T]\log(\hat{y}^*)$, where g is the rate of technological growth and $\hat{y}^*$ is the steady-state level of income per capita in efficiency units.

[9] See Barro and Sala-i-Martin (1995).

[10] See the discussion in Durlauf and Quah (2001) for more details. Also, Solow (2000) contains an interesting discussion on the differences and similarities between the neoclassical model and the endogenous growth models.

[11] More specifically, the parameter β is the stable eigenvalue of the log-linearized dynamical system in consumption and physical capital.

If the estimated parameter β is found to be positive, then we say that there is evidence of income convergence. Alternatively, a negative estimate implies divergence of incomes[12]. Note that the model implies that countries with the same tastes and technology should converge to the same steady-state level of income. Therefore, we should observe a negative relationship between initial income and rate of growth. That is, the model implies that the coefficient on the initial income coefficient should be negative.

Baumol (1986) and Barro (1991, 1997), estimate a linear version of equation (1). In this case, we have $(1/T)\log(y_{i,T}/y_{i,0}) = a + b\log y_{i,0} + u_{i,0,T}$. Here a negative slope coefficient b is consistent with convergence. However, this interpretation, as well as the one in the non-linear model, is subject to the caveats of the regression fallacy discussed in the introduction. Equation (1) can be easily modified to allow for conditioning variables to control for differences in the steady state income:

$$(1/T)\log(y_{i,T}/y_{i,0}) = a - [(1-e^{-bT})/T]\log(y_{i,0}) + y' X_0 + u_{i,0,T} \qquad (2)$$

where X_0 is a set of initial period policy variables conditioning for differences in the steady state level of income. Conditional β-convergence occurs whenever the estimated β is found to be positive.

The final concept of convergence we consider is σ-convergence. It can be shown that in the neoclassical model that the cross-sectional variance of the logarithm of income per capita at time t is related to the parameter that governs the speed of income convergence. This concept of convergence requires that the cross-sectional variance of the logarithm of income per capita to diminish over time.

---

[12] The parameter β can be used to estimate the time that it takes for an economy to transit from its current income level to its steady-state equilibrium income level. For instance, Barro and Sala-i-Martin (1992) find that the speed of convergence is approximately 1.8% for the 48 contiguous U.S. states for the period 1880-1990.

$$s_t^2 = e^{-2b} \cdot s_{t-1}^2 + s_{ut}^2 \qquad (3)$$

The above equation is a first-order linear difference equation in s_t^2. From the solution of (3), we can see how σ-convergence is related to β-convergence. Assume that the variance of the disturbance, s_{ut}^2, in equation (3) is constant over time. Then the solution to equation (3):

$$s_t^2 = \frac{s_u^2}{1-e^{-2b}} + (s_0^2 - \frac{s_u^2}{1-e^{-2b}}) \cdot e^{-2bt} \qquad (4)$$

Equation (4) shows that β-convergence (that is, β>0) is necessary but not sufficient for σ-convergence. It is possible that $(s_0^2 - \frac{s_u^2}{1-e^{-2b}}) < 0$, and therefore, even in the presence of β-convergence, income per capita can disperse over time.

One difficulty in interpreting conditional β-convergence is that it is not clear whether the conditioning variables entering the convergence equation are part of a reduced form growth model. Therefore, with few exceptions, notably MRW, these regressions cannot be interpreted in the light of a particular growth model. This fact also raises the question of what policy variables should be used as conditioning factors. Clearly, it becomes harder to interpret statistical significance if policy variables are chosen arbitrarily without any economic motivation[13]. Another potentially serious problem is that the conditioning variables usually are correlated with one another, and are endogenous variables themselves. The latter property can generate large standard errors and bias the least squares estimates. Furthermore, conditional β-convergence is subject to

---

[13] Levine and Renelt (1992) find that the presence of policy variables that are not strongly motivated by economic theory can have a substantial impact on the statistical significance of other control variables. They also find that the only robust conditioning variables are investment in physical capital and initial income per capita. However, the interested reader should consult the survey by Durlauf and Quah (2000) for a critique of Levine and Renelt's procedure.

the same pitfalls of unconditional convergence, namely, it is uninformative with respect to the catching up process. On the other hand, as pointed out in Durlauf and Quah (2000), evidence of σ-convergence cannot identify the formation of clusters within the cross-section distribution, for instance, a bimodal distribution. Although Friedman (1992) suggests that σ-convergence is the real test of convergence.

3. Quantile Regression: Estimation

Consider the simple linear regression model $y_i = x_i^{'}\boldsymbol{b} + u_i$ for i=1,…,n, where $\boldsymbol{b}$ is a Kx1 vector of coefficients[14], $x_i^{'}$ is nxK matrix of explanatory variables, y_t is the dependent variable and u_i is the error term with distribution not necessarily known. The least square estimator can be found by choosing the vector $\boldsymbol{b}$ that minimizes the sum of the squares residuals, that is $\min_{b \in \Re^K} \sum_{i=1}^{n} (y_i - x_i^{'}\boldsymbol{b})^2$. In contrast, the $\boldsymbol{t}^{th}$, $0 < \boldsymbol{t} < 1$, quantile regression estimator solves the following minimization problem:

$$\min_{b \in \Re^K} \left[\sum_{i \in \{i : y_i \geq x_i b\}} \boldsymbol{t} \left| y_i - x_i \boldsymbol{b} \right| + \sum_{i \in \{i : y_i < x_i b\}} (1 - \boldsymbol{t}) \left| y_i - x_i \boldsymbol{b} \right| \right] \qquad (5)$$

The objective function above is a weighted sum of absolute deviations, which can be interpreted as an asymmetric linear penalty function. It can be shown that the estimator for $\boldsymbol{b}$ is consistent and asymptotically normal[15]. An important special case of the quantile regression estimator is the least absolute deviation estimator (LAD) or median regressor.

---

[14] Here we will abuse the notation a little bit, since we are using the same Greek letter, β, to denote two different variables. In section 2 it denoted the speed of income convergence, here it denotes the vector of coefficient estimates.
[15] See Koenker and Bassett (1978) for the seminal article and Buchisnky (1998) for a recent survey.

One can see from (1) that if we vary the parameter τ on the [0,1] interval we can generate the entire conditional distribution of GDP growth rates. The coefficient $b_i(t)$ can be interpreted as the marginal impact on the t^{th} conditional quantile due to a marginal change in the i^{th} policy variable. Thus, the quantile regression approach allows us to identify the effects of the covariates at different points on the conditional distribution of the dependent variable. For instance, suppose that the dependent variable is the average GDP growth rate and consider $t = 0.10$, that is, countries that are in the left tail of the conditional distribution of GDP growth rate (slow-growing countries). And $t = 0.90$, that is, countries that are in the upper tail of the conditional distribution of GDP growth rate (fast-growing countries). We may have different estimates for the slope coefficients $b_i(0.10)$ and $b_i(0.90)$. If this is the case, a marginal change in a particular policy variable will affect the GDP growth rate differently, depending on being on the 10^{th} or 90^{th} quantile of the GDP growth rate conditional distribution. Under mean regression methods the slope coefficient is constrained to be the same for all quantiles. In this case, we are not able to see if policy variables affect countries differently.

It is important to understand when quantile regression is superior to traditional conditional mean estimation procedures. In particular, if the error term has the same distribution independent of the policy variables, then estimation of the conditional mean function can be informative about the conditional distribution of the GDP growth rate[16]. Implicit in this formulation is the idea that the policy variables affect only the location of the conditional distribution of the GDP growth rates. In this case, the least squares procedure is adequate for estimation purposes. However, when policy variables affect the

[16] One would probably like to complement the estimation of the conditional with the estimation of the conditional variance.

conditional distribution of the GDP growth rate in other ways, such as its dispersion, the skewness or kurtosis, estimation of the conditional mean is no longer adequate. In this case, quantile regression provides a more complete picture of the relationship between the GDP growth rate and the policy variables. Our results suggest that, in fact, policy variables affect the conditional distribution of the GDP growth rates in more complex ways than a simple location shift. Furthermore, we find that the ability to distinguish the effects of policy variables among different quantiles is important empirically.

4. Inference on the Regression Quantile Process

In this section we give the heuristics of the test procedure proposed by Koenker and Xiao (2001). They proposed test is based on the entire regression quantile process. It allows one to test if policy variables affect the location and/or the scale of the conditional distribution of the GDP growth rate. To motivate their test, consider the "treatment-control" problem in the statistics literature. Consider a random sample of size *n* drawn from a homogeneous population, which is randomized into n_1 treatment observations and n_0 control observations. We want to consider the effects of the treatment on the response variable Y_i. In our setting, we can imagine that the response variable is the GDP growth rate and the treatment is a policy variable such as initial income or human capital.

Lehman (1974) provides the general formulation for the treatment-control problem. He suggests that the distribution of the response variable is given by the response of the untreated subject, denoted by X, plus the amount that the treatment adds to the subject, denoted by $\Delta(X)$. Then, the distribution G of the treatment response is that of the random variable $X+\Delta(X)$. Assume that the distribution of X is given by H. Doksum

(1974) provides an axiomatic analysis of Lehman's formulation. Define Δ(X) as the "horizontal distance" between H and G at x as:

$$H(x) = G(x + \Delta(x)) \qquad (6)$$

Then $\Delta(x)$ can be expressed uniquely as $\Delta(x) = G^{-1}(H(x)) - x$. Let $t = H(x)$ and substitute it in the previous equation. We obtain a new variable, $d(t)$, that is called the "quantile treatment effect".

$$d(t) = \Delta(H^{-1}(t)) = G^{-1}(t) - H^{-1}(t) \qquad (7)$$

These two distributions can differ in several ways. Koenker and Xiao (2001) analyze two possibilities. First, they consider a pure location shift, that is, when $G^{-1}(t) = H^{-1}(t) + d_0$. In this case, the treatment affects the location of the distribution of the control variable and not any other aspect of the distribution. Second, they consider a location-scale shift, that is, when $G^{-1}(t) = d_1 H^{-1}(t) + d_0$. In this case, the treatment affects not only the location but also the dispersion of the distribution of the control variable.

It is illustrative to consider the pure location and location-scale shift models in the context of the linear regression model. The pure location shift model can be thought to come from the linear regression model with homoskedastic error terms. Consider the linear model $y_i = b_0 + b_1 x_i + u_i$, for i=1,…,n, where b_0 is the intercept, b_1 is the slope coefficient, y_i denotes the dependent variable and x_i is the explanatory variable. Denote by $Q(t|x)$ the t^{th} conditional quantile of y given x. We can write $P(y_i < Q|x) = F(Q - x_i' b|x) = t$ or $Q_i(t|x) = b_0 + b_1 x_i + F^{-1}(t|x)$, where F(.) is the distribution of the error term. We can simplify it even further to obtain $Q(t|x) = x' b(t)$, where $x^{'}$ is a nx2 matrix with a column of ones, and $\beta(\tau) = (\beta_0 + F^{-1}(\tau|x), \beta_1)^{'}$. In this case, the slope coefficient is independent of the quantiles. Consequently, a change in the policy variables has the same effect on countries, independent of their position on the conditional distribution of GDP growth rates. That is, if they are fast-growing countries or slow-growing countries. Note that the intercept coefficient is increasing in the quantiles since $F^{-1}(.)$ is increasing in τ. If the location shift hypothesis is valid then conditional mean estimation methods such as least squares can estimate well the relationship between the GDP growth rate and the policy variables.

The location-scale shift model can be thought to come from the linear regression model with heteroskedastic errors. Consider the linear model $y_i = \beta_0 + \beta_1 x_i + \sigma(x_i)u_i$, for i=1,…,n, where we use the same notation as above. Assume, for simplicity, that the form of heteroskedasticity is linear in the explanatory variable, $\sigma(x_i) = \gamma x_i$. Following the same steps as before we can easily obtain the conditional quantile function $Q_i(\tau|x) = \beta_0 + \beta_1 x_i + \gamma x_i F^{-1}(\tau|x)$. We can write it more concisely as $Q(\tau|x) = x^{'}\beta(\tau)$, where $x^{'}$ is a nx2 matrix with a column of ones, and $\beta(\tau) = (\beta_0, \beta_1 + \gamma F^{-1}(\tau|x))^{'}$. One can clearly see that in the location-scale shift model the slope coefficient depends on the quantile. Thus, changes in the policy variables affect countries differently depending on their position in the conditional distribution of GDP growth rates. Estimation of the conditional mean in this context would not uncover this effect, therefore, providing an incomplete picture of the relationship between growth and policy variables.

5. The Empirics of Economic Growth with Quantile Regression

We use Barro-Lee's data set available at http://www.nber.org/pub/barro.lee/ZIP/. The sample consists of 98 countries for which data was available for the period 1960-1985.

The first model we estimate is the Barro (1991) unconditional growth equation. The regression model is given by, $(1/T)\ln(Y_{85}/Y_{60}) = b_0 + b_1 \ln Y_{60} + e_i$, where the dependent variable is the average rate of growth of real GDP per capita for the period 1960-1985, the explanatory variable is the 1960 value of real GDP per capita and e_i is the error term. Barro (1991) finds that the correlation between the real per capita GDP in 1960 and the average rate of growth of real GDP per capita is 0.09, and not statistically significant. He interprets this finding as evidence in favor of endogenous growth models predicting income divergence among the economies such as Lucas (1988) and Rebelo (1991). Estimation of the regression quantiles gives a very different picture. Table 1 displays estimates for the 10th, 25th, 50th, 75th and 90th quantiles. The first observation is that, as discussed in section 3, we face some form of heteroskedastic error terms since the estimated coefficients vary with the quantiles. This result suggests that the quantile regression approach is important in order to identify empirically the different impacts of the initial income on the average GDP growth rate.

The estimated coefficient on initial income is positive for lower quantiles. This suggests that initial income per capita is positively correlated with the average GDP growth rate for countries in the lower tail of the distribution. The coefficient on initial income is decreasing in the quantiles turning negative at the 65th quantile (not shown in table 1). It is approximately –0.0008 at the 75th quantile and reaches –0.002 at the 90th quantile. Thus, the data suggests that there is evidence of income convergence for countries in the top 35% of the conditional distribution of GDP growth rates. One has to be careful in interpreting this result; it says that we have evidence of unconditional convergence *conditional* on countries being in the upper quantiles of the conditional distribution of GDP growth rates. This finding suggests that the growth performance for countries in the upper tail of the distribution can be well described by an exogenous growth model while the growth performance for countries in the bottom 65% of the distribution is better described an endogenous growth model.

The above finding is further explored in Mello (2002). He observes that the empirical growth literature find evidence of convergence for OECD samples (Barro, 1991, and MRW), that there is no convergence for broadly constituted samples (Barro, 1991) and that there is no or weak convergence for non-OECD samples. These empirical regularities make it clear that the result of convergence is sensitive to the sample choice as was also pointed out in De Long (1988). Mello (2002) argues that these apparently conflicting results of income convergence can be reconciled in the quantile regression framework using the ideas of sample selection bias as in Heckman (1979).

Table 1: Coefficient on the initial income per capita for selected quantiles.

Variable\Quantile	0.10	0.25	0.50	0.75	0.90
Intercept	-0.00795	0.00345	0.01498	0.03564	0.04676
Log of Initial GDP per capita (Y_{60})	0.00272	0.00188	0.00155	-0.00079	-0.00221

Note: The estimated equation is given by $\ln(Y_{85}/Y_{60}) = b_0 + b_1 \ln Y_{60} + e_i$.

Table 2 displays the calculated test statistic for location-scale hypothesis and for the pure location shift hypothesis for the unconditional growth equation, denoted by K_n and T_n, respectively.

Table 2: Location-scale tests on the unconditional growth equation

Variable	Location-Scale K_n	Location T_n
Initial GDP per capita (Y_{60})	2.6733	2.7505

Note: The estimated equation is the same as in table 1. The critical values are: 2.483, 1.986, 1.730, at 1%, 5% and 10% level of significance, respectively. The test is performed for ε=0.20. This parameter trims the lower and upper quantile 20th quantile in the computation of the test statistic.

The critical values are taken from Koenker and Xiao (2001, p.41). For the location-scale test the calculated value of K_n is approximately 2.67 and the critical value at 1% level of significance is 2.483, so that the null hypothesis is rejected. For the pure location shift hypothesis, the calculated test statistic is 2.75 and the critical values are the same, so that we reject the pure location shift for the unconditional growth regression. This finding suggests that initial income affects the conditional distribution of GDP growth rate in more complex ways than a location and/or scale shift. For instance, the initial income may affect the skewness, the kurtosis or the tails of the distribution. Moreover, this result also suggests that mean regression estimation methods such as least squares are inadequate for the study of growth equations.

The second model that we estimate follows MRW. We consider two functional forms. The first one is given by:

$$\ln(Y_{85}/Y_{60}) = \pi_0 + \pi_1 \ln Y_{60} + \pi_2 \ln(I/GDP) + \pi_3 \ln(n + g + \delta) + \xi_i$$

where Y_{85} is the real GDP per working-age individual in 1985, Y_{60} is the real GDP per working-age individual in 1960, I/GDP is the average share of real investment to real GDP over the period 1960-1985, n is the average rate of growth in the working-age population (defined as those individuals between ages of 15 and 64 years) and $g+\delta$ are, respectively, the rate of technological growth and the rate of depreciation. MRW assume that a reasonable value for the parameters $g+\delta$ is 0.05. This equation establishes out-of-steady-state dynamics for economies under the augmented Solow growth model by MRW.

We also estimate an augmented version of the above equation that includes a proxy for human capital.

$$\ln(Y_{85}/Y_{60}) = \beta_0 + \beta_1 \ln Y_{60} + \beta_2 \ln(I/GDP) + \beta_3 \ln(n+g+\delta) + \beta_4 \ln(h) + \varepsilon_i$$

where the notation is the same as before and h is a proxy for human capital. As in the previous equation, this regression model captures the local dynamics of economies under MRW's model. We use two different proxies for human capital, *Prim60*, primary school enrolment in the initial period and *Sec60*, secondary school enrollment at the initial period[17]. Table 3 displays our estimates for selected quantiles.

Table 3: Slope coefficients for selected quantiles for the MRW equations.

Variable \Quantile	0.10	0.25	0.50	0.75	0.90
Intercept	-0.0536	0.52608	1.75881	3.76151	5.99236
Ln(Y_{60})	-0.1353	-0.17131	-0.17552	-0.29747	-0.32080
Ln(I/GDP)	0.4985	0.51612	0.53579	0.50645	0.54131
Ln(n+g+δ)	-0.7836	-0.74325	-0.38106	-0.05772	0.62194
Intercept	-1.4076	-0.5362	1.01586	3.18936	5.2219
Ln(Y_{60})	-0.1377	-0.2297	-0.30442	-0.34137	-0.3667
Ln(I/GDP)	0.3584	0.4559	0.46376	0.40998	0.4106
Ln(n+g+δ)	-1.0689	-1.07063	-0.71586	0.04069	0.7488
Ln(Prim60)	0.1024	0.1304	0.17941	0.24221	0.3032
Intercept	-0.2983	0.9632	3.45054	4.87010	6.9434
Ln(Y_{60})	-0.1848	-0.2918	-0.36683	-0.54731	-0.4661
Ln(I/GDP)	0.2417	0.3822	0.32065	0.40488	0.3965
Ln(n+g+δ)	-0.7416	-0.68347	0.01323	-0.10033	0.8012
Ln(Sec60)	0.0907	0.1259	0.21001	0.21959	0.1690

Note: The first equation estimated is given by: $\ln(Y_{85}/Y_{60}) = \pi_0 + \pi_1 \ln Y_{60} + \pi_2 \ln(I/GDP) + \pi_3 \ln(n+g+\delta) + \xi_i$. The second equation estimated is given by: $\ln(Y_{85}/Y_{60}) = \beta_0 + \beta_1 \ln Y_{60} + \beta_2 \ln(I/GDP) + \beta_3 \ln(n+g+\delta) + \beta_4 \ln(h) + \varepsilon_i$, where ln(h) denotes the proxy for human capital.

[17] We also estimated an equation with the two proxies together, however we saw no improvement in the fit and omitted the results of this specification.

The first equation in table 3 indicates that the coefficient on the initial GDP per working-person is negative at all quantiles, suggesting evidence of conditional convergence. The results for the entire quantile regression process (not shown here) indicate that for the top 30% fastest-growing countries, the estimated coefficient is well below zero and statistically different from the least squares estimate. For instance, for the 10^{th} quantile the coefficient on the initial income per capita is -0.1353 while for the 90^{th} quantile the coefficient is –0.3208. MRW's estimate for the coefficient on the initial income is –0.141 for the large sample (98 countries) and –0.351 for the OECD sample. First, one should note that our estimates are consistent with those of MRW. More interestingly, our estimate for the bottom 10% slow-growing countries is similar to MRW's estimate for the large sample and our estimate for the top 10% fast-growing countries is similar to his estimate for the OECD sample. This suggests that the forces of convergence come from countries in the upper tail of the conditional distribution of GDP growth rates.

This observation suggests that conditional convergence is "stronger", in some sense, for the top 30% fastest-growing countries in the sample. One way to give a numerical measure of the strength of the convergence forces is to calculate from the estimated coefficient on the initial income the speed of convergence and the implied half-life[18]. The half-life gives the time that it takes for an economy to transit half way to its steady state. MRW's estimate of the speed of convergence for the large sample is 0.6% per year. This estimate implies a half-life of approximately 116 years. For the OECD sample, the speed of convergence is 1.7% per year with a half-life of approximately 41 years. Our results suggests that for the bottom 10% slow-growing countries the speed of convergence is

0.6% per year with a half-life of approximately 116 years, virtually the same numbers as MRW, and for the top 10% fast-growing countries we have a speed of convergence of 1.5% per year with a half-life of approximately 46 years. If one takes these estimates as benchmarks, a half-life of 116 years can be considered extremely slow. This finding could be used as a strong argument for rich fast-growing countries to increase their help to poor slow-growing countries.

The coefficient on the investment rate is remarkably stable across the quantiles and is not statistically different from its least squares estimate. For instance, for the bottom 10% of distribution it is equal to 0.4985 increasing up to 0.54131 for the top 10% of the distribution. This coefficient gives the ratio of share of wealth invested in physical capital to the share of wealth invested in the other inputs and its predicted value should be 0.5 (see MRW details). The empirical value for this coefficient has the sign and magnitude consistent with the predicted value. Finally, the estimates for the effects of population growth, rate of technological growth and depreciation rate are positively correlated with the rate of growth for the 20% fastest growing countries in our sample. This suggests that for countries in the upper tail of the distribution population growth has a stronger association economic growth[19].

The estimated coefficients for the second equation in table 3 are significantly different from zero and are consistent with the previous equation. MRW use as a proxy for human capital the percentage of the working-age population that is in secondary school. Since in this equation we use a different proxy for human capital our results are

---

[18] The formula for the speed of convergence is given in MRW. The half-life can be easily calculated as $\ln 2/\lambda$, where λ is the speed of convergence.

[19] Although, in the MRW model population growth is considered exogenous, inferring about causation here may be a bit subtle. One can easily think of models where population growth is endogenous and rate of GDP growth is exogenous.

not directly compared to those in MRW. However, it is interesting to look at the numbers for the speed of convergence and the implied half-life for our second equation. The coefficient on the initial income is negative for all quantiles and increases in absolute value from 0.14 for the 10th quantile to 0.37 for the 90th quantile. For the bottom 10% slow-growing countries the speed of convergence is virtually the same as in MRW large sample estimates, 0.6% per year with an implied half-life if approximately 116 years. For the top 10% fastest-growing countries the speed of convergence is 1.8% per year with an implied half-life of 38 years. Our estimates are consistent with previous studies suggesting evidence of conditional convergence for broadly constituted samples.

The estimated coefficient for *Prim60* increases considerably for the upper quantiles, reaching 0.30 in the 90th quantile from 0.10 in the 10th quantile. These estimates suggest that human capital has a "stronger", in some sense, impact on fast-growing countries than in slow-growing countries. Although, no one disputes the importance of investment in human capital as an engine of long-run growth and social development, this finding should serve to call the attention of policy makers to the fact that we need a deeper understanding of the relationship between human capital and growth before prescribing economic policy to slow-growing countries.

Similar results are obtained when the variable *Sec60* is used as a proxy for the human capital. The coefficient estimates are shown as the third equation in table 3. As in the previous specification, there is evidence of "stronger", in some sense, conditional convergence for rapid growing countries. This regression specification can be compared with MRW's. Their estimate of the coefficient of the initial income for the large sample is –0.289 and for the OECD sample is –0.398. The speed of convergence is 1.4% and 2.00% per year, respectively. The implied half-life for the large sample is approximate 50 years, and approximately 34 years for the OECD sample. Our estimates suggest that the coefficient for the bottom 10% slow growing countries is –0.1848, with a convergence speed of 0.8% and half-life of approximately 85 years. For the top 10% fast-growing countries the initial income coefficient estimate is –0.4661, with a convergence speed of 2.5% and an implied half-life of approximately 28 years.

The quantile regression processes for the human capital variable, investment ratio and population growth are not significantly different from the least squares estimates. At baked eye one might, therefore, conjecture that conditional mean methods are adequate to estimate MRW's regression model. However, we next provide evidence that the data reject the pure location shift hypothesis.

Table 4 displays the calculated test statistics for the pure location shift and the location-scale shift hypothesis for MRW model. We performed the joint tests and the individual tests on the coefficients. It is important to highlight that the individual tests violate the assumption of independence among the coordinates. Nevertheless, it is still informative to see which policy variables contribute the most to the test statistic K_n and T_n. When we look at the individual tests we find that population growth is the policy variable that contributes the most to the location-scale test statistic, while the initial income contributes the most to the location test statistic.

The location-scale hypothesis for the first equation in table 3 yields a test statistic of 6.987. The critical value at 1% level of significance is 4.893, so that we reject the null of a location and scale shift. The pure location shift test yields a test statistic of 7.29 and since the critical values are the same as stated above, we clearly reject the null hypothesis at 1% level of significance. These results suggest that the policy variables affect the conditional distribution of GDP growth rates in more complex ways than the location and/or scale shift hypothesis predict. The tests also indicate that mean regression methods such as least squares are not adequate to assess the effect of the policy variables in the GDP growth rate and of convergence.

Table 4: Location-scale tests on the MRW model

Variable/Equation	Location-scale	Location	Location-scale	Location
	Equation 1		Equation 2	
$Ln(Y_{60})$	0.739	1.287	0.561	1.047
Ln(I/GDP)	0.428	0.721	0.101	0.821
$Ln(n+g+\delta)$	0.791	0.812	1.061	0.965
Ln(Prim60)	--	--	0.867	1.181
Joint test	6.987	8.129	4.201	5.868
	Equation 3			
$Ln(Y_{60})$	0.650	1.224		
Ln(I/GDP)	0.980	1.753		
$Ln(n+g+\delta)$	0.749	0.603		
Ln(Sec60)	0.799	0.961		
Joint test	6.354	13.121		

Note: The test is based on the estimated equations in table 3. The critical values for the tests on individual coefficients are given by: 2.483, 1.986, 1.730, at 1%, 5% and 10%, respectively. The critical values for equation 1 are given by 4.893, 4.133 and 3.749 at 1%, 5% and 10%, respectively. The critical values for the joint test for equation 2 and 3 are given by: 6.023, 5.091, 4.684, at 1%, 5% and 10%, respectively.

The test statistic for the location-scale hypothesis for the second equation in table 3 is approximately 4.20 and the critical value at 10% level of significance is 4.684, so that we fail to reject the null hypothesis of a location and scale shift. However, for the same equation we reject the pure location shift hypothesis. The policy variable that contributes the most for the test statistic for the location and scale hypothesis is, as in equation 1, population growth. And the policy variable that contributes the most to the pure location shift test statistic is initial income. Finally, for the third equation, when *Sec60* is used as a proxy for human, we reject both the location-scale shift and pure location shift hypotheses. Overall our results reveal that policy variables such as human capital, investment rate, population growth affect the conditional distribution of the GDP growth rates in more complex ways than prescribed by a location and/or scale shift. The final model we estimate follows the general specification in Barro (1991):

$$\ln Y_{85}/Y_{60} = \beta_0 + \beta_1 \ln Y_{60} + \beta_2 \ln Sec60 + \beta_3 \ln Prim60 + \beta_4 \ln G^c/Y + \beta_5 \ln Rev$$
$$+ \beta_6 \ln Assass + \beta_7 \ln PPIDev60 + \varepsilon_i$$

where the dependent variable is the real GDP per capita growth rate for the period 1960-1985, Y_{60} is the initial level of GDP, *Sec60* and *Prim60* are, respectively, the values of school-enrollment rates for secondary and primary levels, G^c/Y is the average from 1970-1984 of the ratio of real government consumption (exclusive of defense and education) to real GDP, the variable *Rev* is the number of revolutions and coups per year for 1960-1985, the variable *Assass* is the number of assassinations per million population for 1960-1985 and *PPIDev60* is the magnitude of the deviation of the 1960 purchasing power parity value of investment deflator[20].

Barro (1991), using ordinary least squares in a sample of 98 countries, finds that the growth rate of income per capita for the 1960-1985 period is negatively correlated with initial GDP per capita, positively correlated with the two measures of human capital (*Prim60* and *Sec60*) and negatively correlated with governmental consumption, number of revolutions per year, number of assassinations per million population and with the deviation from the purchasing power parity index. The equation below gives the estimated coefficients for the Barro equation. All variables are significant at conventional levels of significance. Table 5 below displays the quantile regression estimates for the Barro equation.

[20] This variable is a proxy for market distortions. In particular, it captures a situation where one could observe artificially high investment prices or artificially low investment prices. This variable should be negatively related with economic growth. For more details on the variables, the reader is referred to Barro (1991).

$$\ln(Y_{85}/Y_{60}) = 0.0302 - 0.0075\ln Y_{60} + 0.0305\ln Sec60 + 0.025\ln \Pr im60 - 0.119\ln G^c/Y$$
$$- 0.0195\ln \operatorname{Re} v - 0.0333\ln Assass - 0.0143\ln PPIDev60$$

Table 5: Slope coefficients for selected quantiles for the Barro equations.

Variable \Quantile	0.10	0.25	0.50	0.75	0.90
Intercept	0.0125	0.0220	0.01089	0.01911	0.0225
Y_{60}	-0.0040	-0.0034	-0.00357	-0.00511	-0.0058
Prim60	0.0161	0.0186	0.01793	0.03196	0.0491
Sec60	0.0382	0.0172	0.03244	0.02983	0.0092
G^c/Y	-0.0618	-0.12591	-0.03867	-0.04294	-0.0424
Rev	-0.0127	-0.01699	-0.01081	-0.01053	-0.0055
Assass	-0.1586	-0.05511	-0.03673	-0.02111	-0.00341
PPIDev	-0.0178	-0.00453	-0.00452	0.00008	-0.0079

Note: The estimated equation is given by:

$$\ln Y_{85}/Y_{60} = b_0 + b_1 \ln Y_{60} + b_2 \ln Sec60 + b_3 \ln \Pr im60 + b_4 \ln G^c/Y + b_5 \ln \operatorname{Re} v$$
$$+ b_6 \ln Assass + b_7 \ln PPIDev60 + e_i$$

The estimates on the coefficient on the initial income per capita suggest that there is evidence of conditional convergence. The magnitude of the coefficient indicates that convergence occurs at a slow pace. Although the estimated coefficients vary with the quantiles, they are not significantly different from the least squares estimate. The quantile regression process for the variable *Sec60* is positive and it varies cyclically with the quantiles. According to the estimates this variable is strongly correlated with the rate of growth for the first and third quartiles. The variable *Prim60* is positively correlated with the growth rate at all quantiles and it shows an upward trend as the quantile increases. This suggests that human capital is “strongly” correlated with average GDP growth rates for countries in the upper tail of the distribution. This result is consistent with our estimates in table 3.

Government consumption is negatively correlated with the growth rate and its effects appear to be stronger for the bottom half of the fastest growing countries. In the same manner, the variable *Rev*, which measures of political stability, is negatively related with the rate of growth and this effect is more pronounced for the bottom 50% of the fastest growing countries. The variables, *Assass* and *PPIDev60*, have a negative association with the rate of growth. However, in both cases the coefficient is not statistically significantly different from zero. We tested their joint significance using the rankscore-based test proposed by Gutenbrunner *et al*. (1993) and fail to reject the null that their coefficients are jointly equal to zero. This result contrasts with Barro's, where he finds that both variables are statistically significant with the predicted signs.

Table 6 contains the tests for the location-scale hypothesis for Barro's equation. The calculated test statistic for the location and scale shift is 91.33 and the critical value at 1% level of significance is 9.094, so that the null hypothesis of a location and scale shift is rejected. We also reject the null hypothesis of a pure location shift effect. The calculated test statistic is 18.88 against the same critical values as in the location-scale shift. The policy variable that contributes the most to both tests statistic is the *PPIDev60*. Our results for the Barro model are in accordance with the previous ones, these policy variables affect the conditional distribution of GDP growth rates in more complex ways than a location and/or scale shift. Since the pure location shift is rejected, we again have evidence that mean regression methods are inadequate to estimate growth equations.

Table 6: Location-scale tests on Barro's regression

Variable	Location-scale	Location
Y_{60}	1.9405409	1.92920562
Sec60	1.2444785	1.02716272
Prim60	0.7666357	0.83176546
G^c/Y	1.4146483	1.54562329
Rev	0.7311822	0.93488999
Assass	1.2090607	0.84505831
PPIDev	2.5509280	2.76771075
Joint test	18.50341	14.88212

Note: The estimated equation is the same as in table 3. The critical values for the joint test are: 9.094, 7.887 and 7.299 at 1%, 5% and 10% level of significance, respectively. The critical values for the individual coefficients are 2.483, 1.986 and 1.730 at 1%, 5% and 10% level of significance, respectively.

6. Conclusion

This paper suggests a new statistical framework to assess income convergence and the relationship between GDP growth rate and policy variables. Rather than relying on mean regression estimation methods such as least squares we propose using quantile regression to estimate and make inference from growth equations. Estimation of the regression quantiles generates a family of conditional quantile functions giving a more complete picture of the conditional distribution of the GDP growth rate. Moreover, the solution to the objective function of the quantile regression estimator potentially yields different solutions to different quantiles allowing us to examine how the GDP growth rate responds to the policy variables at different points in its conditional distribution. Furthermore, recent developments in inferential methods for quantile regression allow us to test whether the policy variables affect the location and/or the scale of the conditional distribution of the GDP growth rate.

Estimation of the regression quantiles for the unconditional growth equation suggest that there is evidence of income per capita convergence for the top 35% fastest growing countries in our sample. Consequently, we have evidence of divergence for the bottom 65% slow-growing countries. This finding suggests new directions for theoretical growth models. In particular, it would be interesting to examine what is required from a growth model in order to generate these empirical regularities.

Estimation of the MRW model suggests ample evidence of conditional convergence; this is consistent with the previous literature. We find that for the conditional growth equation the coefficient on the initial income per capita is increasing in the quantiles. This suggests that convergence is stronger, in some sense, for the fast-growing countries. Our estimates suggest that countries in the upper tail (top 10%) of the conditional distribution of GDP growth rate exhibit a speed of convergence that is three times faster than countries that are in the lower tail (bottom 10%). For the latter, we estimate that the implied half-life is approximately 116 years. If this number is to be taken seriously, it urges rich countries to increase their help to develop the slow-growing economies.

Tests of the location and scale shift hypothesis suggest that mean regression estimation methods such as least squares are not adequate to study growth equations. We reject the location-scale and pure location hypothesis for all, except one, regression model that we estimated. Thus, policy variables may affect the distribution of GDP growth rates in alternatives ways, other than the location and/or the scale of the distribution. For instance, they can affect the tails, the skewness, the kurtosis, or virtually, in any other way we can think of.

One important feature of our article is that it explicitly shows the failure of traditional conditional mean estimation methods as a statistical framework for the analysis of economic growth. Not only that but the article also provides an alternative statistical framework capable of providing a more complete and systematic analysis of the countries' growth experience. Furthermore, our finding that policy variables affect countries differently depending on their position on the conditional distribution of GDP growth rates suggests that parameter heterogeneity is a central characteristic of the growth process. Although, in theory researches acknowledge the importance of country heterogeneity, only few empirical studies, notably Durlauf et al. (2000), take into account heterogeneity explicitly. These two observations together should be used to stimulate researchers to look for alternative statistical methods that can meaningfully address the issue of country heterogeneity.

References:

Barro, R.J. 1991. Economic growth in a cross-section of countries. *Quarterly Journal of Economics* **106**: 407-443.

_______, and Sala-i-Martin, X. 1992. Convergence. *Journal of Political Economy* **100**: 223-251.

_______, and Sala-i-Martin, X. 1995. *Economic Growth*. Mc-Graw Hill.

Baumol, W. 1986. Productivity, convergence and welfare: what the long run data show. *The American Economic Review* **76**: 1072-1085.

Buchinsky, Moshe. 1998. Recent Advances in Quantile Regression Models: a Practical Guideline for Empirical Research. *Journal of Human Resources*, **33**: 88-126.

De Long, J.B. 1988. Productivity, convergence and welfare: comment. *The American Economic Review* **78**: 1138-1154.

Durlauf, S. and Bernard, A.1995. Convergence in international output. *Journal of Applied Econometrics* **10**: 97-108

______, and Bernard, A. 1996. Interpreting tests of the convergence hypothesis. *Journal of Econometrics* **71**: 161-173

_____, and D. Quah. 1998. The new empirics of economic growth. *Center for Economic Policy Performance*, discussion paper # 384.

______, 2000. "Econometric analysis and the study of growth: a skeptical perspective", department of economics, U. of Wisconsin at Madison.

______, A. Kourtellos and A. Minkin. 2000. "The local Solow Growth Model", department of economics, U. of Wisconsin at Madison.

Friedman, Milton. 1992. Do Old Fallacies Ever Die? *Journal of Economic Literature* **30**: 2129-2132.

Hampel, F. 1968. Contributions to the theory of Robust estimation. Ph.D. Thesis, University of California at Berkeley.

Ihaka, Ross and R. Gentleman. 1996. R: A language for Data Analysis and Graphs. *Journal of Computation and Graphical Statistics* **5-3**: 299-314.

International Financial Statistics. 1999. International Monetary Fund, Washington D.C.

Islam, N.1995. Growth empirics: a panel data approach. *Quarterly Journal of Economics* **110**: 1127-1170.

Koenker, R. and G. Bassett. 1978. Regression Quantiles. *Econometrica*, **46**:33-50

_____, and S. Portnoy. 1996. Quantile Regresion. Working paper #97-0100, University of Illinois at Urbana-Champaign.

_____, and K. Hallock. 2000. Quantile Regression: an Introduction. *Journal of Economic Perspective*, forthcoming.

_____, and Z. Xiao. 2002. Inference on the Regression Quantile Process. *Econometrica*, forthcoming.

_____, and J. Machado. 1999. Goodness of fit and related inference processes for Quantile Regression. *Journal of the American Statistical Association*, pp.1296-1310.

Khmaladze, E. V. 1981. Martingale Approach in the Theory of Godness-of-fit Tests. *Theory of Probability and its Applications*. 26:240-257.

Lucas, Robert E. 1988. On the Mechanics of Economic Development. *Journal of Monetary Economics*.

Levine, Ross and D. Renelt. 1992. A Sensitivity Analysis of Cross-Country Growth Regressions. *American Economic Review*, **84**:942-963.

Mankiw, N.G., Romer, D. and Weil, D. 1992. A contribution to the empirics of economic growth. *Quarterly Journal of Economics* **107**: 407-437.

Mello, Marcelo. 2002. On the Conditional Distribution of Growth Rates: Convergence Reconciled. University of Illinois at Urbana-Champaign.

Quah, D. 1993a. Galton's fallacy and the tests of the convergence hypothesis. *Scandinavian Journal of Economics* **95**: 427-443.

Solow, R. 1956. A Contribution to the theory of economic growth. *Quarterly Journal of Economics* **70**: 65-94.

_____, 2000. Growth Theory: An Exposition. Oxford University Press.