Job Ladder and Business Cycles*

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Abstract

I study the aggregate implications of job-to-job flows in a Heterogeneous Agents New Keynesian model. Workers search on-the-job and cannot directly insure against the earnings risk stemming from climbing and falling off the ladder. The state of the economy depends on the distribution of workers over match productivity, earnings, and wealth. The job ladder is shown to have both supply and demand-side consequences over the business cycle: the employment reallocation over the ladder moves labor productivity in response to aggregate shocks, while workers' demand for consumption reacts to changes in labor market flows. In the wake of an adverse financial shock, reallocation over the job ladder slows down, keeping workers stuck at low-productivity jobs. Aggregate productivity falls gradually over time, and drags down consumption and output even further. These patterns match the behavior of aggregates during and after the Great Recession, with the reduction in labor productivity explaining both the slow recovery and the missing disinflation.

JEL Codes: D31, D52, E21, E24, E31, E32.

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1. Introduction

Labor market frictions matter for the transmission mechanism of aggregate shocks to the economy by shaping both the supply and demand for goods. On the supply side, search frictions give rise to unemployment and allow for good (productive) jobs to coexist in the market alongside bad (unproductive) ones—unemployed workers accept low-quality jobs because they can keep searching for better, more productive jobs while employed. Hence frictions restrict the supply of goods both through unemployment, which constraints the overall amount of labor used in production, and through the misallocation of employed workers, which affects the level of aggregate productivity. On the demand side, the workers' employment history is an important determinant of their income dynamics. Unemployment spells can have long lasting impact on labor earnings, while job-to-job transitions drive earnings growth of the employed. Since labor income is the primary source of workers' overall disposable income and accounts for a significant portion of their income risk, these events directly affects workers' consumption expenditures and precautionary savings decisions. Both supply and demand consequences are mediated by labor market flows (i.e., unemployment to employment and job-to-job flows), which fluctuate over the cycle.

To study how these supply and demand channels stemming from the labor market play out in equilibrium and over the cycle, I develop a Heterogeneous Agents New Keynesian (HANK) model with search and matching frictions. The model features a continuum of risk averse workers who search both off and on-the-job for vacancies posted by firms. Worker–firm matches are heterogeneous in productivity. Matches are destroyed at some exogenous rate, in which case the worker becomes unemployed. This setting gives rise to a *job ladder*: leaving unemployment is just the first rung of this ladder, which employed workers keep climbing by contacting and moving toward more productive jobs. Bertrand competition among employers in the spirit of Postel-Vinay and Robin (2002) determines how wages evolve upon job-to-job transitions and within matches upon the arrival of outside offers that do not trigger a job change. Workers face borrowing constraints and cannot directly insure against labor earnings risks stemming from climbing and falling off the ladder. The remaining blocks of the model closely follow the New Keynesian tradition. The output of the worker–firm match, which I call "labor services," is an input to the production of monopolistically competitive retailers, who face nominal rigidities. Retailers produce specialized goods by combining labor services and intermediate material goods, which they sell to a representative final good producer. A government runs an unemployment insurance program, and monetary policy follows a Taylor rule.

Market incompleteness and the job ladder make the cross-sectional distribution of workers over match productivities, earnings, and wealth part of the equilibrium. Unemployment to employment flows and job-to-job transitions are endogenous and respond to aggregate shocks, moving workers along the ladder with consequences for labor earnings and aggregate productivity. Nominal rigidities render output partly demand-determined, so demand and supply forces interact in equilibrium to determine the response in the labor and goods markets.

I use this environment to study the response to an adverse financial shock, which I calibrate to mimic unemployment dynamics around the Great Recession.[1] I show that the model does well in accounting for the joint behavior of labor market flows, labor productivity, consumption, and inflation. In particular, the model generates a rise in unemployment, a drop in job-to-job transitions, and a persistent contraction in consumption and productivity. Inflation features only a *transitory and moderate drop*, as in the data.[2] This behavior is explained by the offsetting (dis)inflationary pressures coming from the job ladder, whose forces vary along the transition. In the model, nominal rigidities give rise to a New Keynesian Phillips curve, which links inflation to the discounted sum of future marginal costs. During the initial periods following the shock, consumption falls sharply in response to the reduction in future income, which leads to a large contraction of marginal costs. As time passes, however, the decline in job-to-job transitions slows down worker reallocation up the ladder causing labor productivity to fall.[3] This force is persistent and exerts upward pressure on marginal costs at longer horizons, which prevents inflation from falling too much at the onset of the recession.

In the remainder of the paper, I explore in more depth the demand and supply-side channels operating through the job ladder. Turning to the supply-side effects first, I study a counterfactual equilibrium where labor productivity is kept fixed, so the supply of labor services varies only along the unemployment-employment margin. I show that the full job ladder, which takes into account the misallocation among employed workers, increases the persistence of the consumption response and helps account for the slow recovery following the recession. The rationale for this result is simple. When the economy undergoes a recession, the reduction in labor market flows not only increases unemployment but also leaves employed workers stuck at low-productivity jobs. The employment distribution along the job ladder is a slow-moving state that impairs production even after the direct effects of the shock have died out, delaying the return of the economy to steady state. The unemployment-employment margin by itself offers only a restricted view of the state of the labor market, and misses what transpires among employed workers, whose dynamics

[1] Specifically, I shock the discount rate of labor services intermediaries, which reduces their incentives to post vacancies in the labor market.

[2] The absence of significant disinflation during the Great Recession, usually referred to as *missing disinflation*, is seen as a puzzle by some economists. I discuss this fact and why it is surprising in Section 5.

[3] The productivity consequences of job-to-job transitions were first raised by Barlevy (2002), who named it the *sullying effect* of recessions. This contrasts with the so-called "cleansing effect" of recessions, according to which recessions may increase labor productivity through the destruction of the least productive jobs.

are as, if not more, important to production. This point is also highlighted by Moscarini and Postel-Vinay (2019), which I further discuss in the literature review.

To understand the ladder's demand-side implications, I study the transmission mechanism of the shock to consumption. I start by decomposing the consumption response to the shock in the same way as Kaplan, Moll, and Violante (2018).[4] In the context of a monetary policy shock, the authors found that changes in household disposable income are the main drivers of the consumption response.[5] Here, income changes can materialize through (i) changes in the aggregate component of wages, dividends, and lump-sum transfers that affect the *current income* of all workers, and (ii) changes in labor market transition rates that affect the expectation of *future income growth*. For the financial shock I consider, I find that the bulk of the movement in aggregate consumption comes from fluctuations in labor market transition rates. In particular, this channel operates mainly through changes in the job-to-job flows and not in the job finding rate of unemployed workers.

I also study the model's cross-sectional consumption response upon the impact of the shock. Interestingly, I find that workers who reduce their consumption the most are the *non hand-to-mouth* located at the lower rungs of the ladder (mainly the unemployed and recently hired employed workers). This result contrasts with the existing HANK literature, which has thus far mainly emphasized the role played by *constrained hand-to-mouth* agents in the transmission of aggregate shocks to consumption. The reason for this difference is the following. Workers standing on the first rungs of the ladder rely on labor market transitions to grow their labor earnings, and there are the workers most impacted by the collapse of the job ladder. This fall in reallocation impacts workers by decreasing their expected future labor earnings growth, but not their current disposable earnings. While *hand-to-mouth* workers have a unit marginal propensity to consume (MPC) out of the latter, they are not sensitive to changes in future earnings. Conversely, unconstrained workers respond to the decline in expected future labor earnings growth by adjusting their consumption expenditure plans.

I further explore the contribution of incomplete markets by solving a version of the model that shares the same supply-side structure, but features perfect consumption insurance (*complete markets*) on the worker side. I study the complete markets economy response to the same shock

[4]Specifically, the exercise computes the partial equilibrium consumption in which some variables in the worker's problem adjust as in equilibrium, while others are kept fixed at their steady-state level. This exercise is useful because it sheds light on the transmission mechanism of the shock to consumption by indicating which variables in the worker's problem most account for the consumption response.

[5]The authors state the result in terms of direct and indirect effects. The direct effects of the monetary policy shock are those stemming from changes in the real rate alone; that is, those that operate even in the absence of any change in household disposable labor income. The indirect effect is the change in consumption coming from the movements in household income that arise in general equilibrium, which mostly operate through an increase in labor demand. They show, in the context of their two-asset HANK model, that most of the consumption response to monetary policy shock comes from the indirect effects.

and I find that incomplete markets *dampens* the consumption response. I analyze how this result connects with the existing literature trying to explain the differences between HANK and RANK economies and discuss the forces that can account for the dampened response. Finally, I also study the economy's response to other typical macro shocks, such as monetary and TFP shocks. The job ladder demand and supply channels also play a role under these more conventional shocks. A monetary shock, for instance, transmits to consumption by raising both current (through aggregate wages) and expected future labor income (through labor market transitions).

Literature. Job-to-job flows are abundant in the data and represent over half of new hires each month.[6] Besides its contribution to overall flows, job-to-job transitions constitute a major source of productivity and earnings growth, making them important for the transmission of aggregate shocks more generally. In this section, I start by discussing some of the empirical evidence on the (cyclical) job ladder and its consequences for worker allocation and earnings. Later, I describe how this paper connects to the literature.

The defining characteristic of the job ladder is that "workers agree on a common ranking of available jobs which they aspire to climb through job search, while being occasionally thrown back into unemployment".[7] Whenever given the opportunity, workers tend to move toward "better jobs". Therefore, a robust implication of the ladder is that higher ranked firms should be more successful in attracting and retaining workers. Bagger and Lentz (2018) use this insight to rank firms in Danish matched employer-employee data by the fraction of their hires filled by workers coming from other jobs, as opposed to unemployment. They show that firms' position in this "poaching rank" is stable over time and positively correlated with the firm's value added per worker, suggesting that firms high up in the ladder are also more productive. Looking at the US, Haltiwanger et al. (2018) documents the presence of a robust wage ladder.[8] They find that net flows from low-wage to high-wage firms is highly procyclical, with movements from bottom to high rungs declining by 85% during the Great Recession. Finally, Crane, Hyatt, and Murray (2019) implement four different methods to rank firms by productivity using matched employer-employee data for the US and find, irrespectively of the method used, that the firm productivity distribution shifts down in recessions. Taken together, this evidence highlights the role of the job ladder shaping employment allocation over the business cycle.

As for the impact of job-to-job transitions on earnings, there is extensive empirical evidence docu-

[6] Job-to-job transition probabilities fluctuate around 2.4%, an order of magnitude smaller than the job finding probabilities, but since the measure of employed worker is also much bigger than the measure of unemployed agents, gross flows are similar.

[7] Citation from Moscarini and Postel-Vinay (2017).

[8] See Moscarini and Postel-Vinay (2017) for different ways one can try to identify job ladder rungs.

menting that workers experience wage increases when they undergo a job-to-job transition.[9] Just as important, even employed workers who do not switch jobs may still benefit from outside offers, as those can be used to increase their wages at their current jobs. As evidence of the latter mechanism, Moscarini and Postel-Vinay (2017) find, using longitudinal microdata from the Survey of Income and Program Participation (SIPP), that earnings growth covaries with "predicted" job-to-job transitions even among workers who do not actually experience one. The "predicted" rate means to capture how likely it is for a worker to undergo a job-to-job transitions based on effective transitions experienced by observationally similar workers. The authors interpret the positive correlation as evidence of worker's gaining surplus via outside offers, as they would in a sequential auction model like that of Postel-Vinay and Robin (2002).

Next, I discuss how this paper relates to the literature. By featuring risk averse workers making consumption and savings decisions in an environment with search frictions and on-the-job search, this paper relates to Lise (2012). His partial equilibrium analysis is the building block of the demand-side of my model, as the regular income fluctuation problem in the traditional heterogenous agent incomplete markets model. This paper also relates to the extensive labor literature studying cyclical movements in labor market flows. Papers in this literature tend to feature workers with linear-utility and do not address the impact of the job ladder on aggregate variables outside the labor market (see Menzio and Shi (2011), Robin (2011), Lise and Robin (2017), Moscarini and Postel-Vinay (2018)). In the few cases where labor market frictions are incorporated into business cycle frameworks with consumption decisions and nominal rigidities, models tend to abstract from job-to-job flows (see Christiano, Eichenbaum, and Trabandt (2016)).

An exception is the work of Moscarini and Postel-Vinay (2019), which heavily motivates this paper. They are the first to introduce a job ladder into a DSGE New-Keynesian model and study the aggregate responses to productivity, preference, and monetary shocks. Backed by their previous empirical work uncovering a positive relation between job-to-job transitions and wage inflation,[10] the authors use the model as a laboratory to test the predictive power of labor market flows on future inflation. While I share their motivation to study the role of the job ladder over business cycles, this paper differs from theirs in two respects. First, I examine economy's response to an adverse financial shock and show that the job ladder helps accounting for the aggregate behavior during and after the Great Recession, an exercise they do not consider. Second, on the modeling side, I assume that labor earnings risk is uninsurable. I show that this assumption affects the transmission of aggregate shocks to consumption, with workers reducing their consumption ex-

[9] See, for example Topel and Ward (1992), Hyatt and McEntarfer (2012), Moscarini and Postel-Vinay (2017), Hahn et al. (2017). Gertler, Huckfeldt, and Trigari (2018) estimates the average wage changes of job changers is about plus 4.5%. The average hides lots of heterogeneity, with conditional wage changes equal to plus 30% for workers realizing wage gains and minus 23% for workers realizing wage losses.

[10] See Moscarini and Postel-Vinay (2017).

penditures when the job ladder breaks down. This work also relates to Faccini and Melosi (2019). The authors empirically evaluate a simpler version of Moscarini and Postel-Vinay (2017) model for the US during the post-Great Recession period, but focus mainly on the missing inflation following the recession instead of the missing disinflation during the recession, which is the main focus of this paper.

This paper also contributes to the burgeoning literature on Heterogenous Agent New Keynesian (HANK) models by adding realistic labor market flows to this framework.[11] Den Haan, Rendahl, and Riegler (2017), Gornemann, Kuester, and Nakajima (2016) and Kekre (2019) also study HANK models with labor market frictions, but none considers that the employed also face search frictions through on-the-job search. In an analytically tractable HANK model with unemployment, Ravn and Sterk (2018) highlight that the precautionary savings response to *countercyclical* unemployment risk amplifies the consumption response to shocks compared to a complete market economy. This result contrasts with the dampening in consumption I find in response to the financial shock. There are two main differences between the model I develop here and their analysis. First, the cyclicality of the earnings risk in this paper is much more complex and takes into account wage fluctuations while employed, as well as unemployment risk.[12] Moreover, the model features a full distribution of marginal propensities to consume (MPCs), introducing a *redistribution channel* (Auclert, 2018) to any aggregate shock that unevenly affect workers.

The rest of the paper proceeds as follows. Section 2 outlines the model and Section 3 defines the equilibrium. Section 4 explains the calibration strategy. Section 5 presents the results for the Great Recession exercise, while Section 6 unpacks how the job ladder and incomplete markets affect the equilibrium. Section 7 concludes.

2. Model

In this section, I lay out the Heterogeneous Agent New Keynesian (HANK) model I use to study the aggregate implications of labor market flows.

Goods, Technology, Agents. Time is continuous. There are three vertically integrated sectors in the economy, each producing a different type of good that can be used either as an input by other

[11]The recent literature that incorporates micro heterogeneity into New Keynesian models of the macroeconomy include among others Guerrieri and Lorenzoni (2017), Bayer et al. (2019), McKay and Reis (2016), Auclert (2018), McKay, Nakamura, and Steinsson (2016), Ravn and Sterk (2017). Auclert and Rognlie (2018), Kaplan et al. (2018).

[12]Ravn and Sterk (2018) also feature *aggregate* wage fluctuations that impact the cyclicality of earnings risk in their model. My point here refers to the piece-rate wage changes induced by the job ladder, which introduces a complex mapping between labor market flows and workers' labor income process that varies over the cycle.

sectors or consumed.[13]

At the bottom of this supply chain, *labor intermediaries* hire workers in a frictional labor market. Technology is linear in labor, with a unit of labor mapping to z units of labor services (thought as an intermediate input), which is then sold in a competitive market at price φ_t. Productivity z is specific to the worker–firm match and is drawn at origination from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0,1]$.

A measure one of *retailers* indexed by $j \in [0,1]$ lies above the intermediate sector. Each retailer produces a *specialized input* $\tilde{Y}_j$ with a constant returns to scale technology in two inputs: labor services and materials.[14] The specialized inputs are then aggregated by a competitive representative firm to produce the final good $\tilde{Y}_t$.

The economy is populated by a continuum of *ex-ante* identical risk averse workers indexed by $i \in [0,1]$. Labor market risk makes workers heterogeneous in their employment status, labor income, and wealth. A government issues debt and taxes labor income in order to finance government expenditures and an unemployment insurance program. I start by describing the worker's problem.

Workers. Workers receive utility flow u from consuming c_{it} and do not value leisure. Preferences are time-separable, and the future is discounted at rate ρ

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt, \tag{1}$$

where the expectation reflects individual-level uncertainty in labor income.

An unemployed worker receives unemployment insurance (UI) benefits in the amount of $b \times \varphi_t$. An employed worker in a match of productivity z receives as a wage $y \times \varphi_t$, where the piece-rate $y \leq z$ depends on the worker's history in the labor market. I delay the discussion on the piece-rate wage determination for later.

Workers receive lump-sum dividends in the amount of d_{it}, save through a riskless government bond at flow real rate r_t, and are subject to a no-borrowing constraint. Wealth a_{it} evolves according

[13] See Christiano et al. (2016) and Moscarini and Postel-Vinay (2019) for a similar supply-side structure.

[14] Materials are converted one-for-one from the final good. I discuss the importance of materials in the retailer's problem. See Christiano, Trabandt, and Walentin (2010) for a standard New Keynesian model with materials.

to

$$\dot{a}_{it} = (1-\tau)\varphi_t\Big(\mathbb{1}^u_{it}b + (1-\mathbb{1}^u_{it})y_{it}\Big) + r_t a_{it} + d_{it} - c_{it} - \tau^0_t, \qquad (2)$$
$$a_{it} \geq 0$$

where $\mathbb{1}^u_{it}$ is an indicator for unemployment status, τ^0_t is a government lump-sum transfer and τ is a proportional tax. The distribution of dividends across workers is a crucial determinant of the aggregate consumption response in HANK models (e.g., Bilbiie, 2018; Broer, Hansen, and Krusell, 2018; Werning, 2015). I follow Kaplan et al. (2018) and distribute profits in proportion to individuals' labor income

$$d_{it} = \frac{\mathbb{1}^u_{it}b + (1-\mathbb{1}^u_{it})y_{it}}{\int \Big(\mathbb{1}^u_{it}b + (1-\mathbb{1}^u_{it})y_{it}\Big)di} D_t, \qquad (3)$$

where D_t denotes aggregate profits.[15]

Workers maximize their lifetime utility given in (1) subject to the wealth accumulation process in (2), the labor income process $\{\mathbb{1}^u_{it}, y_{it}\}_{t\geq 0}$, dividends payouts $\{d_{it}\}_{t\geq 0}$, and paths of $\{r_t, \varphi_t, \tau^0_t\}_{t\geq 0}$, which they take as given. In Appendix A.2, I write the Hamilton–Jacobi–Bellman equation associated with the household problem and discuss the impact of the job ladder on consumption and savings decisions, following the insights from Lise (2012). At steady state, the recursive solution to this problem consists of value functions and consumption decision rules for the unemployed and the employed worker $\{c^u(a), c^e(a,y)\}$.[16] The worker's consumption policy function together with labor market transition rates and wage contracts induce a stationary distribution over wealth, labor income, and match productivities $\Psi(a,y,z)$. With a slight abuse of notation, I denote marginal distributions by Ψ as well. Outside steady state, distributions and policies are time varying and described by a Kolmogorov forward and a Hamilton–Jacobi–Bellman equations. I indicate that dependence when necessary by adding a t subscript to equilibrium variables.

[15] Aggregate profits include profits earned both by monopolistically competitive firms and labor intermediaries. Rewriting the worker's budget constraint under this profit distribution rule, we get

$$\dot{a}_{it} = \left((1-\tau)\varphi_t + \frac{D_t}{\int \Big(\mathbb{1}^u_{it} + (1-\mathbb{1}^u_{it})y_{it}\Big)di}\right)\Big(\mathbb{1}^u_{it}b + (1-\mathbb{1}^u_{it})y_{it}\Big) + r_t a_{it} - c_{it} - \tau^0_t$$

Hence, distributing profits in proportion to labor earnings neutralizes the redistribution effects by making all workers equally exposed to its fluctuations. Overall dividends D_t and price of labor services φ_t enter in the same way in the budget constraint by multiplying the idiosyncratic worker labor market state $(\mathbb{1}^u_{it}, y_{it})$.

[16] Note that policy functions depend on wealth and the piece rate wage only. The attentive reader may notice the lack of match productivity z in the worker's state space, that, even if not a direct payoff relevant variable, still contains information about future labor income distribution. As I discuss below, the worker does not observe the productivity of its current match, making income and wealth the only state variables in the worker problem.

Search Frictions in the Labor Market. The labor market features search frictions. Labor intermediaries post vacancies v_t to match with workers. Employed and unemployed workers search for open job vacancies. The searching effort of unemployed workers is normalized to one, while employed workers search with lower intensity s_e. Combined, they produce a search effort of

$$\mathcal{S}_t = u_t + s_e(1 - u_t). \tag{4}$$

Effective job market tightness is therefore

$$\theta_t = \frac{v_t}{u_t + s_e(1 - u_t)}. \tag{5}$$

The flow of meetings at time t is given a by constant returns to scale matching function $\mathcal{M}(v_t, \mathcal{S}_t)$. Define $\lambda_t := \frac{\mathcal{M}(v_t,\mathcal{S}_t)}{\mathcal{S}_t}$ as the rate at which an unemployed worker meets a vacancy, while employed workers contact outside firms at a rate $\lambda_{et} = s_e\lambda_t$. A vacancy contacts a worker with intensity $q_t := \lambda_t/\theta_t$. Once a worker and firm meet, the firm makes a wage offer (details below) that may or may not be accepted by the worker. Finally, all matches are subject to a destruction shock at an exogenous flow probability δ.

Wage Contract. Firms are restricted to offer workers piece-rate wage contracts that can be renegotiated only if the worker receives a better outside offer.[17] When the firm contacts a worker, it observes the worker's employment status and incumbent match productivity in case the worker is already employed. In contrast, workers are uninformed about their match productivity, but learn about it from labor market transitions and wage offers—I discuss this assumption in the next section. In what follows, I describe wage offers to employed and unemployed workers.

Employed Worker— Consider a worker employed at a match of productivity z who contacts an outside firm with which the match productivity draw is z'. The two firms Bertrand compete for the worker's services over piece-rate wage contracts, with the more productive firm winning the bidding for the worker.

First, let me consider the case where $z' > z$; that is, when the poacher is more productive than the incumbent firm. The incumbent's maximum wage offer is to promise the worker the whole output flow of the match—i.e., offer a piece-rate $y = z$. The poaching firm z' attracts the worker

[17] Note that piece-rates are usually defined in terms of a share of the match output flow, so if the match produces X, a piece-rate p would entail a wage of pX with $p \leq 1$. In the presentation here, I instead define the piece-rate in terms of the price of labor services. So the wage of a worker in match z with piece-rate of y is $y\varphi_t$, with the restriction $y \leq z$. See Bagger et al. (2014) for an implementation of piece rate version of the sequential auction framework in a standard labor market model that abstracts from incomplete markets and consumption and savings decisions.

by outbidding incumbent's piece-rate wage offer by ϵ, which results in the worker moving to firm z' at a piece-rate wage of $z + \epsilon$. In the solution of the model, I take ϵ to be an arbitrarily small number.[18]

Now, suppose instead that $z' < z$. The competition between the two firms has the worker staying with the incumbent, but the wage contract can still be renegotiated if the poaching firm's maximum wage offer is above the worker's current piece-rate (i.e., if $z' > y$). In this case, the worker's piece-rate wage from the incumbent firm increases to $z' + \epsilon$.

Unemployed Worker— Upon meeting an unemployed worker, I assume that the firm makes a piece-rate offer of $\underline{z}$; that is, the firm offers the unemployed the full production of the least productive firm. In the calibration, I choose the unemployment insurance replacement rate b to be equal to $\underline{z}$, so firms effectively offer the unemployment insurance rate to unemployed workers.

In the description above, I have treated the worker's acceptance decision as given. In particular, I implicitly assume that (i) the unemployed worker accepts the initial wage offer coming from any firm, and (ii) the employed worker always moves/stays in the firm offering the highest wage. While (ii) is a natural assumption in the current setup, where more productive matches also offer higher wages, it is not clear that (i) would hold without any additional assumptions. In what follows, I discuss the unemployed worker's reservation strategy in the presence of such wage contracts.

Worker's Reservation Strategy— While firms offer the same initial wage contract to workers coming out of unemployment, the unemployed workers' value of meeting a vacancy increases with the productivity of the match. This is because being hired by a firm with greater productivity implies a better (in the first-order sense) distribution of future wages.[19]

Because the unemployed search intensity is greater that of the employed ($\lambda > \lambda_e$), there is an *option value* associated with waiting to meet more productive firms. The value of remaining in unemployment and waiting for better matches versus accepting an offer at a match of productivity z will depend on the worker's assets, leading to a reservation productivity policy that depends on wealth.

[18] Note that this assumption departs from Postel-Vinay and Robin (2002) as the more productive firm attracts the worker by matching the *wage offer* of the less productive firm, as opposed to matching the worker's value of staying at the incumbent firm under the maximum wage offer. The same assumption is made by Graber and Lise (2015) and is intended to keep the problem tractable in the presence of a non-degenerate wealth distribution on the worker side.

[19] To see this, consider the future path of wages for a recently hired worker at matches of productivity z_1, z_2, with $z_1 > z_2$, in the circumstance where he meets an outside firm of productivity $z_3 \in [z_1, z_2]$. If employed at firm z_1, the worker switches jobs and his piece-rate wage changes to z_1. If employed at firm z_2, however, the worker stays in the firm and the wage increases to $z_3 > z_1$.

The extent to which search decisions depend on worker's wealth is certainly an important question.[20] My main interest here, however, is not to analyze how incomplete markets impact search decisions but instead to study how a "realistic" model of the labor market transmits aggregate shocks to consumption. Therefore, I simplify workers' reservation decisions by assuming that the worker never gets to observe the productivity z of its own match.[21] This transforms the reservation decision of the unemployed into a trivial one: by making all offers coming out of unemployment identical— meaning that all firms offer the same wage, so they all look the same to the unemployed worker—they are either all accepted or all rejected. Since being employed entails a higher present value of earnings than being unemployed, all offers will be all accepted by the worker.

Making the productivity a hidden state adds a learning/filtering dimension to the worker's problem, who still gets to observe his wage history in the labor market. I describe this problem in Appendix A.1. Next I turn to the supply side of the economy.

Final Good Producer. A competitive representative final good producer aggregates a continuum of specialized inputs, $\tilde{Y}_{j,t}$, using the technology

$$\tilde{Y}_t = \left(\int_0^1 \tilde{Y}_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $\epsilon > 0$ is the elasticity of substitution across goods. The firm's first-order condition for the jth input is

$$\tilde{Y}_{j,t}(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} \tilde{Y}_t, \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{7}$$

Retailers. The jth input good in (6) is produced by a *retailer*, who is a monopolist in the product market. Following Basu (1995) and Nakamura and Steinsson (2010), each retailer produces their specialized good by combining materials $M_{j,t}$ and labor services $N^e_{j,t}$ according to the production function

$$\tilde{Y}_{j,t} = M_{j,t}^{\gamma} (Z_t N^e_{j,t})^{1-\gamma}, \tag{8}$$

where Z_t is an aggregate productivity component. Materials are converted one-for-one from the final good $\tilde{Y}_t$ in (6), so each retailer effectively uses the output of all other retailers as input to production. Retailers buy labor services at the competitive price φ_t and materials for the real price

[20]For examples of papers that study this, see Lentz and Tranæs (2005) and Eeckhout and Sepahsalari (2018).

[21]A simpler way to eliminate the option value would be to assume that the search intensity is the same for the employed and unemployed, $s_e = 1$. This, however, would preclude the model from matching the small flow of employer to employer transitions relative to unemployment to employment. But, as I show in the experiments, it is the slow reallocation along the ladder that generates long-lasting impacts of misallocation—one of the main points of the paper.

of one.

Cost minimization implies a common marginal cost across all retailers, given by

$$m_t = \left(\frac{1}{\gamma}\right)^{\gamma} \left(\frac{\varphi_t / Z_t}{1-\gamma}\right)^{1-\gamma}. \tag{9}$$

Cost minimization also implies that the relative price of labor services and materials inputs must be equal to the ratio of their marginal productivities

$$\frac{\varphi_t / Z_t}{1} \frac{\gamma}{1-\gamma} = \frac{M_{jt}}{Z_t N_{jt}}. \tag{10}$$

Each retailer must also choose a price $P_{j,t}$ to maximize profits subject to demand curve (7) and price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the firm's rate of price change $\dot{P}_{j,t}/P_{j,t}$ and expressed as a fraction of gross output $\tilde{Y}_t$ as

$$\Theta_t \left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right) = \frac{\theta}{2} \left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right)^2 \tilde{Y}_t, \tag{11}$$

where $\theta > 0$.[22] Therefore, each retailer chooses $\{P_{j,t}\}_{t \geq 0}$ to maximize

$$\int_0^{\infty} e^{-\int_0^t r_s ds} \left\{ \tilde{\Pi}_t(P_{j,t}) - \Theta_t \left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right) \right\} dt,$$

where retailers discount profits at the real rate $\{r_t\}_{t \geq 0}$ and

$$\tilde{\Pi}_t(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} - m_t\right) \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t$$

are flow profits before price adjustment costs.

In a symmetric equilibrium, all firms choose the same price $P_{j,t} = P_t$ and produce the same amount of goods $\tilde{Y}_{j,t} = \tilde{Y}_t$. Moreover, as shown in Kaplan et al. (2018), the quadratic price adjustment costs in continuous-time setting yields a simple equation characterizing the evolution of aggregate inflation $\pi_t := \dot{P}_t / P_t$

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon}. \tag{12}$$

[22] I follow Hagedorn, Manovskii, and Mitman (2019) in assuming that price adjustment costs are "virtual", meaning that they affect optimal choices but do not cause real resources to be expended. That is why pricing costs do not appear in the goods market clearing condition in the definition of equilibrium.

Equation (12) is the New Keynesian Phillips curve, which can also be represented in present-value form as

$$\pi_t = \frac{\varepsilon}{\theta}\int_t^\infty e^{-\int_t^s r_\tau d\tau}\frac{\tilde{Y}_s}{\tilde{Y}_t}(m_s - m^*)\,ds. \tag{13}$$

The presence of materials adds a flexible factor input into production, which allows output to change immediately (at time-0) upon aggregate shocks. To see this, substitute (10) into (8) and (6) evaluated at the symmetric equilibrium. This gives an aggregate *restriction* between aggregate production $\tilde{Y}_t$, marginal costs m_t and labor services N_t^e

$$\tilde{Y}_t = (m_t\gamma)^{\frac{\gamma}{1-\gamma}} Z_t N_t^e. \tag{14}$$

So production changes in equilibrium if (i) productivity Z changes, (ii) marginal costs m changes or (iii) labor services inputs N^e change. Market clearing in the market for labor services—see equilibrium definition in Section 3—imposes that all the supply of labor service must be employed by retailers. This is a stock (state variable), however, so it cannot adjust at the impact of an aggregate shock—retailers, while individually allowed to reduce their usage of labor services, cannot do so in the aggregate immediately following the shock. Labor service competitive price φ_0 must therefore adjust to make retailers willing to hire the labor service stock in the economy. As φ_0 changes to clear the labor market, retailers adjust their materials-labor ratio according to (10), which leads to production to adjust.

Labor Intermediaries. A firm in the intermediate sector can post a vacancy at a flow cost of κ^f, expressed in units of the consumption good. Upon meeting a worker, the firm must pay an additional fixed screening/training cost to learn the match productivity and start producing.[23] This cost is allowed to depend on the employment status of the worker, but different from the vacancy cost, it does not expend any real resources and does not show up in any budget constraint.[24]

Firms discount their profit flow at the rate $r_t + \chi_t$, where χ_t is a (exogenous) spread between the return of vacancy posting investments and the risk-free rate.[25] Let $\mathcal{J}_t(z, y)$ denote the expected present discounted value of dividends for a firm with match productivity z currently offering the worker a piece-rate contract y. The firm's value function is defined recursively in Appendix A.3.

[23] As suggested by Pissarides (2009) and exploited by Christiano et al. (2016) in an estimated model without OJS, screening costs raise amplification of unemployment fluctuations to aggregate shocks by insulating hiring costs from vacancy congestion coming from the matching function.

[24] These costs can be thought of as utility costs associated with the training/screening of workers. See Moscarini and Postel-Vinay (2019) for a similar assumption.

[25] At steady state, I set χ to zero. Outside steady state, I interpret shocks to χ_t as a reduced form financial shock.

The measure of vacancies v_t is pinned down in equilibrium by the following free-entry condition

$$\frac{\kappa^f}{q_t} = \int \Bigg\{ \frac{u_t}{u_t + s_e(1-u_t)} \left[\mathcal{J}_t(z, \underline{z}) - \tilde{\kappa}^{su} \right] + \\ + \frac{s_e(1-u_t)}{u_t + s_e(1-u_t)} \left[\int_{\underline{z}}^{z} \mathcal{J}_t(z, z') \frac{d\Psi_t(z')}{1-u_t} - \tilde{\kappa}^{se} \right] \Bigg\} d\Gamma(z), \tag{15}$$

which equates the expected flow cost of hiring a worker, $\frac{\kappa^f}{q_t}$, to the expected value of a match. The latter accounts for the probability of meeting an unemployed worker, an event which the firm values by $\mathcal{J}(z, \underline{z})$, and the probability of meeting an employed worker matched with a firm of productivity z', which has a value of $\mathcal{J}(z, z')$ if $z > z'$ and otherwise, is zero.

The distribution of workers in the labor market—the measure u_t of unemployed workers and the distribution $\Psi_t(z)$ of employed workers—affects firms' incentives by changing their expectations of the type of worker they will encounter. At the same time, the distribution of employment depends on the measure of vacancies posted through the market tightness.

Monetary Authority. The monetary authority sets the nominal interest rate on nominal government bonds i_t according to a Taylor rule

$$i_t = \bar{r} + \phi_\pi \pi_t + \epsilon_t, \tag{16}$$

where $\phi > 1$ and ϵ_t is a monetary policy shock. Given inflation and the nominal interest rate, the real return on the government bonds r_t is determined by the Fisher equation $r^b = i_t - \pi_t$.

Government. The government issues *real* bonds of infinitesimal maturity B_t^g, with positive values denoting debt. This is the only savings instrument available to workers.

The government taxes workers' labor income at rate τ and uses this revenue to finance unemployment insurance, government expenditures G_t, and real rate payments on its debt. The government fiscal policy must satisfy the sequence of budget constraints

$$\dot{B}_t^g = r_t B_t^g + G_t + u_t(1-\tau)\varphi_t b - \tau \varphi_t \int y d\Psi_t(y) - \tau_t^0, \text{ for all } t. \tag{17}$$

At steady state, lump-sum transfers τ^0 are set to zero. Outside steady state, I let lump-sum transfers be the fiscal instrument that adjusts in order to keep government debt B_t^g constant at the steady-state level.[26]

[26]See Kaplan and Violante (2018) for a discussion on the importance of fiscal adjustment in HANK models.

3. Equilibrium

The rich worker heterogeneity over jobs, earnings and wealth shows up in the equilibrium definition below only through a small number of *functions* that integrate workers' decisions and states over its distribution.[27] For example, while the consumption of workers varies across earnings and wealth, equilibrium conditions only depend on an *aggregate consumption function*

$$\mathcal{C}_t := \int c_t(a,y) d\Psi_t(a,y)$$

where the time index t subsumes the dependency of policies and distributions on the whole sequence of equilibrium prices and quantities entering the worker's problem.[28] In a similar way, the *aggregate labor services supply*

$$\mathcal{N}_t^e := \int z d\Psi_t(z)$$

is all that enters the market clearing of labor services. Notwithstanding all the complexity involved in *evaluating* those functions,[29] they still constitute a mapping from *aggregate* sequences of equilibrium prices and quantities (like real rate) into other *aggregate* sequences (like consumption), which in turn must satisfy certain equilibrium conditions.[30] This observation is the basis of the numerical algorithm used to solve the model – see Appendix C for details. I now turn to the equilibrium definition.

Definition 1 *(Equilibium)*
Given an initial government debt B^g, an initial distribution Ψ_0 over wealth, labor income and match productivity, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, an general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N_t^e, M_t, u_t, v_t, D_t\}_{t\geq 0}$, labor market transition rates $\{\lambda_t, \lambda_{et}\}_{t\geq 0}$, government policies $\{G_t, B_t^g, \tau_t, \tau_t^0, i_t\}_{t\geq 0}$, labor income process $\{\mathbb{1}_{it}^u, y_{it}\}_{i\in[0,1], t\geq 0}$, worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t, \mathcal{N}_t^e\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that workers optimize, firms optimize, monetary and fiscal policy follow their rules, the labor income process is the result of labor market transitions and wage-setting, worker aggregate functions and distributions are consistent with labor market transition rates and worker's decision rules,

- *the free-entry condition* (15) *holds,*
- *and all markets clear:*

[27] See Auclert et al. (2019) for this insight, who call these functions by *heterogeneous-agent block*.

[28] Specifically, the worker cares about the evolution of $\{r_t, \varphi_t, d_t, \tau_t, \tau_t^0, \lambda_t, \lambda_{et}\}_{t\geq 0}$.

[29] Aggregate consumption at time t, for instance, is the summation of consumption decisions $c_t(a,y)$, itself a function of the whole sequence of prices, labor market transitions and fiscal policy, across wealth and earnings distribution, the evolution of which depends on the consumption decisions and labor market transitions up to time t.

[30] Even though continuous time perfect-foresight transition equilibrium objects consists of *real valued functions* $X : [0,\infty) \to \mathbb{R}$ and not really *sequences* $Y : \mathbb{N} \to \mathbb{R}$, I use sequences when describing those in the text since this agrees with the more commonly used discrete time convention.

- *asset market*

$$\mathcal{A}_t = B_t^g$$

- *labor services market*

$$N_t^e = \mathcal{N}_t^e$$

- *goods market*

$$\mathcal{C}_t + G_t + \kappa^f v_t = Y_t = \tilde{Y}_t - M_t$$

4. Calibration

I calibrate the model at a monthly frequency. The calibration strategy is divided into four main steps. First, I calibrate the labor market transition rates to match estimated flows and choose the firm productivity distribution to match the dispersion in the residual wage distribution. Second, I choose the vacancy costs and the relative importance of screening versus flow costs. Third, I use the overall amount of liquidity, which in the economy takes the form of government bonds, to directly target average MPC in the data. Finally, I calibrate the parameters of the production and monetary side to standard values used in the New Keynesian literature. The full list of parameter values and targeted moments is given in Table 1.

Labor Market (Transitions and Productivity): I assume a standard Cobb–Douglas matching function $\mathcal{M}(v, \mathcal{S}) = v^\alpha \mathcal{S}^{1-\alpha}$, with $\alpha = 0.5$, as in Moscarini and Postel-Vinay (2018). I target a job finding rate λ of 0.45, which implies a monthly job finding probability of $1 - exp(\lambda) = 0.36$. I set δ to match the monthly probability of transitioning from employment to unemployment. These two flows imply a steady-state rate of $\frac{\delta}{\delta+\lambda} = 5\%$. The relative search efficiency of employed worker s_e is set so the steady-state monthly job-to-job transition rate equals 2.4%.

The productivity distribution Γ is assumed to be an affine transformation of Beta distribution; that is, a match productivity $z = c_0 + c_1 X$, where $X \sim \text{Beta}(\beta^1, \beta^2)$. This introduces four parameters $(c_0, c_1, \beta^1, \beta^2)$. The output flow in the least productive match c_0 is normalized to 0.3. I also set β^1 to 1, so that the Beta distribution has an exponential-like shape. To pin down the remaining two parameters, (c_1, β^2), I target empirical 90/10 and 50/10 percentiles of the residual log wages distribution. In the data, these are defined by the residual from a Mincerian wage regression with as many control variables as possible. The residual is intended to capture the wage dispersion stemming from search frictions, which in my model are the only reason why workers have different labor income. The chosen values of $c_1 = 2.61$ and $\beta^2 = 10.0$ deliver 50/10,90/10 percentiles of (0.64 and 1.10). These are in line with the estimates reported by Lemieux (2006) (Figure 1A) and Autor, Katz, and Kearney (2008) (Figure 8). Finally, I set $b = \underline{z}$ so the unemployed earns as much as a recently employed agent. This delivers a UI replacement rate of approximately 50%, which is within the range of values used in the literature.

Table 1: List of parameter values and targeted moments

Variable		Value	Target
Labor market			
$\mathcal{M}$	matching function	$v^{0.5}\mathcal{S}^{0.5}$	—
δ	destruction rate	0.024	—
λ	job finding prob.	0.412	unemployment of 5%
s_e	employed search intensity	0.127	ee transition of 0.024
b	replacement rate	$\underline{z}$	UI replacement rate of 50%
$z = c_0 + c_1 X$ $X \sim \text{Beta}(\beta^1, \beta^2)$	productivity grid	$(0.30, 2.61)$ $(1.0, 10.0)$	*residual wage dispersion* $p^{50}/p^{10}, p^{90}/p^{10} = (0.64, 1.10)$[1]
$\kappa^f, \kappa^{su}, \kappa^{se}$	vacancy costs	$0.34, 3.4, 1.0$	see text
Preferences and Liquidity			
ρ	discount rate	$0.08/12.0$	$r^{ann} = 0.02$
$u(\bullet)$	utility function	$\log(\bullet)$	—
B^g/Y^{ann}		≈ 0.30	*target quarterly MPC of 0.25*[2]
Retailers, Final Good and Government			
γ	material share	0.50	share of materials in gross output
ϵ	elasticity of substitution	10.0	—
ϵ/θ	slope of Phillips curve	0.0067	price rigidity of 12 months
τ	tax rate	0.25	$G/Y \approx 0.20$
ϕ_π	Taylor rule coefficient	1.50	—

[1] Lemieux (2006) and Autor, Katz, and Kearney (2008).
[2] Johnson, Parker, and Souleles (2006); Parker et al. (2013) report quarterly MPC estimates around $[0.15, 0.30]$.

Labor Market (Vacancy Costs): The canonical search and matching model fails to match the cyclical volatility in the job finding rate—a point initially noted by Shimer (2005). The same difficulty is also present in a model with on-the-job search—see Moscarini and Postel-Vinay (2018) for a detailed comparison of the canonical model versus a model with on-the-job search. Since one of my objectives is to study the impact of labor market fluctuations on consumption, it is crucial to get fluctuations in unemployment and earnings risks that approximate those in the data.

I achieve this by resorting to high fixed screening costs. Specifically, I need three restrictions to pin down the values of vacancy posting κ^f and screening costs $(\tilde{\kappa}^{se}, \tilde{\kappa}^{su})$. The targeted job finding rate $\lambda = 0.45$ imposes the first restriction—through the matching function, this implies a steady-state level of market tightness θ that must be consistent with the free-entry condition. I impose two additional restrictions by (i) making the firm indifferent between hiring an employed worker and hiring an unemployed worker at steady state [31] and (ii) making screening costs 90% of the total hiring cost. The fixed cost's share of total cost is in line with Christiano, Eichenbaum, and Trabandt (2016), who estimate this to be 94%.

To understand the rationale behind (i), suppose I did not make the screening costs dependent on employment status. As the value of meeting an unemployed worker is greater than that of meeting an employed worker, firms would be more willing to post vacancies whenever unemployment is high because these are periods when firms face a higher probability of meeting an unemployed worker. This force, which is quite powerful in the model, accelerates transitions back to steady state and reduces the unemployment response to shocks. Hence, having the screening cost depend on the employment status of workers and satisfying restriction (i) mitigates this effect.

Liquidity and Preferences: I assume that steady-state inflation is equal to zero and that the steady-state real interest rate equals 2%. Workers have log utility over consumption, and their annual discount rate is 8%. As discussed in Kaplan et al. (2018), one-asset HANK models feature a tension between matching the high observed aggregate wealth-to-output ratio and generating a large average MPC, as in the data. If the model is calibrated to target the former, it implies small MPCs; if we directly target the MPCs in the data, the model must feature a low aggregate wealth. Given the importance of the MPCs to the demand response to aggregate shocks, as outlined by Auclert, Rognlie, and Straub (2018), I set B^g to directly target MPCs. Specifically, I target an average quarterly MPC out of a $500 unexpected transfer of 0.25. The estimate lies within the range of values reported by Johnson, Parker, and Souleles (2006); Parker et al. (2013). This target yields a government debt B^g in the amount of 28% of annual GDP.

Production: The elasticity of substitution for the inputs produced by retailers ϵ is set to 10. The input share of materials γ is set to 0.5, which lies in the interval of values considered by Nakamura and Steinsson (2010). I set the price adjustment cost θ coefficient to 1500, so the slope of the Phillips curve is given by 0.0067. The Phillips curve under Rotemberg or Calvo price rigidities has the same log-linear representation, so we can map the slope of the Rotemberg Phillips curve to the implied Calvo parameter determining the time between price changes. In that case, the slope of 0.0067 implies prices change once every 12 months, which is close to the Bayesian estimates from Smets and Wouters (2007) and Christiano, Eichenbaum, and Trabandt (2014).[32]

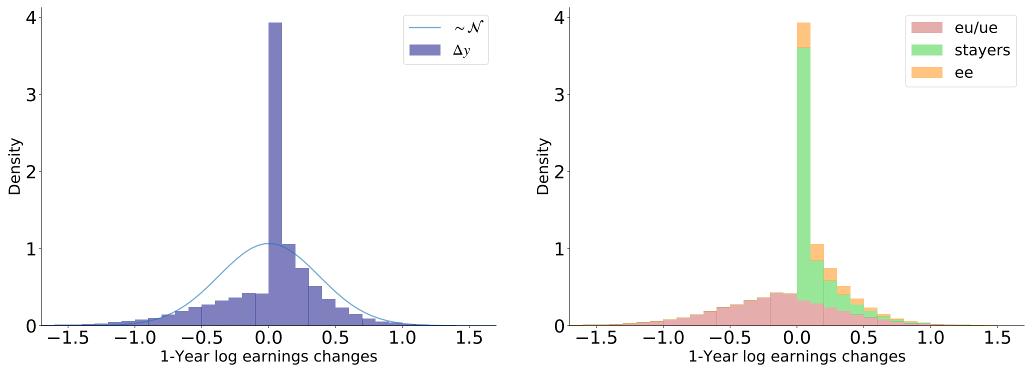
[31] In terms of the values defined before, this restriction writes as

$$\int \left[\mathcal{J}(z,\underline{z}) - \kappa^u\right] d\Gamma(z) = \int \left\{ \left[\int_{\underline{z}}^{z} \mathcal{J}(z,z') \frac{d\Psi(z')}{1-u} - \tilde{\kappa}^{se} \right] \right\} d\Gamma(z).$$

[32] This mapping is given by

$$\frac{\epsilon}{\theta} = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha},$$

Figure 1: Histogram of earnings changes



(A) Unconditional (B) Conditional

Fiscal and Monetary Policy: I set the labor income tax to 25%. Government expenditures are determined residually from the government budget constraint and amounts to around 20% of GDP. The Taylor rule coefficient is set to 1.5.

4.1. Earnings Dynamics

In the current environment, worker labor earnings follow an *endogenous* process determined by labor market transition rates and competition among employers.[33] This setting contrasts with the usual heterogenous agent models in which earnings follow an exogenous process for idiosyncratic productivity.

In this section, I explore the model's implied distribution of worker earnings growth and compare it to the evidence from the Master Earnings File of the Social Security Administration (SSA) reported by Guvenen, Ozkan, and Song (2014), who documented substantial deviations of earnings changes from lognormality.

I consider the economy to be at its steady state. Let T stand for the beginning of a calendar year, so yearly earnings accrue from T to $T+12$. Workers' gross labor earnings flows at time t are given by φy_{it}, so yearly labor earnings are

$$y_T^A = \varphi \int_T^{T+12} y_{is} ds.$$

where β is the household discount factor, and $1-\alpha$ denotes the probability with which the firm gets to reset prices in the month. Setting $\beta = exp(-12.0r)$, $\epsilon/\theta = 0.0067$ which leads to $\alpha = 0.92$, meaning an expected price rigidity of $(1-\alpha)^{-1} \approx 12$ months.

[33]See Appendix A.1 for a formalization of the piece-rate wage process.

I denote annual log-earnings by $\tilde{y}^A_T \equiv \log y^A_T$, so *annual log earnings changes* are $\Delta\tilde{y}^A \equiv \tilde{y}^A_{T+1} - \tilde{y}^A_T$. I start by simulating a panel of workers in the model and recording their annual earnings changes $\Delta\tilde{y}^A$, as well as their labor market transition events, in case they experience any. Figure 1, Panel (A) plots the histogram generated from the model. The earnings changes distribution features more mass around zero and on the left tail than what would be predicted by a normal distribution with the same mean and variance (blue line).

In Figure 1, Panel (B), I condition earnings changes on the type of labor flow experienced by the worker: "ue,eu" indicates workers who experience transitions in and out of unemployment; "stayers" are workers who neither change jobs nor experience an unemployment spell; and "ee" workers experience at least one job change in year $T+1$. First, I note that 43% undergo an "eu,ue" type of transition, while the remaining workers experience a continuous spell of employment over $T, T+1$. Among the latter group, 17% experience a job-to-job transition in year $T+1$. Earnings changes for workers who experience an unemployment spell feature a heavy left tail, which is the result of a lack of earnings during unemployment and the low re-entry wages. Workers who do not suffer an unemployment spell experience a positive expected earnings growth, but the gains are higher for workers who experience a job-to-job transition. Carrillo-Tudela, Visschers, and Wiczer (2019) investigate the relationship between the distribution of earnings changes and worker mobility in the SIPP and find similar patterns.[34]

Table 2 reports the model-implied moments for the log earnings changes along with the SSA data on male earnings from Guvenen et al. (2014). Earnings growth data conflates the influence of all variables affecting wages, such as tenure effects, human capital accumulation, reallocation shocks, and so on. The wage dispersion and earnings growth in the model are the result of search frictions alone, so we should not expect it to capture all the risk contained in the data. In fact, looking at the variance of log earnings changes $Var[\Delta\tilde{y}^A]$, we see that the dispersion in the model is only half of that seen in the data. Instead, the objective is to demonstrate that the earnings process implied by the simple job ladder model considered herein is at least consistent with the main facts on earnings risk documented by Guvenen et al. (2014).

The model replicates two key facts from Guvenen et al. (2014): negative skewness and excess kurtosis. Additionally, the model is close to the data with respect to the fraction of small earnings changes of less than 5% and 10%. These two facts are natural consequences of the job ladder structure: for the employed worker, earnings grow only due to outside offers (either matched or accepted), which occurs infrequently through job search, while unemployment shocks entail large

[34]See Figure 1 in their paper for the conditional distribution of earnings changes in the data. The main difference between the model and the data is that a fraction of earnings changes following job-to-job transitions, or no transitions are associated with earnings losses, which the model cannot generate by construction.

Table 2: Moments of earnings change distribution

Moment	Data	Model
$Var(\tilde{y}^A)$	0.700	0.178
$Var[\Delta\tilde{y}^A]$	0.260	0.140
$Skew[\Delta\tilde{y}^A]$	-1.07	-0.721
$Kurt[\Delta\tilde{y}^A]$	14.93	5.907
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.05$	0.310	0.337
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.10$	0.490	0.434
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.20$	0.670	0.578
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.50$	0.830	0.838

Notes: $\tilde{y}^A$ denotes annual log-earnings. Moments from Data column are taken from Guvenen et al. (2016). The model implied moments are computed by simulating a panel of 50,000 workers for a 2 year period. As in the data, I exclude unemployment insurance from the model measure of earnings.

earnings losses.[35]

4.2. Consumption of the Unemployed

In what follows, I assess the model implications for MPC differences between the employed and unemployed, as well as wage and consumption dynamics following a job loss event. These (untargeted) moments highlight important dimensions of the consumption reaction in the face of income changes. Table 3 reports the results.

The first line has the results on the MPCs. Using the Italian 2010 Survey of Household Income and Wealth (SHIW), Kekre (2019) finds that the annual (self-reported) MPC is 25 percentage points higher for unemployed individuals. In the model, this difference is 18 percentage points. Second, I evaluate the model's prediction for consumption drop upon unemployment. The model predicts a consumption drop of 23% in the first month of unemployment. This outcome is in line with available empirical evidence, albeit toward the high end of estimates. Using scanner data, Aguiar and Hurst (2005) report a 19% decline in food expenditures among unemployed workers. Chodorow-Reich and Karabarbounis (2016) report a 25% drop in expenditures in the categories of food, clothing, entertainment, and travel during unemployment in the Consumption Expenditure Survey (CE). Even when they examine overall expenditures on nondurable goods and services, they still find a sizable drop of 21%.

[35]The ability of a job ladder model to reproduce the negative skewness and excess kurtosis documented by Guvenen

Table 3: Additional moments

	Estimate	Source	Model
Annual MPC unemp./emp.	0.25	Kekre (2019)	0.66-0.48=0.18
Relative consumption of unemp./emp.	0.81, 0.75	Aguiar and Hurst (2005) Chodorow-Reich and Karabarbounis (2016)	0.77

Notes: Annual MPCs are computed as the fraction consumed out of a $500 unexpected transfer. The $500 rebate is translated into the model by scaling annual gross labor income of $69,100 from the 2004 SCF to model units.

5. Results

In what follows, I conduct and analyze the main quantitative exercise of the paper: an adverse (reduced form) financial shock aimed to capture labor market movements during the Great Recession (GR). I also study the economy's response to monetary and TFP shocks, but I leave these results and discussion to the Appendix D. The numerical implementation is discussed in Appendix C. In all cases, I consider the perfect-foresight solution to an unanticipated aggregate shocks, starting from the steady state with no aggregate risk ("MIT shocks").

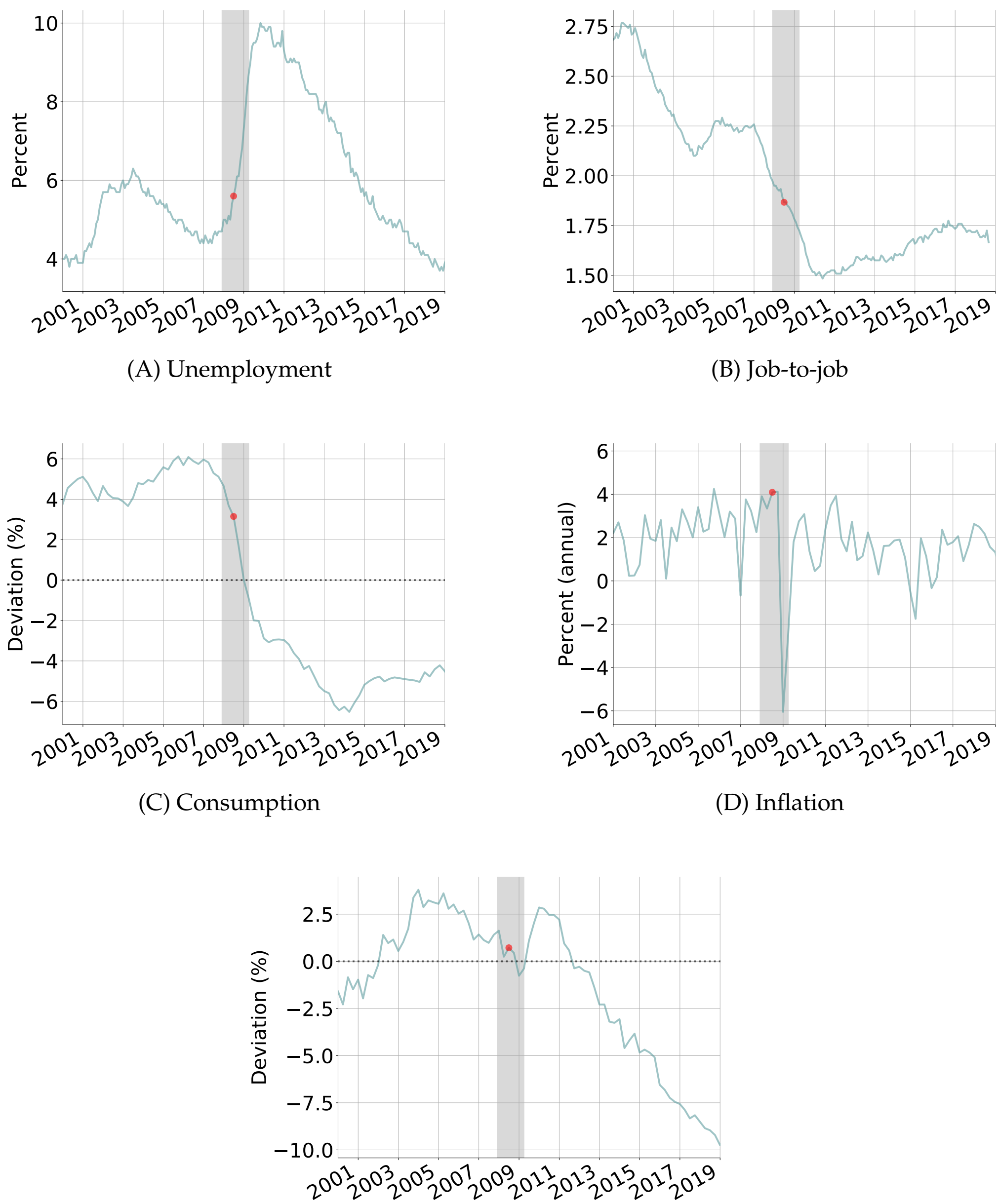
5.1. Financial Shock

Great Recession. Figure 2 shows the behavior of some aggregate variables during and after the GR. From the last quarter of 2007 until the second quarter of 2009, the US experienced a severe economic downturn: unemployment rate more than doubled, reaching 10 percent, job-to-job transitions fell by 0.6 percentage points and consumption dropped by almost 4%. Recovery has been really slow. Unemployment took 6 years to go back to its steady-state level, while job-to-job transitions failed to do so to this date. Figure 2, Panel (C), which plots log-deviations of consumption from a linear trend estimated from 1984, shows that consumption growth during the recovery has not been high enough to close the negative gap opened during the GR. Despite the depth of the downturn, inflation only fell modestly – with the exception of last quarter of 2008, when prices fell by 6%, inflation has fluctuated in the range of 1-3% for most of the recovery. The limited amount of disinflation in face of the large contraction in economic activity was seen as puzzling.[36] In particular, inflation behavior is surprising if viewed thought lens of the Phillips curve, here thought both

et al. (2014) is highlighted in Hubmer (2018).

[36] Hall (2011), for instance, argues that popular DSGE models based on the simple New Keynesian Phillips curve "cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack".

Figure 2: Great Recession series



(A) Unemployment

(B) Job-to-job

(C) Consumption

(D) Inflation

(E) Labor productivity

Notes: Consumption and labor productivity are log-linearly detrended, while other variables are in levels. The red dot marks the second quarter of 2008, which I will use as the time-0 steady state when comparing model IRFs to the data. See Appendix B for data sources.

as an empirical and theoretical relation connecting real variables, like unemployment, marginal cost or other measure of "slackness", to inflation. Coibion and Gorodnichenko (2013) make this point by showing that a Phillips curve relating inflation and unemployment estimated from 1960 to 2007 consistently underpredicts inflation by 2-3% in the years following the GR. This fact is usually referred to as *missing disinflation*.

Labor productivity, Figure 2, Panel (E), starts to decrease sometime before the Great Recession, features short-lived spike in 2009/2010, only to slow down again around 2012. The slowdown in labor productivity, also highlighted by Christiano et al. (2014), Reifschneider, Wascher, and Wilcox (2015) and Fernald et al. (2017), is often cited as contributing to the slow recovery following the recession. The causes behind it are a matter of debate. One view, considers that the productivity behavior could be a direct result of the crisis, which led firms to reduce their productivity-enhancing investments.[37] A second view, articulated by Fernald et al. (2017), considers the fall to be unrelated to the factors leading to the GR and simply the result of poor luck (i.e., of exogenous negative shocks to TFP). As I discuss next, the job ladder provides an alternative (complementary) explanation that ties the fall in labor productivity to the slowdown in labor reallocation.

Financial Shock. In what follows, I hit the economy with a reduced form financial shock calibrated to target unemployment dynamics during the Great Recession.[38] While I do not model financial frictions explicitly, I consider a shock that transmits through the economy in manner similar to that of a financial shock. Specifically, I shock the spread χ_t in the discount rate of labor intermediaries. The shock raises the required rate of return for their vacancy-posting investment decisions, directly reducing firms' incentives to enter the labor market. In a similar exercise, likewise trying to understand the GR, Christiano et al. (2014) model a financial shock as a "wedge" to the household intertemporal Euler equation for capital investment, which drives a spread between the rate of return of capital and the risk-free rate. More generally, this shock relates to the investment wedges from business cycle accounting literature explored by Chari, Kehoe, and McGrattan (2007), who show that popular theories of financial frictions, such as Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999), manifest themselves as wedges to investment Euler equation. In my model, investment occurs through vacancy creation: firms must expend resources to post vacancies, which can lead to the creation of a worker–firm match providing a long-lived profit stream to the firm. The financial shock then raises the required rate of return for

[37] An example of such is Anzoategui et al. (2019), who develop a model of R&D and technology adoption. In this environment, the fall in TFP becomes an endogenous outcome of a financial shock.

[38] Although the fundamental cause of the GR is still a matter of debate, it is clear that a shock to the financial sector played a crucial role.

this investment, as would the investment wedge in a model with capital.[39]

Figure 3 shows the impulse response to a increase in the spread of labor intermediaries. The shock is calibrated to target unemployment dynamics during the Great Recession.[40] The shock directly affects vacancy-posting incentives by reducing the value of a match for the firms. Through the free-entry condition (15), vacancies collapse, making unemployment surge (Panel (A)) and job-to-job transitions fall (Panel (B)). In equilibrium, unemployment increases by 5 percentage points, consumption falls 8% at the trough, and labor productivity – measured as output divided by the measure of employed workers – falls by 4%. The overall behavior predicted by the model is similar to that during the Great Recession. Figure 3 also shows the behavior of marginal costs and inflation. The model predicts a sharp initial drop of marginal costs. Inflation, however, falls only momentarily and quickly reverts above steady state. The dotted lines in the inflation graph denote the data points from Figure 2, starting from the second quarter of 2008.

What explains these results? The fall in job-to-job transitions keeps employed workers stuck at the lower rungs of the productivity ladder. This misallocation in the employment distribution explains the aggregate labor productivity movements in Panel (B), which fall even though total factor productivity Z_t has not changed.[41] The effects of misallocation are persistent and prevail even after the unemployment rate returns to its steady-state value. Similar to an adverse technological shock, the misallocation exerts upward pressures on marginal costs, which explains the inflationary pressures during the recovery.

[39] Versions of the search and matching model in which firms' discount factor fluctuates in response to aggregate shocks have been recently explored by Hall (2017), Kehoe, Midrigan, and Pastorino (2017) and Borovicka and Borovickova (2018). Time-varying discount rates considerably increase the model's unemployment volatility compared with the risk-neutral textbook search and matching model. In these examples, however, the firm's discount rate varies endogenously in response to technological shock. Here, I consider exogenous variations in the wedge χ and interpret those as standing for a financial shock.

[40] I consider paths for χ_t of the form

$$\begin{cases} \chi_0 & \text{if } t < \bar{T} \\ \chi_0 \exp(-\chi_1 t) & \text{if } t > \bar{T} \end{cases} \tag{18}$$

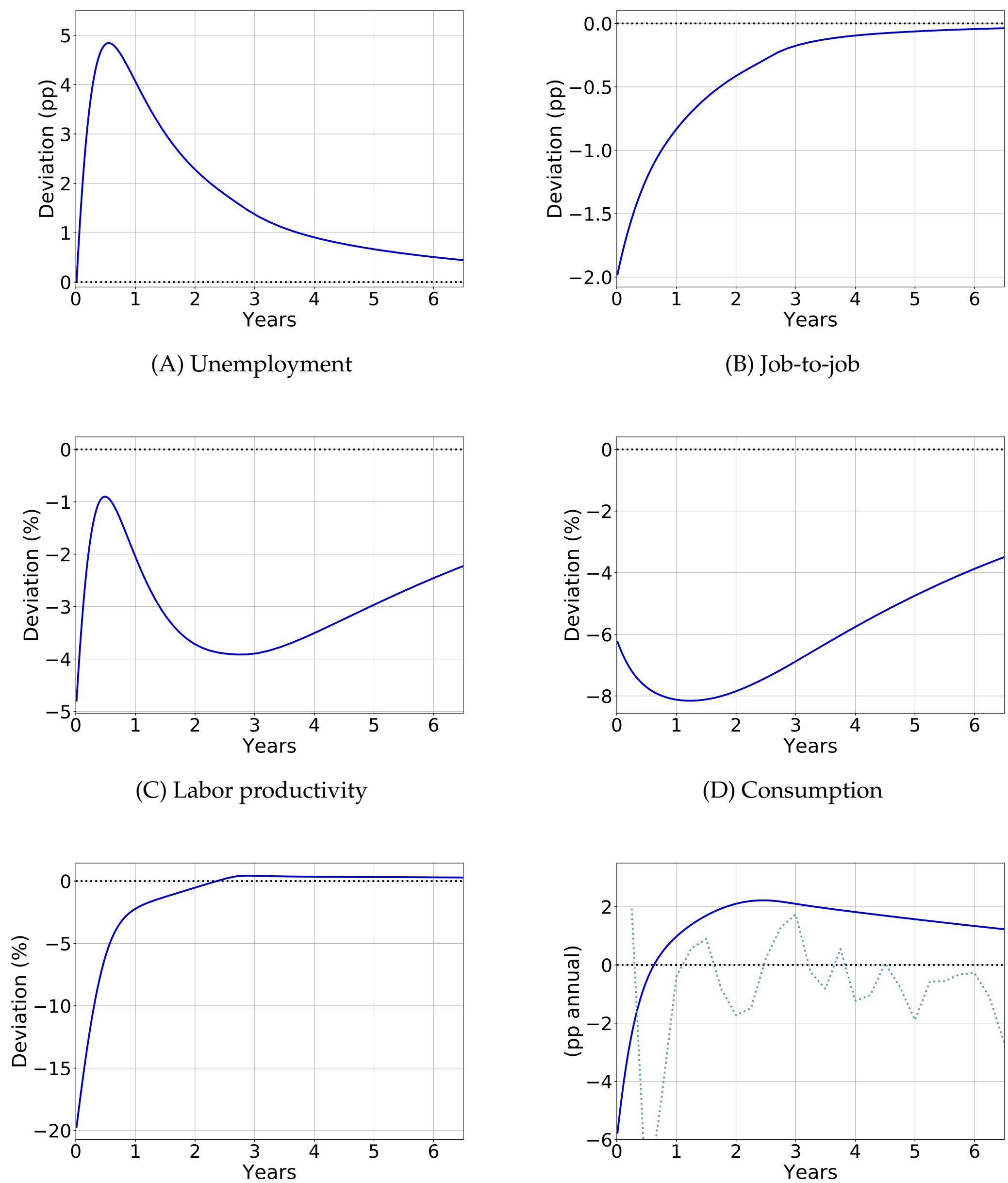
I explore different combinations of $\bar{T}, \chi_0, \chi_1$ and choose the one that more closely matches the unemployment dynamics during the GR. Getting the persistence of unemployment is particularly hard, since the misallocation induced by the shock is itself a force that pushes unemployment back to steady state. See calibration section for an explanation of this point.

[41] The labor productivity measure captures changes both in materials input usage and to the average match productivity of employed workers. Using the production function of retailers, one can show that model implied labor productivity is given by

$$\frac{Y_t}{1-u_t} = (1-\gamma mc_t)(m_c\gamma)^{\frac{\gamma}{1-\gamma}} Z_t \frac{\mathcal{N}_t^e}{1-u_t}.$$

So labor productivity can fall either due to (i) fall in TFP component Z_t; (ii) decline in marginal costs, which induces a decline in materials; (ii) decline in the average match productivity of employed workers $\mathcal{N}_t^e/1-u_t$. Since $\mathcal{N}^e$ is a state variable in the model, the initial drop in labor productivity comes entirely through a reduction in materials. Along the recovery, marginal costs rise *above* steady state, so the labor productivity fall is entirely due to the lower average match productivity of employed workers.

Figure 3: Response to an increase in the spread of labor intermediaries



(A) Unemployment

(B) Job-to-job

(C) Labor productivity

(D) Consumption

(E) Marginal costs

(F) Inflation

Notes: The blue line denotes the model response to the financial shock. The background dotted line in the inflation graph represents the data from Figure 2 in deviations from steady-state 2% inflation. Inflation is shown as annual percentage point deviations from steady state, unemployment and job-to-job transitions are in percentage point deviations, while other variables are shown as log deviations from steady state.

At the moment of the shock, however, the supply of labor services has not yet changed.[42] So the response over initial periods is mainly driven by a fall in aggregate demand that responds to the lower future incomes and higher real interest rates. Since the supply of labor services takes time to adjust, most of the initial reaction occurs via the usage of material inputs, driving down price of labor services and of marginal costs.[43] This does not result in a major disinflation because inflation depends on the whole discounted sum of future marginal costs – recall equation (13). Higher future marginal costs during the recovery therefore prevent inflation from falling too much at the outset. Several other papers offered related explanations for the missing disinflation.[44] Similar to those, I relate the missing disinflation to a fall in productivity. But in my case, the fall in labor productivity comes from the slowdown in employment reallocation in the labor market.

Understanding the Consumption Response. Heterogeneous agent incomplete markets models feature consumers with (i) a sizable MPC out of transitory income changes and (ii) precautionary savings motive.[45] These differences have been shown to matter for how monetary and fiscal shocks are transmitted to consumption — see Kaplan et al. (2018) and Auclert et al. (2018). The main insight gained from these exercises is that changes in disposable income, to which high-MPC agents are very sensitive, are the main driver of the consumption response in HANK models. In contrast, consumption in Representative Agent New-Keynesian (RANK) models is driven almost entirely by changes in the real rate through intertemporal substitution.

In a standard HANK model with no frictions in the labor market, the income channel operates through changes in competitive prices (like wage) and quantities (hours, dividends), but not through changes in higher moments of the income process.[46] The frictional labor market adds another channel through which consumption may be affected: changes in the transition rates impact the distribution of *future labor income*. In particular, recessions increase the duration of unemployment and dampen the expected wage growth of employed workers. In what follows, I study the

[42]Remember that the supply of labor services is given by $\int z \, d\Psi_t(z)$. At $t = 0$, the distribution Ψ_0 is a state variable so labor services are equal to their steady-state value.

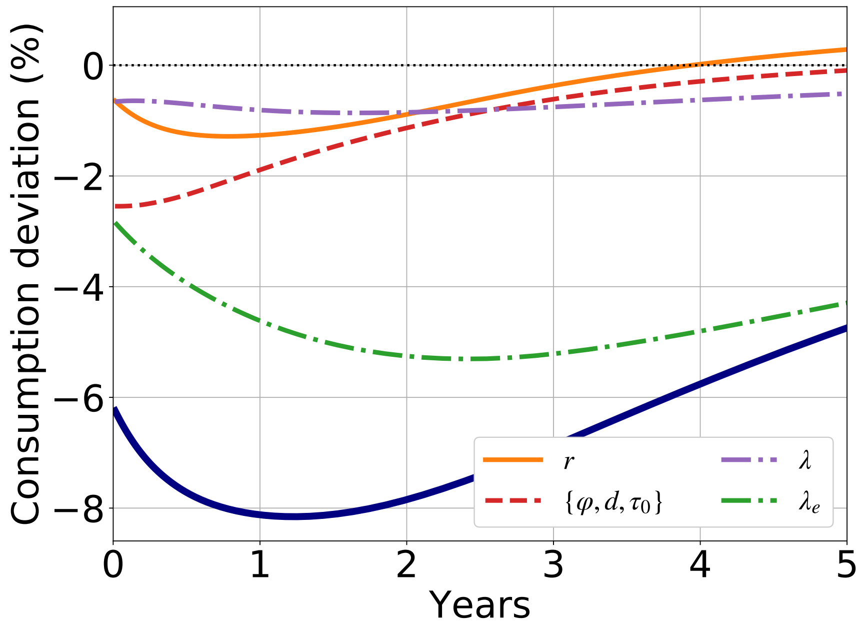
[43]The dynamic of the response is similar to the response of new shocks explored in Christiano (2010) and Barsky and Sims (2011). As explained in Christiano (2010): "News that technology will worsen in the future creates the expectation that future inflation will be high and this leads an inflation forecast targeting monetary authority to increase the real rate of interest. This policy reaction creates an immediate contraction in the economy which reduce marginal costs."

[44]See Christiano et al. (2014) and Anzoategui et al. (2019) for explanations that rely on the slowdown on productivity growth, and Del Negro, Giannoni, and Schorfheide (2015) for an explanation that does not rely on supply-side considerations, but on monetary policy instead.

[45]See Kaplan and Violante (2018) for a discussion of these features and Acharya and Dogra (2019) for a analytically tractable HANK model that isolates the impact of (i) from (ii).

[46]Bayer et al. (2019) studies the impact of second moment risk shocks, but do not consider those as endogenous responses to common aggregate shocks. Gornemann et al. (2016) and Den Haan et al. (2017) feature a incomplete market model where unemployment risk fluctuates in response to aggregate shocks, but they do not decompose the consumption response as I do here.

Figure 4: Consumption response decomposition



Notes: The blue line denotes the consumption response in equilibrium. All other lines are counterfactual consumption responses that allow for some equilibrium variable to adjust as in equilibrium while others are kept at their steady-state values.

role of this new channel to the consumption response following the financial shock.

The aggregate consumption function $\mathcal{C}_t$ is constructed by integrating workers' optimal consumption response $\{c_{it}\}_{i\in[0,1],t\geq 0}$, which is a function of the sequence of equilibrium prices, quantities and labor market transition rates. I make this dependence explicit by expressing aggregate consumption as a direct function of these equilibrium paths

$$\mathcal{C}_t(\{r_s, \varphi_s, d_s, \tau_s, \tau_s^0, \lambda_s, \lambda_{es}\}_{s\geq 0}) := \int_i c_{it} di \tag{19}$$

To evaluate the impact of the different channels, I compute the partial equilibrium consumption response to paths that let some variables adjust as in equilibrium while keeping others at their steady-state value. In particular, I divide variables entering the worker's problem into three groups: (i) the real rate (r); (ii) the competitive price of labor services, dividends and government transfers (φ, d, τ_0), which I jointly refer below by *disposable income*; (iii) labor market transition rates – in other words, the job finding rate (λ) and on-the-job contact rate (λ_e).[47]

[47] In the context of a monetary policy shock, Kaplan et al. (2018) distinguish between direct (real rate) and indirect (general equilibrium) effects. In my exercise, all variables entering the worker's problem are indirect general equilibrium effects.

Totally differentiating (19), we can write the change in consumption at date t, denoted by $d\mathcal{C}_t$, as

$$d\mathcal{C}_t = \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial r_\tau} dr_\tau d\tau + \sum_{i\in(\varphi,d,\tau_0)} \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial i_\tau} di_\tau d\tau + \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \lambda_\tau} d\lambda_\tau d\tau + \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \lambda_{e\tau}} d\lambda_{e\tau} d\tau \tag{20}$$

Figure 4 plots this decomposition together with the equilibrium consumption response (blue line). In line with what others have found, consumption response is driven mainly by changes in income (both current and future) rather than changes in the real rate. Among the variables affecting workers' income, changes in the on-the-job contact rate, λ_e, account for most of the response, especially at longer horizons. Changes in the price of labor services, dividends and government transfers constitute the second most relevant channel, while the job finding rate accounts for a small fraction of the overall consumption adjustment.[48] The contribution of worker contact rate λ_e to overall consumption response highlights the importance of going beyond unemployment and incorporating job-to-job transitions if one wants to understand the impact of shocks that significantly move labor market flows.

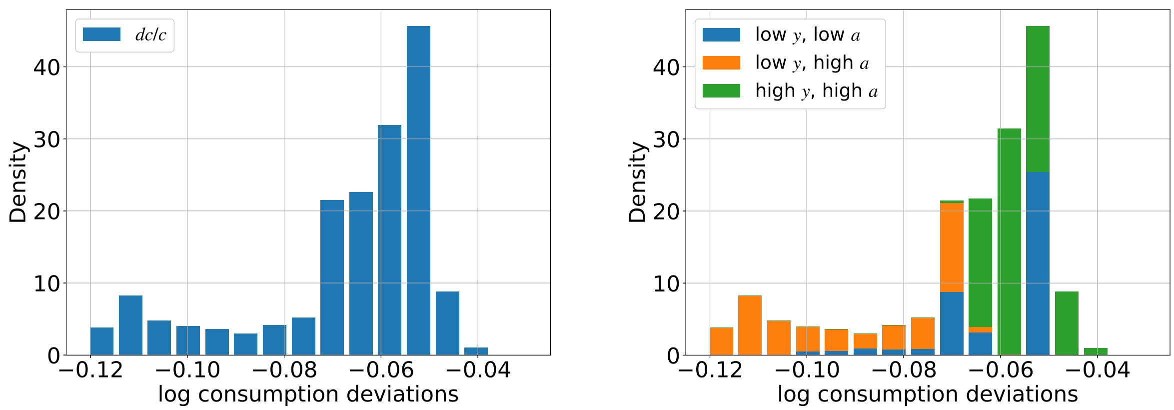
The Consumption Response Across the Distribution. The aggregate consumption response hides a significant amount of heterogeneity that takes place across the worker's distribution. To show this, I concentrate on the time-zero consumption response (i.e., the consumption adjustment that takes place immediately after the shock). Figure 5, Panel (A) plots the distribution of consumption log-deviations from steady-state upon the financial shock. While aggregate consumption falls by approximately 6%, the cross-sectional consumption response shows a significant dispersion, with percentage changes ranging from -4% to -11%.

To explain the dispersion in responses, I examine the initial consumption drop along the wealth distribution (Figure 6, Panel A) and the labor earnings distribution (Figure 6, Panel B). Each panel plots the overall consumption drop (blue line) along with the decompositions at each point of the distribution.

I first consider the consumption responses across the wealth distribution. While consumption response is relatively flat over most of the distribution (ranging from 5 to 7%), its decomposition is far from uniform. The fall in consumption for workers with zero wealth (the initial flat section of the figure) is almost entirely due to the drop in disposable income (red line). As we move along the wealth distribution, the response to changes in disposable income is dampened (consumption falls by less) and workers become more reactive to the changes in the real rate. These observations are consistent with Kaplan and Violante (2018), who also report similar decompositions. The

[48] As I show below, while unemployed workers are the sensitive to the fall in the job finding probability, they represent a small fraction of the population, so their reaction contributes little to overall consumption fluctuation.

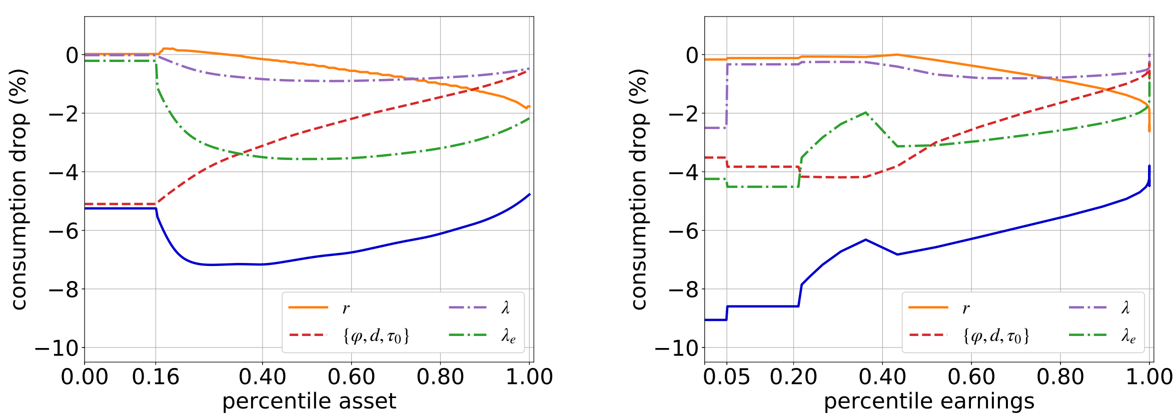
Figure 5: Histogram for time-0 log-deviations of consumption



(A) (B)

Notes: the left panel has the histogram for time-0 consumption log-deviation from steady state for the cross-section of workers; the right panel partitions the histogram in three different groups defined by their joint labor earnings y and wealth holdings a.

Figure 6: Decomposition through the distribution



(A) (B)

Notes: the left panel plots time-0 consumption percentage deviation from steady state along the wealth distribution; the right panel does the same exercise for the earnings distribution. The blue line denotes the overall equilibrium response. All other lines are counterfactual consumption responses that allow for some equilibrium variable to adjust as in equilibrium while others are kept at their steady-state values.

response to labor market rates (green and purple lines), the new element here, is U-shaped in the wealth distribution. Workers at the borrowing constraint have a low sensitivity to labor market rates, while workers in the middle of the distribution react markedly to it. Unlike disposable income, the consumption reaction to labor market rates is still significant at the top of the asset distribution. Hence, even workers who have a large buffer-stock of savings and who are well insured to changes in disposable income react to movements in labor market rates.

Panel B shows how the consumption response varies across the labor earnings distribution. The flat portions of the graph, from 0 to 5% and from 5% to 20%, represent unemployed and recently employed workers respectively. The equilibrium consumption response falls mostly for unemployed workers and less and less as we move along the income distribution. Turning to the decompositions, the unemployed react mostly to the changes in labor market rates, while workers with low earnings (more likely to be employed at lower rungs of the ladder) are mostly sensitive to changes in disposable income (red line). This is mainly because recently hired workers keep dissaving at steady state, which causes a large fraction of them to be low-wealth (hand-to-mouth) agents. As we move right in the distribution (starting around 40^{th} percentile), workers increase their response to labor market rates (green line decreases) but quickly become less sensitive to changes in disposable (red line increases). As before, interest rate sensitivity is weak for most workers, but higher for the upper rungs.

Jointly considering the responses across the two distributions suggests that workers with mid-high levels of wealth and currently low earnings (the unemployed or recently hired) are the ones adjusting their consumption the most upon impact. I verify this conjecture by conditioning the consumption response in Figure 5, Panel B on worker's wealth and earnings. Specifically, I split households into three groups: (i) low wealth and low earnings, (ii) mid-high wealth and low earnings and (iii) mid-high wealth and high earnings.[49] In terms of their MPCs, the quarterly marginal propensity to consume out a $500 lump-sum transfer for each group is 0.68, 0.15 and 0.07 respectively. The results are displayed in Panel B of Figure 5. Indeed, the group with mid-high wealth and low earnings is the one whose consumption falls the most upon impact.

Most of the HANK literature emphasizes the presence of high-MPC agents, and more specifically the covariance of agents' MPCs and income changes, as amplifying the aggregate demand response.[50] In contrast, I find here that those who reduce consumption the most are not the constrained low-wealth agents, but instead unconstrained workers with mid-high levels of wealth. A fall in labor market transition rates does not change workers' *current earnings*, which are determined instead by the price of labor services. Piece-rate wage changes are triggered by outside

[49] I ignore the low wealth and high earnings since those are nearly a mass zero group in the equilibrium distribution.
[50] See Auclert (2018) for a formalization of this intuition.

offers, an event that takes time to come about. So labor market transition rates affect workers by changing their *expected future income growth*.

While a high MPC denotes a strong sensitivity to changes in current income, it also implies a low sensitivity to changes in expected future income. On the other hand, unconstrained workers have a flexible savings margin that adjusts to the fall in the latter. Moreover, the decrease in labor transition rates at the time of the shock affects the *whole future path of earnings* and, therefore, can have substantial impact over their present value.

Workers' reaction to future income changes is also highlighted by Auclert et al. (2018). The authors extend the notion of MPC out of current income changes to an intertemporal MPC *matrix* describing how agents' consumption adjusts to changes in income at any point in the future or in the past. They demonstrate that the general equilibrium response to a fiscal policy shock under a constant real-rate rule is fully determined by the properties of this matrix. In a setting with a Taylor rule determining the real rate or in response to different shocks, like is the case here, intertemporal MPCs, while not sufficient, are still crucial in determining the overall aggregate effects.

6. Unpacking the Mechanism

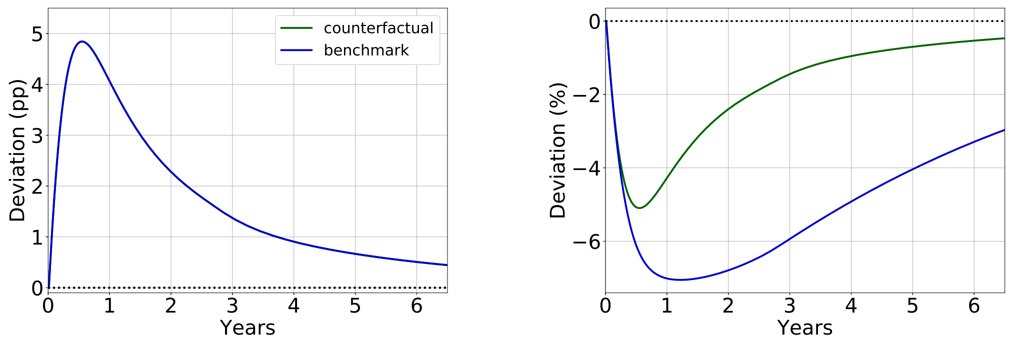
Macro models studying aggregate fluctuations usually abstract from the job ladder. In the introduction, I argued that this element should have both supply and demand-side consequences: the distribution of employed workers moves in response to aggregate shocks and drives aggregate labor productivity, while workers' demand for consumption reacts to earnings changes induced by labor market flows.

In what follows, I try to evaluate the relevance of each component by comparing the response of the full model to counterfactuals that "switch off" these elements. Section 6.1 investigates the supply consequences of the job ladder, while Section 6.2 explores how incomplete markets matter for the demand/consumption block.

6.1. Aggregate Productivity Effects of the Job Ladder

What are the supply-side effects induced by worker reallocation in the job ladder? To answer this question, I will consider a different notion of equilibrium, which I denote as *exogenous-*Λ *equilibrium*. The definition is analogous to the original equilibrium, except for the following modifications: I drop the free-entry condition (15) and treat both the supply of labor services $\mathcal{N}_t^e$ and

Figure 7: Response to an increase in the spread of labor intermediaries



(A) Unemployment (B) Labor services

Notes: The blue line denotes the benchmark model response to the adverse financial shock, while the green line (counterfactual) is the exogenous-Λ equilibrium where labor supply varies with unemployment only.

workers' income process $\{\mathbb{1}^u_{it}, y_{it}\}$ as exogenous.[51] The full definition is laid out in the appendix.

To isolate the productivity effects coming from the job ladder, I compare the benchmark response with the exogenous-Λ equilibrium in which workers face the same equilibrium labor income processes $\{\mathbb{1}^u_{it}, y_{it}\}_{t\geq 0}$, but where the supply of labor services $\mathcal{N}^e_t$ varies only with the measure of employed workers, according to $(1-u_t)\mathcal{N}^{e,SS}$. This counterfactual neutralizes the impact of the job ladder on supply of labor services by treating all employed workers as equally productive, while keeping the job ladder implications for labor earnings unchanged.[52]

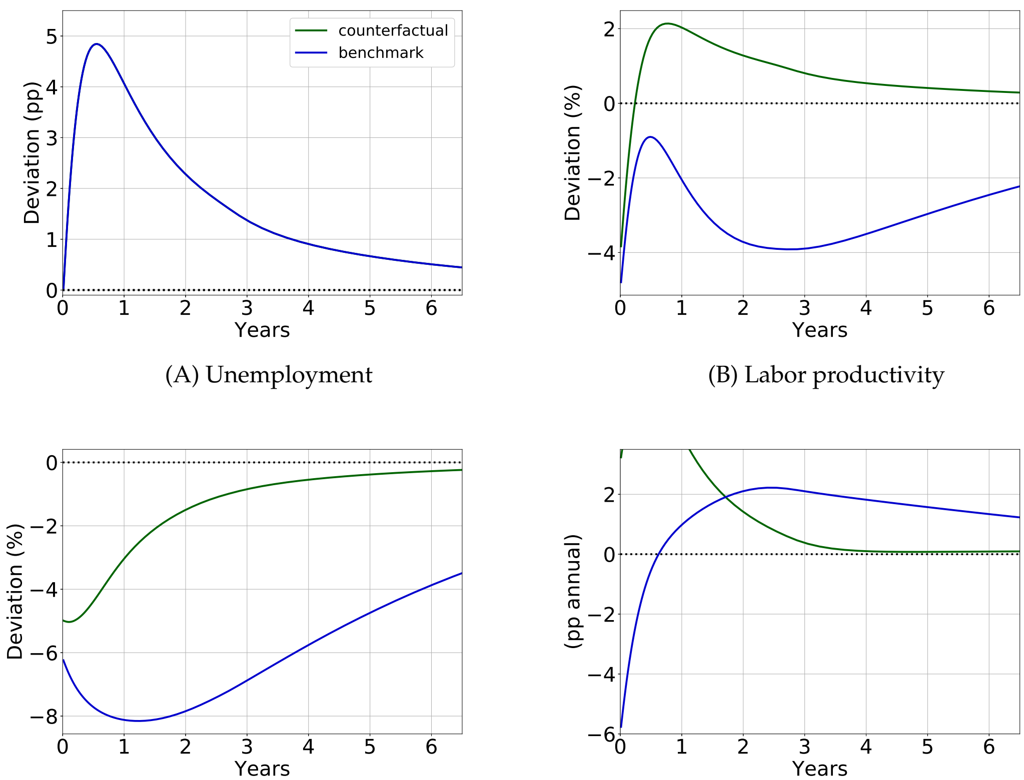
Figure 7 plots the evolution of labor market stocks for the benchmark and the counterfactual equilibrium. Panel (A) shows that unemployment fluctuations are the same in the two economies, as expected. Panel (B) plots the overall supply of labor services $\mathcal{N}^e_t$. In the counterfactual equilibrium (green line), labor services mirror the movements in unemployment. In the benchmark (blue line), the stock of labor services suffer a larger and more persistent decline than unemployment, reflecting the misallocation that occurs among employed workers.

Figure 8 shows that this difference matters tremendously for the response of other aggregates. Consumption in the counterfactual economy is much less persistent and quickly recovers toward

[51] Importantly, I do not impose that the exogenous paths/processes $\{\mathcal{N}^e_t, \mathbb{1}^u_{it}, y_{it}\}$ must be the outcome of a feasible path of transition rates Λ_t.

[52] An alternative way to answer this would be write down a model without on-the-job search and compute the economy's response to the same underlying shocks. There are a couple of difficulties with this strategy though. First, it is not clear how to incorporate the benchmark earnings process into a model without a job ladder. Second, the model without on-the-job search would feature a different response for variables in and out of the labor market, making the comparison of variables like inflation and consumption less transparent.

Figure 8: Response to an increase in the spread of labor intermediaries



(A) Unemployment

(B) Labor productivity

(C) Consumption

(D) Inflation

The blue line denotes the benchmark model response to the adverse financial shock, while the green line (counterfactual) is the exogenous-Λ equilibrium where labor supply varies with unemployment only.

steady state. The counterfactual also predicts inflation throughout the whole transition, with measured labor productivity rising above steady state instead of falling.

6.2. Aggregate Demand Effects of the Job Ladder

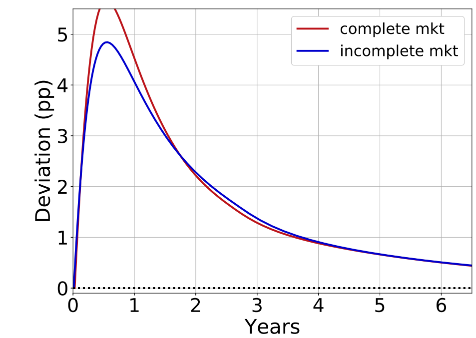
In Section 5.1, I highlighted the importance of the job ladder, and specifically of the on-the-job contact rate, for the transmission of the shock to consumption. In this section, I explore the role of market incompleteness. To that end, I compare the response of my HANK model with a complete markets model, in which workers' consumption is perfectly insured against idiosyncratic labor market outcomes.[53] This version of the model resembles the one discussed by Moscarini and

[53] In the complete markets economy, workers are assumed to belong to a family that pools labor earnings from all its members and decides their consumption. See Appendix A.5 for a statement of the family problem. While the family

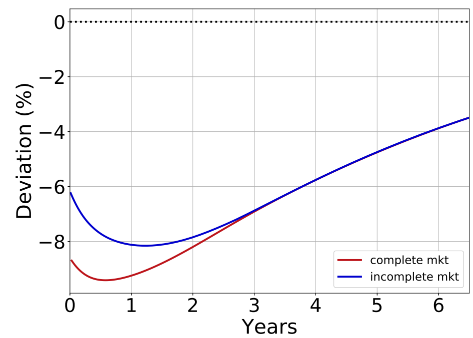
Postel-Vinay (2019).[54]

Figure 9 plots the incomplete (blue) and complete markets (red) response to the same financial shock. Notice that market incompleteness *dampens* the consumption response at impact—in the first quarter, consumption drops by 3% more under complete markets, while unemployment moves by one more percentage point.[55] After 2 years, however, responses are nearly identical except for inflation, which is higher under complete markets.

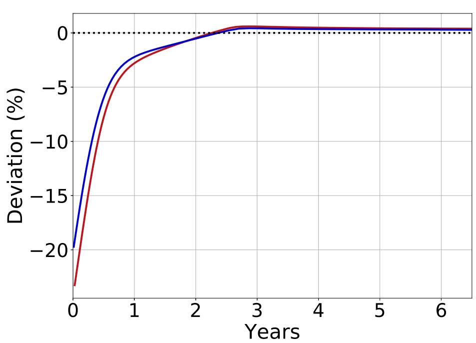
Figure 9: Response to an increase in the spread of labor intermediaries



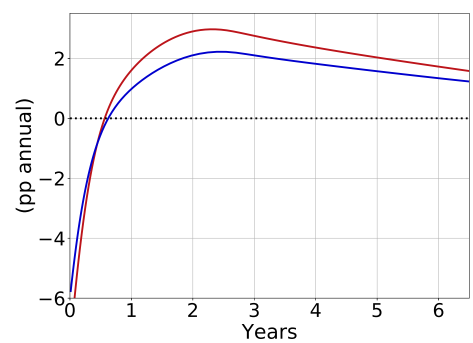
(A) Unemployment



(B) Consumption



(C) Marginal cost



(D) Inflation

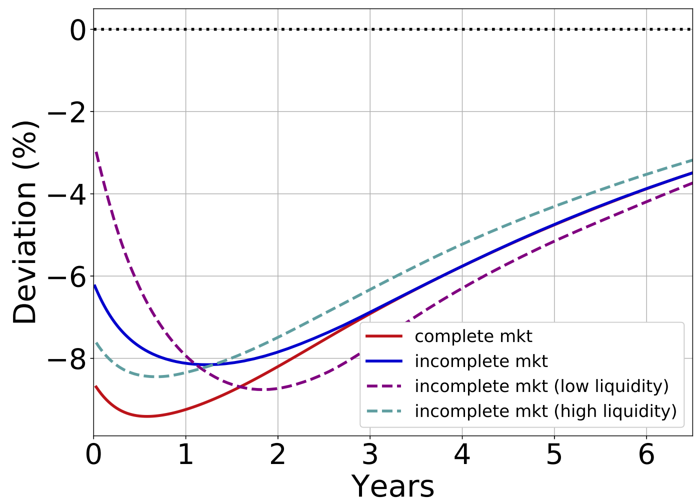
Notes: The blue and red lines are the incomplete and complete markets equilibrium response to the adverse financial shock.

provides insurance to idiosyncratic labor market risk, aggregate shocks cannot be diversified away and must be borne by the family.

[54]There are a couple of differences though. Moscarini and Postel-Vinay (2019) wage setting implements Postel-Vinay and Robin (2002), while my implementation, discussed in Section 2, makes some simplifying assumptions. Moreover, I add materials to allow for a variable production input, while Moscarini and Postel-Vinay (2019) specify the model in discrete time and allow for unemployment and employment to adjust inside the quarter.

[55]I discuss the dampening result next section.

Figure 10: Low x high liquidity



Notes: The blue and red lines are the incomplete and complete markets equilibrium response to the adverse financial shock. The dashed lines represent different calibrations of the incomplete markets model.

Amplification versus Dampening. The extent to which consumption dynamics in HANK models differ from their RANK counterparts is a topic of intense research. Werning (2015) derives an important "as if" result under which the aggregate consumption equilibrium response to changes in the real rate is the same under a HANK and RANK model. As he emphasizes, this result does not ignore the incomplete markets consumption lower sensitivity to real rate changes or the presence of high-MPC households. Instead, the result holds exactly because these two forces cancel each other out: the weaker partial equilibrium intertemporal substitution response to real rate changes is exactly balanced by a stronger consumption response to general equilibrium income effects. Following this result, the HANK literature has emphasized three forces affecting whether and how HANK differs from RANK (i) the government fiscal response to aggregate fluctuations, (ii) the income sensitivity to the aggregate of high-MPC agents, and (iii) the precautionary savings response to cyclical variations in uninsurable risk.

The importance of fiscal response is discussed by Kaplan and Violante (2018). The second point, discussed by Bilbiie (2018) in the context of a monetary shock, shows that amplification occurs when high-MPC agents are also the ones whose income changes the most with the aggregate.[56] The precautionary savings channel is emphasized by Acharya and Dogra (2019); Challe et al. (2017); Ravn and Sterk (2018); Werning (2015). In particular, Ravn and Sterk (2018), who also

[56]See Patterson (2019) for an empirical evidence of this channel and Alves et al. (2019) for an quantitative exploration of this channel in an two-asset HANK model.

consider an incomplete market with labor market frictions, argue that *countercyclical income risk* stemming from unemployment risk *amplifies* fluctuations. The intuition for their result goes as follows: worsening labor market conditions increase households' desire for precautionary savings and reduces aggregate demand. In the presence of price rigidities, firms respond to lower demand by cutting hires which worsens labor markets even further, inducing a further reaction by households, and so on.

The model developed in this paper departs along several dimensions from the simpler environments studied by those authors. Thus there is no reason to believe that their results concerning amplification or dampening should also hold here. However, it is still useful to think about the forces they identify as driving the differences between these two environments to understand my results. In order to explore the relative importance of MPC heterogeneity and precautionary savings response, I consider two alternative calibrations of the incomplete markets model: a high and a low liquidity calibration, achieved by varying the amount of liquid assets available for workers to self insure. The response under these two economies are displayed along with the ones from complete and incomplete markets in Figure 10.

The low liquidity economy magnifies the differences between complete and incomplete markets—aggregate consumption reacts much less at impact than its complete markets version. Recall from the decomposition in Section 5.1 that consumption expenditures fell mainly because of the changes in the on-the-job contact rate. Thus, through the lens of point (ii), responses would be amplified in HANK if the agents whose income are most affected by shock were also the ones whose consumption are most sensitive to the same income changes. The low liquidity economy differs from the benchmark by featuring a higher fraction of hand-to-mouth workers, most of whom stand at the lower rungs of the ladder. These hand-to-mouth workers, as I discussed previously, are not sensitive to the changes in their *expected future income* induced by a decrease in the on-the-job contact rate. This seems to contribute for the dampened response—the larger the fraction of hand-to-mouth workers at the lower rungs of the ladder, the weaker seems to be the aggregate consumption response at impact.

The high liquidity economy diminish the importance of this channel by having workers hold a large buffer-stock of savings, no matter their position in the ladder. Different from the complete markets, however, workers are still subject to income risk coming from the job ladder and change their demand for precautionary savings in face of changes in income risk. These, however, don't seem to matter as much—looking at Figure 10, we can see that differences between the high liquidity and complete markets are much more muted, and point toward dampening instead of amplification, as was suggested by Ravn and Sterk (2018). In my setting, a reduction in labor market flows affects both downside and upside income risk. Longer unemployment spells increase the downside income risk of moving to unemployment, but the lower rate at which workers climb the ladder also changes their upside income risk.[57]

Further understanding of how these forces play out in my setting and their relative importance for the dampening in consumption is an interesting future exercise to pursue.

7. Conclusion

The shutdown of the job ladder during recessions has demand and supply implication that go way beyond the equilibrium in the labor market. As Moscarini and Postel-Vinay (2017) puts it, the *cyclical job ladder* shapes business cycles. In this paper, I developed a Heterogeneous Agents New Keynesian (HANK) model with search frictions in the labor market. Workers search on-the-job as well as through unemployment, which gives rise to a job ladder structure. The job ladder plays a critical role in transmitting aggregate shocks to aggregate labor productivity and consumption demand—the allocation of workers over the ladder partially determines production at any given point in time, while workers' labor income and consumption expenditures varies with the intensity of labor market flows. An adverse financial shock calibrated to mimic the dynamics around the Great Recession generates both the missing disinflation and slow recovery.

[57]Some caution in the use of the term "precautionary savings" and "earnings risk" is warranted. The type of earnings risk considered here is a first-order change—the possibility of going through longer unemployment spells and lower upside gains deteriorates the distribution of future labor income via first-order stochastic dominance. As highlighted by Eeckhoudt and Schlesinger (2008), even a model with quadratic utility generates a "precautionary demand" for saving under first-degree risk increases.

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A. Model derivation

A.1. Filtering problem

I start by writing down the process for productivity and piece-rate wage $\{z_t, y_t\}$ for the worker. For the derivations in this section, I consider the economy to be at steady state, so transitions rates are not indexed by t. Productivity z_t is specific to the worker-firm match and is drawn at origination from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0,1]$. The type of contract offered by firms and labor market transitions determine the evolution of piece-rate y_t. Let $(0,0)$ stand in for the status of unemployed agent and $X = \{0\} \cup [\underline{z}, \bar{z}]$, so the state space for the Markov process $\{z_t, y_t\}$ is X^2. In what follows I describe this process in recursive notation, letting $\cdot^*$ denote the new state.

The rate at which worker leaves state (z, y) to a new state (z^*, y^*) depend only on the employment status and the type of transition: workers leave unemployment state $(0,0)$ with intensity λ, employed workers contact other firms with intensity λ_e and suffer exogenous destruction shocks with intensity δ. Upon any of those events, the distribution of the worker's new state (z^*, y^*) is given by a *stochastic kernel funtion* $T_i : X^2 \times X^2 \to \mathbb{R}$, where $i \in \{ue, ee, eu\}$ indexes the different type of transitions.

The kernel when finding a job from unemployment T_{ue} or receiving a match destruction shock T_{eu} do not depend on the current state (z, y). I write then as

$$T_{ue}(z^*, y^*) = \gamma(z^*)\delta(y^* - \underline{z}) \tag{A.1}$$

$$T_{eu}(z^*, y^*) = \delta(z^* - 0)\delta(y^* - 0) \tag{A.2}$$

where $\delta(\cdot)$ is a Dirac delta function.[58] The match productivity of a worker moving out of unemployment is drawn from exogenous distribution Γ and its piece-rate wage is $\underline{z}$ no matter which firm he goes to. An employed worker that receives a destruction shock moves to unemployment

[58] The Dirac delta can be loosely thought of as a object with the following properties

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} g(x)\delta(x)\, dx = g(0)$$

I use the Dirac delta in the derivation whenever the worker transition is deterministic — for instance, when the worker looses his job he transitions to state $(0,0)$ with certainty.

state $(0,0) \in S$.

The stochastic kernel for an employed worker T_{ee} is more complicated as it depends on the worker's current state (z, y). Remember from the discussion in the main text that an employed worker with state (z, y) who receives an offer from outside firm will: (i) with probability $\Gamma(y)$ discard the offer since it is smaller than its current wage; (ii) with probability $\gamma(y')$ receive an wage offer of $y' \in (y, z)$ which is matched by its current firm, who offers $y' + \epsilon$ to the worker; (iii) with probability $1 - \Gamma(z)$ the worker meets a firm $z^* > z$ which poaches the worker by offering $z + \epsilon > z$, the maximum wage offer of the incumbent. So, taking $\epsilon \downarrow 0$, I write

$$T_{ee}(z^*, y^*|z, y) = \begin{cases} \gamma(z^*) \times \delta(y^* - z) & \text{for } z^* > z, \\ \gamma(y^*) \times \delta(z^* - z) & \text{for } y^* \in (y, z) \\ \Gamma(y) \times \delta(y^* - y) \times \delta(z^* - z) & \text{ow} \end{cases}$$

Integrating out firm productivity z^* from stochastic kernel T_i, we recover the conditional density f_i for piece-rate wage y^*

$$f_{ue}(y^*) = \delta(y^* - \underline{z}), \quad f_{eu}(y^*) = \delta(y^* - 0)$$

$$f_{ee}(y^*|z, y) = \begin{cases} \bar{\Gamma}(z) \times \delta(y^* - z) & \text{for } y^* = z \\ \gamma(y^*) & \text{for } y^* \in (y, z) \\ \Gamma(y) \times \delta(y^* - y) & \text{for } y^* = y \end{cases}$$

which ultimately is the distribution workers care about when deciding how much to consume. Importantly, f_i is a function of current match productivity z, which the worker is uninformed about.

Therefore, when making its consumption/savings decisions he must hold beliefs about z to evaluate the probability distribution for the future piece-rate wages f_i. Let Φ be worker's belief distribution regarding the firm's productivity, with ϕ denoting the (generalized) density function.[59] This distribution is a function of the whole history of job transition and wage offers experienced by the worker. Fortunately, Bayes's rule gives us a way to update this distribution in response to a new signal, so we can treat this problem recursively.[60] Using the same notation as before, let ϕ denote the pre-transition belief density and ϕ^* the updated belief following a labor market event (transition or a wage gain inside the firm). Again, transitions in and out of unemployment involve

[59] I use the generalized classification because some densities will be degenerate.

[60] The derivation in this section draws upon Hansen (2007).

simple updates that are independent of the previous belief

$$\phi^*_{ue}(z^*) = \gamma(z^*) \tag{A.3}$$

$$\phi^*_{eu}(z^*) = \delta(z^* - 0) \tag{A.4}$$

where δ is again the Dirac delta function. An unemployed worker who meets a firm holds as belief the exogenous

For an employed worker who meets an outside firm, the updated ϕ^* density function given new wage offer y^* and transition status is determined by Bayes's rule according to

$$\phi^*_{ee}(z^*) = \begin{cases} \dfrac{\int_{z<z^*} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int \left[\int_{z<z^*} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker switch jobs} \\ \\ \dfrac{\int_{z^*=z} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int \left[\int_{z^*=z} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker does not switch jobs} \end{cases} \tag{A.5}$$

Note that an employed worker gets to observe two signals: whether the highest wage offer came from the incumbent or the poacher — in the former, he realizes that $z^* > z$, while if he stays in the same match $z^* = z$ — and the new piece-rate offer y^*. The filtering problem can thus be thought as a substituting the original Markov process $\{z_t, y_t\}$ by a new one where the hidden match productivity z is replaced by a distribution Φ over possible values, the evolution of which is determined by equations (A.3) to (A.5).

In this case, the conditional density for piece-rate wages y^* becomes a *compound lottery*

$$\bar{f}_i(y^*|y, \phi) = \int f_i(y^*|z, y) d\Phi(z) \tag{A.6}$$

The following proposition show that distribution $\Phi(z)$ is fully characterized by the current piece-rate wage.

Proposition
The belief ϕ for an unemployed is degenerate at $z = 0$. Piece-rate densities in case of a job destruction and job finding from unemployment are independent from z and agree with the full information case, i.e. $\bar{f}_{ue} = f_{ue}$ and $\bar{f}_{eu} = f_{eu}$.

The belief ϕ for an employed worker is a function of piece-rate wage y only

$$\phi(z; y) = \frac{\gamma(z)}{\bar{\Gamma}(y)} \text{ for } z > y \tag{A.7}$$

with the condition piece-rate density in case of job-to-job transition given by

$$\bar{f}_{ee}(y^*|y) = \int f_{ee}(y^*|z,y) \times \frac{\gamma(z)}{\bar{\Gamma}(y)} dz \tag{A.8}$$

PROOF: Conditional density in the case of transitions in and out of unemployment f_{ue}, f_{eu} follow directly from the discussion in the text.

For the employed worker, the proof simply apply Bayes rule for each possible transitions.

coming from unemployment — When the worker is hired from unemployment he receives wage $y = \underline{z}$ and holds belief $\phi = \gamma$ equal to exogenous distribution of match productivity. Note that this satisfies (A.7).

employed worker with job transition — Consider an employed worker with belief distribution Φ and piece-rate y who contacts an outside firm. Suppose that as an outcome of this contact, the worker receives an offer $y_1(+\epsilon)$ from the outside firm, while the incumbent offer is y_1. The worker accepts the offer from the poacher and his belief over productivity z^* of the new match is given by (A.5)

$$\begin{aligned}
\phi^*(z^*) &= \frac{\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z,y) d\Phi(z)}{\int \left[\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z,y) d\Phi(z) \right] dz^*} \\
&= \frac{\int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z) d\Phi(z)}{\int \int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z) d\Phi(z) dz^*} \\
&= \frac{\phi(y_1)\gamma(z^*)}{\phi(y_1) \int \mathbb{1}\{z^* > y_1\}\gamma(z^*) dz^*} \\
&= \frac{\gamma(z^*)}{\bar{\Gamma}(y_1)} \text{ for } z^* > y_1
\end{aligned}$$

where the second line substitutes T_{ee}, the third integrates with respect to z^*.

employed worker with wage increase in the firm — Consider an employed worker with belief distribution Φ and piece-rate wage y. Suppose a poaching firm comes along with the following outcome: the incumbent firm offers a wage increase $y_2(+\epsilon)$ above the poacher's offer of y_2. The worker stays in the incumbent under a higher wage and its belief evolves as

$$
\begin{aligned}
\phi^*(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z,y)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z,y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z=z^*\}} \mathbb{1}\{z > y_2\}\gamma(y_2)\delta(z^* - z)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \mathbb{1}\{z > y_2\}\gamma(y_2)\delta(z^* - z)d\Phi(z)\right] dz^*} \\
&= \frac{\gamma(y_2)\phi(z^*)\mathbb{1}\{z^* > y_2\}}{\gamma(y_2)\int \mathbb{1}\{z^* > y_2\}\phi(z^*)dz^*} \\
&= \frac{\phi(z^*)}{\bar{\Phi}(y_2)} \text{ for } z^* > y_2
\end{aligned}
$$

employed worker with discarded wage offer — Consider an employed worker with belief distribution Φ and piece-rate wage y. Suppose a poaching firm comes along and offer a wage smaller than the current piece-rate y, which does induce a counteroffer from the incumbent, i.e. $y^* = y$. Applying Bayes rule one more time,

$$
\begin{aligned}
\phi(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y|z,y)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y|z,y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z=z^*\}} \Gamma(y)\delta(y^* - y)\delta(z^* - z)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \Gamma(y)\delta(y^* - y)\delta(z^* - z)d\Phi(z)\right] dz^*} \\
&= \frac{\phi(z^*)}{\int \phi(z^*)dz^*} = \phi(z^*)
\end{aligned}
$$

which is the expected result as the signal does not reveal any new information regarding the productivity of the current match.

conclusion — Whenever the worker moves from jobs or when he finds a job from unemployment, his belief ϕ satisfies (A.7). When he receives a wage increase by the incumbent, the updated density ϕ^* a function of the previous belief ϕ, which does not satisfy (A.7) for a generic ϕ. Note however that if we assume that ϕ if of form (A.7), then we get

$$
\phi^*(z^*) = \frac{\phi(z^*)}{\bar{\Phi}(y_2)} = \frac{\gamma(z^*)/\bar{\Gamma}(y)}{\bar{\Gamma}(y_2)/\bar{\Gamma}(y)} = \frac{\gamma(z^*)}{\bar{\Gamma}(y_2)} = \phi(z; y_2)
$$

Since, worker arrives at a firm either by job-to-job transition or from unemployment, ϕ must be of form (A.7). This proves the result. □

Comments on Wage Setting — I end this section by discussing the wage contracts.

First, note that the specification adopted satisfies worker and firm's *individual rationality* as both parts always prefer following the contract to dissolving the match. Second, match origination and job-to-job transitions are efficient with workers always moving toward more productive matches.

However, contracts are not optimally designed.[61] First, since workers are risk averse, firms would be willing to offer some insurance against aggregate fluctuations in the price of labor services. Moreover, different from the current contract has firms overpaying workers when poaching. Since expected future earnings are increasing in the match productivity, more productive firms could in principle poach workers from less productive firms by offering less than the incumbent's maximum wage offer. By how much workers value smoother income paths and the timing of payments depends on their wealth, which makes the optimal contract a function of worker's assets. Not only I regard wage contracts conditional on workers' asset holdings a poor representation of reality, implementing a wealth dependent wage would greatly complicate the determination of wages.

A.2. Worker Problem – Recursive Formulation

I present household's Hamilton-Jacobi-Bellman (HJB) in this section. I focus on the stationary versions of these equations. Let $V^u(a), V(a,y)$ denote the optimal value of unemployed, employed worker's original problem — see the description on the main text — starting from initial level of assets a and, in the case of employed worker, from earnings y. Furthermore, let $s^u(a,c), s(a,y,c)$ denote the savings of employed and unemployed worker with assets a, labor earnings y (b for the unemployed) when he consumes a flow c

$$s^u(a,c) := (1-\tau)\varphi b + ra + d(b) - c, \qquad s(a,y,c) := (1-\tau)\varphi y + ra + d(y) - c$$

where I have already incorporated the fact that dividends are distributed in proportion to labor earnings. The HJB is thus given by

$$\Big(\rho + \lambda_u\Big) V^u(a) = \max_c \Big\{ u(c) + \partial_b V^u(a)\, s^u(a;c) \Big\} + \lambda_u V(a, \underline{z}) \tag{A.9}$$

[61] See Lentz (2014) for the derivation of the optimal contracts in a environment with risk averse workers and where firms are allowed to make counter offers.

$$
\begin{aligned}
\rho V(a,y) = & \max_c \left\{ u(c) + \partial_a V(a,y)\, s(a,y;c) \right\} + \delta \left[V^u(b) - V(a,y) \right] \\
& + \lambda_e \int_y \left[\int_y^z \left[V(a,y^*) - V(a,y) \right] \mathrm{d}\Gamma(y^*) + \bar{\Gamma}(z) \left[V(a,z) - V(a,y) \right] \right] \phi(z;y)\, \mathrm{d}z
\end{aligned} \tag{A.10}
$$

where $\phi(z;y)$ is the household belief regarding the current match productivity. Remembering the definition of $\bar{f}$ in (A.8) we can rewrite the HJB of the employed as

$$
\begin{aligned}
\rho V(a,y) = & \max_c \left\{ u(c) + \partial_a V(a,y)\, s(a,y;c) \right\} + \delta \left[V^u(a) - V(a,y) \right] \\
& + \lambda_e \int_y \left[V(a,y^*) - V(a,y) \right] \bar{f}(y^*|y) dy^*
\end{aligned} \tag{A.11}
$$

Insights from Lise (2012). Lise (2012) also develops a model of on-the-job search in which risk averse workers decide how much to save. He derives an Euler equation describing how consumption growth depends on the preference fundamentals and labor market transitions rates. I follow his derivation in what follows.

Differentiating the value of employment (A.11) with respect to assets we have

$$
(\rho - r) V_a(a,y) = V_{aa}(a,y) s(a,y,c) + \delta \left[V_a^u(a) - V_a(a,y) \right] + \lambda_e \int_y \left[V_a(a,y^*) - V_a(a,y) \right] \bar{f}(y^*|y) dy^*
$$

Substituting the foc $u'(c(a,y)) = V_a(a,y)$

$$
\begin{aligned}
(\rho - r) u'(c(a,y)) = & \, u''(c(a,y)) c_a(a,y) s(a,y,c) + \delta \left[u'(c^u(a)) - u'(c(a,y)) \right] \\
& + \lambda_e \int_y \left[u'(c(a,y^*)) - u'(c(a,y)) \right] \bar{f}(y^*|y) dy^*
\end{aligned}
$$

which is the Euler equation for employed workers describing how consumption evolves between job transitions. We can express it in a more standard form by: (i) dividing everything by $u'(c(a,y))$; (ii) noting that $c_a(a,y)s(a,y,c)dt = dc$, i.e. the change in consumption absent any change to the worker's employment status. In that case, we get

$$
\frac{dc}{c(a,y)} = \frac{1}{\gamma} \left(r - \rho + \delta \left(\frac{u'(c^u(a))}{u'(c(a,y))} - 1 \right) - \lambda_e \int_y \left(1 - \frac{u'(c(a,y^*))}{u'(c(a,y))} \right) \bar{f}(y^*|y) dy^* \right) dt \tag{A.12}
$$

where γ is the coefficient of relative risk aversion. Equation (A.12) illustrate the impact of labor market frictions λ, δ on the worker's desire for saving or dissaving.

- The $r - \rho$ term contrast the rate of return on savings versus the discount rate and drives the usual intertemporal substitution savings motive.

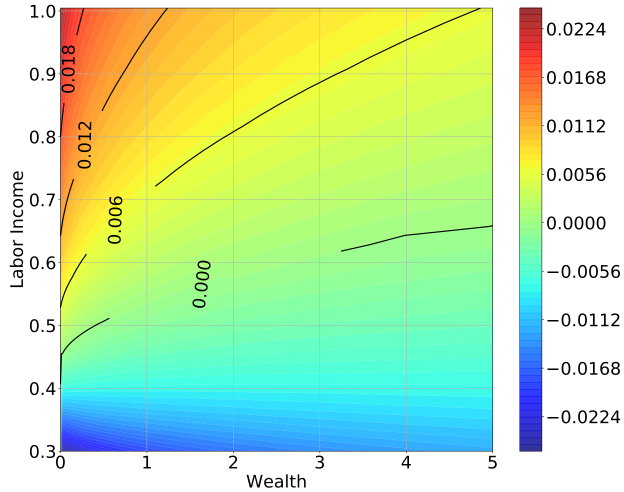
- The term that appears multiplying δ induces precautionary savings.[62] The effect is stronger the larger is the consumption decrease upon an job loss, that is the larger is the difference between consumption of employed worker $c(a, y)$ and the unemployed worker with same assets $c^u(a)$.
- The term multiplying the intensity at which wages grow while on the job (through job-to-job transitions and matched outside offer), captured by λ_e, induces additional *impatience* over and above discount rate ρ. In particular, this force is stronger the higher is the integral that multiplies it. This term is bigger (i) the smaller are the worker earnings y, that is the lower he stands in the wage ladder; (ii) the smaller is $u'(c(a, y^*))/u'(c(a, y)$, that is the more worker consumption increases when moving from y to y^*.

Figure 11 plots dc/c for different points of the state space for an employed worker. For any given level of assets, worker with low earnings tend to dissave, as we would expect from the intertemporal substitution and λ_e terms in (A.12), while workers with high earnings tend to save, as their precautionary savings motive coming from job loss term becomes more relevant. Moreover, differences in savings behavior are most stark at low level of assets. This is also consistent with the comments above, as the strength of labor market transitions δ, λ_e depended on consumption changes upon those events. The wealthier the worker is, less sensitive is his consumption to these labor market events, which makes their consumption growth depend mainly on intertemporal substitution.

[62]Some caution in the use of the term "precautionary savings" is warranted. At loosely level, the empirical literature usually associates changes in intertemporal consumption behavior driven by changes in labor-income risk as due to "precautionary savings". Theoretically, this was first made formal by Kimball (1990) who showed that additional savings induced by moving from a non-stochastic future labor income to a stochastic future income of *equal mean* depended on the third derivative of utility, which he labelled as *prudence*. This result, however, relies on a very specific change in risk and it is not clear how it generalizes for different sources of risk.

The job loss risk considered here is a first-order change in risk — the possibility of going through unemployment deteriorates the distribution of future labor income via first-order stochastic dominance. As highlighted by Eeckhoudt and Schlesinger (2008): "If one considers the increased risk in future labor income to be a higher probability of unemployment, then one cannot use theoretical conclusions based upon prudence." As the author shows, even a model with quadratic utility generates a precautionary demand for saving under first-degree risk increases.

Figure 11: Consumption policy: dc/c



A.3. Intermediate firm Problem – Recursive Formulation

Consider a firm with productivity z under a piece-rate wage contract of y. The value of a match to the firm $\mathcal{J}_t(z,y)$ satisfies the following HJB

$$\begin{aligned}\Big(r_t + \chi_t\Big)\mathcal{J}_t(z,y) = \varphi_t(z-y) + \lambda_{et}\int_y^z \Big[\mathcal{J}_t(z,z') - \mathcal{J}_t(z,y)\Big] d\Gamma(z') \\ + (\lambda_{et}\bar{\Gamma}(z) + \delta)[0 - \mathcal{J}_t(z,y)] + \partial_t \mathcal{J}_t\end{aligned} \tag{A.13}$$

The firm discount profit flows at rate $(r_t + \chi_t)$. With intensity λ_{et}, the worker meets an outside firms. If productivity z' of the poaching firm is between the current piece-rate y and the productivity of the match z, the firm makes a counteroffer z' to the worker who stays in the incumbent firm. This event changes the firm value to $\mathcal{J}(z,z')$. If the productivity z' of the poaching firm is above the productivity of the match, i.e. $z' > z$, the firms looses its worker and the match is dissolved, which leaves the firm with value of 0. The same happens if the match is hit by a destruction shock δ. Finally, the value of the match changes with calendar time t by $\partial_t \mathcal{J}_t$.

We can interpret this HJB equation by treating the function $\mathcal{J}$ as the value of an asset with flow dividends $\{\varphi_t(z-y_t)\}$, where y is indexed by time to reflect that it evolves in the history of the match. [63] The returns in this asset comes from two sources. The first is the flow dividends $\varphi_t(z-y_t)$. The second comes from capital and losses, which in the current context incorporate all right hand-side terms after dividends in (A.13). The asset looses value whenever the firm

[63] See Acemoglu (2007), Chapter 7 for a similar argument.

has to renegotiate the contract y, or the match is destroyed. The asset also appreciate/depreciate depending on the evolution of aggregate variables, which is captured by $\partial_t \mathcal{J}_t$. The HJB equation then states that return on this asset – right-hand side of (A.13) – must be equal to the required rate of return – $(r_t + \chi_t)\mathcal{J}$.

In order to derive some properties of the value of the firm, let me consider a stationary environment where $\lambda_{et}, r_t, \chi_t, \varphi_t$ do not vary with time. In this case, (A.13) simplifies to

$$\Big((r+\chi) + \delta + \lambda_e \bar{\Gamma}(z)\Big)\mathcal{J}(z,y) = \varphi(z-y) + \lambda_{et}\int_y^z \Big[\mathcal{J}(z,z') - \mathcal{J}(z,y)\Big] d\Gamma(z') \tag{A.14}$$

The derivative of $\mathcal{J}$ with respect to piece-rate y is

$$\begin{aligned}(r+\delta+\lambda_e\bar{\Gamma}(y))\mathcal{J}_y(z,y) - \lambda_e\gamma(y)\mathcal{J}(z,y) &= (-\varphi - \lambda_e\gamma(y)\mathcal{J}(z,y))\\ \mathcal{J}_y(z,y) &= -\frac{\varphi}{(r+\delta+\lambda_e\bar{\Gamma}(y))\mathcal{J}_y(z,y)}.\end{aligned}$$

Doing to same for productivity z, I can write

$$\mathcal{J}_z(z,y) = -\frac{\varphi + \lambda_e\mathcal{J}(z,z)}{\Big(r+\delta+\lambda_e\bar{\Gamma}(y)\Big)}.$$

Hence, I conclude $\mathcal{J}_z > 0, \mathcal{J}_y < 0$, that is the value of the match is decreasing in piece-rate wage and increasing in match productivity.

A.4. Exogenous-Λ equilibrium

I spell out the full definition of the exogenous-Λ equilibrium.

Equilibrium *(Exogenous-**Λ**)*
Given an initial government debt B^g, an initial distribution Ψ_0 over assets and labor income, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, exogenous labor income process $\{\mathbb{1}^u_{it}, y_{it}\}_{i\in[0,1],t\geq 0}$ and an exogenous path of labor services supply $\{\mathcal{N}^e_t\}_{t\geq 0}$, a general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N^e_t, M_t, u_t, D_t\}_{t\geq 0}$, government policies $\{G_t, B^g_t, T_t, \tau_t, \tau^0_t, i_t\}_{t\geq 0}$, worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that households optimize, firms optimize, monetary and fiscal policy follow their rules, the worker aggregates and distribution are consistent with the worker's decision rules and exogenous process for income, and all markets clear

- *Asset market clearing*

$$\mathcal{A}_t = B^g_t$$

- *Labor services market clearing*

$$N_t^e = \mathcal{N}_t^e$$

- *Goods market clearing*

$$\mathcal{C}_t + G_t = Y_t$$

A.5. Complete Market Family

The complete market version of the model follow Merz (1995) in adopting a representative family construct, which allows for perfect consumption insurance. The family is composed by a continuum of workers who are either employed or unemployed. At time t, a measure u_t of its workers is unemployed and receives unemployment insurance in the amount of $b\varphi_t$ from the government. The distribution of employed workers inside a family is given $\Psi_t(z,y)/(1-u_t)$, where again z denotes the productivity of the match and y the piece-rate contract earned by employed workers. The family pools all income earned by workers in the form of unemployment insurance and wages. Additionally, the firm receive profits D_t from its ownership of firms. The family then decides on consumption C_t to members and saves through government bonds at rate of return r_t. The problem of the family is then

$$\max_{\{C_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(C_t) dt$$
$$\text{S.t. } \dot{A}_t = r_t A_t + (1-\tau_t)\varphi_t \left(\int y d\Psi_t(y) + b u_t \right) + D_t - C_t + \tau_t^0$$

where τ_t^0 are lump-sum transfers from the government.

B. Data

Consumption is given by real personal consumption expenditures (GDPC), inflation is the PCE deflator (PCECTPI). Both are produced by the Bureau of Economic Analysis (BEA) at quarterly frequency. For labor productivity, I use Nonfarm Business Sector Real Output Per Hour of All Persons from the Bureau of Labor Statistics (BLS). Job-to-job transitions data comes from Fallick and Fleischman (2004).

C. Numerical Implementation

The numerical solution adapts Auclert et al. (2019) method for solving nonlinear perfect-foresight transitions to a continuous time setting.

The perfect-foresight equilibrium defined in Section 3 can be framed in the form of a *functional equation*. Let X be a the space of real-valued functions $x : [0, \infty) \mapsto \mathbb{R}$. Equilibrium restrictions form an operator $\mathcal{H} : X^n \to X^n$ for $n \in \mathbb{N}$ and an equilibrium is a set of real-values functions $y^* \in X^n$ such that $\mathcal{H}(y^*) = \mathbf{0}$. For instance, the real rate path $\{r_t\}_{t\geq 0} \in X$ is one of the n dimensions of the equilibrium vector y^*, while the asset market $\{\mathcal{A}_t - B_t^g\}_{t\geq 0}$ is one of the n dimensions (say $i \leq n$) of the image of y^* under $\mathcal{H}$, that is $\mathcal{H}(y^*)$. In equilibrium, the restriction that asset markets must clear is equivalent to the statement that $\mathcal{H}_i(y^*)(t) = 0$ for all $t \geq 0$.

Solving this in a computer involves discretizing and truncating the time dimension, in which case the X turns into $\mathbb{R}^K$ for some finite K and $\mathcal{H}$ becomes a nonlinear system of equations $H : \mathbb{R}^{nK} \to \mathbb{R}^{nK}$. So solving for the equilibrium is equivalent to solving a root-finding problem of a conventional (although potentially big) nonlinear system of equations. Spreading time points effectively and reducing dimension n by substituting equilibrium conditions makes solving this problem possible.

I will add details on the implementation soon.

D. Other shocks

In this section, I present the impulse response to other aggregate shocks as well as additional series to the financial shock exercise conducted in the main text.

D.1. Monetary

I consider an experiment in which at time $t = 0$ there is a innovation to the Taylor rule (16) of $\epsilon_0 = -1.00/12$ percent (i.e., -1 percent annually) that mean-reverts at rate η. I set $\eta = 0.5/3$, corresponding to a quarterly autocorrelation of $\exp(-3\eta) = 0.61$. Figure 12 shows the responses.

The expansionary monetary shock lowers nominal rates by 40 basis points, decreases unemployment by 0.2 percentage points, and increases consumption by 0.6%. Labor productivity goes up by 0.2% an year after the shock and reverts slowly back to steady state.[64] While inflation moves significantly at impact, the response quickly dies out and is much less persistent than real rate and consumption. Unemployment response is in line with empirical VAR responses to identified monetary shocks in Christiano, Eichenbaum, and Trabandt (2016), Figure 1. Christiano, Eichenbaum, and Evans (2005) estimate that a monetary policy shock that lowers the federal funds rate by 30 basis points raises aggregate productivity by 0.1-0.3% two years after the shock. Meier and Reinelt (2019) reports similar results, with aggregate productivity going up 0.4-0.8% two years after the shock.

Figure 13 plots the decomposition of consumption response in an exercise similar to the one conducted for the financial shock in the main text. The main insights discussed in the main text also applies here: consumption responds mainly to changes in income, with a relevant share being attributed to the rate at which employed workers contact outside firms.

Comparison with a "standard" HANK. It is useful to compare the responses above against a model where the stock labor services do not adjust in response to a monetary shock, that is a model in which $\mathcal{N}_t^e = \mathcal{N}^{e,SS}$. This is meant to stand for a HANK model where households have a stochastic endowment of labor which they supply inelastic at competitive labor markets. So all the adjustment in production must occur via utilization fo the material good M_t.

Formally, the equilibrium is an exogenous-Λ with $\{\mathcal{N}_t^e, \mathbb{1}_{it}^u, y_{it}\}_{i\in[0,1],t\geq 0}$ hold fixed at steady state. Figure 14 shows the response for the two cases. In the benchmark equilibrium, where labor markets flows respond to the monetary shock, we have a less inflation, bigger and more persistent consumption increase as well as persistent impact over labor productivity.

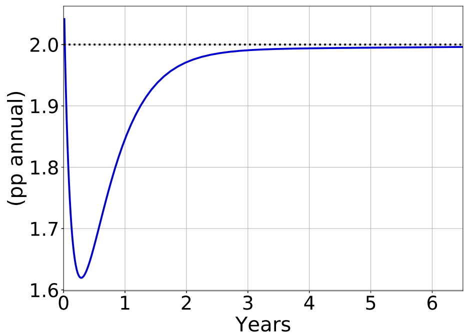
Complete vs Incomplete Markets. Figure 15 shows the response to a expansionary monetary shock under incomplete (blue) and complete markets (red). Consumption response is amplified under incomplete markets, and responses of both consumption and labor productivity are more persistent.

D.2. TFP

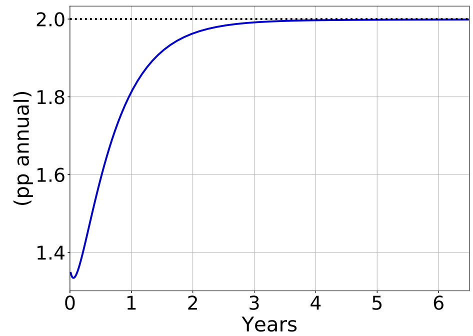
I consider an experiment in which at time $t = 0$, there is a 1% drop to TFP Z_t in production function (8) that mean reverts a rate of 0.02. Figure 16 shows the responses.

[64] As in the case of financial shock, the impact productivity response comes mainly from adjustment of materials input.

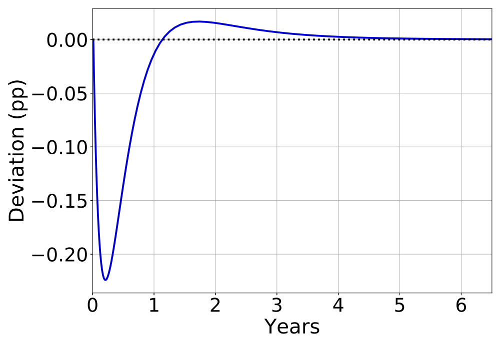
Figure 12: Response to an expansionary monetary policy shock



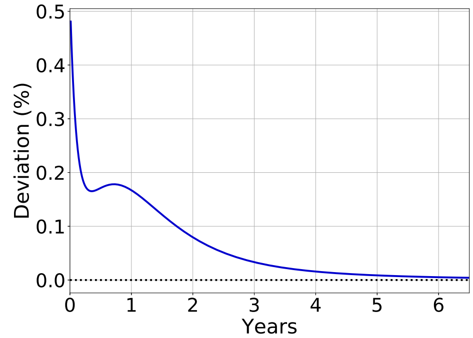
(A) Nominal rate



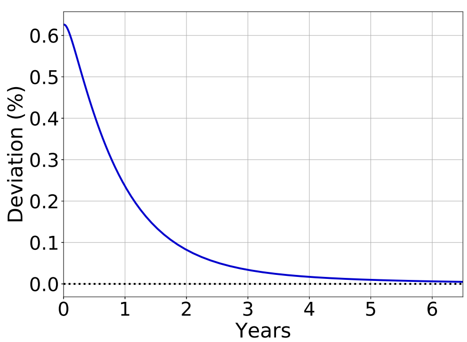
(B) Real rate



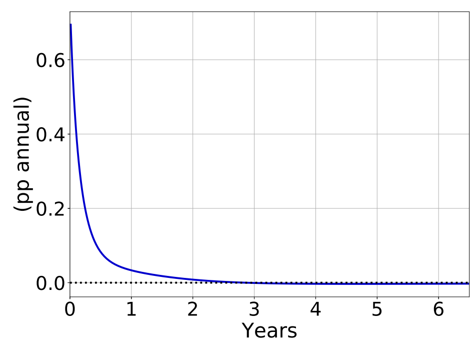
(C) Unemployment



(D) Labor productivity



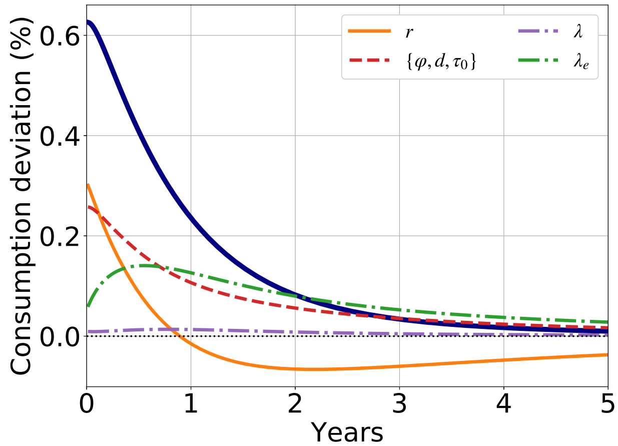
(E) Consumption



(F) Inflation

The negative TFP shock causes consumption to fall by 1.5% and unemployment to increase by 0.15 percentage points. As expected from a New Keynesian model, the negative TFP shock is inflationary. However, in this case inflation is the consequence of both the direct TFP shock as well as the reduction in the supply of labor services that operates through the reduction of employment

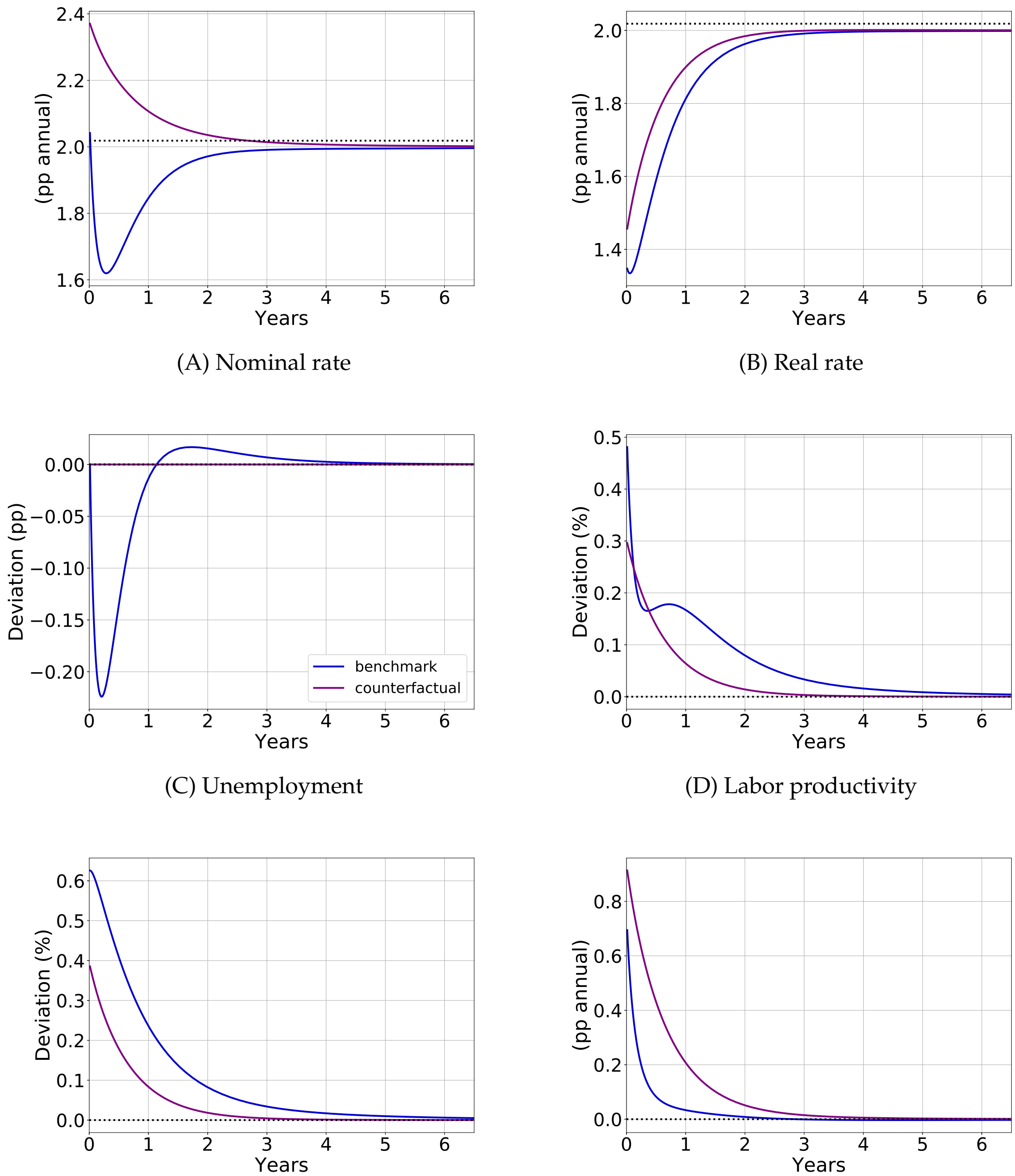
Figure 13: Consumption response decomposition



Notes: The blue line denotes the consumption response in equilibrium. All other lines are counterfactual consumption responses that allow for some equilibrium variable to adjust as in equilibrium while others are kept at their steady state values.

and increased misallocation of employed workers.

Figure 14: Response to an expansionary monetary policy shock



(A) Nominal rate

(B) Real rate

(C) Unemployment

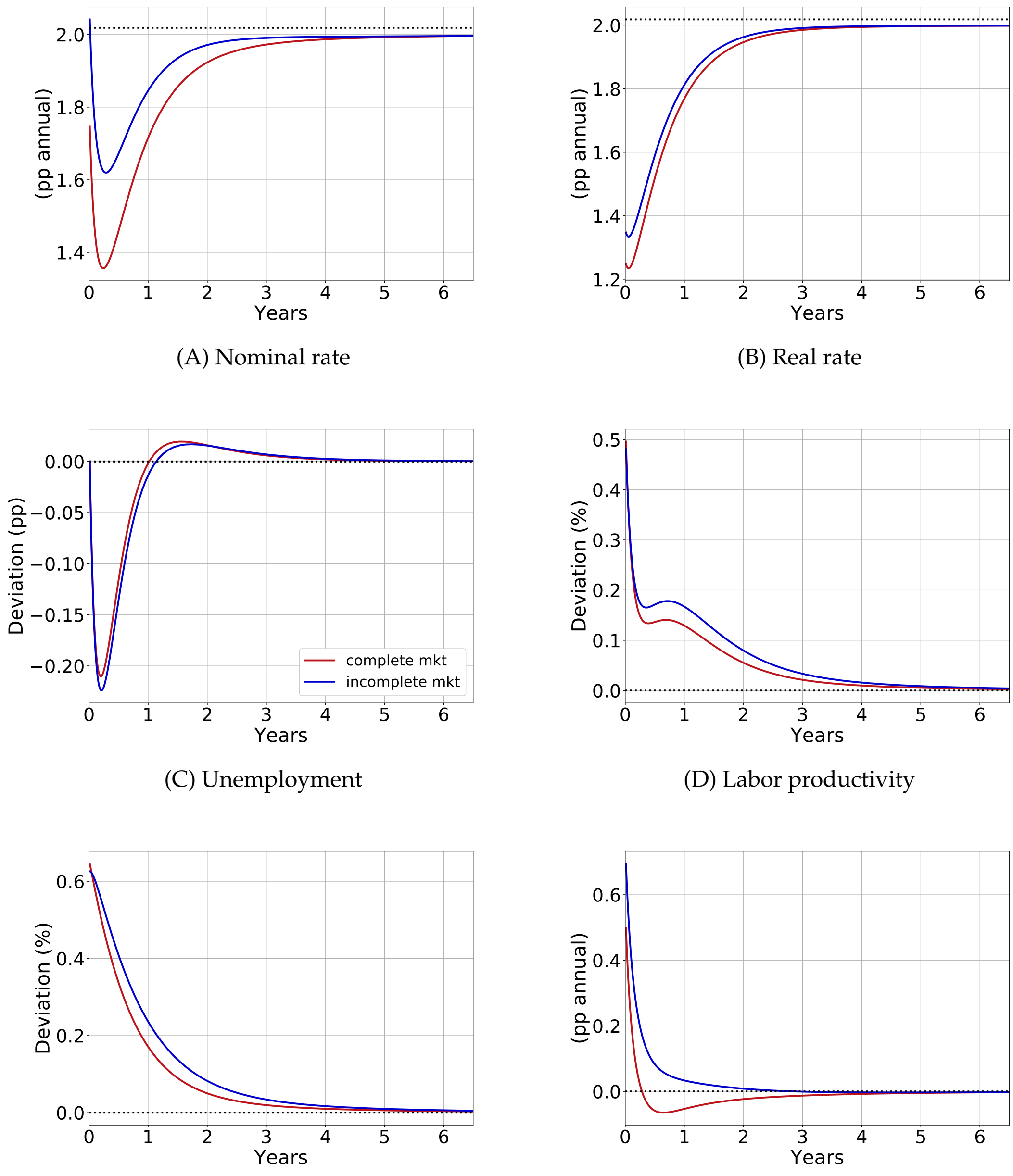
(D) Labor productivity

(E) Consumption

(F) Inflation

Notes: The blue and purple lines are the benchmark and counterfactual equilibrium response to an expansionary monetary shock. The counterfactual economy is one in which the stock of labor services $\mathcal{N}_t$ do not adjust in response to the shock.

Figure 15: Response to an expansionary monetary policy shock



(A) Nominal rate

(B) Real rate

(C) Unemployment

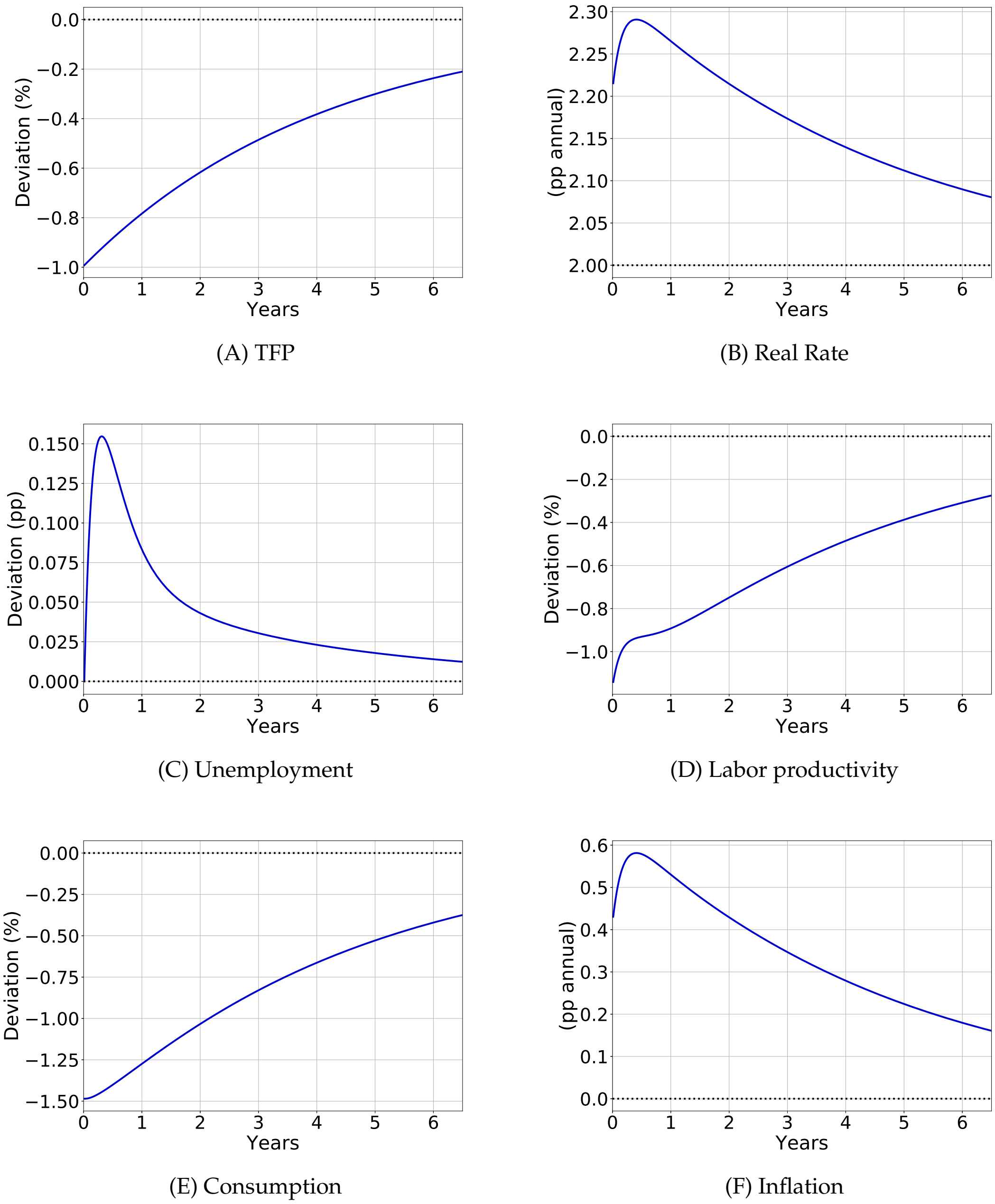
(D) Labor productivity

(E) Consumption

(F) Inflation

Notes: The blue and red lines are the equilibrium incomplete and complete markets response to an expansionary monetary shock.

Figure 16: Response to an contractionary TFP shock



(A) TFP

(B) Real Rate

(C) Unemployment

(D) Labor productivity

(E) Consumption

(F) Inflation

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