Diogo Luiz Duarte

Home Bias in a Monetary Union: How Financial Frictions Affect Output and Monetary Policy Decisions

Dissertação de Mestrado

Dissertation presented to the Programa de Pós-graduação em Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Diogo Abry Guillén

Rio de Janeiro
December 2018
Diogo Luiz Duarte

Home Bias in a Monetary Union: How Financial Frictions Affect Output and Monetary Policy Decisions

Dissertação de Mestrado

Dissertation presented to the Programa de Pós-graduação em Macroeconomia e Finanças of the Departamento de Economia, PUC-Rio as a partial fulfillment of the requirements for the degree of Mestre em Macroeconomia e Finanças. Approved by the following Comission

Prof. Diogo Abry Guillén
Advisor
Departamento de Economia – PUC-Rio

Prof. Carlos Viana de Carvalho
Departamento de Economia – PUC-Rio

Prof. Waldyr Dutra Areosa
Departamento de Economia – PUC-Rio

Prof. Augusto Cesar Pinheiro da Silva
Vice-Decano Setorial de Pós-Graduação do Centro de Ciências Sociais

Rio de Janeiro, December, 20th, 2018
All rights reserved.

Diogo Luiz Duarte

Graduated in Engineering from the Instituto Militar de Engenharia, Brazil.

Bibliographic data

Duarte, Diogo Luiz

42 f. : il. color. ; 30 cm

Dissertação (mestrado)–Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia, 2019.
Inclui bibliografia


CDD: 330
Acknowledgements

First of all, I would like to thank my wife, Luisa, for all the love and support received during these years dedicated to the Master’s degree. I would also like to thank my family, Lucia, Flavio and Walter for their support and for being a lifetime example of dedication and determination.

In all respects, I benefitted decisively from all the advice of my mentor Diogo Abry Guillén during classes and while working on my thesis, and I would like to thank him for the privilege of having his time and energy dedicated to this work.

I will never cease to appreciate the work done by the professors of the Macroeconomics and Finance master study program at PUC-Rio. Their thorough, inspiring work enabled me to gain access to a body of knowledge I would not be able to acquire otherwise. In particular, I would like to thank Carlos Viana de Carvalho and Waldyr Dutra Areosa for their comments about this work.
Abstract


This study lays-out a model with two countries that follow the DSGE framework with financial intermediaries set by Gertler-Karadi (2011) and form a monetary union. We study the impact of financial frictions and the effects of union-wide and country-specific unconventional monetary policies in the union’s member countries. We show that, if the parameters used to limit balance sheet size are calibrated in a way to allow for higher leverage in the banking system, the easier access to capital leads to an Output level that is, at the same time, higher in the Steady State and more fragile to Capital Quality Shocks. It’s also shown that high levels of home-bias lead to lower risk-sharing and lower dissemination of idiosyncratic shocks, which helps explaining why idiosyncratic shocks may cause highly persistent effects in the member countries. Finally, this study also shows that country-specific unconventional monetary policies can be considerably welfare increasing when home-bias in the financial system is high.

Keywords

Home Bias; Monetary Union; Unconventional Monetary Policy; Open Economy.
Resumo


Este estudo define um modelo de dois países que seguem a estrutura exposta em Gertler-Karadi (2011) e formam uma união monetária. Estudamos o impacto de fricções financeiras e os efeitos de políticas monetárias não convencionais implementadas com escopo individual e geral nos países membros desta união. Mostramos que, se os parâmetros usados para limitar o balanço das instituições financeiras forem calibrados para permitir uma alavancagem mais alta, o maior acesso a capital leva a um produto que é, ao mesmo tempo, mais alto no steady state e mais frágil a choques de qualidade de capital. Também mostramos que níveis elevados de Home Bias levam a menos compartilhamento de riscos e a uma disseminação menor de choques idiosincráticos. Por fim, esse estudo também mostra que políticas monetárias não convencionais com escopo individual podem aumentar o bem-estar consideravelmente quando o Home Bias no sistema financeiro é elevado.

Palavras-chave
Home Bias; União Monetária; Política Monetária Não Convencional; Economia Aberta.
Table of contents

1. Introduction ........................................................................................................... 10

2. Model ...................................................................................................................... 14
   2.1. Households ...................................................................................................... 15
   2.2. Banks ................................................................................................................ 16
   2.3. Intermediate Goods Firms .............................................................................. 19
   2.4. Capital Goods Firms ...................................................................................... 20
   2.5. Final Goods Firms ........................................................................................... 21
   2.6. Market Clearing .............................................................................................. 23
   2.7. Monetary Policy ............................................................................................... 23
       2.7.1. Interest Rate Policy .................................................................................. 23
       2.7.2. Unconventional Monetary Policy ............................................................ 24

3. Calibration ............................................................................................................... 26

4. Experiments ............................................................................................................ 28
   4.1. Welfare .............................................................................................................. 30

5. Conclusion .............................................................................................................. 33

6. Bibliographic references ......................................................................................... 35

Appendix .................................................................................................................... 42
List of figures

Figure 1: Low Home-Bias vs. High Home-Bias .................................................. 39
Figure 2: Country-Specific intervention ............................................................... 40
Figure 3: Union-Wide intervention .................................................................. 41
List of tables

Table 1: Parameters.................................................................37
Table 2: Higher Leverage Experiment........................................37
Table 3: Welfare Experiment....................................................38
1. Introduction

There are numerous studies about conventional and unconventional monetary policy in closed economies. Also, other works\(^1\) have developed the field of monetary policy and fiscal policy in monetary unions. The use of unconventional monetary policies, macro-prudential rules and other tools in monetary unions, however, is still a field under development.

The objective of this study is to understand the impact of idiosyncratic shocks in a union's member countries, as well as to compare the efficiency of economic policies in response to these shocks.

In this specific case, this study focuses in understanding if frictions as the existence of home-bias in the banking system might lead to imperfect risk sharing in these unions, and if this can make traditional economic policy less efficient.

This article is formed by 5 sections. In section 2 the baseline model is described. A two-country model is set up, each of them based in the model in Gertler, Karadi [11]. The difference from the original model is the insertion of shared goods and assets markets for both economies. Also, a home-bias factor is inserted that regulates how much of the capital in banks is invested locally and abroad. Another home-bias factor is inserted in the households' consumption basket, but for the purpose of this article we assume households preferences are equal among local and foreign goods for the sake of simplicity.

In section 3, we describe the calibration of the model.\(^2\)

In section 4, we detail a group of three experiments that were executed. The first experiment was to verify how the model is affected when we allow for higher leverage in the financial system. In the original model of Gertler, Karadi [11], leverage is regulated by a group of parameters, the most important of which is \(\lambda_b\), the amount of assets that can be diverted by banks.

---

1 Please see Benigno [1], Ferrero [9], Gali, Monacelli [10] and Farhi [8]
2 The parameters used were taken from Primiceri and Al [15], Gertler, Karadi [11], Devereux, Sutherland[6] and Poutineau, Vermandel [14].
In this study, we slightly change this parameter to a lower value (from 0.3814 to 0.37) to induce higher leverage. The consequence is that banks now can divert a smaller amount of assets in each period, so their incentive to divert assets is lower, and depositors start accepting a higher leverage up to which they are willing to deposit funds in the banking system. What follows is that companies get easier access to funding, and this leads to a higher output level in the steady state. There is a trade-off, however - the higher leverage in financial firms makes them more susceptible to capital quality shocks. The higher the leverage in the financial system, the bigger the magnitude of the effect of an equal-sized capital quality shock on output. In other words, this shows that the expected trade-off in the adoption of regulations allowing for higher (lower) leverage is correctly simulated in the model.

The second experiment was to study how different levels of home-bias in the banking sector affect the dynamics of variables in the model after a capital quality shock. For this purpose, we assume bankers in both countries can hold assets (make loans to companies) from both countries, and that preferences for holding local assets compared to foreign assets are determined in a CES Aggregator as in Poutineau, Vermandel [14] and Krenz [13]. Here, if we have a home-bias factor of 0.5, bankers have no preferences for local assets compared to foreign assets. If the home-bias factor is 1, however, bankers have balance sheets composed exclusively by home assets regardless of the expected returns on these assets. Although Home-Bias may seem counter-intuitive, country regulations, reduced knowledge about foreign companies (compared to local companies), legal limitations, among others may explain the existence of this friction.

This experiment finds that lower levels of home-bias lead to higher risk sharing, causing initial negative effects in the foreign country after a shock in the home country. Negative effects are compensated throughout time by easier monetary policy, and both countries return more quickly to Output levels prevalent in the Steady State. However, a different dynamics happens when home-bias is high. Most of the negative effects of the shock are kept in the local banking system, and the foreign country's banking system is barely affected. The central bank lowers interest rates to compensate for the negative output gap in the
home country, however the output gap is larger in the home country and smaller in the foreign country than when home-bias was low. This takes to a different dynamics in the recovery process, in which the home country keeps output smaller than potential and the foreign country keeps output higher than potential for a longer period. This is indicative that the lack of integration in the financial markets due to home-bias can be one of the causes of highly persistent shocks in a monetary union.

It's important to say that if these countries did not make part of a monetary-union, the flexibility of having a local Central Bank that observes exclusively local economic variables and sets interest rates accordingly would make convergence to potential output considerably faster.

The third experiment was to test two different unconventional monetary policy approaches in the model, as a response to a capital quality shock. The first approach was to use union-wide policies, regardless of the dynamics in country-specific variables. The second was to use country-specific policies, in which the Central Bank observes credit spreads in each country, as opposed to union-wide averages, and buys or sells private debt with the intent of bringing spread levels back to levels prevalent in the steady state.

The result of this experiment was that union-wide interventions had effects very similar to interest rate cuts, affecting union-wide variables observed by the Central Bank (inflation and output) and consequently diminishing the amount of interest rate cuts necessary to bring the Monetary Union back to equilibrium on aggregate. Note that in this case the zero lower bound was not imposed, but this exercise indicates unconventional monetary policy would have effects similar to interest rate cuts once the zero lower bound is binding. Also, because of this substitution effect with interest rates, welfare increases were relatively small when this kind of policy was used in the absence of the ZLB. Capital quality shocks had a more persistent negative effect in the home country output, and a corresponding persistent positive effect in the foreign country.

Country-specific interventions, on the other hand, have positive effects even in the absence of the ZLB. In the case of high home-bias, the use of country-
specific interventions are shown to be welfare increasing, to reduce the persistence of negative(positive) shocks in both countries and to present a lesser substitutive relationship with the Central Bank's interest rate policy. In the case of low home-bias, the use of country-specific interventions is shown to be welfare increasing when interventions are made strictly in the home country. In this case, the flexibility of Country-Specific policies makes them more efficient compared to Union-Wide Policies. In section 5, we finish with our concluding remarks.
2. Model

In this model, we set up a monetary union composed by two symmetric countries. Both economies are isomorphic to the model in Gertler, Karadi [11], except for the fact that these countries are not closed economies. As in GK2011, both countries feature a banking sector that transfers funds between households and non-financial firms.

Both countries share integrated markets for goods and banking loans (assets), however household deposits are kept only in local banks, to better mimics the dynamics in existing monetary unions. Households can consume products from both countries, though, and banks can lend both to local and foreign non-financial firms.

Member countries share a single Central Bank that chooses the appropriate interest rate level according to a Taylor Rule, observing union-wide inflation and the output gap in both countries.

A set of other decisions had to be made concerning the modeling process of this monetary union. For the sake of simplicity, we chose to build a model with only two member countries defined symmetrically, although some monetary unions have many member countries, which are different in size and economic profiles. As countries are identical, the interaction between them follows a Large Open Economy approach.

Also for the sake of simplicity, the integration in the goods markets is made on the final good firms level - so all the production process is done locally and distributed afterwards in the local and in the foreign country. The law of one price holds in this model, so prices for one given good are the same in both countries. The producer currency prices vs. local currency prices issue is not relevant in this case because both countries have a single currency.

Only home country equations are displayed in this study, as foreign equations are identical. Foreign variables used in home country equations are denoted with a star.
2.1. Households

Following Gertler, Karadi [11], in each country there is a continuum of identical households of measure unity. There are two types of members in each household: workers and bankers. Workers provide labor exclusively to local (not to foreign) firms, and bankers in a household are exclusively local bankers. At any moment in time, a fraction $1 - f$ of the household members are workers and a fraction $f$ are bankers. Over time, bankers may become workers and workers may become bankers. A banker stays a banker in the next period with probability $\theta_B$, so every period $(1 - \theta_B)f$ bankers become workers. To keep the proportion among members in households, each period the same amount $(1 - \theta_B)f$ of workers randomly become new bankers. Bankers who become workers pay retained earnings to their respective household. New bankers from a household are provided with an amount of start up funds by their respective household, as shown in the next section. Let $C_t$ be consumption and $L_t$ family labor supply. Household preferences are given by

$$ E_t \sum_{k=0}^{\infty} \Theta_k \left( ln(C_{t+k} - hC_{t+k-1}) - \frac{L_{t+k-1}^{1+\phi}}{1+\phi} \right) $$

with $0 < h < 1$, and $\chi, \phi > 0$. $h$ is the habit formation factor and $\chi$ is the relative utility weight of labor. $\phi$ is the inverse Frisch elasticity of labor supply. $\Theta_k$ is an endogenous discount factor\(^3\)\(^4\).

Households consume, provide labor to firms, receive profits from firms, pay taxes to the government and save, lending capital to financial firms. The Household budget constraint, thus, is given by

$$ C_t + B_t + T_t = R_{t-1}B_{t-1} + w_t L_t + \Pi_t $$

\(^3\) In order to guarantee convergence in this model we use an endogenous discount factor following Uribe, Smith-Grohe (2003) [16] and Bodenstein (2011) [3].

\(^4\) As in Cristiano, Eichenbaum and Evans (2005) [4] and Smets and Wouters(2007) [18], habit formation is allowed to capture consumption dynamics.
$R_t$ is the gross real return paid by deposits in financial intermediaries in the home country and by home government debt. $B_t$ is the amount of one-period deposits made by households in financial intermediaries, $w_t$ is the real wage paid to workers, $\Pi_t$ is the net payout from financial and non-financial firms. $T_t$ are lump sum taxes.

Households have similar preferences for goods produced locally and abroad\(^5\). Consumption $C_t$ is given by a CES aggregator similar to the one in Coeurdacier, Rey [5] as follows.

\[
C_t = \left( 0.5^{\frac{\gamma-1}{2}} C_{H,t}^{\frac{\gamma-1}{\gamma}} + 0.5^{\frac{1}{2}} C_{F,t}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{1}{1-\gamma}}
\]

(3)

Where $C_{H,t}$ is the household consumption of local goods and $C_{F,t}$ is the consumption of foreign goods. Conversely, the consumer price index that corresponds to these preferences is given by

\[
P_t = \left( 0.5 P_{H,t}^{\frac{1}{1-\gamma}} + 0.5 P_{F,t}^{\frac{1}{1-\gamma}} \right)^{\frac{1}{1-\gamma}}
\]

(4)

Where $P_{H,t}$ denotes the price of the local good and $P_{F,t}$ denotes the local price of the foreign good. In this study, the law of one price holds, so the price of one product is the same in both countries.

### 2.2. Banks

The financial sector in this model is also similar to the one in Gertler, Karadi [11]. Banks obtain funds from households via deposits and lend them to non-financial firms. Let $N_{i,t}$ be the amount of Net Worth held by a banker/intermediary at the end of period $t$, $A_{i,t}$ the assets held in the banker’s balance sheet, and $B_{iH,t}$ the amount of deposits. Then, a banker’s balance sheet is given by

\(^5\) Here we follow [13] - similar preferences for local and foreign goods leads to the absence of real exchange fluctuations and keeps the analysis of results simple.
The main difference in this section from GK2011 is that bankers can make loans to local and foreign companies. Here, following Krenz [13] and Poutineau, Vermandel [14] the asset side of the balance sheet is defined as a CES Aggregator that combines local and foreign loans as follows.

\[ A_{iH,t} = B_{iH,t} + N_{iH,t} \]  

(5)

Where \( Q_{H,t}(Q_{F,t}) \) is the price of local(foreign) loans and \( S_{iH,t}(S_{iF,t}) \) is the amount of local(foreign) loans held in the bank’s balance sheet. Conversely, the return on the bank’s assets is given by.

\[ R_t^A = \left( \mu_A \left( R_{k,t} \right)^{1-i_A} + (1 - \mu_A) \left( R_{k,t}^{*} \right)^{1-i_A} \right)^{\frac{1}{1-i_A}} \]  

(7)

Using a CES aggregator to define the composition of the balance sheet of financial intermediaries is a simple and convenient shortcut for defining banker’s asset preferences and introducing the existence of home-bias in portfolio selection. Besides, it permits us to solve this problem without using an endogenous portfolio selection method.

In the beginning of a given period, the financial intermediary’s net worth is given by the difference between the return on assets held and the interest paid over liabilities due in the previous period.

\[ N_{iH,t} = R_t^A A_{iH,t-1} - R_{t-1} B_{iH,t-1} \]  

(8)

The banker’s objective is to maximize expected terminal wealth, given by the present value of expected earnings in the future.
Here, $\Lambda_{t,t+1}$ is defined as

$$
\Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t}
$$

where $\lambda_t$ is the household’s marginal utility of consumption.

This condition alone would make bankers increase leverage indefinitely, provided that the risk adjusted return is greater than or equal to the return on household’s deposits. To limit that leverage, we introduce the same friction used in Gertler, Karadi[11]: at the beginning of the period the banker can choose to divert a fraction $\lambda_B$ of available assets and transfer them to the household of which he or she is a member. Households do not want to lose their deposits, so for depositors to be willing to make deposits in the bank, the value for the bank of keeping in business must be larger than the value obtained by diverting a share of assets. This is given by the following incentive constraint

$$
V_{i,t} \geq \lambda_B A_{i,t}
$$

This constraint limits endogenously the leverage of the financial intermediaries. The limited amount of leverage is a friction that takes the model to an equilibrium where the returns on the Bank’s loans is different from zero.

From the above conditions it can be shown that, if the incentive constraint binds, the Balance Sheet is related to Net Worth by the following equation

$$
A_t = \frac{\eta_t}{\lambda_B - \nu_t} N_t = \phi_t N_t
$$
where $\phi_t$ is the bank’s leverage ratio. Equations for $\nu_t$ and $\eta_t$ are defined in the Appendix.

The motion equation for $N_t$ is defined based on its constituent parts, the net worth of existing bankers in a period $t$ and the net worth of new bankers

$$N_t = (N_{n,t} + N_{e,t})$$

The net worth of existing bankers is given by

$$N_{e,t} = \theta \beta [(R_t^A - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1}$$

and the net worth of new banks is chosen to be a small fraction of the assets intermediated by existing banks in the previous period

$$N_{n,t} = \omega A_{t-1}$$

This is done to make sure bankers don’t put together an amount of retained earnings that is big enough to make loans without the use of deposits. Also, the amount of capital given to new Bankers is set as a fraction of assets in the previous period based on the fact that in order to be able to finance part of these assets the initial amount of capital must be related to this variable.

In this article, following Gertler, Karadi [11], the value of $\omega$ is very small, and chosen carefully to calibrate the steady state’s leverage ratio in the financial system. The parameter $\omega$ is chosen so that a leverage of 4 is obtained in the banking system.

### 2.3. Intermediate Goods Firms

Following Gertler, Karadi [11], there are three kinds of firms in this model: intermediate goods firms, final goods firms and capital goods firms. Intermediate goods firms are non-financial firms that produce goods that are sold to final goods firms. Intermediate goods are only repackaged by final goods firms. In theory, this separation is not necessary - this is only used to separate actual production from the product differentiation and price adjustment setup that allows
for sticky prices. In each period, intermediate goods firms obtain funds from financial intermediaries through the issuance of $S_t$ claims, acquire capital, and use it to produce goods in the following period. Each claim is priced at $Q_t$. The production function follows a Cobb-Douglas equation

$$Y_{m,t} = a_t(U_t^\alpha K_t^{1-\alpha})$$  \hspace{1cm} (16)

where $U_t$ is the fraction of capacity utilization, $\xi_t$ is the capital quality shock, and $K_t$ is the amount of capital. $a_t$ is the total factor productivity.

The firms earn zero profits, and all the payoffs from producing these goods are used to pay the financial intermediaries. The return on financial intermediaries loans thus is defined as

$$R_{k,t+1} = \frac{\alpha \frac{P_{m,t+1}Y_{m,t+1}}{K_t} + (Q_{t+1} - \delta(U_t))\xi_{t+1}}{Q_t}$$  \hspace{1cm} (17)

Conversely, firms choose the utilization rate and labor demand as defined by the following equations

$$W_t = \frac{P_t}{P_t} = (1 - \alpha) \frac{P_{m,t}Y_{m,t}}{L_t}$$  \hspace{1cm} (18)

$$\delta'(U_t)\xi_t K_{t-1} = P_{m,t} \alpha \frac{Y_{m,t}}{U_t}$$  \hspace{1cm} (19)

2.4.
Capital Goods Firms

Capital goods firms buy capital from intermediate goods firms, repair depreciated capital and create new capital. There are adjustment costs associated with the production process. The functional form of adjustment costs is the same used in Gertler, Karadi (2011) [11]
where $I$ is the steady state investment and $I_{n,t}$ is the investment net of depreciation.

$$ I_{n,t} = I_t - \delta(U_t)\xi_t K_t $$

(21)

The firms maximize profits, given by

$$ E_t \sum_{k=0}^{\infty} \Theta_k \Lambda_{t+k} \{ Q_{t+k} I_{t+k} - [1 + f(.)] I_{t+k} \} $$

(22)

The law of motion of capital is given by

$$ K_t = I_t + (1 - \delta(U_t))\xi_t K_{t-1} $$

(23)

From the first order condition for investment it follows that

$$ Q_t = 1 + f(.) + \frac{I_{n,t} + I}{I_{n,t-1} + I} f'(.) - E_t \beta(C_t) \Lambda_{t+1} \left( \frac{I_{n,t+1} + I}{I_{n,t} + I} \right)^2 f'(.) $$

(24)

2.5. Final Goods Firms

Final output is a CES composite of a continuum of mass unity of differentiated final firms

$$ Y_t = \left[ \int_0^1 Y_t(f) \frac{1}{\xi_t} df \right]^\frac{1}{\xi_t} $$

(25)

where $Y_t(f)$ is the output by retailer $f$. 
The local producer price index is given by

\[ P_t = \left[ \int_0^1 P_{H,t}(f)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}} \]  \hspace{1cm} (26)

and from cost minimization by households, we have that the demand by the product of each producer is given by

\[ Y_t(f) = \left( \frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\epsilon} Y_t \]  \hspace{1cm} (27)

Final goods firms re-package the output from intermediate goods firms. One unit of intermediate output is used to produce one unit of the final product. The marginal cost, then, is given by \( P_{mt} \). Each period in time firms are able to adjust prices with probability \((1 - \gamma)\) (Calvo pricing). In between these periods, prices are indexed to the lagged inflation rate. Exactly as in Krenz [11], the final goods firms problem is to set \( P_{H,t} \) to maximize profits, which is given by

\[ \dot{P}_{H,t} = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} \theta^{k} \gamma \lambda_{t+k} \Pi_{H,t+k}^{(1-\epsilon)\gamma_k} Y_{t+k} \Pi_{mt+k} P_{H,t+k} \]  \hspace{1cm} (28)

where \( p_{H,t} = \frac{P_{H,t}}{P_{t}} \) is the relative price of home goods, \( \Pi_{H,t+k} = \frac{P_{H,t+k}}{P_{H,t}} \) is the inflation between periods \( t \) and \( t + k \), and \( 0 < \gamma_k < 1 \) is the degree of price indexation. From the law of large numbers, we obtain the following relation for the price level

\[ P_{H,t} = \left( \theta \Pi_{H,t-2}^{(1-\epsilon)\gamma_k} P_{H,t-1}^{1-\epsilon} + (1 - \theta) \dot{P}_{H,t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \]  \hspace{1cm} (29)
2.6. Market Clearing

Market clearing conditions are given by the following equations. The amount of capital acquired by intermediate firms must be the same as the amount of claims issued by these firms

\[ Q_t K_t = Q_t (S_{h,t} + S_{h,t}^*) \]  \hspace{1cm} (30)

\[ Q_{t}^* K_{t}^* = Q_{t}^* (S_{f,t} + S_{f,t}^*) \]  \hspace{1cm} (31)

And the goods markets clear as follows

\[ Y_t = C_{H,t} + C_{H,t}^* + \frac{P_t}{P_{H,t}} [I_t + f \left( \frac{I_{n,t} + I}{I_{n,t-1} + I} \right) (I_{n,t} + I)] \]  \hspace{1cm} (32)

\[ Y_t^* = C_{F,t} + C_{F,t}^* + \frac{P_t^*}{P_{F,t}} [I_t^* + f \left( \frac{I_{n,t}^* + I^*}{I_{n,t-1}^* + I^*} \right) (I_{n,t} + I^*)] \]  \hspace{1cm} (33)

2.7. Monetary Policy

2.7.1. Interest Rate Policy

Following Krenz [13], the interest rates are defined by a sole Central Bank following a Taylor rule and observing output gaps in both countries. As consumption baskets are similar in both countries and the law of one price holds, inflation for consumers is the same in both countries.

\[ i_t^{CB} = \left( \beta \Pi_t^\kappa_x \bar{y}_t^{0.5 \kappa_y} y_t^{0.5 \kappa_y} \right)^{1-\rho_i} (i_{t-1}^{CB})^{\rho_i} \bar{\epsilon}_{M,t} \]  \hspace{1cm} (34)
Finally, nominal and real interest rates are linked by the Fisher equation

\[ \hat{i}_t^{CB} = R_t E_t \Pi_{t+1} \]  

(35)

2.7.2. Unconventional Monetary Policy

In this part of the paper, we follow and extend the policy rule used in Gertler, Karadi [11] to model unconventional monetary policy implemented by the Central Bank. On that paper, in a given period in time, the Central Bank funds a fraction \( \psi_t \) of intermediated assets, the remaining \( 1 - \psi_t \) being funded by bankers.

\[ Q_t S_{gt} = \psi_t Q_t K_{t+1} \]  

(36)

The Central bank injects(withdraws) liquidity in(from) the system whenever this fraction \( \psi_t \) is increased (decreased). The Central Banks target is to use this intervention to stabilize credit spreads whenever there is a deviation from steady state levels. The Unconventional Monetary Policy mechanism here is that the Central Bank increases the pool of money available for firms thorough the funding of part of these loans. This lowers credit spreads, and helps stabilize the economy after a shock. \( \psi_t \) is given by

\[ \psi_t = \psi + \nu [(R_{kt+1} - R_{t+1}) - (R_k - R)] \]  

(37)

Where \( (R_k - R) \) is the steady state premium, \( \psi \) is the fraction of assets held in the steady state and \( \nu \) is a parameter that defines the Central Bank’s unconventional monetary policy response function.

In our study the approach is similar - the Central Bank funds, at a given moment in time, a fraction \( \psi_t \) of local assets and a fraction \( \psi_{t}^{*} \) of foreign assets. There are two possible types of intervention, though.
In the first one, Country-Specific intervention, the Central Bank observes only the local (foreign) spread to decide the fraction of local (foreign) assets that will be funded.

\[ \psi_t = \psi + \nu_H[(R_{ht+1} - R_{t+1}) - (R_k - R)] \]  

(37a)

\[ \psi_t^* = \psi + \nu_F[(R_{ht+1}^* - R_{t+1}^*) - (R_k^* - R^*)] \]  

(38)

In the second one, Union-wide intervention, the Central Bank observes the monetary union average spread to decide the fraction of local and foreign assets that will be funded.

\[ \psi_t = \psi_t^* = \psi + \nu[0.5((R_{ht+1} - R_{t+1}) + (R_{ht+1}^* - R_{t+1}^*)) - 0.5 * ((R_k^* - R^*) + (R_k - R))] \]  

(39)
3. Calibration

Most of the parameters in this model use standard values used in the literature and are the same used in Gertler, Karadi [11]. Although these parameters were calibrated for a pre-2007 world, using the same parameters in this study makes it possible to compare results from both models, focusing in the effect of extensions added to the original model. The values chosen for each parameter are listed in Table 1.

For the habit parameter $h$, the elasticity of marginal depreciation with respect to the utilization rate $\xi$, the inverse elasticity of net investment to the price of capital $\eta$, the relative utility weight of labor $\chi$, the Frisch elasticity of labor supply $\varphi^{-1}$, the rigidity parameter $\gamma$, the price indexing parameter $\gamma_{\pi}$, the coefficients in the monetary policy rule, $\kappa_{x}$ and $\kappa_{y}$ and the monetary policy smoothing parameter $\rho$ we use estimates from Primicieri et al., [15]. The autoregressive factor used for the capital quality shock is also the same used in Gertler,Karadi.

Conventional values are chosen for the capital share $\alpha$, the depreciation rate $\delta$, and the elasticity of substitution between goods $\varepsilon$.

The parameters used for the financial intermediaries, namely the fraction of assets that can be diverted $\lambda_{B}$, the factor that regulates the proportional transfer to new bankers $\omega$ and the bankers survival probability $\theta_{B}$ are chosen to achieve an interest spread of 100 basis points, a leverage of four, and an average live span for bankers of 10 years. The interest rate spread is compatible to BAA corporate bond spreads before 2007. The leverage ratio of four is compatible with aggregate data and corresponds to an average ratio considering investment and commercial banks, that had leverage levels from fifteen to thirty, and corporate and non-corporate business sectors, for which this ratio was closer to 2 in the aggregate.

Out of the remaining parameters, Home Bias in asset holdings $\mu_{a}$ and the elasticity between local and foreign assets $\iota_{A}$ were taken from Poutineau, Vermandel [14] and Krenz [13] and are calibrated for the Eurozone. These values were chosen to help understanding how home-bias in assets could affect an
economy similar to the most important monetary union in the world. The savings propensity \( \eta \) used in the endogenous discount factor was chosen differently from other publications to have an even smaller value\(^6\) - this was done because the way Welfare is defined later in this paper makes the Welfare level subject to the endogenous discount factor fluctuations. Fluctuations are larger when \( \eta \) is larger, and the consequence is that an otherwise arbitrary parameter \( \eta \) leads to different outcomes in welfare estimations. As the purpose of the endogenous discount factor is only to keep model convergence, then, we chose a value small enough to have little impact on welfare calculations, and at the same time keeping convergence in the model.

\(^6\) In Krenz [13] and others, \( \eta \) is set arbitrarily to a value of 0.01
4. Experiments

In this section we outline the experiments done with this model. As stated before, the objective is to understand the effects of policies in this two-country monetary union model. The first experiment was to verify the impact of allowing for higher leverage in the banking system of both countries. In a way, this experiment is similar but opposite in direction of creating policies that limit the amount of leverage in the banking system via regulation. Here, the method chosen to permit for higher leverage was to decrease the fraction of assets that can be diverted by the banks in the beginning of a given period, $\lambda_B$. The value of this parameter is marginally decreased and the impacts are verified in the other model variables.

This method was chosen because it achieves the purpose of getting to a new equilibrium with higher leverage in the banking system but keeps the original features of the model.

Results are presented on table 2.

The results are in line with the effects that would be expected in this situation - higher leverage leads to a higher amount of available loans, a higher amount of capital, and a higher level of output in the steady state. Also, note that in a model calibrated in such a way that there is a limitation in the amount of loans and, conversely, interest spreads are different from zero, a higher amount of loans allows for lower interest rate spreads.

Allowing for a higher leverage in the financial system, however, does not have only positive effects. We compared the effects in output of a capital quality shock in the financial system of the monetary union in both scenarios. The result was that, as expected, output was more negatively affected in the scenario in which the leverage was higher, which makes clear there is a trade-off in allowing for a higher leverage in the financial system and it’s pros and cons.

Our second experiment was to compare the effects of a Capital Quality Shock suffered by the home country in the members of the monetary union
considering two different scenarios. In the first scenario, the home bias in the
banking system is low ($\mu_A = 0.55$) - meaning banks in both countries have slight
preferences for financing local companies compared to foreign companies. In the
second scenario, we repeat the same exercise in a model where home-bias in
assets is the same as the level in the Eurozone ($\mu_A = 0.91$).

The first thing to be noted is how relevant this friction is compared to a
non-friction scenario - the home-bias level in assets in the Eurozone is so high that
it is not very far from one, level at which local firms would exclusively hold local
assets, regardless of how different expected returns might be among local and
foreign assets.

The results of these experiments are shown in figure 1. The first thing to
note is how consumption is affected in both exercises - in a low home-bias
scenario the consumption in both countries is negatively affected, although the
impact in the home country is stronger than in the foreign country. In a high home
bias scenario, however, the capital quality shock is initially felt only by the home
country, and effects on the foreign country are even slightly positive. The message
here is that contagion in the consumption side of the monetary union is very low
when home bias is high.

In both scenarios, goods prices are affected in a similar way - negatively.
In the face of falling prices and a negative union-wide output gap, the Central
Bank cuts interest rates in the first moment, but what follows is a much stronger
dynamics in the less affected foreign country (specially in the high home bias
scenario), which gets to output levels higher than before the shock. On aggregate,
the Central Bank soon starts to observe prices are increasing again and the output
gap is not that negative anymore. For this reason, interest rates are elevated again.
At this moment, the recovery is still not complete in the home country, but it is
already complete on aggregate, looking in an union-wide perspective.

This experiment shows how shocks that are idiosyncratic to a specific
country can have highly persistent effects in the members of a monetary union,
which can be negative in the country affected by the shock, and smaller or even
positive in the foreign country. In a closed economy, it would not be possible for
the foreign country to keep working persistently above the initial output level in a non-inflationary way - in a monetary union, however, the consumption basket is formed by local and foreign products and this is the channel through which the normalization process in both economies keeps going ahead after the central bank has finished it’s work.

As we know, from economic theory, a group of risks appear once these persistent effects hold true, such as hysteresis (workers unemployed for a long time might lose the capacity to come back working again) or even the migration of workers from a higher unemployment region to a lower unemployment one in search for jobs, as commented in Blanchard, Katz [2]. Both situations could transform highly persistent effects into permanent effects.

4.1. Welfare

In this section a welfare measure is defined in order to compare different characteristics of central bank intervention via unconventional monetary policy. We follow Faia, Monacelli [7] and Uribe, Smith-Grohe [17] and define Welfare associated with an hypothetical policy regime \( r \) as the present value of household’s expected lifetime utility

\[
W_t^r = E_0 \sum_{t=0}^{\infty} \Theta_t^r U(c_t^r, h_t^r) \tag{40}
\]

where \( c_t^r \) and \( h_t^r \) denote contingent plans for consumption and labor under regime \( r \). Conversely, suppose welfare associated with regime \( a \) is defined as

\[
W_t^a = E_0 \sum_{t=0}^{\infty} \Theta_t^a U(c_t^a, h_t^a) \tag{41}
\]

In order to compare both regimes, we define \( \lambda \) as the welfare cost of adopting regime \( a \) instead of regime \( r \), and define it as the fraction of consumption
households would be willing to give up to be as well off in regime $a$ as under regime $r$.

$$W_t^a = E_0 \sum_{t=0}^{\infty} \Theta_B^t U((1 - \lambda_c)c_t^r, h_t^r)$$ (42)

In this study, we estimate the conditional welfare taking the non-stochastic steady state as a starting point. This assures the economy will start from the same starting point under all policy regimes. Welfare is estimated using Dynare, taking a second order approximation of the whole model around the steady state.

Results are shown on table 3. The first thing to note is how Low Home-Bias is Welfare-improving compared to a High Home-Bias scenario, not only for the home country, but also for the foreign country. When unconventional monetary policy is set taking union-wide variables in consideration and the ZLB is absent, the effects in welfare are remarkably small. This happens because conventional and unconventional monetary policy are affecting union-wide variables in a similar way, and for this reason these policies have a substitutive relationship - when the central bank acts via UMP, the interest policy can be considerably less aggressive to achieve it’s targets. On average, welfare improves with the size of the intervention parameter $\nu_H = \nu_F$, however effects are nearly negligible in most variables, as can be seen in figure 3.

When country-specific UMP is implemented, however, the results are fairly different. In a low Home-Bias situation, Country-Specific UMP is most efficient when executed only in the home country - in other words, welfare increases on $\nu_H$, but decreases on $\nu_F$. In the case of intervention on the home country and no intervention in the foreign country, welfare results are very similar to the results of Union-Wide UMP in the home Country, but the negative effects in the foreign country are considerably smaller.

In a High Home-Bias setup, the results show there are strong positive impacts in both countries - Welfare increases on both $\nu_H$ and $\nu_F$. Country-specific intervention in a high home-bias situation is so welfare improving it takes welfare to levels comparable to levels present in a low Home-Bias scenario, if the Central
Bank UMP is aggressive enough ($\nu > 100$). The impact in the models variables is much more significant, as can be seen in figure 2.
5. Conclusion

Monetary Unions are historically seen as an alternative with Pros and Cons. The loss of the national currency as a shock absorber can help idiosyncratic shocks cause negative effects over member countries when country-specific policies are absent.

This study lays-out a two-country monetary union model, that embeds a banking system. We show there is a trade-off between higher output and the robustness of the banking system to capital quality shocks depending on financial leverage levels, and that consequently regulation can help preventing leverage to get to excessive high levels.

We also show that the existence of frictions like the home-bias in the banking system can help explain the persistence of shocks in member countries, even if equilibrium is reached on aggregate after the capital quality shocks.

Finally, we show that different unconventional monetary policy strategies can be more or less welfare-improving whenever these frictions hold, and that in high home-bias monetary unions like the Eurozone a country-specific unconventional monetary policy could be more welfare improving than union-wide policies.

In further studies, a model including banking systems where financial entities can hold government debt and in which Central Banks can use public debt as a monetary policy instrument (please see Gertler, Karadi(2013)[12]) would be an interesting extension.

Also, for the future, the study of other kinds of frictions and their impact in the efficiency of monetary policy is recommendable, for the way such studies make clear how the elimination/reduction of such frictions throughout time may be welfare improving for households in Monetary Unions.

In the case of home-bias in banking loans, many welfare increasing solutions could be considered. Higher international banking activity and a
consequently lower home-bias could increase the effectiveness of monetary policy for member countries. For this reason, policies designed to stimulate international banking activity and tools created to increase risk-sharing among international banks could be a promising step forward in the process of deeper integration among the members of monetary unions.
6. Bibliographic references


## Tables

### Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.815</td>
<td>Habit parameter</td>
</tr>
<tr>
<td>$\chi$</td>
<td>3.409</td>
<td>Relative Utility Weight of Labor</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.276</td>
<td>Inverse Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.001</td>
<td>Elasticity of the discount factor</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>0.990</td>
<td>Saving propensity parameter</td>
</tr>
<tr>
<td>$l$</td>
<td>4.000</td>
<td>Elasticity of Substitution between local and foreign goods</td>
</tr>
</tbody>
</table>

### Financial Intermediaries

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.381</td>
<td>Fraction of Capital that can be diverted</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.002</td>
<td>Proportional transfer to new bankers</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.972</td>
<td>Survival rate of the bankers</td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>0.910</td>
<td>Bankers Home bias between local and foreign loans</td>
</tr>
</tbody>
</table>

### Intermediate Good Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.330</td>
<td>Effective Capital Share</td>
</tr>
<tr>
<td>$U$</td>
<td>1.000</td>
<td>Steady State capital utilization rate</td>
</tr>
<tr>
<td>$\delta(U)$</td>
<td>0.025</td>
<td>Steady State depreciation rate</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>7.200</td>
<td>Elasticity of marginal depreciation with respect to utilization rate</td>
</tr>
</tbody>
</table>

### Capital Producing Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_i$</td>
<td>1.728</td>
<td>Inverse Elasticity of net investment to the price of capital</td>
</tr>
</tbody>
</table>

### Retail Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>4.167</td>
<td>Elasticity of Substitution</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.779</td>
<td>Probability of keeping prices fixed</td>
</tr>
<tr>
<td>$\theta_\pi$</td>
<td>0.241</td>
<td>Measure of price indexation</td>
</tr>
</tbody>
</table>

### Government

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_\pi$</td>
<td>2.043</td>
<td>Inflation Coefficient of the Taylor Rule</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>-0.500</td>
<td>Output Gap coefficient of the Taylor Rule</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.800</td>
<td>Smoothing parameter of the Taylor Rule</td>
</tr>
<tr>
<td>$\bar{G}$</td>
<td>0.200</td>
<td>Steady State proportion of government expenditures</td>
</tr>
</tbody>
</table>

### Table 2: Higher Leverage Experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_B$</td>
<td>0.3814</td>
<td>Fraction of Capital that can be diverted</td>
</tr>
<tr>
<td>$\epsilon^o$</td>
<td>4.145</td>
<td>Financial Firms Leverage</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.848565</td>
<td>Output in the Steady State</td>
</tr>
<tr>
<td>$K$</td>
<td>5.653473</td>
<td>Capital in the Steady State</td>
</tr>
<tr>
<td>$R_K$</td>
<td>0.012563</td>
<td>Return on Financial Intermediaries Loans</td>
</tr>
<tr>
<td>$R_K - R$</td>
<td>0.002546</td>
<td>Financial Intermediaries Spread</td>
</tr>
<tr>
<td>$Y - Y_{SS}$</td>
<td>-6.114</td>
<td>Percentage Output impact after 10 periods</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.800</td>
<td>Smoothing parameter of the Taylor Rule</td>
</tr>
<tr>
<td>$\bar{G}$</td>
<td>0.200</td>
<td>Steady State proportion of government expenditures</td>
</tr>
</tbody>
</table>
Table 3: Welfare Experiment

<table>
<thead>
<tr>
<th>Unconventional Monetary Policy</th>
<th>Welfare Home</th>
<th>Welfare Foreign</th>
<th>Welfare Average</th>
<th>Welfare Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_A$</td>
<td>$\nu_H$</td>
<td>$\nu_F$</td>
<td>Home</td>
<td>Foreign</td>
</tr>
<tr>
<td>Union-Wide Intervention, Low Home-Bias</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Intervention</td>
<td>0.550</td>
<td>0</td>
<td>0</td>
<td>-306.6539</td>
</tr>
<tr>
<td>Weak Intervention</td>
<td>0.550</td>
<td>10</td>
<td>10</td>
<td>-305.0281</td>
</tr>
<tr>
<td>Strong Intervention</td>
<td>0.550</td>
<td>100</td>
<td>100</td>
<td>-304.0760</td>
</tr>
<tr>
<td>Union-Wide Intervention, High Home-Bias</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Intervention</td>
<td>0.910</td>
<td>0</td>
<td>0</td>
<td>-319.5309</td>
</tr>
<tr>
<td>Weak Intervention</td>
<td>0.910</td>
<td>10</td>
<td>10</td>
<td>-319.7411</td>
</tr>
<tr>
<td>Strong Intervention</td>
<td>0.910</td>
<td>100</td>
<td>100</td>
<td>-319.5364</td>
</tr>
<tr>
<td>Country-Specific Intervention, Low Home-Bias</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Intervention</td>
<td>0.550</td>
<td>0</td>
<td>0</td>
<td>-306.6539</td>
</tr>
<tr>
<td>Weak Intervention</td>
<td>0.550</td>
<td>10</td>
<td>0</td>
<td>-305.1036</td>
</tr>
<tr>
<td>Strong Intervention</td>
<td>0.550</td>
<td>100</td>
<td>0</td>
<td>-304.0442</td>
</tr>
<tr>
<td>Country-Specific Intervention, High Home-Bias</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Intervention</td>
<td>0.910</td>
<td>0</td>
<td>0</td>
<td>-319.5309</td>
</tr>
<tr>
<td>Weak Intervention</td>
<td>0.910</td>
<td>10</td>
<td>10</td>
<td>-311.3109</td>
</tr>
<tr>
<td>Strong Intervention</td>
<td>0.910</td>
<td>100</td>
<td>100</td>
<td>-305.9601</td>
</tr>
</tbody>
</table>

Notes: Welfare Home (Foreign) is the welfare calculated for the Home (Foreign) Country. Welfare average is calculated as the numerical average between the Welfare in the Home Country and the Welfare in the Foreign Country.
Figures

Figure 1: Low Home-Bias vs. High Home-Bias

Notes: High Home-Bias is represented by the solid line. Low Home-Bias is represented by the dashed line
Figure 2: Country-Specific intervention

Notes: No intervention is represented by the solid line, $\nu = 10$ is represented by the dashed line, and $\nu = 100$ is represented by the dotted line.
Figure 3: Union-Wide intervention

Notes: No intervention is represented by the solid line, \( \nu = 10 \) is represented by the dashed line, and \( \nu = 100 \) is represented by the dotted line.
Appendix

Model’s Additional Equations

Households CPOs

\[ \lambda_t = (C_t - hC_{t-1})^{-1} - \beta(C_{A,t})h(E_tC_{t+1} - hC_t)^{-1} \]  
(43)

where \( \lambda_t \) is the marginal utility of consumption,

\[ w_t = \chi \frac{\dot{q}}{\dot{\lambda}_t} \]  
(44)

and

\[ 1 = \beta(C_{A,t})E_t(\Lambda_{t,t+1}R_t) \]  
(45)

Banks

Following Gertler, Karadi[11], \( V_{i,t} \) (from equation 9) can be written as

\[ V_{i,t} = v_tA_{iH,t} + \eta_tN_{iH,t} \]  
(46)

with

\[ v_t = E_t\{(1 - \theta_B)\Theta_tA_{i,t+1}(R^i t_{i+1} - R_{i+1}) + \Theta_t\Lambda_{i,t+1}\theta_Bx_{i,t+1}v_{t+1}\} \]  
(47)

\[ \eta_t = E_t\{(1 - \theta_B) + \Theta_t\Lambda_{i,t+1}\theta_Bz_{i,t+1}\eta_{t+1}\} \]  
(48)

where \( x_{i,t+1} = A_{ih,t+1}/A_{ih,t} \) is the gross growth rate in assets between \( t \) and \( t+1 \), and \( z_{i,t+1} = N_{ih,t+1}/N_{ih,t} \) is the gross growth rate of net worth.

The incentive constraint, therefore, can be expressed as

\[ v_tA_{iH,t} + \eta_tN_{iH,t} \geq \lambda_BA_{i,t} \]  
(49)

And conversely, if the constraint binds, we get to equation (12)

\[ A_t = \frac{\eta_t}{\lambda_B - v_t}N_t = \varphi_{i,t}N \]