Pontifícia Universidade Católica do Rio de Janeiro

Final Course Monograph

A Survey about Insurance

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1810400

Advisor professor: Timo Hiller

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ECONOMICS DEPARTMENT

FINAL COURSE MONOGRAPH

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I declare that this work is from my own authorship and that I haven't had any external assistance, except when authorized by the advisor professor.

For my mother, Fabiana, whom I have no words to describe

Thankings

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Abstract

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Insurance economics started to be studied in the 1960's by Arrow and Borch. This work reviews the main aspects of insurance. It starts talking about equivalence principle and then reviews insurance product, demand and resource allocation. In addition to that, the study talks about informational asymmetries, especially moral hazard and adverse selection, which reduces insure efficiency. Also, it talks about key factors for returns, drivers of value and allocation principles.

Keywords: insurance, equivalence principle, risk sharing, insurance demand, resource allocation, information asymmetry, moral hazard, adverse selection, financial pricing of insurance, insurance companies

Index

1. Motivation

Insurance has been around for a long time. Writing records show the first ever pure insurance contract to be dated in 1347, in Genoa, Italy, for a pool backed of pledges in landed estates.

Since that, insurance has evolved. Nowadays, they are present in many economics' segments: from the health area, with life and health insurance - these commonly called health plans - to the financial market, with options and derivatives, and socioeconomic field, with unemployment insurance, for example.

The most interesting about insurance is its unique dynamic. Insurance is the only product a consumer buys but doesn't know if will ever use it – actually, probably he is willing to never make it useful.

Being such an important and life changing product at the same time it is an economically exotic instrument, it seems intriguing to better understand insurance and its characteristics.

2. Objective

Although risk has been frequently discussed, insurance never had much importance. In the early 1960's, Borch (1961, 1962) and Arrow (1963, 1965) produced articles that started to change this situation. Arrow focused his works on economics of insurance, uncertainty, and information. Borch, in the other hand, focused more on insurance specifically. Prior to him, insurance was viewed as an actuarial problem only and he was the first one to introduce utility hypothesis [see Borch (1961)] and game theory [see Borch (1962)] in insurance analysis. Using these concepts, Borch made several contributions on insurance optimal contracts and risk sharing between parties. Boyle (1990) goes deeper in this manner, showing almost all and how important Borch's contributions were to insurance.

A more broad and complete work is Dionne and Harrington (2017) paper about insurance and insurance markets. Their work is a very good introductory dive into insurance world, since they go through the most relevant papers published in the area. It has worked either as a primary and a secondary reference for my project – as it has innumerous references.

However, this work is neither a complete study about insurance nor an in-depth study about any specific insurance topic. Instead, the focus here is to have a reasonable– not too deep, but not too broad - view about many - not just a few, but not all - aspects regarding insurance. Thus, the study is divided in sections, where in each of them the following topics are discussed: (a) introduction to insurance theory, (b) insurance product, (c) insurance demand, (d) insurance and resource allocation, (e) information asymmetry, (f) financial premium pricing, (g) insurance returns and valuation.

The main objective is to understand how insurance works, how economy theory model its functioning, what drives its demand, supply and price, what are the incentives behind it, how to price them and others.

3. Introduction

Insurance prices are given by the equivalence principle. According to the equivalence principle, premiums paid – the amount paid for the insurance – must be equal to the expected value of the claims – the costs associated to losses – plus an administrative cost for the insurance. Equivalence principle is said to be the "actuarial fair value of insurance". Borch (1967) elaborates an example regarding equivalence principle and its approach.

Consider an insurance contract where claims are either 1 with probability p or 0 with probabilities of (1 – p). The loss expectations of such a contract is p. Thus, an insurance company offers insurance if the premium charged x is greater than the expected claims plus an administrative cost for the insurance. We will get:

$$x = p + \frac{1}{n} C(n)$$

where x is premiums, C(n) is the administrative cost and n is the number of contracts.

The number of contracts n depends on the premium x charged by the insurer. Thus, we denote n(x) the insurance demand function.

Just like Borch, some authors consider administrative costs as a part of the premium. However, it is seen that some authors consider a loading factor that multiplies expected losses instead.

The optimization problem is that both sides of the equation increase as x increases. In this way, there is not only one optimal solution: it is not possible to determine the lowest amount x that the insurer must charge. To obtain an optimal solution, it would be necessary to adopt some assumptions.

Hence, this is a choice of insurer's preference and depends on its willingness to assume risks. The company decides its desired profit distribution by choosing the amount charged for premium x. This profit distribution is bounded by the maximum profit nx, n contracts sold at x premium price with no claims, and the maximum loss n(1-x), n contracts sold at x premium price, but all of them associated with a claim = 1.

By choosing the premium it will charge, we can assume:

Considering the fixed premium chosen by the company – according to its risk preferences-, we can formulate a utility function for such a decision. The company must then maximize its utility function choosing s

$$\int_0^\infty u(S + nP - s - x)\, dF^{(n)}(x)$$

where u is the utility function, S is the initial capital S, n is the number of contracts, P is the premium per contract P, s is the administrative and sales costs – that determines n = n(s) contracts sold - and F(x) is the claim distribution.

From the functions u(x), n(s) and F(x), we know both n(s) and F(x) are familiar to any actuary. However, u(x) represents company's risk policy, and it is a subject much more likely to be studied in economy field rather than actuarial.

4. Insurance Product

Muller (1981) have studied specifically about insurance product. His efforts were focused on answering what really is the product sold by insurance companies.

Many authors have tried to elaborate definitions of insurance. Pfeffer and Klock (1974) said:

> "Insurance is a device for the reduction of uncertainty of one party, called the insured, through the transfer of particular risks to another party, called the insurer, who offers a restoration, at least in part, of economic losses suffered by the insured."

Insurance is about risk exchange. However, this is very abstract and insurance industry and businesses don't use this definition when selling or advertising their product to consumers. Thus, definitions about insurance usually describe it as an operational process of covering possible economic losses. This definition implicitly makes consumers think they are not getting the best use of the product if they are not receiving any claim back. That means, they think they don't get a service equivalent to the premiums they pay. This bad acknowledge about the insurance product can lead to moral hazard, which is typically cited as a consequence for dishonesty or indifference - but this will be discussed later on section 7.

So, we know insurance is about risk exchange. But how does this work? Muller has a good definition:

> "Risk designates the situation of decision making under incomplete information"

Let’s say we are a decision maker in a decision situation. For each possible action (a) we take, we will have an outcome (r), given a specifically state of the world (s) with a certain probability (p).

If we arrange these elements in a decision matrix:

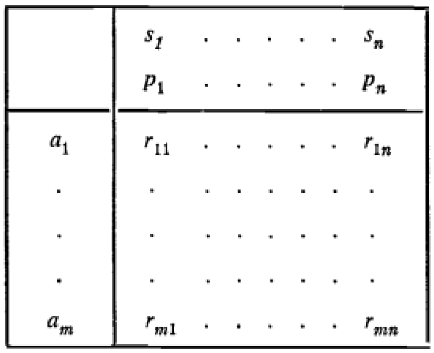

Figure 1: decision matrix in Muller (1981)

There is risk in this decision matrix: we have incomplete information about probabilities of each state of world. Thus, uncertainties about which outcome we will get for each action taken.

Buying insurance change the decision matrix. Now, for a certain action (a*), the outcome is known (r*), no matter which state of the world is faced. Risk is mitigated because there is complete information – probabilities equal to 1 – about an outcome for a certain action.

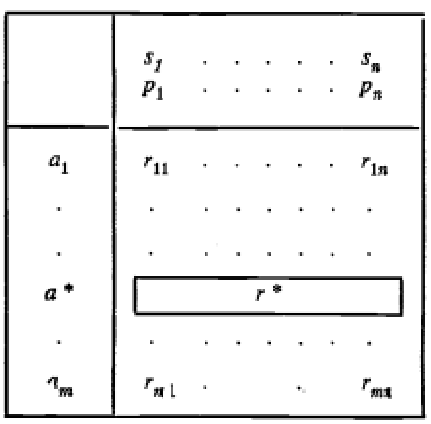

Figure 2: decision matrix under insurance in Muller (1981)

After explaining through the matrix, Muller concludes what he thinks is the product insurer companies sell:

> “The insurer does deliver to the decision maker certain types of information which reduce his information deficit and enable him to form more reliable expectations about the future state of the insured object.”

That is the insurance product after all.

5. Insurance Demand

Since we have an idea about the insurance product, now we analyze what would be and how to determine the demand for such a product.

Mossin (1968) proposed a model for estimating an individual’s demand for insurance, given his risk situation and initial wealth.

Before exploring Mossin’s models – and others that follow -, it is necessary to bring in some concepts:

a. Expected Utility Model, by von Neumann and Morgenstern (1947).

   Although frequently criticized, the linear expected utility model is the main tool used to analyze economic situations, especially under uncertainty.

   Given some axioms and assumptions, a specific action (or insurance policy) will be chosen over other if $E_A U > E_B U$, that means if expected utility associated with policy A is bigger than expected utility associated with policy B.

b. Risk-Aversion, by Pratt (1964) and Arrow (1965)

   Given a von Neumann-Morgenstern utility function for money, with $U'(x) > 0$ and $U''(x) < 0$ – thus being a concave function-, risk aversion $r(x)$ is measured by $-U''(x) / U'(x)$.

Measuring risk aversion is important to determine the risk premium, the maximum amount an individual is willing to spend above E(L) – expected loss – to avoid a risk.

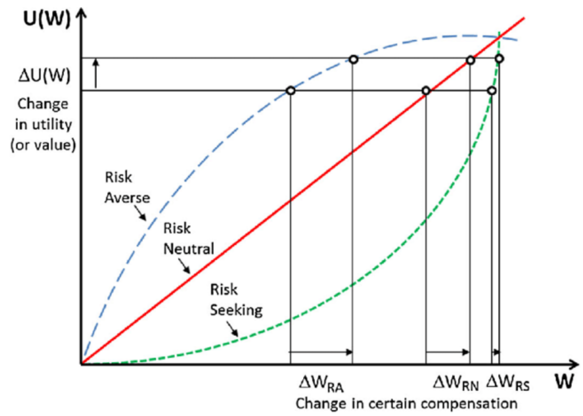

Figure 3: comparing a risk averse utility function to risk neutral and risk seeking functions

Coming back to Mossin (1968), the author studied what would be the optimal coverage at a given premium, a random wealth and a random loss – with given distribution function.

Given a wealth (W), a random loss (L), a premium (P), and a coverage rate (c) – the demand for insurance -, final wealth (Y) will be either:

$E[\,Y = W - L - cP + cL\,]$, if insurance is bought

$E[\,Y = W - L\,]$, if insurance is **not** bought

Expectations are used because there is a probability of loss occurring or not.

As the premium is equal to the expected losses amplified by a loading factor ($\lambda \geq 1$), we can reorganize the equations above regarding utility and expectations:

$$P = \lambda E(L)$$

$$E[\,U(W - L - c\,\lambda E(L) + cL)\,] = E[\,U(W - L)\,]$$

If $\bar{\lambda} > \lambda > 1$, the optimal coverage rate is obtained by maximizing E[U(Y)] for the coverage rate c (the demand for insurance), subject to the condition $0 \leq c \leq 1$. Since risk aversion conditions (U' < 0 and U'' >0) are given, there is a global maximum.

If $\lambda = 1$, optimal coverage rate (c*) is equal to one.

If $\lambda > \bar{\lambda}$, no coverage is demanded.

As Arrow (1965) proposed risk-aversion to be decreasing in function of wealth, Mossin (1968) shows that insurance coverage is an inferior good: its demand decreases as wealth increases. It is intuitively: as your wealth increases you care each time less for a determined (fixed) loss.

Hoy and Robson (1981) went into further: not only insurance is an inferior good, but insurance good can be a Giffen Good. If there is a constant relative risk-aversion, wealth effect can dominate the substitute effect. The higher possible losses are, the higher the plausibility of this happening. In a hypothetical situation, assuming the possible losses are equal to the whole wealth, insurance may be Giffen as probability of loss decreases or the insurance rate increases.

Borch (1985), however, contrasted both authors. For him, both works achieve these conclusions based on two-point probability distributions and on special assumptions. Presenting a more general framework for the problem, Borch showed:

> If $P > \bar{P}$, no insurance will be bought: negative coverage rate is optimal. Actually, you are acting as an insurer – and that is why Hoy and Robson (1981) found that insurance can be a Giffen good. Even greater premiums will lead to even greater – in absolute term - coverage rates.
>
> If $E(L) \leq P \leq \bar{P}$, insurance will be bought and will be an inferior good.
>
> IF $P < E(L)$, insurance is an asset – as it expects a positive yield profit – and it becomes a normal good.

Even though he found insurance is an inferior good in normal situations, instead of modeling it as Mossin (1968) did, he thinks is more realistic to model insurance as a function of fixed costs. Loading factor, which multiplies the expected loss, is substituted by a fixed charge to cover administrative and marketing expenses.

Mossin: P = λE(L)

Borch: P = E(L) + c , where c stands for costs

In this analysis, either you get full insurance cover or no insurance at all. Thus, it makes no sense to wonder whether insurance is an inferior good (or even a Giffen).

Deductibles are very common in insurance policies – especially in automobile and healthcare industry. Instead of fully or partially covering the losses, the insurance company only cover the excess losses from the deductible, which is a fixed amount D stablished in contract that the insured must cover himself. Mossim (1968) also studied them.

Under this type of arrangement, premium becomes a function of the deductibles chosen and the amount paid by insurer is a random variable I that depends on level of deductible D chosen:

$P(D) = \bar{\lambda}E(I)$

$$I = \begin{cases} 0 & if\ L - D < 0 \\ L - D & if\ L - D \geq 0 \end{cases}$$

It is seen that optimal deductibles D* will always be positive. Also, as intuitively thought due to decreasing risk averse utility functions, optimal deductibles increase as wealth increases. A higher level of wealth means individuals is less risk-averse, so he is willing to pay a higher level of deductibles.

6. Insurance and Resource Allocation

General discussion about general equilibrium models started in 1953. Allais (1953), Arrow (1953) and Debreu (1953) discussed about the subject.

In their framework, they redefined the common certainty nature of problems by adding uncertainty about the states of the world. Individuals' preference reflects beliefs about the likelihood of a specific state of the world and behavior toward risk. reflect their preferences. The solution for the problem is trivial and a Pareto optimal allocation of is achieved.

Inspired by these three authors, Karl Borch (1962) was the first to formulate a model for insurance. He models a reinsurance market with n companies, each one holding a portfolio of insurance; each company risk situation is defined by its risk distribution – a probability function for total amount of claims - and its funds – total capital available to pay claims.

Borch studies separately the demand and supply for reinsurance cover. To find a general equilibrium, after some manipulation and assumptions, he maximizes the utility function of a company that buys and sells insurance.

His findings include that under risk-averse and specific conditions there is an optimal arrangement such that risk is spread between companies and that partial coverage will be the optimal solution. Even though Borch considers his results unsatisfactory, he was the first to model insurance without a give price for insurance. His work, therefore, was the reference for many subsequent - that develop his model.

Arrow (1965), for example, was one of them. Under the same framework, he arrives at similar conclusions to Borch regarding partial coverage: it is optimal. Intuitively, as no insurance coverage is bad for companies, because inhibits risky actions, and full insurance coverage is bad, because it discourages success, partial coverage may really be the best.

Arrow (1974) develops further on deductibles and finds that if premiums value is based on actuarial approach plus a loading factor the insurance will cover full losses above the deductible.

Few years later, Raviv (1979) went even deeper in Borch’s work and developed Arrow’s. He shows that Pareto optimal contracts involves both a deductible and a coinsurance above deductible level. He says that deductible feature depends on the insurance costs while coinsurance is due to cost and/or risk sharing between parties.

Borch (1982), in a later study, concludes that deductibles are a practical device to prevent expenses related to administrative costs, especially checking, monitoring and paying claims.

Eeckhoudt and Gollier (1999) made especial case study: they analyzed the insurance of lower probability events. Under Expected Utility conditions, they find that optimal deductible for low probability may even be equal to deductible of the high probability, but usually it will be lower. Actually, he notes that this is consistent with the practice adopted by insurers who reduces price efficiency of low probability policies to be protected in case of a huge - even though improbable- loss.

7. Information Asymmetry

Problems involving information asymmetry are extremely relevant in insurance literature. They are divided in two main categories: moral hazard and adverse selection. Generally speaking, both acts as bad incentives and harms insurance efficiency.

a. Moral Hazard

When insured people behave riskier just because they have insurance, there is moral hazard. Also, frauds or intending actions are related to moral hazard attitude. There is enormous literature around this topic. An entire survey could be written on this subject.

Moral hazard problems can be divided in two categories: ex-ante and ex-post moral hazard. The difference is related to when individuals take actions: before the state of the world for the former or after for the latter.

Shavell (1979a) introduces a principal-agent problem. Principal-agent problem refers to a situation where an agent takes an action that impacts both himself and the principal, but the principal cannot visualize his actions.

This situation is analogous to insurance policies, where insurers are the principal and insured the agents.

In another paper, Shavell (1979b) proposed a mode where probabilities of the states of the world depend on individual's effort and individuals' actions are not known for the insurer. In this case, partial coverage still holds to be optimal, but it varies as effort varies. The amount of effort is negatively related to partial coverage and if an individual cares a lot, which means the cost of care for him is low, partial coverage is desirable.

Shavell also showed that moral hazard doesn't completely eliminate insurance benefits – if an appropriate pricing rule is implemented – and that if insurer can partially know insured actions, moral hazard is reduced – as intuitively is understandable since information asymmetry is lower.

Grossman and Hart (1983) analyzed moral hazard in a principal-agent problem framework.

Usually [see Shavell (1979a)], authors have approached this problem by maximizing principal's utility under a constraint that agent's utility is at certain specific and minimal level. The intuition is that agents are the ones who take the action and therefore will look at his "first order" preferences.

Grossman and Hart (1983) didn't use this approach: instead, they used a cost versus benefits approach. Agent's action incurs either a cost to principal and a benefit to principal. Hence, he avoids the necessity for a "first order" choice of the agent.

Results found show that is never optimal for an incentive scheme to maintain principal's and agent's payoff negatively related through the whole outcome range. Also, conditions for a monotonic incentive scheme function were found.

Spence and Zeckhauser (1971) were the first study about ex-post moral hazard. In the paper, instead of knowing the nature of the accident, as occurs in ex-ante moral hazard, authors proposed that an optimal contract depends on how well the principal can observe the state of nature and the nature of the accident. As shown in one of the cases he demonstrates, when the principal has limited monitoring capacity, choosing the optimal contract will be a second-best exercise.

Derrig (2002) shows that frauds are another type of ex-post moral hazard. The author defines fraud as:

> "Criminal acts, provable beyond a reasonable doubt, making the willful act of obtaining money or value from an insurer under false pretenses or material misrepresentations"

In the literature, Bond and Crocker (1997) defined fraud costs associated with the insurer getting additional information about as costly state verification. Audition process by insurance companies is an example of that. Picard (2013) shows that there is another cost involved with fraud: costly state falsification. This cost is associated with insured costs in preventing insurer will know his claim is a fraud. Collusion schemes is an example of that.

Picard (2013) provides interesting results. Firstly, he shows that fraud impacts the design of optimal contracts. In some cases, a deductibles contract is optimal as it reduces audit costs. Some type of coinsurance contract can also be optimal because it reduces incentives to fraud. Secondly, cooperation among insurers will hamper fraudulent claims. Common agencies and shared data bases, for example, help insurance companies to mitigate adverse selection risk. In addition, contractual relationships between insurer and third-parties hamper collusion, as their incentives are aligned. If sales commission is related to a loss-premium ratio, brokers are discouraged to enter in collusions.

b. Adverse Selection

When an insurer doesn't know an individual's risk characteristics for whom is issuing a policy, there is adverse selection, especially considering the person to be insured is better informed about himself. It is very common in health insurance, for example: smokers or chronic illness people tend to look for this insurance more than normal people. Insurers promote selective underwriting strategies to prevent this.

Akerlof (1970) talks about adverse selection in his famous paper "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism". Even though the study is not about insurance specifically, the author's contributions on adverse selection are very important to our relevant study area. Actually, he exemplifies how similar insurance is to the automobile market the proposed. Analyzing health insurance, he asks why people over 65 have difficulties in buying insurance and the answer is due to adverse selection. The individuals willing to buy insurance are those that know they will probably need. If insurance company rises prices, this problem just gets amplified: now only people certain they will need are willing to buy. As the individuals have more information about themselves than insurance companies, they know whether buying such an insurance is worth for them or not.

From the background Akerlof (1970) proposed, Pauly (1974) builds a model to understand how equilibrium behaves under lack of perfect information. Insurers cannot know whether the insured is a good or a bad risk, so they will charge the same amount for both. This amount will be somewhere between what a bad risk would have to pay and what the good risk would. Pauly (1974) notes that an equilibrium can be reached if excess payments by good risk equals deficiency payments by bad risk. Also, he notes there is possibility for no equilibrium to be hold: if for every premium charged, excess payments are lower than deficiency payments. When this happens, good risk insured are driven out of the market.

Under assumption that insurance company can't differentiate insured and thus the only think known is that bad-risk individuals will demand more insurance than good-risk ones, Rothschild and Stiglitz (1976) models their insurance equilibrium hypothesis. They demonstrate that there is no equilibrium in an environment of pooling contract between good-risk and bad-risk, as insurance companies either will have profit - which means no equilibrium, because profits equal zero on equilibrium - or there will be a better contract which is preferred by good-risk consumers. Under this situation, bad-risk individuals will choose full-coverage while good-risk will choose partial-coverage.

For an equilibrium to hold, instead of only one average contract, there must be two contracts: one for the bad-risk and one for the good-risk insured. This is a sorting technique. Then, company sets the equilibrium contracts separately.

However, as insurance company operates under imperfect information, it doesn't know whether the insured is a good or a bad. Thus, all consumers will buy the lowest premium contract, as if they all were good-risk insured, and the company will be in prejudice situation, which means there is no equilibrium.

The big importance of the paper, however, is showing that equilibrium may not exist at all. If insurance set two contracts where bad-risk insured are indifferent between them, this situation could be an equilibrium. However, there will be a contract where both are better off: this will not be Pareto optimal.

Picard (2009) analyzed the situation where insurance companies offer participating contracts, that means dividend is paid when insurer profits or supplementary money is put when insurer loses, and arrived at an interesting conclusion. If insurance companies offer this type of contract, equilibrium can exist in Rothschild and Stiglitz (1976) model.

Insurance companies usually differentiate people by race, age, sex, or any other characteristic. This is called risk categorization. Some of these are observable in costless manner – like race, age and sex - by insurance companies, thus they charge more or less based on that. Male young drives, for example, will have a more expensive automobile insurance than old women, because it is known that they are less cautious driving. As shown by Crocker and Snow (1986) this discrimination enhances efficiency, but it has been controversial if this is a desirable policy concerning society aspects. Categorization - using particular variables - is actually prohibited in some markets.

Just to note that the authors findings show that not costless observations will not make the market more efficiently; its effects are ambiguous, as there is no negligible cost involved. Therefore, this practice would only be really economically desirable when costs for categorization are irrisory.

Dionne (1983) proposed an infinite-period model where insurer motivates insured to reveal his real risk profile in the first period of the long-term contract by letting the insured to choose his premium. If the insured lies, however, about his risk profile, we will be penalized through the following years of the contract. If he lies, he will have a premium that certain giver a lower utility than utility under no insurance would. Dionne shows, then, that the optimal strategy for the insured will be to tell the truth.

However, there are two shortcomings to the model: firstly, it can’t be extrapolated to competitive markets, and secondly it doesn’t discount future payments.

Cooper and Hayes (1987) partially solve the first one, extending the Rothschild and Stiglitz (1976) model to a two-period model. If the two-period contract is binding, the solution will be just the same as monopoly case - but with insurers constrained to zero-profit. If the contract is not binding, welfare situation of good-risk individuals gets lower - because they can’t commit to a two-period directly and there is a chance they have an accident -, but the situation for others agents remains the same. Hence, not binding two-period reduces experience rating – but it still exists.

8. Financial Pricing

There are several ways of modeling the financial price of an insurance. Bauer et al (2013) provides a great survey about this topic

Actuarial theory pricing is only a matter of expected discounted values. The results, usually stated as an Equivalence Principle, approaches an equilibrium price under perfect competition and no information asymmetries. However, even if independent, if a financial risk is associated to an actuarial risk, a financial pricing theory needs to be incorporated.

Consider the following example, proposed by Bauer et al (2005), on equity linked endowments with a guarantee:

> An insurance policy pays the maximum between the stock price and the initial investment at expiration time 1 if risk materializes and nothing if it does not.
>
> In a one-period Binomial model where risk-free rate is R = 25%, the stock priced at S_0 = \$100 at time 0 can take two values at time 1: $S_1(u)$ = \$200 and $S_1(d)$ = \$50 with probabilities p_s = 50% and (1 - p_s) = 50%, respectively. As we have proposed, the insurance policy will pay the maximum of the stock price and the initial investment G_0 = S_0 = \$100 at time 1 in the case the risk materializes, which happens with a probability of p_x = 75%.

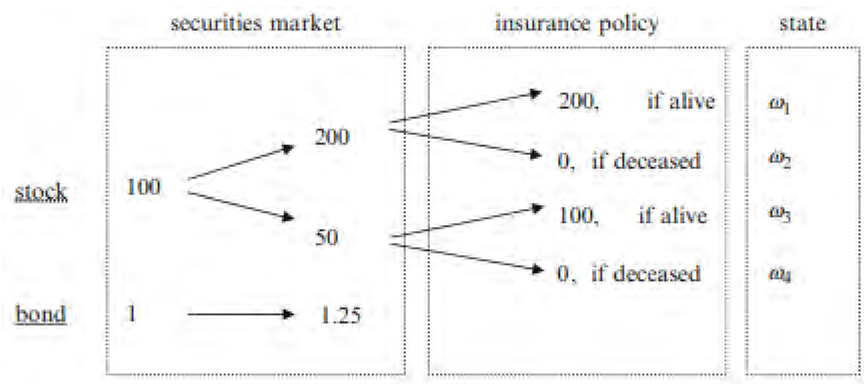

Figure 4: policy rules and states of nature from Bauer et al (2005).

The price for this security will be limited by an arbitrage-free interval between minimum and maximum expected payoff:

$$\left(\inf_Q E^Q \left[\frac{1}{1+R} \text{Payoff} \right]; \sup_Q E^Q \left[\frac{1}{1+R} \text{Payoff} \right] \right) = (0; 120)$$

Assuming L is the number of survivors, N is the total policies sold, and the expected payoff for an individual, the payoff per policy is:

$$\frac{L\, x\, (200\, x\ \ I_{\{w_1,w_2\}} + 100\, x\, I_{\{w_3,w_4\}})}{N}$$

As N approaches infinity, $\frac{L}{N} \to p_x$ by the law of large numbers and thus the expected payoff will be:

$$0{,}75\ x\ \ (200\ x\ \ I_{\{w_1,w_2\}} + 100\ x\ I_{\{w_3,w_4\}}) = 90$$

By this example, we see that the price of an insurance contract is derived by 2 factors:

(i) a product-specific measure of risk-neutral

(ii) actuarial probabilities for independent financial and actuarial risks

Prices are given by cashflow bundles weighted by probabilities. This means financial risk is priced according to financial pricing and actuarial risk is priced according to actuarial method - expected discounted value given actuarial probabilities.

In this last example, markets were complete. When markets are not, but almost, complete, a similar solution is found. Now, there is not an infinite number of insured policies, thus perfect diversification doesn't apply anymore.

Assuming the underlying financial market is complete and insurance risk doesn't affect payoffs at financial markets, the price, given the same expected conditions, will be:

$$\frac{\hat{\beta}_0}{1+R} + \hat{\beta}_1 \times 100 = 90 = E^Q\left[\frac{1}{1+R}\text{Payoff}\right]$$

β_0 and β_1 are calculated by quadratic hedging problem in the form of weighted least-squares problem, where the first denotes the amount in bonds and the latter the amount in stocks.

We can see that results show the same conclusions: financial risk priced by financial pricing and actuarial risk by actuarial pricing. Even though now financial risk is not perfectly diversifiable, diversification is still key for the results.

Another way to price insurance is by using the famous CAPM. The key idea under this hypothesis is that insurance companies' stock price reflects market and actuarial risks.

Company returns on equity is defined by:

$$R_{t+1}^{(e)} = R_{t+1}^{(a)}\,(R_t/G_t + 1) + R_{t+1}^{(u)}\,(P_t/G_t) = R_{t+1}^{(a)}(k_t\ s_t + 1) + R_{t+1}^{(u)}(s_t)$$

Applying some algebra, we get:

$$R_{t+1}^{(e)} = R_{t+1}^{(a)}\,(R_t/G_t + 1) + R_{t+1}^{(u)}\,(P_t/G_t) = R_{t+1}^{(a)}(k_t\ s_t + 1) + R_{t+1}^{(u)}(s_t),$$

where $\frac{P_t}{G_t} = s_t$ is the premiums-to-equity ratio and $\frac{R_t}{P_t} = k_t$ is the liabilities-to-premiums ratio.

If we assume return on equity is based on a CAPM model[1], [1]we get by equalizing them the equilibrium expected return on underwriting:

$$E_t\left[R_{t+1}^{(u)}\right] = -k_t e^{r_t} + \beta^{(u)}\left(E_t\left[R_{t+1}^{(m)}\right] - e^{r_t}\right)$$

where $\beta^{(u)} = \frac{Cov_t\left[R_{t+1}^{(u)}, R_{t+1}^{(m)}\right]}{Var_t\left[R_{t+1}^{(m)}\right]}$ is the company's underwriting beta, i.e., how much it profits on underwriting process.

There is evidence, however, that insurance prices are marked-up above what would be expected from this correlation to market risk factor. Bauer et al (2013) supposes that problems regarding the model potentially arrive from the existence of risk premiums linked to specific insurance risks and/or some financial frictions.

In relation to insurance specific risks, Cummins and Phillips (2005) noted that estimating the cost of capital by the Fama-French Three Factor model results in a significantly higher cost of equity, especially due to the financial distress factor. Bauer argues longevity and catastrophic risk are non-diversifiable and thus are examples to explain that. Mitchell et al. (1999) and Froot (2001) found evidences for Bauer thesis: annuities and catastrophic insurance have premiums much higher than "actuarial fair value".

Actually, Borch (1962) notes that insurance pricing increases as the total amount of risk in the market increases, which indicates a positive risk premium.

In relation to financial frictions, most studies about frictions try to understand the impact they have on supplying insurance, i.e., the marginal cost of offering

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[1] Capital Asset Pricing Model, developed by Sharpe (1964), is the most used method to estimate fair returns.The CAPM equation is as follows, where R(e) is the return on equity; e(r) is the risk-free rate; R(m) is the market return and B is a coefficient calculated by Cov [R(e), R(m)] / Var [R(m)], i.e., a standardized measure for variations of an asset returns compared to the market returns

$$E_t\left[R_{t+1}^{(e)}\right] = e^{r_t} + \beta^{(e)}\left(E_t\left[R_{t+1}^{(m)}\right] - e^{r_t}\right),$$

insurance. There are three ways this can occur: (i) insurer care about solvency, so they carry excessive capital, (ii) raising or carrying capital is costly – which means no insurer will hold infinite capital -, (iii) securities markets are incomplete, because otherwise insurance liabilities could be perfectly hedged.

An example of friction in holding capital is taxation of dividends, which implies double taxation, and therefore may induce insurers to hold excessive cash. This is showed by Jensen (1986).

An example of friction in raising capital, as showed by Myers and Majluf (1984), is asymmetric information, which may hamper access to capital.

An example of incomplete securities markets is reported in Cummins and Weiss (2009), where they show it catastrophe bonds trade at spreads that correspond to premiums two to three times higher than the expected loss.

9. Insurance Companies: returns, value drivers and allocations

Return on Assets and Return on Equity have a relationship determined by financial leverage. Leverage amplifies returns on assets, for both negative and positive sides.

Insurance companies operate with levered capital structure, even though this levered capital is not due to debt capital – as a levered company in other industry -, but for an "insurance leverage".

Insurance leverage comes from the nature of insurance business. Any insurance company has an inflated balance sheet because of deferred expenses – expenses yet to be expense. Insured individuals pay the insurance before using it.

Ferrari (1968) provided an interesting algebraic manipulation to rearrange return to stockholders. He analyzed that it is a combination of return on assets multiplied by insurance leverage and summed with underwriting profit multiplied by insurance exposure.

Using the following notation, we get:

T	Total return – after tax
I	Investment Profit (or loss) – after tax
U	Underwriting Profit (or loss) – after tax
P	Premium Income
A	Assets
R	Reserves (and other liabilities)
S	Stockholder's equity

Return on Equity = $\frac{T}{S}$, where S = A – R and T = I + U

So, we get: $\frac{I+U}{S}$ = Return on Equity.

Applying even more algebra, we will find:

$$\frac{T}{S} = \frac{AI + AU + IR - IR}{AS}$$
$$= \frac{I\,(A-R)}{AS} + \frac{IR}{AS} + \frac{AU}{AS}$$
$$= \frac{IS}{AS} + \frac{IR}{AS} + \frac{U}{S}$$
$$= \frac{I}{A} + \frac{IR}{AS} + \frac{U}{S} \cdot \frac{P}{P}$$
$$\frac{T}{S} = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P} \cdot \frac{P}{S}$$

With this last equation, it is possible to see that Return on Equity depends upon two terms. The first term represents the investment return, based on investment return on assets amplified by the insurance leverage - that depends on the relative size of reserves in relation to equity. The second term is the return on underwriting, obtained by the multiplication of underwriting profits and insurance exposure. Note that if we were comparing an insurance company to a manufacturing one, insurance underwriting profit is just like sales margin and insurance exposure is like the turnover ratio.

If the formulas are manipulated in another way,

$$\text{from (1)} \quad \frac{T}{S} = \frac{I}{A} + \frac{IR}{AS} + \frac{U}{S} \cdot \frac{R}{R}$$

$$\text{therefore} \quad \frac{T}{S} = \frac{I}{A} + \frac{R}{S} \quad \left(\frac{I}{A} + \frac{U}{R}\right)$$

This means return on equity is the sum of the investment return on assets and the sum of investment return on assets and underwriting profits related to reserves amplified by insurance leverage. It is interesting to note that as long as investment return on assets are bigger than underwriting losses, it still makes sense to underwrite policies.

As it is possible to visualize from the equation above, insurance leverage has a huge impact on return on equity. Since return on equity is the main driver in the valuation of an insurance company, it is thus important to optimize the capital structure, so the value of the insurance company is maximized. A too big insurance leverage is not desirable as it increases the variability of earnings: any change in underwriting profits or investment return will cause a big impact in returns to equity.

The management of reserve capital is much more difficult than debt capital for some reasons.

1) The former has an expected value with a variance while the latter is (almost always) fixed or given.
2) Debt capital cost increases as leverage increases. Creditors demand higher interest rates – and pledges. With reserve capital cost doesn't change if increasing or decreasing leverage.

A less volatile earning allows a more levered capital structure. That way, investment policy is dependent on the optimal reserve level a company chooses. Therefore, investment returns must be included as well in actuarial analysis.

Approaching aspects of accounting and analysis of insurance companies, Nissim (2010) has a very broad work. There is plenty of useful accountability explanations, however valuation analysis is interesting, because it derives from answering: what drives value in an insurance company?

The value of an insurance company is determined by the discounted cash flow to shareholders.

$$EV_0 = \frac{E[NEF_1]}{(1+r_e)^{.5}} + \frac{E[NEF_2]}{(1+r_e)^{1.5}} + \ldots = \sum_{t=1}^{\infty} E[NEF_t] \times (1+r_e)^{-t+.5}$$

Since Net Equity Flow (NEF) can be stated:

$$NEF_t = CI_t - CE_t + CE_{t-1}$$

where CI is comprehensive income, and CE refers to common equity.

If we substitute this last equation into the first one, we will find that:

$$EV_0 = CE_0 + \sum_{t=1}^{\infty} E_0[CI_t - r_e CE_{t-1}] \times (1+r_e)^{-t+.5}$$

That means, Equity Value today (t=0) is equal to common equity today plus the present value of expected residual income for years to come.

Residual income is the excess income in relation to the required returns investor demand given common equity.

If we add to the second term $\frac{CE_{t-1}}{CE_{t-1}} x \frac{CE_0}{CE_0}$ and rearrange the equation we will get

$$EV_0 = CE_0 \times \left(1 + \sum_{t=1}^{\infty} E[(ROE_t - r_e) \times CUM\_CE\_G_{t-1}] \times (1+r_e)^{-t+.5}\right)$$

where $CUM\_CE\_GE_{t-1} = \frac{CE_{t-1}}{CE_0}$ and means the cumulative growth of common equity

Dividing both sides by CE_0 it is possible to identify the drivers for the value-to-book ratio:

$$\frac{EV_0}{CE_0} = 1 + \sum_{t=1}^{\infty} \frac{E[(ROE_t - r_e) \times CUM\_CE\_G_{t-1}]}{(1+r_e)^{t-.5}}$$

Value-to-book ratio will be greater than one if the second term of the equation is positive. This happens only if ROE is greater than the cost of equity.

If we want to find drivers for value-to-earnings ratio, we need to redefine NEF.

NEF will be stated then as:

$$NEF_t = CI_0 \times \frac{CI_t}{CI_0} \times \frac{NEF_t}{CI_t} = CI_0 \times CUM\_EAR\_G_t \times PAYOUT_t$$

where $CUM\_EAR\_G_t = \frac{CI_t}{CI_0}$ and $PAYOUT_t = \frac{NEF_t}{CI_t}$

This way we get:

$$\frac{EV_0}{CI_0} = \sum_{t=1}^{\infty} \frac{E[CUM\_EAR\_G_t \times PAYOUT_t]}{(1+r_e)^{t-.5}}$$

Equity-to-earnings depends on cost of equity and expectations about growth and payout ratio.

Knowing the drivers for both ratios, it is important to dive into what each parameter specifically means.

Return on Equity (ROE) determines whether an insurance company trades above or below its book (equity) value. Just like cost of equity depends on the risk, ROE should be analyzed regarding risk.

ROE has a mean-reversion tendency; if it is too high, it tends to come back to a lower and mean level, and if it is too low, it as well reverts towards a mean level. In a situation where ROE is above mean, for example, probably new invested capital will not be invested in such rates. Hence, ROE in the following decreases. Economically speaking, probably new competitors will enter the market, reducing expected return for incumbents.

In accounting, losses are stated immediately but profits are recognized as time flows. This is due to a convention in accounting called conservatism. In year 0 of a big impairment charge, for example, income, the numerator, decreases, lowering the

ROE. In the following year, the denominator is impacted by the impairment charge, as the profit(loss) goes into equity. With a lower denominator and bigger numerator, because there is no impairment in year 1, ROE will be greater.

The ROE reversion will be faster or stronger under some circumstances:

- The larger the gap between current and "normal" profitability – just like the given example of impairment charges
- Ther grater the amount of reinvested earnings – ROE on new investments weights more on total ROE
- The more volatile ROE is – due to temporary shocks or circumstances.

Besides looking at profitability by itself, it is important decompose it - as Ferrari (1969) did - to understand how this profitability is being achieved and, therefore, how sustainable it is. Some metrics to evaluate this purpose are:

a. recurring revenue-to-equity: revenue without one-offs and/or transitory items to equity – sometimes it is insightful to adjust equity, excluding other comprehensive income and goodwill. It gives a better insight about the efficient use of equity. A high ratio is positive.
b. revenue mix ratios: looking at the representation of each source of revenue (premiums earned, investment income and fee income) to total revenue, excluding realized gains. It helps to visualized where the revenue is coming from. The higher the proportion of revenue from premiums, the better – it is the most persistent source of revenue.
c. book-tax difference ratio: the difference between after tax income and tax earnings (taxable income after tax) to equity. It shows how accruals are impacting the earnings, as company has limited ability to manipulate taxable income with them. A low ratio is positive – as signals earnings are more truthful.

In addition to profitability, growth is a fundamental driver of value. As insurance companies have two sources of revenues and variable expenses (variance of claims), looking at historical earnings growth is not a good method for estimating it. Instead, we must focus on revenue growth and, being more precise, in recurring revenue growth – excluding one-off items and realized gains/losses from investments. It is even a much better method than looking at assets or equity's growth, as items on the balance sheet are immediately affected by any action incurred while revenue is adjusted as time passes. For example, a new investment or acquisition impacts instantly the balance sheet while revenue is impacted slowly.

Recurring revenue can be divided in two categories: premiums earned and investment returns. Premiums' growth rate is a good way of estimating growth, but there are important notes one should take when looking at it. Premiums can grow due to more policy writings or to higher price. If policy writings are increasing, it is key to understand if policies are not being underpriced, especially if it is a competitive market – where a company should not grow more than the market. If prices are increasing, it is elementary to know if customers are just willing to pay more or if there is change in risk exposure.

The last fundamental driver of value is cost of equity. Cost of equity is the return investor require to invest. Riskier investments entail higher costs of equity. In financial literature, the widely used approach is the Capital Asset Pricing Model (CAPM). According to the author, idiosyncratic risk is not a real price factor, because diversification eliminates this risk. Hence, only systemic risk in considered, that means, undiversifiable risk. By looking at the sensitivity of a stock's return to overall market's return, βeta is obtained. βeta amplifies the market premium, the higher return an investor requires to be invested in the equity market instead of risk-free bonds. Then, this is summed up to the risk-free bond.

A more empirical, but as useful, approach is to derive the cost of equity by the stock prices in the market. Given the stock price and with estimations for ROE and growth in hands, inverting the equation will give us the implied cost of equity capital.

As we have seen in section 8, no insurance company is certain about how much is paying for claims in a specific year. Therefore, companies hold more capital than the expected claims to be certain they can meet any unexpected claims they might have. However, this incurs in a capital allocation problem: how much more capital is needed? If the company decides to cover all possible claims, it will be holding too much cash. If decides to run the business with little capital, however, it will be dealing with ruin risk. Mumford et al (2005) explores this dynamic.

After deciding how much capital to hold, companies need to decide how to allocate it through its insurance businesses. One of the simplest and most used methods to do that is the Tail Value at Risk (TVaR) allocation principle. In TVaR method, capital allocated to a specific business line is:

$$C_i = C \times \frac{E[X_i \mid X \geq q^{\alpha}(X)]}{E[X \mid X \geq q^{\alpha}(X)]}.$$

C is total capital - exogenously given. X is the present value of unexpected losses, which are losses above the expected claims, and $q^{a}(X)$ is the a-quantile of X – which means the minimum loss incurred in (1-a) percentage of cases. Xi is the unexpected claim of i business. Ci is the capital allocated to this i business.

It is possible to obtain a more robust TVaR allocation if we assume capital can flow from a business line to other business line if needed. In this perspective, we allocate capital to a line according to its loss knowing that the whole company can cover it:

$$C_i = C \times \frac{E[X_i \mid X \leq C]}{E[X \mid X \leq C]}$$

This modified TVaR method will lead to more stable allocations compared to traditional TVaR. This can be explained because this method doesn't use quartile measures and therefore depends less on the behavior of distribution tails.

Having allocated the capital, it is important to measure how well it is being used. Total capital in an insurance company can be divided in two categories. "Underwriting capital" is referred to capital needed to support reserves, i.e., future claims the company will suffer due to premiums that are being paid. Thus, adequate levels of “underwriting capital” are always held. Any excess capital an insurance company holds is used for new underwritings.

Return on capital usually is the main focus of shareholder and, because of that, of managers - who get bonus related to this metric commonly. By focusing merely on return on capital, however, managers aren’t incentivized to efficiently run their insurance portfolios, but only to make a high return. And they can spoof these returns for some time. If managers hold very little capital for old underwritings, they free up capital for new underwritings. If taking a strong pricing strategy, this means the company will write many new policies, which will be reflected in a spike in premiums and growth. Until the second or third year and if they get luck – little capital held for old underwriting is sufficient - managers will probably produce fantastic returns - and get fantastic compensation bonus in return. However, this isn’t a sustainable strategy. When the new writings start to call for its claims, sooner or later the company will not have sufficient capital to deal with all the claims it is susceptible to. The company is running on high risk of ruin. This risk seems to benefit only the insurance manager.

How to prevent this from happening? If one considers returns on new underwriting, one will see no big difference in return and thus this will disincentive using too much capital for new underwriting – as it will only be used when indeed imply in good return.

Froot and Stein (1998) say an insurance company should hedge all tradable financial risks. A financial institution maximizes shareholder value by not taking any other financial risk and fully hedging its exposure to tradable risks. Insurance companies should avoid financial markets risks, as claims and the natural insurance business are not related to any stock index, for example.

10. Conclusion

Insurance was very behind the curve with respect to economic theory. To this date, insurance remains out of limelight in economics. However, this is changing since

Borch and Arrow opened this whole new field of researching. And insurance economics has been ever since evolving faster.

Models on insurance demand have become more accurate and detailed, including much deeper studies about information asymmetry, moral hazard, adverse selection, hedging techniques, resource allocation, uninsurable risk, transaction costs and others.

Works and developments on insurance supply are also being made. Pricing and capital allocation have already a vast literature. However not many can be found yet about regulation and organizational structure, for example. There will be even more growth in insurance supply studies than insurance demand.

Empirical studies are even more lagged; they are almost non-existent, but technology availability and innovations, like big data, AI, 5G internet speeds and other will change that. Field studies will become easier.

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