Asset Prices and Monetary Policy - a Sticky-Dispersed Information Model^{*}

Marta Areosa a,b,† Waldyr Areosa a,b

 $^a{\rm Banco}$ Central do Brasil $^b{\rm Department}$ of Economics, PUC-Rio, Brazil

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Abstract

We present a DSGE model with heterogeneously informed agents and two investment opportunities—stocks and bonds—to study the interaction between monetary policy and asset prices. The information is both sticky, as in Mankiw and Reis (2002), and dispersed, as in Morris and Shin (2002). This framework allows us to (i) show that variations in stock market wealth affect consumption, (ii) demonstrate that a central bank can prevent the creation of boom-bust episodes in the economy, (iii) determine the moment of a bust occurrence and (iv) study the impulse responses to dividend and informational shocks.

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[†]Corresponding author: marta@econ.puc-rio.br.

1 Introduction

This paper is concerned with the interaction between monetary policy and asset pricing. We present a DSGE model that considers a portfolio allocation—stocks and bonds—made by agents with heterogeneous information to analyze (i) how asset prices exert real effects in the economy, (ii) how central banks should respond to macroeconomic conditions to prevent asset price boom-bust episodes within the economy, (iii) how informational factors determine the moment of a bust and (iv) how real and informational shocks alter asset prices.

Analogously to Areosa, Areosa, and Carrasco (2010), our model combines sticky and dispersed information. The model incorporates informational stickiness by assuming, as in Mankiw and Reis (2002), that only a fixed fraction of agents update their information sets each period, and the model includes informational dispersion by positing, as in Morris and Shin (2002), that public and private signals about current shocks, which describe the state of the economy, are available to all agents in each period, even to those who have not updated their information set. In addition to giving imperfect information about the state of the economy, public signals cause agents to coordinate their actions by helping them to predict one another's actions, as shown in Morris and Shin (2002), while stickiness adds a dynamic dimension to the strategic use of information by allowing agents to predict how information diffuses over time. For this reason, we believe that a sticky-dispersed information (SDI) model creates the perfect environment for studying dynamic models that incorporate a game of incomplete information.

We consider a model with two assets traded in different stages within a period—morning and afternoon. In the morning, on the basis of new information, households negotiate stocks while firms set their prices. In the afternoon, after observing the outcome of the first stage of the game, households determine their consumption, working hours and the amount to invest in the bond market while the central bank sets the interest rate. The decision to break up each period into two different stages has two important implications. First, as households invest in stock and bond markets at different time points, they are not necessarily indifferent regarding these two markets. Second, as households make their decisions regarding consumption and saving after observing the results of their stock investments, there is a wealth effect on consumption.

From this framework, we prove that, after a shock, asset prices can persistently increase over many periods and suddenly drop, a result that is consistent with the general idea of a boom-bust episode. The model also shows that as asset prices increase, inflation stays low and the output gap becomes positive. This long period of exuberance is consistent with the recent empirical literature. After the bust, both the stock market index and the output gap become negative and slowly recover until reaching a steady state. These dynamics arise from the agents' decisions and depend on how the central bank responds to expected inflation and asset price variations. Informational factors—the degree of informational stickiness and the relative importance of public information—are important in determining the moment of the bust as well as in explaining why not all shocks generate asset price booms. We dissociate the occurrence of boom-bust episodes from the argument that asset prices may not reflect fundamentals. As the central bank may want to prevent extreme movements, even if this dynamic does reflect fundamentals, we derive a criterion, which relates the coefficients of the monetary policy rule to the other parameters of the model, to show how the central bank should react to prevent the creation of boom-bust episodes in the economy. When the central bank is permissive of the creation of boom-bust episodes, the economy becomes much more volatile, given that the effects of a shock are amplified, even if it does not trigger any stock market booms. We also show that, in equilibrium, there is no trade in the stock market, a result that is consistent with Milgrom and Stokey (1982).

Methodological Contribution. Since the seminal paper of Grossman and Stiglitz (1980), many studies have considered the use of dispersed information in asset pricing.¹ Nevertheless, most of these studies consider static models. Even those that incorporate dynamics, such as Allen, Morris, and Shin (2006) and Albagli, Hellwig, and Tsyvinski (2011), as far as we know, build small models that do not allow one to consider the implications for monetary policy. The degree to which the strategic use of information in asset pricing relates to monetary policy is a perennial topic of discussion among scholars because of the difficulty of solving a dynamic model when agents take asset prices as a signal of the current state of the economy, a framework that has become standard in the literature.

In opposition to Grossman and Stiglitz (1980), we posit that agents do not take information from the aggregate price index or the stock market index to avoid endogenous signaling. This assumption is also present in Angeletos and La'O(2011b, 2011a) and Mankiw and Reis (2007) for DSGE models with heterogeneously informed agents that do not consider asset pricing.² Even considering only exogenous signaling, solving a model with information het-

¹See section 2 for a more detailed literature review.

²Areosa, Areosa, and Carrasco (2010) shows that the trajectory obtained for the aggregate price index in a sticky-dispersed model that does not consider endogenous signaling converges to that stated in Morris and Shin (2002) for the limit values of some parameters. This result proves that the equilibrium derived in

erogeneity is challenging because it is not possible to rely on computational methods, as Blanchard and Kahn (1980), to obtain the dynamics of the aggregate variables. Although we have not been the first to solve a DSGE model that incorporate informational heterogeneity, our approach departs from what have been done before. Therefore, our contribution is also methodological.

We derive the equilibrium from the aggregation of agents' decisions, which depend on their (individual) information set. We proved a proposition that shows how to compute aggregate expectations when agents have heterogeneous information. This proposition is not model specific and depends only on the informational structure, meaning the it can be used to solve any other model that incorporates sticky-dispersed information. By means of this proposition, we analytically derive the equilibrium dynamics of the aggregate variables. In equilibrium, aggregate variables are linear combinations of current and past shocks. The complexity of these combinations makes it extremely difficult for agents to extract information about any shock from the aggregate variables, which reinforces the idea that agents do not derive information from these variables.

Organization. The next section briefly describes the literature. We introduce the model in Section 3 and its log-linear version in Section 9.1. We describe agents' information set in Section 5 and derive the equilibrium in Section 6. We calibrate the model to evaluate its quantitative implications in Section 7 and provide concluding remarks in Section 8. Details of all the derivations are available from the authors.

2 Related Literature

Theoretical Literature. Most models in the monetary policy literature assume a representative agent and complete information. Clarida, Gali, and Gertler (1999) describes the workhorse New Keynesian model. This basic model has been extended in many different directions. In particular, there has been an increasing interest in how financial friction interacts with monetary policy. Bernanke, Gertler, and Gilchrist (1999) is regarded as being the most influential paper on this subject. Other important contributions that present models featuring both nominal rigidities and financial friction are Kiyotaki and Moore (1997), Christiano, Motto, and Rostagno (2007), Gertler and Karadi (2011) and

Morris and Shin (2002) does not consider the agents to extract any information from the aggregate price index.

Cúrdia and Woodford(2010b, 2010a, 2011).

To some extent, our work revisits some of the questions addressed in Bernanke and Gertler (1999). Nevertheless, we choose a completely different approach to analyze those questions. While we focus on the strategic use of information in asset pricing, they modify the model proposed in Bernanke, Gertler, and Gilchrist (1999) by allowing asset prices to deviate from fundamentals and rely on the balance sheet channel to show that asset price fluctuations can create real effects in the economy. Because of these differences, we can derive the conditions for the occurrence of a boom-bust episode. Furthermore, although the central bank does not need to react to changes in asset prices, such a "lean against the wind" policy can help the central bank prevent disruptive episodes in the economy without becoming excessively reactive to expected inflation.

A distinct strand of the literature, one that dates back to Phelps (1968) and Lucas (1972) and is revisited in Woodford (2003), studies the various macroeconomic implications of heterogeneous information without considering the interaction with asset pricing. Sims (2003) studies the case in which agents have a limited capacity for processing information. More recently, Angeletos and La'O (2011b) studies optimal monetary policy in an environment in which firms have incomplete information. Christiano, Ilut, Motto, and Rostagno (2008) introduces informational friction in a homogeneous information model to show that a boom phase in capital (or stock) price starts when agents obtain new information about a future improvement in technology and ends some periods later when another informational shock leads agents to realize that they have been overly optimistic.

Bubbles, measured as deviations from the fundamental price, have also been the subject of many theoretical papers. Rational bubbles occur in a setting in which all agents have rational expectations and share the same information. As in Blanchard and Watson (1983) and Bernanke and Gertler (1999), in each period, the bubble persists with some probability or bursts with the complementary probability. As a consequence, the bubble necessarily grows with expectations at a rate that depends on the interest rate. The literature on limits to arbitrage challenges the view that rational investors should go against a bubble even before it emerges. Abreu and Brunnermeier (2003) considers a model in which an asset bubble can persist despite the presence of rational arbitrageurs because rational traders face a synchronization risk in knowing that coordination is required to bring the market down. Bubbles can also emerge when investors have heterogeneous beliefs and face shortsale constraints or when investors have different information, but still share a common prior distribution. Miller (1977) analyzes the former case while Allen, Morris, and Postlewaite (1993) presents a good example of the latter. In these models, an investor might hold an overpriced asset if he thinks he can resell it in the future to a less informed trader or to someone who holds biased beliefs.

Empirical Literature. The empirical literature addresses many of the issues mentioned in our study. One line of research provides evidence for the existence of a stock market wealth effect on consumption. Funke (2004) finds a small but statistically significant effect in 16 emerging markets over 1985–2000. ? and Carroll, Otsuka, and Slacalek (2011) find similar results for advanced economies—Japan, European countries and the United States.

Another line of research documents some stylized facts that are associated with stock market booms. Bordo and Wheelock (2004, 2007) draws attention to the fact that stock market booms are periods of low inflation. In particular, the work of Adalid and Detken (2007) documents that, in boom-bust episodes, inflation is low in the boom phase and then rises slightly at the end. On the basis of vector autoregression evidence, Barsky and Sims (2011) argues that information shocks drive stock prices and economic activity up and inflation down.

3 The Model

The model encompasses a continuum of infinitely-lived households, indexed by $z \in [0, 1]$, a continuum of firms, indexed by $k \in [0, 1]$, a central bank and the government. The households invest in bonds and stocks. They also offer labor in a perfectly competitive market with fully flexible wages and purchase differentiated goods in a retail market with monopolistically competitive firms. The households and firms have heterogeneous information about the state of the economy. We assume that each household z owns a firm k. This assumption ensures that information is distributed among firms in exactly the same way that it is distributed among households.

Timing. Each period has two stages: morning and afternoon. In the morning, new private and public signals, which give imperfect information about the state of the economy, become available to the households and firms. On the basis of this information, households buy or sell a stock portfolio and firms decide upon their prices. In the afternoon, after goods

and stock prices become publicly known, households decide on consumption, working hours and how much to invest in the bond market while the central bank sets the interest rate. We do not consider a cash-in-advance constraint as in Christiano, Eichenbaum, and Evans (2005) and Ravenna and Walsh (2006). All payments occur in the afternoon, although in the morning, agents commit themselves with some payments while they negotiate stocks. Firms pay dividends at the end of the period.

3.1 Households

The preference of household z at period t is represented by an expected utility function that is separable in consumption, $C_{\tau}(z)$, and working hours, $H_{\tau}(z)$:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} E\left[\frac{\left(C_{\tau}\left(z\right)\right)^{1-\sigma}}{1-\sigma} - \frac{\left(H_{\tau}\left(z\right)\right)^{1+\omega}}{1+\omega} \right| \mathfrak{S}_{t}\left(z\right) \right].$$
(1)

We use $E[\cdot|\mathfrak{S}_t(z)]$ to represent the expectation conditioned at the information set $\mathfrak{S}_t(z)$ of household z and $\beta \in (0, 1)$ to denote the discount factor.

Household z uses a Dixit and Stiglitz (1977) aggregator to combine the goods purchased from each firm k, $C_t(z, k)$, in a consumption bundle, $C_t(z)$,

$$C_t(z) = \left[\int \left(C_t(z,k)\right)^{\frac{1}{\mu}} dk\right]^{\mu}$$

Therefore, the price index of the economy, P_t , and demand for each good k are given by

$$P_{t} = \left[\int (P_{t}(k))^{\frac{1}{1-\mu}} dk \right]^{1-\mu}, \qquad (2)$$
$$C_{t}(z,k) = C_{t}(z) \left(\frac{P_{t}(k)}{P_{t}} \right)^{\frac{\mu}{1-\mu}},$$

where $\mu > 1$.

Household z chooses how much to invest in bonds, $B_t(z)$, and in a portfolio that holds the same share of each firm's capital stock, $B_t^S(z)$, considering the budget constraint

$$B_{t}(z) + B_{t}^{S}(z) Q_{t} = B_{t-1}(z) (1 + r_{t-1}) + B_{t-1}^{S}(z) (Q_{t} + D_{t-1}) + W_{t} H_{t}(z) - P_{t} C_{t}(z) - T_{t}(z) - V_{t}(z),$$

where W_t is the wage per hour received by each agent, r_t is the nominal interest rate, $T_t(z)$ is a lump-sum tax, Q_t is the price of the portfolio per share, D_t is the amount of dividends paid per share and, in a similar manner to Agénor, Alper, and da Silva (2011), $V_t(z)$ is a transaction costs associated with changes in stock holdings. In this context $B_t^S(z)$ represents the proportion of the stock market owned by household z.

3.1.1 First Stage: Morning

In the morning, households decide how much to invest in the stock market. Considering

$$V_t(z) = rac{K}{2} rac{\left(B_t^S(z)\right)^2}{1 + r_t},$$

we get from the first order conditions associated to $B_t^S(z)$ and $B_t(z)$ that

$$B_t^S(z) = \frac{1}{K} E\left[Q_{t+1} + D_t - Q_t \left(1 + r_t\right) | \mathfrak{S}_t(z)\right].$$
(3)

This relation tells that households hold stock shares as long as they believe that the return they will get in the stock market will exceed the opportunity costs of not investing in the bond market. The constant K, besides converting the transaction cost into monetary units, captures the sensitiveness of stock holdings to the return agents expect to obtain on stocks relative to the return offered by the bond market

To some extent, this relation resembles the demand for assets stated in Grossman and Stiglitz (1980), considering that $(Q_{t+1} + D_t)$ represents the payoff households will receive on the risky asset. In their work, K = aV where a and $V = V[Q_{t+1} + D_t - Q_t (1 + r_t)| \mathfrak{S}_t (z)]$ are the coefficient of absolute risk aversion and the conditional variance, that would be equal for all agents.³ In our work, however, we cannot give the same interpretation since the conditional variance differs between agents due to informational stickiness.

We use a simplifying assumption by stipulating that dividends follow

$$D_t = \varphi + \tau Q_t + v_t. \tag{4}$$

We justify this assumption in terms of the steady-state levels.⁴

³In fact, they write $V = V[Q_{t+1} + D_t | \Im_t(z)]$ because agents observe $Q_t(1 + r_t)$.

⁴In Subsection (9.1.1), we show that $\overline{D} = \varphi + \tau \overline{Q}$ in steady-state. Therefore, equation (4) considers small departures from the steady-state relation.

3.1.2 Second Stage: Afternoon

In the afternoon, households choose consumption, working hours and how much to invest in the bond market. This choice occurs after households observing the outcome of the first stage. We argue that househods incorporate this new information in two different ways. First, as they observe P_t and Q_t , they do not have to make predictions about it. This approach differs from Grossman and Stiglitz (1980) as households do not try to decompose these variables to extract information about the state of the economy. Second, as households already know how much was allocated in stocks, they will choose $s_t(z) \equiv (B_t(z) + B_t^S(z)Q_t) / B_t^S(z)Q_t$, which represents the amount of wealth household z wants to hold at period t, expressed relative to the amount invested in stocks. Although choosing $s_t(z)$ indirectly determines how much household z spends on bonds when $B_t^S(z)Q_t$ is known, it is not equivalent to choosing $B_t(z)$. Choosing $s_t(z)$ means that households consider what has occurred in the stock market when they decide how much to invest in bonds, rather than facing two independent investment decisions.

Therefore, in each period t, each household z chooses $C_t(z)$, $H_t(z)$ and $s_t(z)$ in order to solve

$$\max \sum_{\tau=t}^{\infty} \beta^{\tau-t} E\left[\frac{(C_{\tau}(z))^{1-\sigma}}{1-\sigma} - \frac{(H_{\tau}(z))^{1+\omega}}{1+\omega} \middle| \mathfrak{S}_{t}^{*}(z)\right]$$

s.t. $s_{\tau}(z) B_{\tau}^{S}(z) Q_{\tau} = s_{\tau-1}(z) B_{\tau-1}^{S}(z) Q_{\tau-1}(1+r_{\tau-1})$
 $+ B_{\tau-1}^{S}(z) (Q_{\tau} + D_{\tau} - (1+r_{\tau-1}) Q_{\tau-1})$
 $+ W_{\tau} H_{\tau}(z) - P_{\tau} C_{\tau}(z) - T_{\tau}(z) - V_{\tau}(z)$

where $\mathfrak{S}_t^*(z)$ denotes the information set of household z after the outcome of the previous stage becomes publicly available.

From the first-order conditions of this problem, we obtain a labor supply relation and an Euler equation,

$$\frac{W_t}{P_t} = (H_t(z))^{\omega} (C_t(z))^{\sigma}, \qquad (5)$$

$$\frac{(C_t(z))^{-\sigma}}{P_t} = \beta E \left[\frac{B_{t+1}^S(z) Q_{t+1}}{B_t^S(z) Q_t} \frac{(C_{t+1}(z))^{-\sigma}}{P_{t+1}} (1+r_t) \middle| \Im_t^*(z) \right].$$
(6)

The Euler equation, expressed in (6), incorporates a new term that represents the expected variation in the wealth allocated in the stock market. This augmented Euler equation exposes the fact that, in a dynamic optimization program, transformations of the choice variable induces the other variables to evolve differently to reach the same point of maximum.

We justify the use of $s_t(z)$ as the relevant choice variable when agents face investment opportunities in two different markets on the basis of three arguments. First, households may be willing to incorporate the outcome of the first stage when they take their decisions at the second stage. This information is implicit in $s_t(z)$. Second, although the new term does not represent total wealth, $\tilde{W}_t(z) \equiv B_t(z) + B_t^S(z) Q_t$, it captures the wealth effect on consumption through variations in the stock market index. Mathematically,

$$\frac{\partial C_{t}\left(z\right)}{\partial \tilde{W}_{t}\left(z\right)} = \frac{\partial C_{t}\left(z\right)}{\partial Q_{t}} \frac{\partial Q_{t}}{\partial \tilde{W}_{t}\left(z\right)} \neq 0.$$

Finally, $s_t(z)$ incorporates a portfolio allocation decision. In order to make this point clear, we rewrite the budget constraint as

$$\tilde{W}_{t}(z) = \tilde{W}_{t-1}(z)(1 + R_{t}(z)) + W_{t}H_{t}(z) - P_{t}C_{t}(z) - T_{t}(z)$$

where $R_t(z)$ is the return household z obtains for transferring wealth from one period to another, given by

$$1 + R_t(z) = \left(1 - \frac{1}{s_{t-1}(z)}\right) (1 + r_{t-1}) + \frac{1}{s_{t-1}(z)} \left(\frac{Q_t + D_{t-1}}{Q_{t-1}}\right).$$
(7)

Equation (7) shows that the return household z receives on investments may be expressed as a convex combination of the return it receives on bonds and stocks, proving that $s_t(z)$ includes a portfolio composition decision.

3.2 Firms

In every period t, each firm k chooses $P_t(k)$ in order to maximize the expected profit, $\Pi_t(k)$, given by

$$\Pi_{t}(k) \equiv E\left[\left(1+s\right)P_{t}(k)Y_{t}(k) - W_{t}H_{t}(k)|\Im_{t}(k)\right],$$

where s is the subsidy that offsets the effect of imperfect competition on output, $H_t(k)$ is the total hours hired by firm k and $Y_t(k)$ is the amount produced by firm k. Considering that the production function of each firm is

$$Y_t\left(k\right) = A_t H_t\left(k\right),$$

where $A_t > 0$ is an exogenous technology factor, and that total demand, including government purchases, for the good k is

$$Y_t(k) = Y_t\left(\frac{P_t(k)}{P_t}\right)^{\frac{\mu}{1-\mu}},$$

the price that the firm k would choose to maximize its expected profit is

$$E\left[Y_t(P_t)^{\frac{\mu}{\mu-1}}\left[(1+s)\left(P_t(k)\right)^{\frac{\mu}{1-\mu}} - \mu \frac{W_t}{A_t}\left(P_t(k)\right)^{\frac{\mu}{1-\mu}-1}\right]\right|\mathfrak{S}_t(k)\right] = 0.$$
 (8)

If firms had full information about the state of the economy, they would maximize profits by setting

$$P_t^* = \frac{\mu}{1+s} \frac{W_t}{A_t}.$$
(9)

3.3 Potential Output

We define potential output as the output that would prevail if all agents had complete information about the state of the economy. This definition recognizes that information heterogeneity represents an inefficiency that prevents firms from optimizing their profits and households from maximizing their utility. Moreover, because prices are flexible, under complete information, all firms would maximize their profits by choosing the same price P_t^* , expressed in (9). That is, under complete information, $P_t(k) = P_t = P_t^*$ and consequently

$$1 = \frac{P_t\left(k\right)}{P_t} = \frac{\mu}{1+s} \frac{W_t}{P_t A_t}$$

When all households have the same information, they choose the same level of consumption and working hours. Using (5), we can implicitly define the potential output as

$$1 = \frac{\mu}{1+s} \frac{1}{A_t} \left(\frac{Y_t^n}{A_t}\right)^{\omega} \left(Y_t^n - G_t\right)^{\sigma}.$$
(10)

4 Reduced form

We summarize the log-linear version of the model in two equations that describes dynamics of the aggregate variables, \hat{P}_t and \hat{Q}_t ,⁵

$$\hat{P}_t = \bar{E} \left[\kappa_P \hat{P}_t + \kappa_Q \hat{Q}_t - \left(\frac{\sigma + \omega}{\sigma}\right) U_t + \omega \hat{G}_t - (1 + \omega) \hat{A}_t \right]$$
(11)

$$0 = \bar{E} \left[\left(1 - \beta^{-1} \Phi_Q \right) \left(\hat{Q}_{t+1} - \hat{Q}_t \right) - \beta^{-1} \Phi_P \left(\hat{P}_{t+1} - \hat{P}_t \right) + \tilde{\tau} \hat{v}_t - \beta^{-1} \hat{u}_t \right].$$
(12)

In these equations, \hat{w}_t and $\bar{E}[\cdot]$ denote the percent deviation of a variable w_t and, as in Morris and Shin (2002), the average expectation $\bar{E}[\cdot] = \int E[\cdot|\mathfrak{S}_t(k)] dk$. The constants κ_P , κ_Q and $\tilde{\tau}$ depend on the parameters of the model.⁶

We obtain these equations from the aggregation of individual decisions, considering that the policy rule takes the form of

$$\hat{r}_{t} = \Phi_{P} E_{t}^{CB} \left[\hat{P}_{t+1} - \hat{P}_{t} \right] + \Phi_{Q} E_{t}^{CB} \left[\hat{Q}_{t+1} - \hat{Q}_{t} \right] + \hat{u}_{t},$$
(13)

where $E_t^{CB}[\cdot]$ is the expectation conditioned on central bank's information set. We consider that the central bank updates its information set every period, meaning that it knows as much about the state of the economy as the most informed agent.⁷ Following Bernanke and Gertler (1999), this simple forward-looking policy rule does not include the output gap to focus on price fluctuations. The weights Φ_P and Φ_Q measures the intensity of the policy response to expected inflation and to expected growth in the stock market index. In some cases, we will set $\Phi_Q = 0$ to evalute how the dynamics of the aggregate variables change when the central bank does not "lean against the wind". The term \hat{u}_t represents the monetary shock

Equation (11) comes from the aggregation of pricing decisions, $\hat{P}_t = \int \hat{P}_t(k) dk$, while equation (12) results from the log-linear version of the market clearing condition $\int B_t^S(z) dz =$ 1. These equations make clear that the dynamics of the aggregate variables depend on how agents build their expectations on the state of the economy, described by the shocks \hat{G}_t , \hat{A}_t , \hat{v}_t and \hat{u}_t (given that U_t is a function of \hat{u}_{t+k}).

⁵In the appendix, we discuss the steady-state and the linear model.

⁶More specifically, $\kappa_P \equiv 1 + \left(\frac{\sigma+\omega}{\sigma}\right) (\Phi_P - 1)$, $\kappa_Q \equiv \left(\frac{\sigma+\omega}{\sigma}\right) (1 + \Phi_Q)$ and $\tilde{\tau} = \tau/(\bar{D} - K)$, where Φ_P and Φ_Q are parameters of the policy rule, as shown in (13), and \bar{D} is the stady state value of D_t , as shown in the appendix.

⁷Peek, Rosengren, and Tootell (2003) finds evidence that the Federal Reserve has an informational advantage over the public that helps to improve macroeconomic forecasts.

5 Information

In our model, four exogenous variables affect the economy in each period: productivity (\hat{A}_t) , government spending (\hat{G}_t) , dividend surprise (\hat{v}_t) and monetary surprise (\hat{u}_t) .⁸ The agents know that each of these variables follows an AR(1) process,

$$\begin{split} \hat{A}_{t} &= \rho_{a} \hat{A}_{t-1} + \hat{a}_{t} & \hat{a}_{t} \sim N\left(0, \gamma_{a}^{-1}\right), \\ \hat{G}_{t} &= \rho_{g} \hat{G}_{t-1} + \hat{g}_{t} & \hat{g}_{t} \sim N\left(0, \gamma_{g}^{-1}\right), \\ \hat{v}_{t} &= \rho_{v} \hat{v}_{t-1} + \hat{\varepsilon}_{t} & \hat{\varepsilon}_{t} \sim N\left(0, \gamma_{\varepsilon}^{-1}\right), \\ \hat{u}_{t} &= \rho_{u} \hat{u}_{t-1} + \hat{\eta}_{t} & \hat{\eta}_{t} \sim N\left(0, \gamma_{\eta}^{-1}\right), \end{split}$$

with persistence $\rho_w \in (0,1)$, for $w \in \{a, g, \varepsilon, \eta\}$. The set of shocks, $\theta_t \equiv (\hat{a}_t, \hat{g}_t, \hat{v}_t, \hat{u}_t)$, characterizes the current state of the economy. If agents had complete information, they would observe all states, θ_{t-k} , for all $k \ge 0$. Nevertheless, in our model, the information is both sticky and dispersed.

As in Mankiw and Reis (2002), a fraction $1 - \lambda$ of households in each period receives information about the current and past state of the economy whereas the other fraction λ does not. For simplicity, the probability of being selected to receive information about the state of the economy is the same across agents and is independent of history. We use Λ_j to refer to the set of all households that last received information about the state of the economy at t - j and z_j to refer to $z \in \Lambda_j$. Therefore, because of stickiness, household z_j observes θ_{t-k} for all $k \geq j$.

Similarly to Morris and Shin (2002), public and private signals about the state of the economy are available in every period, even for those who have not been selected to update their information set because of informational stickiness. For $\hat{w}_{t-k} \in \{\hat{a}_{t-k}, \hat{g}_{t-k}, \hat{\varepsilon}_{t-k}, \hat{\eta}_{t-k}\}$, these signals take the form of

$$\begin{aligned} x_{t-k}^{w}(z) &= \hat{w}_{t-k} + \xi_{t-k}^{w}(z), \qquad \xi_{t-k}^{w}(z) \sim N\left(0, \beta_{w}^{-1}\right), \\ y_{t-k}^{w} &= \hat{w}_{t-k} + \zeta_{t-k}^{w}, \qquad \qquad \zeta_{t-k}^{w} \sim N\left(0, \alpha_{w}^{-1}\right). \end{aligned}$$

where $\xi_{t-k}^{w}(z)$ and ζ_{t-k}^{w} are informational shocks respectively associated with private and public signals. For simplicity, all informational shocks are independent across time and between agents. That is, for any $k \neq i$ and any $z \neq \tilde{z}$,

⁸We use the expressions "productivity shock", "fiscal shock", "dividend shock" and "monetary shock" to refer to \hat{a}_t , \hat{g}_t , $\hat{\varepsilon}_t$ and $\hat{\eta}_t$, respectively.

$$\xi_{t-k}^{w}\left(z\right) \perp \xi_{t-i}^{w}\left(z\right) \perp \xi_{t-i}^{w}\left(\tilde{z}\right) \perp \zeta_{t-k}^{w}$$

In summary, the information set of household z_j is given by

$$\mathfrak{S}_{t}(z_{j}) = \left\{ \tilde{X}_{t}(z_{j}), \tilde{Y}_{t}, \Theta_{t-j} \right\},\$$

where $\tilde{X}_t(z_j)$ and \tilde{Y}_t are the set of all private and public signals received by household z_j prior to period t and Θ_{t-j} is a set that encompasses past states of the economy prior to period t-j,

$$\begin{split} \tilde{X}_{t}(z_{j}) &\equiv \{x_{t-k}(z_{j})\}_{k=0}^{\infty}, \quad x_{t-k}(z_{j}) \equiv \left(x_{t-k}^{a}(z_{j}), x_{t-k}^{g}(z_{j}), x_{t-k}^{\varepsilon}(z_{j}), x_{t-k}^{\eta}(z_{j})\right), \\ \tilde{Y}_{t} &\equiv \{y_{t-k}\}_{k=0}^{\infty}, \quad y_{t-k} \equiv \left(y_{t-k}^{a}, y_{t-k}^{g}, y_{t-k}^{\varepsilon}, y_{t-k}^{\eta}\right), \\ \Theta_{t-j} &\equiv \{\theta_{t-k}\}_{k=j}^{\infty}, \quad \theta_{t-k} \equiv \left(\hat{a}_{t-k}, \hat{g}_{t-k}, \hat{v}_{t-k}, \hat{u}_{t-k}\right). \end{split}$$

We specify that the central bank updates its information set every period. Thus, $\Im_t^{CB} = \{\tilde{X}_t^{CB}, \tilde{Y}_t, \Theta_t\}$. Because the central bank observes shocks every period, it does not need to use public and private signals to build its expectations.

6 Equilibrium

In order to compute the equilibrium, it is necessary to obtain \hat{P}_t and \hat{Q}_t that simultaneously satisfy (11) and (12). We determine the equilibrium dynamics of the other aggregate variables as a function of \hat{P}_t , \hat{Q}_t and shocks. The difficulty in finding \hat{P}_t and \hat{Q}_t lies in the fact that, as agents have different expectations, we cannot rely on any computational method to evaluate the solution. In this situation, we need to derive the solution analytically.⁹

We use the method of matching coefficients to obtain the equilibrium. First, we assume that the equilibrium is linear and takes the form of

$$\begin{split} \hat{P}_{t} &= \sum_{k=0}^{\infty} \left[c_{k}^{a} \hat{a}_{t-k} + c_{k}^{g} \hat{g}_{t-k} + c_{k}^{\varepsilon} \hat{\varepsilon}_{t-k} + c_{k}^{\eta} \hat{\eta}_{t-k} \right] + \sum_{k=0}^{\infty} \left[\tilde{c}_{k}^{a} y_{t-k}^{a} + \tilde{c}_{k}^{g} y_{t-k}^{g} + \tilde{c}_{k}^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{c}_{k}^{\eta} y_{t-k}^{\eta} \right] \\ \hat{Q}_{t} &= \sum_{k=0}^{\infty} \left[d_{k}^{a} \hat{a}_{t-k} + d_{k}^{g} \hat{g}_{t-k} + d_{k}^{\varepsilon} \hat{\varepsilon}_{t-k} + d_{k}^{\eta} \hat{\eta}_{t-k} \right] + \sum_{k=0}^{\infty} \left[\tilde{d}_{k}^{a} y_{t-k}^{a} + \tilde{d}_{k}^{g} y_{t-k}^{g} + \tilde{d}_{k}^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{d}_{k}^{\eta} y_{t-k}^{\eta} \right]. \end{split}$$

This solution specifies that only the shocks associated with the state of the economy—

⁹See the appendix for details.

productivity (\hat{a}_{t-k}) , fiscal (\hat{g}_{t-k}) , dividend $(\hat{\varepsilon}_{t-k})$ and monetary $(\hat{\eta}_{t-k})$ shocks—and public signals drive aggregate variables. Private signals interfere with the individual decisions of agents. However, as idiosyncratic shocks die out with aggregation, only the part of the private signals in common to all agents, i.e., the shocks associated with the state of the economy, appears in the solution of \hat{P}_t and \hat{Q}_t .

Then, we prove that, for $w \in \{a, g, \varepsilon, \eta\}$, an agent that last updated its set of past states of the economy at t - j computes expectations as

$$E\left[\hat{w}_{t-k} | \Im_{t}(z_{j})\right] = \begin{cases} \hat{w}_{t-k} & ,k \ge j \\ \frac{\alpha_{w}y_{t-k}^{w} + \beta_{w}x_{t-k}^{w}}{\alpha_{w} + \beta_{w} + \gamma_{w}} & ,0 \le k < j \\ 0 & ,k < 0 \end{cases}$$
(14)

It is important to highlight the fact that expectations depend exclusively on the signals received about shocks. If we had considered an approach similar to that of Grossman and Stiglitz (1980), agents would had tried to extract information from the current and past values of the aggregate variables, $\{\hat{P}_{t-k}, \hat{Q}_{t-k}\}_{k=0}^{\infty}$, as they are also function of the shocks. We derive the following proposition based on this expectation, which tells us how to compute $\bar{E}[\cdot]$.

Proposition 1 In an SDI model, if $E[\hat{w}_{t-k}|\mathfrak{S}_t(z_j)]$ is given by (14) and $\{q_k\}_{k=0}^{\infty}$ is a sequence of real numbers, we obtain

$$\bar{E}\left[\sum_{k=0}^{\infty} q_k \hat{w}_{t-k}\right] = \sum_{k=0}^{\infty} q_k \left(1 - \delta_w \lambda^{k+1}\right) \hat{w}_{t-k} + \theta_w \sum_{k=0}^{\infty} q_k \delta_w \lambda^{k+1} y_{t-k}^w$$

where $\delta_w = \frac{\alpha_w + \gamma_w}{\alpha_w + \beta_w + \gamma_w}$ and $\theta_w = \left(\frac{\alpha_w}{\alpha_w + \gamma_w}\right)$.

From this proposition, we make two corollaries:

Corollary 1 In an SDI model, if $\hat{W}_t = \rho_w \hat{W}_{t-1} + \hat{w}_{t-k}$ and $E[\hat{w}_{t-k}|\mathfrak{S}_t(z_j)]$ is given by (14), we have

$$\bar{E}\left[\hat{W}_{t}\right] = \sum_{k=0}^{\infty} \rho_{w}^{k} \left(1 - \delta_{w} \lambda^{k+1}\right) \hat{w}_{t-k} + \theta_{w} \sum_{k=0}^{\infty} \rho_{w}^{k} \delta_{w} \lambda^{k+1} y_{t-k}^{w}$$

Corollary 2 In an SDI model, if $\hat{W}_t = \rho_w \hat{W}_{t-1} + \hat{w}_{t-k}$ and $E[\hat{w}_{t-k}|\Im_t(z_j)]$ is given by (14), we obtain

$$\int \sum_{i=0}^{\infty} E\left[\left.\hat{W}_{t+i}\right| \mathfrak{S}_t\left(z\right)\right] dz = \frac{1}{1-\rho_w} \sum_{k=0}^{\infty} \rho_w^k \left[\left(1-\delta_w \lambda^{k+1}\right) \hat{w}_{t-k} + \theta_w \delta_w \lambda^{k+1} y_{t-k}^w\right].$$

We use Corollary 1 to compute $\bar{E}[\hat{A}_t]$, $\bar{E}[\hat{G}_t]$, $\bar{E}[\hat{u}_t]$ and $\bar{E}[\hat{v}_t]$, Corollary 2 to compute U_t and Proposition 1 to find $\bar{E}[U_t]$. Applying these results to equations (11) and (12), we can match coefficients to obtain the solution. The manner in which we compute the equilibrium rules out solutions that would result in a permanent effect on the aggregate variables, as these solutions would not be compatible with the steady state.

7 Results

We calibrate the model to illustrate the impulse responses of the variables of interest inflation, output gap, interest rate and stock market index—to both real and informational shocks. We divide the model's structural parameters into two different sets: those that are associated with shocks, $\{\alpha_w, \beta_w, \gamma_w, \rho_w\}$, for $w \in \{a, g, \varepsilon, \eta\}$, and those that are not, $\{\beta, \sigma, \omega, \lambda, \Phi_P, \Phi_Q\}$. The baseline values used for the latter set, shown in Table 1, are standard and are based on Giannoni and Woodford (2004). In our baseline case, we set $\Phi_Q = 0$, meaning that the central bank does not respond to variations in the stock market index. We set $\sigma = 0.67$ to obtain the value of $\sigma^{-1} \approx 1.50$, which is similar to the value obtained Giannoni and Woodford (2004) for their equivalent parameter ϑ^{-1} . The discount factor β is set equal to 0.99, which is appropriate for interpreting the time interval as one quarter. We consider $\lambda = 0.45$, implying that most agents receive new information about the current and past state of the economy. We set $K = \mu - \beta^{-1}$ to get $\bar{Q} = 1$, meaning that $\tilde{\tau} = 1$. A value of 1.2 for μ implies a steady state markup of 20% and makes $K \approx 0.19$.

Table 1: Baseline calibration - parameters of the model

Parameter	Description	Value
eta	Time discount factor	0.99
λ	Degree of informational rigidity	0.45
σ	Risk aversion parameter	0.67
ω	Inverse of elasticity of labor supply	0.33
$ar{Q}$	Steady-state value of the stock market index	1
Φ_P	Degree of policy response to expected inflation	[0, 3.5]
Φ_Q	Degree of policy response to expected variations in the stock price index	[0, 0.2]

Table 2 shows the baseline values used for the former set. As will be clarified below, we can use these parameters to determine whether a shock can generate a boom-bust episode

in the economy. In our benchmark calibration, we specify that only a dividend shock can trigger such an episode. Accordingly, the values associated with $\hat{\varepsilon}_t$ are different from those for the other shocks. By taking $\beta = 0.99$, we posit that \bar{r} is approximately 4% per year, while a one-standard-deviation change is approximately equal to 4 basis points.

Table 2: Baseline calibration - parameters of the shocks							
Parameter	Description	$\hat{w}_t = \hat{a}_t$	$\hat{w}_t = \hat{g}_t$	$\hat{w}_t = \hat{\varepsilon}_t$	$\hat{w}_t = \hat{\eta}_t$		
α_w $(\sigma_y = 1/\sqrt{\alpha_w})$	Precision of the public signal y_t^w	$\underset{(0.0001)}{10^8}$	$\underset{(0.0001)}{10^8}$	$\underset{(0.001)}{10^6}$	$\underset{(0.0001)}{10^8}$		
$\beta_w \\ (\sigma_x = 1/\sqrt{\beta_w})$	Precision of the private signal x_t^w	$\substack{10^{12} \\ (1 \times 10^{-6})}$	$\underset{(1\times 10^{-6})}{10^{12}}$	$\underset{(3,16\times10^{-3})}{10^7}$	$\underset{(1\times 10^{-6})}{10^{12}}$		
$ \begin{pmatrix} \gamma_w \\ \sigma_w = 1/\sqrt{\gamma_w} \end{pmatrix} $	Precision of the shock \hat{w}_t	$\underset{(0.0001)}{10^8}$	$\underset{(0.0001)}{10^8}$	$\underset{(0.0001)}{10^8}$	$\underset{(0.0001)}{10^8}$		
$ ho_w$	Persistence	0.5	0.5	0.9	0.5		

All shocks have the same variance, although the importance of private signals relatively to public signals can be quite different.

7.1 Impulse responses

Here, we show the impulse responses to a one-standard-deviation shock for all four exogenous variables. In our benchmark calibration, represented by a solid line, the central bank is permissive of the occurrence of boom-bust episodes. The scale on the right-hand side should be used for this case. For the other cases, the scale on the left-hand side should be used.

Figure 1 shows the impulse responses to a dividend shock, ε_t . As shown in panel (A), after a dividend shock, the stock market index persistently increases over a long period (8 quarters) and suddenly crashes, which is compatible with a boom-bust episode. The intensity of the peaks and troughs depends on the calibration. We have chosen to consider the case in which the trough is more intense than the peak. In the present case, the stock market index increases by 100% over two years and decreases almost 400% in the following quarter. After the bust, the stock market index slowly returns to its equilibrium value. The boom phase is characterized as a period of "exuberance". The output gap, illustrated in panel (B), influenced by the wealth effect on consumption, remains positive throughout the whole period while inflation, plotted in panel (C), stays low. Inflation rises at the end of the boom phase, when the stock market index and the output gap start to increase more intensively.

As shown in panel (D), the central bank, anticipating this inflation growth, raises the interest rate. This increase in the interest rate occurs just before the bust. After the bust, a great contraction in output occurs, followed by a slow recovery. Inflation drops when the output gap becomes negative, but as the economy recovers, it rises again and slowly returns to its equilibrium level. This analysis is in line with empirical evidence. Contradicting the conventional idea that stock market booms are periods of high inflation, Christiano, Ilut, Motto, and Rostagno (2010) shows that, in each of the 18 US stock market boom episodes that occurred in the past two centuries, inflation was relatively low. Furthermore, this result is also compatible with the conventional wisdom that the boom phase ends when the central bank starts raising the interest rate.

The rationale behind this figure is easy to understand. Due to informational stickiness, households become aware of a positive dividend shock at different times: while some observe the shock, others are unsure about occurs as they only receive imprecise news about the shock. However, as time passes, the number of agents who are uncertain about the shock decreases. During the following periods, stock prices rise as (i) high dividends continue to be paid because of the shock inertia and (ii) the stock market receives more attention as households become more informed about the market. When the expected inflation increases more intensively, forcing the central bank to raise interest rate, households face an increase in the opportunity cost of holding stocks. As the information sets of all agents are very similar at this moment, households can predict one another's actions and anticipate that a great number of agents will be willing to sell stocks at the same time. This situation forces stock prices to collapse.

Mathematically, it is easy to explain how a boom-bust episode emerges from the model. The effect of a dividend shock on \hat{P}_t and \hat{Q}_t depends on the coefficient c_k^{ε} , which may present a singularity if

$$\left(1 - \kappa_P - \frac{\kappa_Q \Phi_P}{(\beta - \Phi_Q)}\right) + \left(\kappa_P + \frac{\kappa_Q \Phi_P}{(\beta - \Phi_Q)}\right) \delta_{\varepsilon} \lambda^{k+1} = 0$$

We can rewrite this condition as $\delta_{\varepsilon} \lambda^{k+1} = \vartheta$, where

$$\vartheta = \frac{(1+\beta)\,\Phi_P + (\Phi_Q - \beta)}{(1+\beta)\,\Phi_P + \left(\frac{\omega}{\sigma+\omega}\right)(\Phi_Q - \beta)}.\tag{15}$$

Using [x] to denote the smallest integer that is larger than x, we find that



Figure 1: Impulse responses to a dividend shock

Conclusion 1 A dividend shock on period t creates a boom-bust episode when $\vartheta \in (0, 1)$ and $\delta_{\varepsilon} \lambda \geq \vartheta$. The bust occurs at period $t + k_{\varepsilon}$, $k_{\varepsilon} = \left\lceil \frac{\ln(\vartheta) - \ln(\delta_{\varepsilon} \lambda)}{\ln(\lambda)} \right\rceil$.

In this case, when k increases, λ^{k+1} decreases, forcing $\delta_{\varepsilon}\lambda^{k+1}$ to become closer to the singularity and causing $|c_k^{\varepsilon}|$ to increase. At some point k_{ε} , $\delta_{\varepsilon}\lambda^{k_{\varepsilon}+1}$ becomes smaller than ϑ . From this point on, c_k^{ε} inverts its signal and $|c_k^{\varepsilon}|$ decays. In this context, k_{ε} marks the occurrence of the bust.

The condition $\delta_{\varepsilon}\lambda \geq \vartheta$ gives us a criterion to analyze the possibility of boom-bust episodes. For the case $\Phi_Q = 0$, which implies that the central bank does not respond to asset price variations, ϑ becomes

$$\vartheta = \frac{(1+\beta) \Phi_P - \beta}{(1+\beta) \Phi_P - \beta \left(\frac{\omega}{\sigma+\omega}\right)}$$

It is clear that $\vartheta \in (0,1)$ if and only if $\Phi_P > \frac{\beta}{1+\beta}$.

Although our analysis does not allow us to say that it is optimal for the central bank to prevent boom-bust episodes, these events clearly create disruptive movements in the economy. A central bank concerned with stabilizing the economy may wish to prevent these events insofar as they bring an excessive volatility to macroeconomic variables. In order to prevent a dividend shock from triggering a boom-bust episode, $\vartheta > \delta_{\varepsilon} \lambda$, a central bank that disregards changes in asset prices must respond to expected inflation according to:

$$\Phi_P \ge \left(\frac{\beta}{1+\beta}\right) \left[1 + \left(\frac{\sigma}{\sigma+\omega}\right) \left(\frac{\delta_{\varepsilon}\lambda}{1-\delta_{\varepsilon}\lambda}\right)\right].$$

In contrast to the Taylor principle, which establishes the manner in which a central bank should react to stabilize the entire economy, this criterion is specific for this shock and tells us nothing about the other shocks. Nevertheless, from (33), we know that all shocks present the same singularity, ϑ . In this case, in order to prevent the occurrence of boom-bust episodes, we must have $\vartheta > \delta_w \lambda$ for all shock $\hat{w}_t, w \in \{a, g, \varepsilon, \eta\}$. The condition $\vartheta > \delta_w \lambda$ makes clear that both informational stickiness, represented by λ , and informational dispersion, represented by δ_w , are important in explaining the creation of boom-bust episodes. While the informational stickiness plays a central role in determining the timing of the busts, informational dispersion explains why some shocks may not trigger these events. If public information concerning the stock market—which is based on companies' balance sheets and financial analysts' reports—is relatively more precise than that concerning fiscal conditions, we will have, for instance, $\delta_{\varepsilon} >> \delta_g$. Therefore, we may have, $\delta_{\varepsilon} \lambda \geq \vartheta > \delta_g \lambda$. In this case, as k increases, $\delta_g \lambda^{k+1}$ becomes increasingly distant from ϑ , which explains why changes in dividends can generate boom-bust episodes while changes in government spending cannot. In order to guarantee that no shock will trigger a boom-bust episode in the economy, the central bank must respond to expected inflation according to

$$\Phi_P \ge \left(\frac{\beta}{1+\beta}\right) \left[1 + \left(\frac{\sigma}{\sigma+\omega}\right) \left(\frac{\bar{\delta}\lambda}{1-\bar{\delta}\lambda}\right)\right],\tag{16}$$

where

 $\bar{\delta} = \max\left\{\delta_a, \delta_g, \delta_\varepsilon, \delta_\eta\right\}.$

Conclusion 2 Even responding only to expected inflation ($\Phi_Q = 0$), monetary policy can prevent the occurrence of boom-bust episodes in the economy. The intensity of the response depends, however, on the greatest relative precision of all shocks, $\bar{\delta}$.

When $\bar{\delta}\lambda$ is close to one, it becomes very difficult for the central bank to avoid boom-bust episodes. According to our baseline calibration, $\bar{\delta}\lambda$ is close to 0.9, meaning that the central bank should raise the interest rate by approximately 3.5% for each 1% increase in expected inflation. In this situation, we must consider whether central banks should respond to asset price variations. When $\Phi_Q \neq 0$, the singularity ϑ is given by (15). It is clear that when $\Phi_Q \geq \beta$, $\vartheta \geq 1$, implying that the condition for no boom-bust episodes, $\vartheta > \bar{\delta}\lambda$, is trivially satisfied for any Φ_P . We have $\vartheta \in (0, 1)$ if and only if $\beta > \Phi_Q > \beta - \Phi_P (1 + \beta)$. In this case, the criterion that guarantees that no shock will trigger a boom-bust episode in the economy, $\vartheta > \bar{\delta}\lambda$, becomes

$$\Phi_P > \left(\frac{\beta - \Phi_Q}{1 + \beta}\right) \left[1 + \left(\frac{\sigma}{\sigma + \omega}\right) \left(\frac{\bar{\delta}\lambda}{1 - \bar{\delta}\lambda}\right)\right] \tag{17}$$

In comparison with (16), Φ_Q helps to make the condition looser. When Φ_Q approaches β , it becomes easier for the central bank to prevent the occurrence of boom-bust episodes. This leads to our next conclusion.

Conclusion 3 For any value of Φ_P , there is a Φ_Q large enough to avoid the occurrence of boom-bust episodes in the economy.

According to this criterion, the central bank does not need to react to asset price variations to prevent boom-bust episodes, although it might be willing to "lean against the wind" to avoid an excessive reaction to expected inflation. This finding is advocated by Pavasuthipaisit (2010), based on empirical evidence, and contradicts the prominent suggestion of several authors that central banks should respond to asset price movements only insofar as the latter affects the expected inflation.¹⁰ For instance, in order to obtain a coefficient compatible with a Taylor rule, typically $\Phi_P = 1.5$, in our baseline calibration, a central bank should set Φ_Q equal to approximately 0.6.

As we have seen so far, central banks can prevent the occurrence of boom-bust episodes by choosing $\Phi_Q > \beta$ or, when $\Phi_Q < \beta$, by obeying the condition established in (17). Nevertheless, as $\vartheta \in (0, 1)$ if and only if $\beta > \Phi_Q > \beta - \Phi_P (1 + \beta)$, there is another alternative:

$$\Phi_P < \frac{\beta - \Phi_Q}{1 + \beta}.\tag{18}$$

In this case, the central bank's response to expected inflation and expected variations in the stock price index is very mild. The rationale behind this finding is similar to the point made in Farhi and Tirole (2012): as the central bank responds to the aggregate variables, which incorporate the average behavior of agents, and influences individual decisions, it creates strategic complementarity among the agents' actions. In this environment, agents

¹⁰See, for instance, Gilchrist and Saito (2008), Faia and Monacelli (2005), Gilchrist and Leahy (2002), and Bernanke and Gertler (2001).

try to predict one another's actions, inducing coordination. Therefore, when the response is very mild, coordination diminishes, which avoids boom-bust episodes.

Conclusion 4 Central banks can prevent the occurrence of boom-bust episodes in the economy by implementing very mild responses to aggregate variables and keeping the interest rate near its steady-state level.

One would expect that this case would not be compatible with economy stabilization. However, the manner in which we computed the equilibrium does not induce any conditions for stabilizing the economy. Even in the limit case, when the central bank sets the interest rate at its steady-state level and does not change it ($\Phi_P = \Phi_Q = 0$), we do not find explosive dynamics.

The argument that the central bank should not respond to asset prices variations for not being capable of identifying bubbles is totally irrelevant to our analysis. From the obtained solution, we have that

$$E_0 \left[\hat{Q}_{t+1} - \hat{Q}_t + \tilde{\tau} \hat{v}_t - \beta^{-1} \hat{r}_t \right] = 0,$$
(19)

where $E_0 \equiv E[\cdot|\mathfrak{S}_t(z_0)]$ represents the expectation of the most informed agent.¹¹ From this equation, it is easy to see that, for all z, $\hat{B}_t^s(z) = 0$, meaning that, at equilibrium, there is no trade in the stock market. This result is in line with Milgrom and Stokey (1982).

Dividing both sides of (17) by Φ_P , we see that there is an infinite number of pairs (Φ_P, Φ_Q) that cause the right-hand side to give the same number. From this observation, we can define an equivalence criterion between two different policies. We say that two policies, (Φ_P^A, Φ_Q^A) and (Φ_P^B, Φ_Q^B) , are equivalent if $(\beta - \Phi_Q^A) \Phi_P^B = (\beta - \Phi_Q^B) \Phi_P^A$ holds.

The two other lines in Figure 1 show that the central bank can prevent the occurrence of boom-bust episodes by reacting to the expected inflation and expected stock market index variations. Regarding the proposed criterion, the policies presented above are almost equivalent. We see that both policies lead to similar results: almost no effect on all four variables of interest. Indeed, there is a slight reaction in the interest rate, which produces a small drop in the stock market index and, by consequence, a small drop in the output gap. However, all of these effects are almost unperceivable.

Although the persistence of the shocks, ρ_w , plays no role in the computation of the singularity, it can be very important for the perception of a boom-bust episode. A low level

¹¹In the appendix, we define the variables f_k^w and \tilde{f}_k^w , for $w \in \{a, g, \varepsilon, \eta\}$, and show that in equilibrium $\tilde{f}_k^w = 0, \forall w$ and $f_k^a = f_k^g = f_k^\varepsilon + \tilde{\tau}\rho_v^k = f_k^\eta - \beta^{-1}\rho_u^k = 0$. This result implies that the equation (19) holds in equilibrium.



Figure 2: Impulse responses to a productivity shock

of persistence can greatly attenuate the boom phase, as well as the effects of the bust. As an earthquake may not be perceivable at locations distant from the epicenter, when the persistence is very small, a boom-bust episode can exist but not be noticeable. In the case of an earthquake, the event occurs when there is a crash between two tectonic plates. In our case, the conditions for the occurrence of a boom-bust episode have already been established in Conclusion 1. In both cases, the occurrence of the event is not related to whether or not it is perceivable. The reason that low persistence can make these episodes unperceivable is easy to understand. Investors may react very little to information about high gains in the stock market if they believe that an opportunity that appeared in the past no longer represents a significant opportunity. Persistence becomes particularly important when the boom phase lasts for a long period.

Figure 2 shows the impulse responses to a productivity shock. A positive productivity shock is consistent with lower future marginal costs, which generate a drop in expected future inflation. Anticipating the fact that inflation will become negative, the interest rate is reduced, diminishing the return on bonds. As a consequence, households fly to the stock market, forcing the stock market index to rise. Because of the wealth effect on consumption, output also rises. Inflation, taken as the first difference of prices, becomes positive when the shock occurs and, as expected, negative afterwards, reflecting an increase in the price index at the moment of the shock followed by a smooth convergence to its steady-state value. This dynamic is consistent with all policy parameters. Nevertheless, when the central bank is permissive of the occurrence of boom-bust episodes, the effects of the shock are greatly amplified. This finding is analogous to the one described in Bernanke, Gertler, and Gilchrist (1999) when they consider financial frictions. It also suggests that agents become more reactive to shocks when there is an environment in which a boom-bust episode can emerge. In other words, the economy becomes less volatile when the central bank creates an environment that calms all agents.

We observe a similar pattern when we analyze the impulse responses to a fiscal shock, as plotted in Figure 3. In this case, all of the lines present a similar behavior, but we find that when the central bank acts to prevent the occurrence of boom-bust episodes in the economy, it also attenuates the effect of a fiscal shock. After a positive fiscal shock, inflation is expected to rise, causing the central bank to increase the interest rate. Investors fly to the bond market, causing the stock market index to drop. As a result of the wealth effect on consumption, output also decreases, and this decrease causes the price index to diminish. As prices slowly return to their steady-state level, inflation, taken as the first difference of prices, becomes positive, as anticipated.

Figure 4 illustrates the impulse responses to a monetary shock. As before, when the central bank permits the creation of boom-bust episodes, the magnitude of the shock increases. After a monetary shock, the interest rate increases, forcing the stock market index to decrease. The output gap also decreases as a result of the wealth effect on consumption. Prices, because of this contraction in consumption, drop. However, as prices slowly recover to its state-state value, inflation increases. This movement is anticipated by the central bank, reinforcing the idea that the interest rate should increase.

8 Conclusions

We incorporated asset prices and information heterogeneity into a very simple DSGE model with neither financial friction nor special channels—and showed that its dynamics are compatible with the creation of boom-bust episodes. In contrast to other models, these episodes are not exogenous processes that force asset prices to deviate from fundamentals, but rather arise from the strategic use of information that guides agents' decisions. In this framework, we show that a central bank can prevent the occurrence of these episodes and determine the



Figure 3: Impulse responses to a fiscal shock



Figure 4: Impulse responses to a monetary shock

timing of a specific bust.

We derive the equilibrium from the aggregation of agents' decisions, which depends on each individual information set. We proved a proposition that shows how to compute aggregate expectations when agents have heterogeneous information. This proposition is not model specific and depends only on the manner in which new information is disseminated among agents.

This fact is useful because we can extend the model in many different directions. First, it is important to endogenize the firms' dividends. Productivity and fiscal shocks may change firms' profits and, consequently, dividends. Another important extension concerns the incorporation of the output gap in policy rule and possibly the study of optimal policies for this model.

9 Appendix

9.1 Linear model

We compute the linear version of the model to derive the linear equilibrium. Throughout the text, \hat{w}_t represents the percent deviation of a variable w_t from its steady-state value $\bar{w}, \ \hat{w}_t \equiv (w_t - \bar{w}) / \bar{w}$. We define $\hat{w}_t \equiv w_t$ for variables whose steady-state value is zero. Nevertheless, it is more convenient to write $\hat{G}_t \equiv G_t / \bar{Y}$, where \bar{Y} represents the steady-state of the output level. With a slight abuse of terminology, we use \hat{r}_t to refer to $(1 + r_t)$.

9.1.1 Steady-State

We consider a steady state without productivity shocks, $\bar{A} = 1$, without government spending, $\bar{G} = 0$, and with zero inflation. We also consider that in steady state all households have the same consumption level, \bar{C} . Although we do not explicitly show a mechanism to prevent their wealth from becoming increasingly dispersed as a result of differing individual informational histories, we could easily do so. As in Cúrdia and Woodford (2009), we could assume, for instance, that households sign state-contingent contracts with one another and that $T_t(z)$ incorporates not only lump-sum taxes but also a net insurance transfer that households receive at the same date upon which they update their information sets.

In a steady state with equal consumption, equation (5) indicates that all agents choose the same number of working hours. In the steady state, we offset any inefficiency that comes from monopolistic competition in the goods market by taking the firms' subsidies as $s = \mu - 1$. Using (10), the steady-state output level is implicitly defined by

$$1 = \left(\bar{Y}\right)^{\omega + \sigma}.$$

We also consider that, in the steady state, the firms distribute all their profits. Therefore, in the steady state,

$$\bar{D} = \bar{\Pi} = (\mu - 1) \frac{W}{\bar{P}} \bar{P} \bar{Y} = (\mu - 1) \bar{P}.$$

We normalize $\bar{P} = 1$ in order to have no difference between real and nominal variables in the steady-state.

The Euler equation, expressed in (6), in the steady state reduces to $(1 + \bar{r}) = \beta^{-1}$.

In order to negotiate stocks, households must find someone who is willing to be the counterpart in the transaction. In other words, the stock market must clear every period. Mathematically, this condition is equivalent to

$$\int B_t^S\left(z\right)dz = 1$$

From the demand function for stocks, expressed in (3), we obtain

$$\int E \left[Q_{t+1} + D_t - Q_t \left(1 + r_t \right) \right] \mathfrak{S}_t (z) dz = K,$$
(20)

which, in the steady state reduces to

$$\bar{D} = K + \left(\beta^{-1} - 1\right) \bar{Q}.$$
(21)

In order to make the steady-state version of (4) compatible with this relation, we must have $\varphi = K$ and $\tau = (\beta^{-1} - 1)$, meaning that (4) represents small departures from the steady-state relation. Furthermore, from (21) we write the steady-state value of \bar{Q} as

$$\bar{Q} = \frac{\bar{D} - K}{\tau}.$$
(22)

9.1.2 Individual Consumption

In the afternoon, when households decide how much they will consume and the central bank sets the interest rate in a simultaneous game, agents have already observed \hat{P}_t and \hat{Q}_t , meaning that they do not need to make predictions about these variables, $E[\hat{P}_t | \Im_t^*(z)] = \hat{P}_t$

and $E[\hat{Q}_t | \mathfrak{S}_t^*(z)] = \hat{Q}_t$. Log-linearizing the Euler equation, (6), we obtain

$$E\left[\sigma\hat{C}_{t+1}(z) + \hat{P}_{t+1} - \hat{r}_t - \hat{B}_{t+1}^S(z) - \hat{Q}_{t+1} \middle| \Im_t^*(z) \right] = \sigma\hat{C}_t(z) + \hat{P}_t - \hat{B}_t^S(z) - \hat{Q}_t.$$
(23)

Because households have not yet observed \hat{r}_t , they use (13) to make predictions about it. Furthermore, as the central bank updates its information set every period, the law of iterated expectations holds for all households individually, $E\left[E_t^{CB}\left[\cdot\right] | \mathfrak{S}_t^*(z)\right] = E\left[\cdot | \mathfrak{S}_t^*(z)\right]$. We can write the log-linearized Euler equation, (23), as

$$E\left[\sigma\hat{C}_{t+1}(z) + (1 - \Phi_P)\hat{P}_{t+1} - (1 + \Phi_Q)\hat{Q}_{t+1} - \hat{B}^S_{t+1}(z)\middle|\,\mathfrak{S}^*_t(z)\right] - E\left[\hat{u}_t\middle|\,\mathfrak{S}^*_t(z)\right] = \sigma\hat{C}_t(z) + (1 - \Phi_P)\hat{P}_t - (1 + \Phi_Q)\hat{Q}_t - \hat{B}^S_t(z).$$

As $\mathfrak{S}_{t}^{*}(z) \subseteq \mathfrak{S}_{t+1}^{*}(z) \subseteq ... \subseteq \mathfrak{S}_{t+i}^{*}(z) \subseteq ...$, we can use the law of iterated expectations in this expression to find

$$\sigma \hat{C}_t(z) + (1 - \Phi_P) \hat{P}_t - \hat{B}_t^S(z) - (1 + \Phi_Q) \hat{Q}_t = -\sum_{i=0}^{\infty} E\left[\hat{u}_{t+i} | \Im_t^*(z)\right].$$
(24)

We know from Morris and Shin (2002) that the law of iterated expectations may not hold when agents have heterogeneous information. However, we can use the law of iterated expectations for agents with different information sets because this case does not involve any aggregation. The difference between this case and the one described in Morris and Shin (2002) lies in the fact that households have to build expectations about their own (individual) consumption in the future, instead of, for instance, aggregate consumption. As agents do not use the aggregate variables, \hat{P}_t and \hat{Q}_t , to take information about the shocks, we have $E\left[\hat{u}_{t+i} | \mathfrak{S}_t^*(z)\right] = E\left[\hat{u}_{t+i} | \mathfrak{S}_t(z)\right].$

9.1.3 Aggregate Consumption

We can use (24) to write the individual consumption as

$$\hat{C}_{t}(z) = -\sigma^{-1} \sum_{i=0}^{\infty} E\left[\hat{u}_{t+i} | \Im_{t}(z)\right] + \sigma^{-1} \left(\Phi_{P} - 1\right) \hat{P}_{t} + \sigma^{-1} \left(1 + \Phi_{Q}\right) \hat{Q}_{t} + \sigma^{-1} \hat{B}_{t}^{S}(z)$$

From this equation, the log-linearized aggregate consumption, $C_t \equiv \int C_t(z) dz$, is

$$\hat{C}_{t} \equiv \int \hat{C}_{t}(z) dz = -\sigma^{-1} U_{t} + \sigma^{-1} \left(\Phi_{P} - 1\right) \hat{P}_{t} + \sigma^{-1} \left(1 + \Phi_{Q}\right) \hat{Q}_{t},$$
(25)

where

$$U_{t} = \int \sum_{i=0}^{\infty} E\left[\hat{u}_{t+i} \middle| \mathfrak{S}_{t}\left(z\right)\right] dz.$$

Equality (25) holds because market clearing in the stock market imposes

$$\int B_t^S(z) \, dz = 1 \Rightarrow \int \hat{B}_t^S(z) \, dz = 0.$$

9.1.4 Price Index and Stock Market Index

Price index. Although firms decide on prices in the first stage, nominal wages can still be adjusted to clear the labor market, $\int H_t(z) dz = H_t = \int H_t(k) dk$. Therefore, the log-linearized version of (5) results in

$$\hat{W}_t - \hat{P}_t = \int \left(\hat{W}_t - \hat{P}_t \right) dz = \int \sigma \hat{C}_t \left(z \right) + \delta \hat{H}_t \left(z \right) dz = \sigma \hat{C}_t + \omega \hat{H}_t.$$
(26)

From the production function, we know that the total production

$$\hat{Y}_{t} = \int \hat{Y}_{t}(k) \, dk = \hat{A}_{t} + \int \hat{H}_{t}(k) \, dk = \hat{A}_{t} + \hat{H}_{t}.$$
(27)

We use (26), (27) and $\hat{Y}_t = \hat{C}_t + \hat{G}_t$ to write the log-linearized version of (8) as

$$\hat{P}_{t}(k) = E\left[\left(\hat{W}_{t} - \hat{P}_{t}\right) + \hat{P}_{t} - \hat{A}_{t} \middle| \mathfrak{S}_{t}(k)\right] \\ = E\left[\hat{P}_{t} + (\sigma + \omega)\hat{C}_{t} + \omega\hat{G}_{t} - (1 + \omega)\hat{A}_{t} \middle| \mathfrak{S}_{t}(k)\right].$$

Using (25), the solution of the second stage, we obtain

$$\hat{P}_{t}(k) = E \begin{bmatrix} \left(\frac{\sigma+\omega}{\sigma}\right) \left[\left(\Phi_{P}-1\right) \hat{P}_{t}+\left(1+\Phi_{Q}\right) \hat{Q}_{t}-U_{t} \right] \\ +\hat{P}_{t}+\omega \hat{G}_{t}-\left(1+\omega\right) \hat{A}_{t} \end{bmatrix} \Im_{t}(k) \end{bmatrix},$$
(28)

From this equation, we can compute the price index as

$$\hat{P}_{t} = \int \hat{P}_{t}(k) \, dk = \bar{E} \left[\kappa_{P} \hat{P}_{t} + \kappa_{Q} \hat{Q}_{t} - \left(\frac{\sigma + \omega}{\sigma}\right) U_{t} + \omega \hat{G}_{t} - (1 + \omega) \, \hat{A}_{t} \right]$$

where $\kappa_P \equiv 1 + \left(\frac{\sigma+\omega}{\sigma}\right) (\Phi_P - 1)$, $\kappa_Q \equiv \left(\frac{\sigma+\omega}{\sigma}\right) (1 + \Phi_Q)$ and $\bar{E}[\cdot]$, as in Morris and Shin (2002), represents the average expectation given by

$$\bar{E}\left[\cdot\right] = \int E\left[\cdot\right] \mathfrak{S}_{t}\left(k\right) dk.$$

Stock Market Index. By plugging the interest rule, (13), into the log-linear version of (20), we obtain

$$0 = \int E\left[\hat{Q}_{t+1} - \hat{Q}_t + \tilde{\tau}\hat{v}_t - \frac{1}{\beta}\hat{r}_t\right| \mathfrak{S}_t(z) dz$$

$$= \bar{E}\left[\left(1 - \beta^{-1}\Phi_Q\right)\left(\hat{Q}_{t+1} - \hat{Q}_t\right) - \beta^{-1}\Phi_P\left(\hat{P}_{t+1} - \hat{P}_t\right) + \tilde{\tau}\hat{v}_t - \beta^{-1}\hat{u}_t\right]$$

where $\tilde{\tau} = 1/\bar{Q} = \tau/(\bar{D} - K)$. This equation makes clear that, besides altering the steadystate value of \bar{Q} , which affects the intensity of the aggregate variable responses to \hat{v}_t , K plays no role on the model.

9.2 Expectations

First, we compute $E\left[\hat{\eta}_{t-k} \middle| \mathfrak{F}_t(z)\right]$. A household z_j that last updated its information set at t-j knows, with certainty, the value of $\hat{\eta}_{t-k}$ when $k \geq j$, as household observes all previous shocks at the moment that it adjusts its information set. Therefore, $E\left[\hat{\eta}_{t-k} \middle| \mathfrak{F}_t(z_j)\right] = \hat{\eta}_{t-k}$. If k < 0, household z_j does not have any information about it. If $0 \leq k \leq j$, household z_j does not observe η_{t-k} . Nevertheless, it has two signals about this shock. That is, $E\left[\hat{\eta}_{t-k} \middle| \mathfrak{F}_t(z_j)\right] = E\left[\hat{\eta}_{t-k} \middle| y_{t-k}^{\eta}, x_{t-k}^{\eta}(z_j)\right]$.

In order to compute $E\left[\hat{\eta}_{t-k} \middle| y_{t-k}^{\eta}, x_{t-k}^{\eta}(z_j)\right]$, we need to obtain the distribution function $f\left(\hat{\eta}_{t-k} \middle| y_{t-k}^{\eta}, x_{t-k}^{\eta}(z_j)\right)$. We use Bayes' theorem to write

$$f\left(\hat{\eta}_{t-k} \middle| y_{t-k}^{\eta}, x_{t-k}^{\eta}(z_{j})\right) = \frac{f\left(y_{t-k}^{\eta}, x_{t-k}^{\eta}(z_{j}) \middle| \hat{\eta}_{t-k}\right) f\left(\hat{\eta}_{t-k}\right)}{\int f\left(y_{t-k}^{\eta}, x_{t-k}^{\eta}(z_{j}) \middle| \hat{\eta}_{t-k}\right) f\left(\eta_{t-k}\right) d\hat{\eta}_{t-k}}$$

However, we know that

$$\begin{split} & f\left(y_{t-k}^{\eta}, x_{t-k}^{\eta}\left(z_{j}\right) \middle| \hat{\eta}_{t-k}\right) f\left(\hat{\eta}_{t-k}\right) \\ &= f\left(y_{t-k}^{\eta} \middle| \eta_{t-k}\right) f\left(x_{t-k}^{\eta}\left(z_{j}\right) \middle| \hat{\eta}_{t-k}\right) f\left(\hat{\eta}_{t-k}\right) \\ &= N\left(\hat{\eta}_{t-k}, \alpha_{\eta}^{-1}\right) N\left(\hat{\eta}_{t-k}, \beta_{\eta}^{-1}\right) N\left(0, \gamma_{\eta}^{-1}\right) \\ &= c \exp\left\{-\frac{1}{2}\left[\frac{\left(y_{t-k}^{\eta} - \hat{\eta}_{t-k}\right)^{2}}{\alpha_{\eta}^{-1}} + \frac{\left(x_{t-k}^{\eta} - \hat{\eta}_{t-k}\right)^{2}}{\beta_{\eta}^{-1}} + \frac{\left(\hat{\eta}_{t-k}\right)^{2}}{\gamma_{\eta}^{-1}}\right]\right\} \\ &= c \exp\left\{-\frac{1}{2}\left[\left(\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}\right)\hat{\eta}_{t-k}^{2} - 2\left(\alpha_{\eta}y_{t-k}^{\eta} + \beta_{\eta}x_{t-k}^{\eta}\right)\hat{\eta}_{t-k} + \alpha_{\eta}y_{t-k}^{\eta} + \beta_{\eta}x_{t-k}^{\eta}\right]\right\} \\ &= c_{2}\sqrt{\frac{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}}{2\pi}}\exp\left\{-\frac{1}{2}\left[\left(\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}\right)\left(\hat{\eta}_{t-k} - \frac{\alpha_{\eta}y_{t-k}^{\eta} + \beta_{\eta}x_{t-k}^{\eta}}{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}}\right)^{2}\right]\right\} \\ &= c_{2}N\left(\frac{\alpha_{\eta}y_{t-k}^{\eta} + \beta_{\eta}x_{t-k}^{\eta}}{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}}, \left(\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}\right)^{-1}\right) \end{split}$$

where

$$c = \sqrt{\frac{\alpha_{\eta}\beta_{\eta}\gamma_{\eta}}{(2\pi)^{3}}}$$

$$c_{2} = c\sqrt{\frac{2\pi}{(\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta})}} \exp\left\{\frac{1}{2}\left[\frac{(\alpha_{\eta}y_{t-k}^{\eta} + \beta_{\eta}x_{t-k}^{\eta})^{2}}{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}} - \alpha_{\eta}y_{t-k}^{\eta} - \beta_{\eta}x_{t-k}^{\eta}\right]\right\}$$

Thus,

$$f\left(\hat{\eta}_{t-k} \middle| y_{t-k}^{\eta}, x_{t-k}^{\eta}(z)\right) = \frac{c_2 N\left(\frac{\alpha_{\eta} y_{t-k}^{\eta} + \beta_{\eta} x_{t-k}^{\eta}}{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}}, \left(\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}\right)^{-1}\right)}{c_2}$$
$$= N\left(\frac{\alpha_{\eta} y_{t-k}^{\eta} + \beta_{\eta} x_{t-k}^{\eta}}{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}}, \left(\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}\right)^{-1}\right).$$

In summary,

$$E\left[\hat{\eta}_{t-k}\middle| \mathfrak{S}_{t}\left(z_{j}\right)\right] = \begin{cases} \hat{\eta}_{t-k} & ,k \geq j \\ \frac{\alpha_{\eta}y_{t-k}^{\eta} + \beta_{\eta}x_{t-k}^{\eta}}{\alpha_{\eta} + \beta_{\eta} + \gamma_{\eta}} & ,0 \leq k < j \\ 0 & ,k < 0 \end{cases}$$

The same argument can be used to compute expectations for the other shocks.

9.3 Proofs

Proof of Proposition 1. The notation introduced in Section 5 makes clear that we can decompose the set of all agents in a union of disjoint sets of agents that last updated its information set at t - j, $[0, 1] = \bigcup_{j=0}^{\infty} \Lambda_j$. As shown in (14), household z_j , that last updated its information set at period t - j, knows for sure \hat{w}_{t-k} , if $k \ge j$, and uses its signals about $\hat{w}_{t-k} - x_{t-k}^w$ and y_{t-k}^w —to build its expectation when k < j. Thereafter,¹²

$$\begin{split} \bar{E}\left[\sum_{k=0}^{\infty} q_k \hat{w}_{t-k}\right] &= \int E\left[\sum_{k=0}^{\infty} q_k \hat{w}_{t-k} \middle| \mathfrak{S}_t\left(z\right)\right] dz \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_j} E\left[\sum_{k=j}^{\infty} q_k \hat{w}_{t-k} + \sum_{k=0}^{j-1} q_k \hat{w}_{t-k} \middle| \mathfrak{S}_t\left(z_j\right)\right] dz_j \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_j} \left[\sum_{k=j}^{\infty} q_k \hat{w}_{t-k} + \sum_{k=0}^{j-1} q_k \left(\frac{\alpha_w y_{t-k}^w + \beta_\eta x_{t-k}^w\left(z_j\right)}{\alpha_w + \beta_w + \gamma_w}\right)\right] dz_j \end{split}$$

Since the Lebesgue measure of Λ_j is $(1 - \lambda) \lambda^j$ and idiosyncratic shocks die out with aggregation, we have that $\int_{\Lambda_j} y_{t-k}^w dz_j = (1 - \lambda) \lambda^j y_{t-k}^w$, $\int_{\Lambda_j} \hat{w}_{t-k} dz_j = (1 - \lambda) \lambda^j \hat{w}_{t-k}$ and $\int_{\Lambda_j} x_{t-k}^w (z_j) dz_j = \int_{\Lambda_j} \hat{w}_{t-k} + \xi_{t-k}^w (z_j) dz_j = (1 - \lambda) \lambda^j \hat{w}_{t-k}$. Therefore,

$$\bar{E}\left[\sum_{k=0}^{\infty} q_k \hat{w}_{t-k}\right] = (1-\lambda) \sum_{k=0}^{\infty} q_k \hat{w}_{t-k} \left(\sum_{j=0}^k \lambda^j\right) = \sum_{k=0}^{\infty} q_k \hat{w}_{t-k} \left(1-\lambda^{k+1}\right)$$

$$\begin{split} \bar{E}\left[\sum_{k=0}^{\infty} q_k \hat{w}_{t-k}\right] &= (1-\lambda) \sum_{j=0}^{\infty} \lambda^j \left[\sum_{k=j}^{\infty} q_k \hat{w}_{t-k} + \sum_{k=0}^{j-1} q_k \left(\frac{\alpha_w y_{t-k}^w + \beta_\eta \hat{w}_{t-k}}{\alpha_w + \beta_w + \gamma_w}\right)\right] \\ &= (1-\lambda) \sum_{k=0}^{\infty} q_k \hat{w}_{t-k} \sum_{j=0}^{k} \lambda^j + (1-\lambda) \sum_{k=0}^{\infty} q_k \left(\frac{\alpha_w y_{t-k}^w + \beta_\eta \hat{w}_{t-k}}{\alpha_w + \beta_w + \gamma_w}\right) \sum_{j=k+1}^{\infty} \lambda^j \\ &= \sum_{k=0}^{\infty} q_k \left(1-\lambda^{k+1}\right) \hat{w}_{t-k} + \sum_{k=0}^{\infty} q_k \lambda^{k+1} \left(\frac{\alpha_w y_{t-k}^w + \beta_\eta \hat{w}_{t-k}}{\alpha_w + \beta_w + \gamma_w}\right) \\ &= \sum_{k=0}^{\infty} q_k \left(1-\delta_w \lambda^{k+1}\right) \hat{w}_{t-k} + \theta_w \sum_{k=0}^{\infty} q_k \delta_w \lambda^{k+1} y_{t-k}^w \end{split}$$

Proof of Corollary 1. Since we have $\hat{W}_t = \sum_{k=0}^{\infty} \rho_w^k \hat{w}_{t-k}$, we apply Proposition 1,

¹²With a slight abuse of terminology, we write $\int_{\Lambda_j} [\cdot] dz_j$ to refer to the Lebesgue integral $\int_{\Lambda_j} [\cdot] d\mu$, where μ represents the Lebesgue measure. We believe the terminology we use clarifies which agents we are considering, although this information is implicit in Λ_j .

considering $q_k = \rho_w^k$, to get the result.

Proof of Corollary 2. From (14), $E[\hat{w}_{t+k}|\mathfrak{S}_t(z)] = 0$ when k > 0. Therefore, $E[\hat{W}_{t+i}|\mathfrak{S}_t(z)] = \rho_w^i E[\hat{W}_t|\mathfrak{S}_t(z)]$, which gives

$$\sum_{i=0}^{\infty} E\left[\left.\hat{W}_{t+i}\right| \mathfrak{S}_{t}\left(z\right)\right] = E\left[\left.\hat{W}_{t}\right| \mathfrak{S}_{t}\left(z\right)\right] \left(\sum_{i=0}^{\infty} \rho_{u}^{i}\right) = \frac{E\left[\left.\hat{W}_{t}\right| \mathfrak{S}_{t}\left(z\right)\right]}{1 - \rho_{w}}$$

Thus, applying corollary 1, we get

$$\int \sum_{i=0}^{\infty} E\left[\hat{W}_{t+i} \middle| \mathfrak{S}_t(z)\right] dz = \frac{\bar{E}\left[\hat{W}_t\right]}{1-\rho_w}$$

9.4 Computing the linear equilibrium.

In order to compute the equilibrium, we must find \hat{P}_t and \hat{Q}_t that simultaneously satisfy (11) and (12). We guess that the solution takes the form of a linear function of the shocks, i.e.,

$$\hat{P}_{t} = \sum_{k=0}^{\infty} \left[c_{k}^{a} a_{t-k} + c_{k}^{g} g_{t-k} + c_{k}^{\varepsilon} \varepsilon_{t-k} + c_{k}^{\eta} \eta_{t-k} \right] + \sum_{k=0}^{\infty} \left[\tilde{c}_{k}^{a} y_{t-k}^{a} + \tilde{c}_{k}^{g} y_{t-k}^{g} + \tilde{c}_{k}^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{c}_{k}^{\eta} y_{t-k}^{\eta} \right] (29)$$

$$\hat{Q}_{t} = \sum_{k=0}^{\infty} \left[d_{k}^{a} a_{t-k} + d_{k}^{g} g_{t-k} + d_{k}^{\varepsilon} \varepsilon_{t-k} + d_{k}^{\eta} \eta_{t-k} \right] + \sum_{k=0}^{\infty} \left[\tilde{d}_{k}^{a} y_{t-k}^{a} + \tilde{d}_{k}^{g} y_{t-k}^{g} + \tilde{d}_{k}^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{d}_{k}^{\eta} y_{t-k}^{\eta} \right] (29)$$

Substituting the solution (29) in (11), we get

$$\begin{split} &\sum_{k=0}^{\infty} \left[c_k^a \hat{a}_{t-k} + c_k^g \hat{g}_{t-k} + c_k^{\varepsilon} \hat{\varepsilon}_{t-k} + c_k^{\eta} \hat{\eta}_{t-k} \right] + \sum_{k=0}^{\infty} \left[\tilde{c}_k^a y_{t-k}^a + \tilde{c}_k^g y_{t-k}^g + \tilde{c}_k^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{c}_k^{\eta} y_{t-k}^{\eta} \right] \\ &= \bar{E} \left[\begin{array}{c} \sum_{k=0}^{\infty} \left[e_k^a \hat{a}_{t-k} + e_k^g \hat{g}_{t-k} + e_k^{\varepsilon} \hat{\varepsilon}_{t-k} + e_k^{\eta} \hat{\eta}_{t-k} \right] \\ &+ \sum_{k=0}^{\infty} \left[\tilde{e}_k^a y_{t-k}^a + \tilde{e}_k^g y_{t-k}^g + \tilde{e}_k^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{e}_k^{\eta} y_{t-k}^{\eta} \right] \right] \\ &+ \bar{E} \left[\omega \hat{G}_t - (1+\omega) \hat{A}_t - \left(\frac{\sigma+\omega}{\sigma} \right) U_t \right] \end{split}$$

where, for $w \in \{a, g, \varepsilon, \eta\}$, $e_k^w = \kappa_P c_k^w + \kappa_Q d_k^w$ and $\tilde{e}_k^w = \kappa_P \tilde{c}_k^w + \kappa_Q \tilde{d}_k^w$. As y_{t-k}^w is on the information set of all agents, for all $k \ge 0$ and $w \in \{a, g, \varepsilon, \eta\}$, $E\left[y_{t-k}^w \middle| \mathfrak{S}_t(k)\right] = y_{t-k}^w$. We

use the results presented in Section 6 to write

$$\begin{split} &\sum_{k=0}^{\infty} \left[c_{k}^{a} \hat{a}_{t-k} + c_{k}^{g} \hat{g}_{t-k} + c_{k}^{\varepsilon} \hat{\varepsilon}_{t-k} + c_{k}^{\eta} \hat{\eta}_{t-k} \right] + \sum_{k=0}^{\infty} \left[\tilde{c}_{k}^{a} y_{t-k}^{a} + \tilde{c}_{k}^{g} y_{t-k}^{g} + \tilde{c}_{k}^{\varepsilon} y_{t-k}^{\varepsilon} + \tilde{c}_{k}^{\eta} y_{t-k}^{\eta} \right] \\ &= \sum_{k=0}^{\infty} \left[e_{k}^{a} - (1+\omega) \rho_{a}^{k} \right] \left(1 - \delta_{a} \lambda^{k+1} \right) \hat{a}_{t-k} + \sum_{k=0}^{\infty} \left[e_{k}^{g} + \omega \rho_{g}^{k} \right] \left(1 - \delta_{g} \lambda^{k+1} \right) \hat{g}_{t-k} \\ &+ \sum_{k=0}^{\infty} e_{k}^{\varepsilon} \left(1 - \delta_{\varepsilon} \lambda^{k+1} \right) \hat{\varepsilon}_{t-k} + \sum_{k=0}^{\infty} \left[e_{k}^{\eta} - \left(\frac{\sigma + \omega}{\sigma} \right) \frac{\rho_{u}^{k}}{1 - \rho_{u}} \left(1 - \delta_{\eta} \lambda^{k+1} \right) \right] \left(1 - \delta_{\eta} \lambda^{k+1} \right) \hat{\eta}_{t-k} \\ &+ \sum_{k=0}^{\infty} \left[\theta_{a} \left[e_{k}^{a} - (1+\omega) \rho_{a}^{k} \right] \delta_{a} \lambda^{k+1} + \tilde{e}_{k}^{a} \right] y_{t-k}^{a} + \sum_{k=0}^{\infty} \left[\theta_{g} \left[e_{k}^{g} + \omega \rho_{g}^{k} \right] \delta_{g} \lambda^{k+1} + \tilde{e}_{k}^{g} \right] y_{t-k}^{g} \\ &+ \sum_{k=0}^{\infty} \left[\theta_{\varepsilon} e_{k}^{\varepsilon} \delta_{\varepsilon} \lambda^{k+1} + \tilde{e}_{k}^{\varepsilon} \right] y_{t-k}^{\varepsilon} \\ &+ \sum_{k=0}^{\infty} \left[\theta_{\eta} \left[e_{k}^{\eta} - \left(\frac{\sigma + \omega}{\sigma} \right) \frac{\rho_{u}^{k} \left[1 + \left(1 - \delta_{\eta} \lambda^{k+1} \right) \right]}{1 - \rho_{u}} \right] \delta_{\eta} \lambda^{k+1} + \tilde{e}_{k}^{\eta} \right] y_{t-k}^{\eta} \end{split}$$

Matching coefficients, we get

$$i) \quad c_{k}^{a} = \begin{bmatrix} e_{k}^{a} - (1+\omega) \rho_{a}^{k} \end{bmatrix} \begin{pmatrix} 1 - \delta_{a} \lambda^{k+1} \end{pmatrix} \quad v) \quad \tilde{c}_{k}^{a} = \phi_{k}^{a} c_{k}^{a} + \tilde{e}_{k}^{a} \\ ii) \quad c_{k}^{g} = \begin{bmatrix} e_{k}^{g} + \omega \rho_{g}^{k} \end{bmatrix} \begin{pmatrix} 1 - \delta_{g} \lambda^{k+1} \end{pmatrix} \quad vi) \quad \tilde{c}_{k}^{g} = \phi_{k}^{g} c_{k}^{g} + \tilde{e}_{k}^{g} \\ iii) \quad c_{k}^{\varepsilon} = e_{k}^{\varepsilon} \begin{pmatrix} 1 - \delta_{\varepsilon} \lambda^{k+1} \end{pmatrix} \quad vii) \quad \tilde{c}_{k}^{\varepsilon} = \phi_{k}^{\varepsilon} c_{i}^{\varepsilon} + \tilde{e}_{k}^{\varepsilon} \\ iv) \quad c_{k}^{\eta} = e_{k}^{\eta} \begin{pmatrix} 1 - \delta_{\eta} \lambda^{k+1} \end{pmatrix} \quad viii) \quad \tilde{c}_{k}^{\eta} = \phi_{k}^{\eta} c_{k}^{\eta} + \tilde{e}_{k}^{\eta} \\ - \begin{pmatrix} \frac{\sigma + \omega}{\sigma} \end{pmatrix} \frac{\rho_{u}^{k}}{1 - \rho_{u}} \begin{pmatrix} 1 - \delta_{\eta} \lambda^{k+1} \end{pmatrix}^{2} \quad -\theta_{\eta} \begin{pmatrix} \frac{\sigma + \omega}{\sigma} \end{pmatrix} \frac{\rho_{u}^{k}}{1 - \rho_{u}} \delta_{\eta} \lambda^{k+1} \end{cases}$$
(31)

where

$$\phi_k^w = \theta_w \left(\frac{\delta_w \lambda^{k+1}}{1 - \delta_w \lambda^{k+1}} \right)$$

Similarly, plugging the solution (30) in (12), we get

$$0 = \bar{E} \left[\sum_{k=0}^{\infty} f_k^a \hat{a}_{t-k} + \sum_{k=0}^{\infty} f_k^g \hat{g}_{t-k} + \sum_{k=0}^{\infty} f_k^\varepsilon \hat{\varepsilon}_{t-k} + \sum_{k=0}^{\infty} f_k^\eta \hat{\eta}_{t-k} \right] \\ + \bar{E} \left[\sum_{k=0}^{\infty} \tilde{f}_k^a y_{t-k}^a + \sum_{k=0}^{\infty} \tilde{f}_k^g y_{t-k}^g + \sum_{k=0}^{\infty} \tilde{f}_{k+1}^\varepsilon y_{t-k}^\varepsilon + \sum_{k=0}^{\infty} \tilde{f}_k^\eta y_{t-k}^\eta \right]$$

where

$$f_{k}^{w} = (1 - \beta^{-1} \Phi_{Q}) (d_{k+1}^{w} - d_{k}^{w}) - \beta^{-1} \Phi_{P} (c_{k+1}^{w} - c_{k}^{w}) \tilde{f}_{k}^{w} = (1 - \beta^{-1} \Phi_{Q}) (\tilde{d}_{k+1}^{w} - \tilde{d}_{k}^{w}) - \beta^{-1} \Phi_{P} (\tilde{c}_{k+1}^{w} - \tilde{c}_{k}^{w})$$

and Z_{t+1} is a function of the shocks that occurs at t+1.¹³ As agents do not receive information about future events, $\bar{E}[Z_{t+1}] = 0$. Therefore, using the results presented in Section 6, we get

$$0 = \sum_{k=0}^{\infty} f_k^a \left(1 - \delta_a \lambda^{k+1}\right) \hat{a}_{t-k} + \sum_{k=0}^{\infty} \left[\theta_a f_k^a \delta_a \lambda^{k+1} + \tilde{f}_k^a\right] y_{t-k}^a + \sum_{k=0}^{\infty} f_k^g \left(1 - \delta_g \lambda^{k+1}\right) \hat{g}_{t-k} + \sum_{k=0}^{\infty} \left[\theta_g f_k^g \delta_g \lambda^{k+1} + \tilde{f}_k^g\right] y_{t-k}^g + \sum_{k=0}^{\infty} \left[f_k^{\varepsilon} + \tilde{\tau} \rho_v^k\right] \left(1 - \delta_{\varepsilon} \lambda^{k+1}\right) \hat{\varepsilon}_{t-k} + \sum_{k=0}^{\infty} \left[\theta_{\varepsilon} \left[f_k^{\varepsilon} + \tilde{\tau} \rho_v^k\right] \delta_{\varepsilon} \lambda^{k+1} + \tilde{f}_k^{\varepsilon}\right] y_{t-k}^{\varepsilon} + \sum_{k=0}^{\infty} \left[f_k^\eta - \beta^{-1} \rho_u^k\right] \left(1 - \delta_\eta \lambda^{k+1}\right) \hat{\eta}_{t-k} + \sum_{k=0}^{\infty} \left[\theta_\eta \left[f_k^\eta - \beta^{-1} \rho_u^k\right] \delta_\eta \lambda^{k+1} + \tilde{f}_k^\eta\right] y_{t-k}^\eta.$$

We can summarize this equation as

$$ix) \quad 0 = f_k^a \left(1 - \delta_a \lambda^{k+1}\right) \quad xiii) \quad 0 = \left[f_k^\varepsilon + \tau \rho_v^k\right] \left(1 - \delta_\varepsilon \lambda^{k+1}\right) x) \quad 0 = \theta_a f_k^a \delta_a \lambda^{k+1} + \tilde{f}_k^a \quad xiv) \quad 0 = \theta_\varepsilon \left[f_k^\varepsilon + \tau \rho_v^k\right] \delta_\varepsilon \lambda^{k+1} + \tilde{f}_k^\varepsilon xi) \quad 0 = f_k^g \left(1 - \delta_g \lambda^{k+1}\right) \quad xv) \quad 0 = \left[f_k^\eta - \beta^{-1} \rho_u^k\right] \left(1 - \delta_\eta \lambda^{k+1}\right) xii) \quad 0 = \theta_g f_k^g \delta_g \lambda^{k+1} + \tilde{f}_k^g \quad xvi) \quad 0 = \theta_\eta \left[f_k^\eta - \beta^{-1} \rho_u^k\right] \delta_\eta \lambda^{k+1} + \tilde{f}_k^\eta$$
(32)

In order to find all 16 coefficients, we must solve (31) and (32).

9.4.1 Coefficients c_{t-k}^a , d_{t-k}^a , \tilde{c}_{t-k}^a , \tilde{d}_{t-k}^a

We start considering the coefficients associated to \hat{a}_{t-k} . As $(1 - \delta_a \lambda^{k+1}) \neq 0$, from (ix) we have

$$f_k^a = 0, \forall k \Leftrightarrow \left(1 - \beta^{-1} \Phi_Q\right) d_{k+1}^a - \beta^{-1} \Phi_P c_{k+1}^a = \left(1 - \beta^{-1} \Phi_Q\right) d_k^a - \beta^{-1} \Phi_P c_k^a = K^a, \forall k \in \mathbb{C}$$

where K^a is a constant. From the last equality, we have

$$d_k^a = \frac{K^a + \beta^{-1} \Phi_P c_k^a}{1 - \beta^{-1} \Phi_Q}$$

 $\frac{1^{13}\text{More precisely}, Z_{t+1} = -\beta^{-1}\Phi_P \left[c_0^a a_{t+1} + c_0^g g_{t+1} + c_0^\varepsilon \varepsilon_{t+1} + c_0^\eta \eta_{t+1} + \tilde{c}_0^a a_{t+1} + \tilde{c}_0^g g_{t+1} + \tilde{c}_0^\varepsilon \varepsilon_{t+1} + \tilde{c}_0^\eta \eta_{t+1}\right] + \left(1 - \beta^{-1}\Phi_Q\right) \left[d_0^a a_{t+1} + d_0^g g_{t+1} + d_0^\varepsilon \varepsilon_{t+1} + d_0^\eta \eta_{t+1} + \tilde{d}_0^g y_{t+1}^a + \tilde{d}_0^g y_{t+1}^g + \tilde{d}_0^\varepsilon y_{t+1}^\varepsilon + \tilde{d}_0^\eta y_{t+1}^\varepsilon\right]$

Plugging this expression in $e_k^a = \kappa_P c_k^a + \kappa_Q d_k^a$ and puting it in (i), we get

$$c_{k}^{a} = \left[\frac{\kappa_{P}c_{k}^{a}\left(1-\beta^{-1}\Phi_{Q}\right)+\kappa_{Q}K^{a}+\kappa_{Q}\beta^{-1}\Phi_{P}c_{k}^{a}}{1-\beta^{-1}\Phi_{Q}}-(1+\omega)\rho_{a}^{k}\right]\left(1-\delta_{a}\lambda^{k+1}\right) \\ = \frac{\left[\kappa_{Q}K^{a}-(1+\omega)\rho_{a}^{k}\left(1-\beta^{-1}\Phi_{Q}\right)\right]\left(1-\delta_{a}\lambda^{k+1}\right)}{\left(1-\beta^{-1}\Phi_{Q}\right)-\kappa_{P}\left(1-\beta^{-1}\Phi_{Q}\right)\left(1-\delta_{a}\lambda^{k+1}\right)-\kappa_{Q}\beta^{-1}\Phi_{P}\left(1-\delta_{a}\lambda^{k+1}\right)}\right]$$

In order to have $\lim_{k\to\infty} c_k^a = 0$, we must have $K^a = 0$. This restriction says that there is no permanent effect after a shock. In this case,

$$c_{k}^{a} = \frac{-(1+\omega)\left(\beta - \Phi_{Q}\right)\rho_{a}^{k}\left(1 - \delta_{a}\lambda^{k+1}\right)}{\left(\beta - \Phi_{Q}\right)\left[1 - \kappa_{P}\left(1 - \delta_{a}\lambda^{k+1}\right)\right] - \kappa_{Q}\Phi_{P}\left(1 - \delta_{a}\lambda^{k+1}\right)}$$

$$d_{k}^{a} = \frac{\Phi_{P}c_{k}^{a}}{\beta - \Phi_{Q}} = \frac{-(1+\omega)\Phi_{P}\rho_{a}^{k}\left(1 - \delta_{a}\lambda^{k+1}\right)}{\left(\beta - \Phi_{Q}\right)\left[1 - \kappa_{P}\left(1 - \delta_{a}\lambda^{k+1}\right)\right] - \kappa_{Q}\Phi_{P}\left(1 - \delta_{a}\lambda^{k+1}\right)}$$

Furthermore, from (x), $f_k^a = 0$ implies $\tilde{f}_k^a = 0$ and, based in the same argument presented before

$$\tilde{d}_k^a = \frac{\tilde{K}^a + \beta^{-1} \Phi_P \tilde{c}_k^a}{1 - \beta^{-1} \Phi_Q}.$$

where \tilde{K}^a is another constant. Putting this expression in $\tilde{e}^w_k = \kappa_P \tilde{c}^w_k + \kappa_Q \tilde{d}^w_k$ and plugging it in (v), we get

$$\tilde{c}_{k}^{a} = \frac{\phi_{k}^{a}c_{k}^{a}\left(1-\beta^{-1}\Phi_{Q}\right)+\kappa_{Q}\tilde{K}^{a}}{\left(1-\kappa_{P}\right)\left(1-\beta^{-1}\Phi_{Q}\right)-\beta^{-1}\kappa_{Q}\Phi_{P}}$$

Once again, in order to have $\lim_{k\to\infty} \tilde{c}_k^a = 0$, we must have $\tilde{K}^a = 0$. Thus

$$\begin{aligned} \tilde{c}^a_k &= & \Psi \phi^a_k c^a_k \\ \tilde{d}^a_k &= & \frac{\Phi_P \tilde{c}^a_k}{\beta - \Phi_Q}. \end{aligned}$$

where

$$\Psi = \left(\frac{\beta - \Phi_Q}{\left(1 - \kappa_P\right)\left(\beta - \Phi_Q\right) - \kappa_Q \Phi_P}\right).$$

9.4.2 Coefficients c_{t-k}^g , d_{t-k}^g , \tilde{c}_{t-k}^g , \tilde{d}_{t-k}^g

The steps followed to derive c_{t-k}^g , d_{t-k}^g , \tilde{c}_{t-k}^g , \tilde{d}_{t-k}^g are exactly the same as those previously described. In summary, from (xi), we also know that for all k, $f_k^g = 0$, implying that

$$\begin{cases} \Rightarrow d_k^g = \frac{K^g + \beta^{-1} \Phi_P c_k^g}{1 - \beta^{-1} \Phi_Q} \quad \stackrel{(ii)}{\Rightarrow} c_k^g = \frac{\left[\beta \kappa_Q K^g + \omega \left(\beta - \Phi_Q\right) \rho_g^k\right] \left(1 - \delta_g \lambda^{k+1}\right)}{\left(\beta - \Phi_Q\right) \left[1 - \kappa_P \left(1 - \delta_g \lambda^{k+1}\right)\right] - \kappa_Q \Phi_P \left(1 - \delta_g \lambda^{k+1}\right)} \\ \stackrel{(xii)}{\Rightarrow} \tilde{f}_k^g = 0 \quad \Rightarrow \tilde{d}_k^g = \frac{\tilde{K}^g + \beta^{-1} \Phi_P \tilde{c}_k^g}{1 - \beta^{-1} \Phi_Q} \quad \stackrel{(vi)}{\Rightarrow} \tilde{c}_k^g = \left(\frac{\beta - \Phi_Q}{\left(\beta - \Phi_Q\right) (1 - \kappa_P) - \kappa_Q \Phi_P}\right) \left[\phi_k^g c_k^g + \frac{\beta \kappa_Q \tilde{K}^g}{\beta - \Phi_Q}\right] \end{cases}$$

where K^g and \tilde{K}^g are constants. To avoid any permanent effects, we must have $\lim_{k\to\infty} c_k^g = 0$ and $\lim_{k\to\infty} \tilde{c}_k^g = 0$. These conditions imply that $K^g = 0$ and $\tilde{K}^g = 0$, which gives

$$c_k^g = \frac{\omega(\beta - \Phi_Q)\rho_g^k(1 - \delta_g \lambda^{k+1})}{(\beta - \Phi_Q)[1 - \kappa_P(1 - \delta_g \lambda^{k+1})] - \kappa_Q \Phi_P(1 - \delta_g \lambda^{k+1})}, \qquad d_k^g = \frac{\Phi_P c_k^g}{\beta - \Phi_Q},$$
$$\tilde{c}_k^g = \Psi \phi_k^g c_k^g \qquad \text{and} \quad \tilde{d}_k^g = \frac{\Phi_P \tilde{c}_k^g}{\beta - \Phi_Q}.$$

9.4.3 Coefficients c_{t-k}^{ε} , d_{t-k}^{ε} , $\tilde{c}_{t-k}^{\varepsilon}$, $\tilde{d}_{t-k}^{\varepsilon}$

From (xiii), we have

$$f_k^{\varepsilon} + \tilde{\tau} \rho_v^k = 0, \forall k \Rightarrow \left[\left(1 - \beta^{-1} \Phi_Q \right) d_k^{\varepsilon} - \beta^{-1} \Phi_P c_k^{\varepsilon} \right] = \left[\left(1 - \beta^{-1} \Phi_Q \right) d_{k+1}^{\varepsilon} - \beta^{-1} \Phi_P c_{k+1}^{\varepsilon} \right] + \tilde{\tau} \rho_v^k, \forall k.$$

Iterating this expression, we obtain

$$\left(1-\beta^{-1}\Phi_Q\right)d_k^{\varepsilon}-\beta^{-1}\Phi_Pc_k^{\varepsilon}=\left(1-\beta^{-1}\Phi_Q\right)d_{k+p}^{\varepsilon}-\beta^{-1}\Phi_Pc_{k+p}^{\varepsilon}+\tilde{\tau}\sum_{i=0}^{p-1}\rho_v^{k+i}$$

By requiring that $\lim_{p\to\infty} (1-\beta^{-1}\Phi_Q) d_{k+p}^{\varepsilon} - \beta^{-1}\Phi_P c_{k+p}^{\varepsilon} = 0$, which is analogous to the condition of no permanent effects on the economy, we obtain

$$\left(1-\beta^{-1}\Phi_Q\right)d_k^{\varepsilon}-\beta^{-1}\Phi_Pc_k^{\varepsilon}=\frac{\tilde{\tau}\rho_v^k}{1-\rho_v}\Rightarrow d_k^{\varepsilon}=\frac{1}{1-\beta^{-1}\Phi_Q}\left(\frac{\tilde{\tau}\rho_v^k}{1-\rho_v}+\beta^{-1}\Phi_Pc_k^{\varepsilon}\right)$$

Plugging the expression for d_k^{ε} into (iii), we obtain

$$c_{k}^{\varepsilon} = \left[\kappa_{P}c_{k}^{\varepsilon} + \kappa_{Q}d_{k}^{\varepsilon}\right]\left(1 - \delta_{\varepsilon}\lambda^{k+1}\right)$$
$$= \left(\frac{\beta\tilde{\tau}}{1 - \rho_{v}}\right)\frac{\kappa_{Q}\rho_{v}^{k}\left(1 - \delta_{\varepsilon}\lambda^{k+1}\right)}{\left(\beta - \Phi_{Q}\right)\left[1 - \kappa_{P}\left(1 - \delta_{\varepsilon}\lambda^{k+1}\right)\right] - \kappa_{Q}\Phi_{P}\left(1 - \delta_{\varepsilon}\lambda^{k+1}\right)}$$

From this expression, d_k^{ε} is

$$d_{k}^{\varepsilon} = \left(\frac{\beta\tilde{\tau}}{1-\rho_{v}}\right) \left[\frac{\rho_{v}^{k}\left[1-\kappa_{P}\left(1-\delta_{\varepsilon}\lambda^{k+1}\right)\right]}{\left(\beta-\Phi_{Q}\right)\left[1-\kappa_{P}\left(1-\delta_{\varepsilon}\lambda^{k+1}\right)\right]-\kappa_{Q}\Phi_{P}\left(1-\delta_{\varepsilon}\lambda^{k+1}\right)}\right].$$

We calculate the two other coefficients, $\tilde{c}_{t-k}^{\varepsilon}$ and $\tilde{d}_{t-k}^{\varepsilon}$, as before:

$$f_k^{\varepsilon} + \tilde{\tau} \rho_v^k = 0 \stackrel{(xiv)}{\Rightarrow} \tilde{f}_k^{\varepsilon} = 0 \Rightarrow \tilde{d}_k^{\varepsilon} = \frac{\tilde{K}^{\varepsilon} + \beta^{-1} \Phi_P \tilde{c}_k^{\varepsilon}}{1 - \beta^{-1} \Phi_Q}$$

where \tilde{K}^{ε} is a constant. Remembering that $\tilde{e}_{k}^{w} = \kappa_{P}\tilde{c}_{k}^{w} + \kappa_{Q}\tilde{d}_{k}^{w}$, we can put the expression for $\tilde{d}_{k}^{\varepsilon}$ into (vii) to obtain

$$\tilde{c}_k^{\varepsilon} = \Psi \left[\phi_k^{\varepsilon} c_k^{\varepsilon} + \left(\frac{\beta \kappa_Q \tilde{K}^{\varepsilon}}{\beta - \Phi_Q} \right) \right]$$

In order to have $\lim_{k\to\infty} \tilde{c}_k^{\varepsilon} = 0$, we must have $\tilde{K}^{\varepsilon} = 0$. Thus,

$$\tilde{c}_k^{\varepsilon} = \Psi \phi_k^{\varepsilon} c_k^g \quad \text{and} \quad \tilde{d}_k^{\varepsilon} = \frac{\Phi_P \tilde{c}_k^{\varepsilon}}{\beta - \Phi_Q}.$$

9.4.4 Coefficients c_{t-k}^{η} , d_{t-k}^{η} , \tilde{c}_{t-k}^{η} , \tilde{d}_{t-k}^{η}

The steps used to derive c_{t-k}^{η} , d_{t-k}^{η} , \tilde{c}_{t-k}^{η} , \tilde{d}_{t-k}^{η} are exactly the same as those described for the previous coefficients. In summary, from (xv), we know that, for all k, $f_k^{\eta} - \beta^{-1}\rho_u^k = 0$, implying that

$$d_k^{\eta} = \frac{1}{\beta - \Phi_Q} \left[-\frac{\rho_u^k}{1 - \rho_u} + \Phi_P c_k^{\eta} \right] \stackrel{(iv)}{\Rightarrow} c_k^{\eta} = - \left[\frac{\kappa_Q + \left(\frac{\sigma + \omega}{\sigma}\right) \left(\beta - \Phi_Q\right) \left(1 - \delta_\eta \lambda^{k+1}\right)}{\left(\beta - \Phi_Q\right) \left[1 - \kappa_P \left(1 - \delta_\eta \lambda^{k+1}\right)\right] - \kappa_Q \Phi_P \left(1 - \delta_\eta \lambda^{k+1}\right)} \right] \frac{\rho_u^k \left(1 - \delta_\eta \lambda^{k+1}\right)}{1 - \rho_u}$$

and, from (xvi), that $\tilde{f}_k^{\eta} = 0$; consequently,

$$\tilde{d}_k^{\eta} = \frac{\tilde{K}^{\eta} + \beta^{-1} \Phi_P \tilde{c}_k^{\eta}}{1 - \beta^{-1} \Phi_Q} \stackrel{(viii)}{\Rightarrow} \tilde{c}_k^{\eta} = \Psi \left[\phi_k^{\eta} c_k^{\eta} - \left(\frac{\sigma + \omega}{\sigma}\right) \frac{\phi_k^{\eta} \rho_u^k \left(1 - \delta_\eta \lambda^{k+1}\right)}{1 - \rho_u} + \frac{\beta \kappa_Q \tilde{K}^{\eta}}{\beta - \Phi_Q} \right]$$

where \tilde{K}^{η} is a constant. Finally, in order to have $\lim_{k\to\infty} \tilde{c}_k^{\eta} = 0$, we must have $\tilde{K}^{\eta} = 0$. Thus,

$$\tilde{d}_k^{\eta} = \frac{\Phi_P \tilde{c}_k^{\eta}}{\beta - \Phi_Q} \stackrel{(viii)}{\Rightarrow} \tilde{c}_k^{\eta} = \Psi \phi_k^{\eta} \left[c_k^{\eta} - \left(\frac{\sigma + \omega}{\sigma} \right) \frac{\rho_u^k \left(1 - \delta_\eta \lambda^{k+1} \right)}{1 - \rho_u} \right]$$

9.4.5 Summary

We can summarize the results we obtained for the coefficients of (29) and (30) as

$$\begin{split} c_k^a &= -\left(1+\omega\right)\left(\beta-\Phi_Q\right)\Gamma_k^a, & d_k^a = \left(\frac{\Phi_P}{\beta-\Phi_Q}\right)c_k^a, \\ c_k^g &= \omega\left(\beta-\Phi_Q\right)\Gamma_k^g, & d_k^g = \left(\frac{\Phi_P}{\beta-\Phi_Q}\right)c_k^g, \\ c_k^\varepsilon &= \left(\frac{\beta\tilde{\tau}}{1-\rho_v}\right)\kappa_Q\Gamma_k^\varepsilon, & d_k^\varepsilon = \frac{1}{\beta-\Phi_Q}\left[\Phi_Pc_k^\varepsilon + \frac{\beta\tilde{\tau}\rho_v^k}{1-\rho_v}\right], \\ c_k^\eta &= -\left[\frac{\kappa_Q + \left(\frac{\sigma+\omega}{\sigma}\right)\left(\beta-\Phi_Q\right)\left(1-\delta_\eta\lambda^{k+1}\right)}{1-\rho_u}\right]\Gamma_k^\eta, & d_k^\eta = \frac{1}{\beta-\Phi_Q}\left[\Phi_Pc_k^\eta - \frac{\rho_u^k}{1-\rho_u}\right], \\ \tilde{c}_k^a &= \Psi\phi_k^a c_k^a & \tilde{d}_k^a = \left(\frac{\Phi_P}{\beta-\Phi_Q}\right)\tilde{c}_k^a, \\ \tilde{c}_k^g &= \Psi\phi_k^g c_k^g & \tilde{d}_k^\varepsilon = \left(\frac{\Phi_P}{\beta-\Phi_Q}\right)\tilde{c}_k^g, \\ \tilde{c}_k^\varepsilon &= \Psi\phi_k^\varepsilon c_k^\varepsilon & \tilde{d}_k^\varepsilon = \left(\frac{\Phi_P}{\beta-\Phi_Q}\right)\tilde{c}_k^\varepsilon, \\ \tilde{c}_k^\eta &= \Psi\phi_k^\eta \left[c_k^\eta - \left(\frac{\sigma+\omega}{\sigma}\right)\frac{\rho_u^k(1-\delta_\eta\lambda^{k+1})}{1-\rho_u}\right] & \tilde{d}_k^\eta = \left(\frac{\Phi_P}{\beta-\Phi_Q}\right)\tilde{c}_k^\eta, \end{split}$$

where

$$\Gamma_{k}^{w} = \frac{\rho_{w}^{k} (1 - \delta_{w} \lambda^{k+1})}{(\beta - \Phi_{Q}) [1 - \kappa_{P} (1 - \delta_{w} \lambda^{k+1})] - \kappa_{Q} \Phi_{P} (1 - \delta_{w} \lambda^{k+1})}$$

$$\phi_{i}^{w} = \theta_{w} \left(\frac{\delta_{w} \lambda^{i+1}}{1 - \delta_{w} \lambda^{i+1}}\right)$$

$$\Psi = \frac{\beta - \Phi_{Q}}{(1 - \kappa_{P}) (\beta - \Phi_{Q}) - \kappa_{Q} \Phi_{P}}$$
(33)

9.4.6 Aggregate veriables

After obtaining the equilibrium dynamics of \hat{P}_t and \hat{Q}_t , we evaluate the dynamics of the other aggregate variables as a function of \hat{P}_t and \hat{Q}_t and shcks. We compute inflation as the first difference of prices, $\pi_t \equiv \hat{P}_t - \hat{P}_{t-1}$, and use (25) to find \hat{Y}_t as

$$\hat{Y}_t \equiv \hat{C}_t + \hat{G}_t = \hat{G}_t - \sigma^{-1} U_t + \sigma^{-1} (\Phi_P - 1) \hat{P}_t + \sigma^{-1} (1 + \Phi_Q) \hat{Q}_t,$$

We can also log-linearize (10) to obtain

$$\hat{Y}_t^n = \frac{\sigma \hat{G}_t + (1+\omega) \,\hat{A}_t}{\sigma + \omega},\tag{34}$$

From this expression, we can derive the output gap, x_t , as

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^n \\ = \frac{\omega \hat{G}_t - (1+\omega) \hat{A}_t}{\sigma + \omega} - \sigma^{-1} U_t + \sigma^{-1} (\Phi_P - 1) \hat{P}_t + \sigma^{-1} (1+\Phi_Q) \hat{Q}_t.$$

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