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**The Fiscal Theory of the Price Level with
Nominal Revenues and Expenditures**

Dissertação de Mestrado

Dissertation presented to the Programa de Pós-graduação em
Economia of PUC-Rio in partial fulfillment of the requirements
for the degree of Mestre em Economia.

Advisor : Prof. Tiago Couto Berriel
Co-Advisor: Prof. Carlos Viana de Carvalho

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Abstract

Kiehl Sonnervig, Marcos; Berriel, Tiago Couto (Advisor); Carvalho, Carlos Viana de (Co-Advisor). **The Fiscal Theory of the Price Level with Nominal Revenues and Expenditures**. Rio de Janeiro, 2017. 52p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

The usual assumption that fiscal policy is set in real terms is neither realistic nor innocuous. In this article, we propose a model that accounts for the existence of nominal revenues and expenditures. This creates an unexplored channel through which monetary and fiscal policies interact. We show that, in this environment, the price level can be fiscally determinate, even when all government debt is real. Also, the effects of monetary and fiscal policies are sensitive to the degree of indexation in the government budget. Using Bayesian techniques, we estimate the model for the US economy and find that the revaluation of these nominal components plays an important role as a source of fiscal financing, under a fiscally dominant regime.

Keywords

Fiscal Theory of the Price Level; Government Debt; Government Revenue; Government Expenditure; Price Level; Inflation;

Resumo

Kiehl Sonnervig, Marcos; Berriel, Tiago Couto; Carvalho, Carlos Viana de. **Receitas e Despesas nominais na Teoria Fiscal do Nível de Preços**. Rio de Janeiro, 2017. 52p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

A hipótese usual de que a política fiscal é definida em termos reais não é nem realista, nem inócua. Neste artigo, propomos um modelo que leva em consideração a existência de receitas e despesas nominais. Essa característica cria um canal inexplorado pelo qual as políticas monetária e fiscal interagem. Nós mostramos que, neste ambiente, o nível de preços pode ser fiscalmente determinado, mesmo quando toda a dívida do governo é real. Além disso, os efeitos das políticas monetária e fiscal são sensíveis ao grau de indexação no orçamento do governo. Usando técnicas de estimação Bayesiana, nós estimamos o modelo para a economia norte-americana e encontramos que a corrosão do valor real das despesas desempenha um papel importante no financiamento dos déficits do governo, sob o regime de dominância fiscal.

Palavras-chave

Teoria Fiscal do Nível de Preços; Dívida Pública; Receita Pública; Despesa Pública; Nível de Preços; Inflação;

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1 Introduction

In monetary macroeconomic models, when the fiscal policy is explicitly modeled, it is often set in real terms - see (1), (2), (3), (4), (5), (6), for some examples. An implicit assumption in these models is the ability of the government to choose the real value of its flow variables (i.e. transfers, revenues, purchases). This assumption, however, is not realistic and, in some circumstances, is not innocuous.

An analysis of the effects of a nominal fiscal policy is developed in (7), who examines the impact of inflation on the real value of the tax revenue. The author asserts that if there are lags between a *taxable event*¹ and the moment the government receives that payment, then changes in the price level would affect the revenue's real value (the "Tanzi Effect"). Similarly, (8, 9) analyse how inflation affects the real value of the government expenditure. Bacha calls attention for the fact that, if expenditures are set in nominal terms (and it is not perfectly contingent to the price level), then inflation erodes its real value (the "Bacha Effect"). This mechanism is used to explain how the Brazilian economy could have, in 1992, an inflation rate of over 1000% and a Federal deficit as low as 1.7% of the GDP.

In a more recent article, (10) analyzes, in a partial equilibrium model, the inflationary effects of a currency crises when the government chooses different ways to finance its deficits (other than a fiscal reform). As "alternative" sources of financing, the authors consider: (i) seigniorage, (ii) reduction, through inflation, of the real value of non-indexed debt and, finally, (iii) the erosion of the real value of non-indexed expenditures. Regarding this third source of financing, the authors assume that a constant fraction of the government expenditure is set in nominal terms and, therefore, have its real value permanently eroded by inflation.

In a different context, but related to the theme, (11) consider a case where the government sets an *ad hoc* rule for the surpluses, which reacts positively to the current inflation. The author shows that if surpluses raises linearly with inflation, then rational expectations equilibria in which inflation explodes are

¹An event that creates a legal financial liability on the part of a taxpayer towards the state

ruled out.

In this paper we bring these ideas to the literature on the Fiscal Theory of the Price Level (FTPL). The theory, developed in (1), (2), (4), (5) and (6), analyses the interaction between fiscal and monetary policies and recognize that the fiscal policy can be the sole determinant of the price level ((11)). Adding the possibility of nominal fiscal policy opens an unexplored channel through which fiscal and monetary policies interact. Furthermore, when inflation directly affects the real value of surpluses, some of the theory predictions can change both quantitatively and qualitatively.

Regarding our theoretical contribution, first, we show that the price level can be fiscally determinate in an economy where all government debt is real, provided that a positive fraction of surpluses are not inflation-contingent. Second, in this model, under a fiscally dominant regime, monetary policy tightening can generate a negative response of inflation in the short run (a result also existent in models with long-term nominal debt²).

In order to assess the role of each source of fiscal financing and to get realistic dynamics, we develop and estimate for the U.S economy a dynamic stochastic general equilibrium model where, in addition to real debt and surpluses, the government also have their nominal counterparts. We find that the role of nominal debt as a source of fiscal financing is virtually null and that, under our mean estimates, is also less relevant than the nominal components of government purchases and transfers.

The article is organized as follows: In section 2 we highlight and make a clear distinction between the different sources of fiscal financing in the FTPL; in section 3 we develop and analyse the properties of a simple model with non-inflation-contingent surpluses; In section 4 we present the complete model; in section 5 we describe our estimation methodology; in section 6 we describe and analyse the results; finally, in section 7 we briefly summarize our results.

²See (12) and (13)

2

FTLP without Nominal Debt

The fiscal theory is usually seen as theory that requires the existence of nominal government debt, used as a “cushion” for fiscal imbalances. As an example of this “misperception”, (14)¹ state that:

A key difference between the fiscal theory and unpleasant arithmetic [(15)] is that the former operates only in an economy with nominal government debt, whereas the latter is typically discussed under the assumption of real debt.

In fact, in the fiscal theory, the requirement that the real value of government debt matches the present value of future primary surpluses can be met by other ways rather than changes in the real value of debt. We will call these other sources of fiscal financing as “channels” through which the theory operates.

Apart from the nominal debt “cushion”, a second source of fiscal financing, whose analysis is the main objective of this article, is the erosion of the real value of government expenditure - which might occur when it is set in nominal terms. The third channel is the variation of the real discount rate, which requires the existence of some form of nominal rigidity².

Being said that, now we consider the case where the government does not have nominal debt. In this environment, the price level can be fiscally determinate, conditional on the existence of non-indexed nominal surpluses and/or nominal rigidities. In Section 3, we analyze a simple model where all government debt is real, prices are flexible and surpluses are not perfectly indexed (in the sense that inflation affects its real value). We prove the existence of a fiscal equilibrium and analyze its properties.

Another interesting case occurs when both debt and surpluses are perfectly indexed, but there is some sort of nominal rigidity (on prices and/or nominal wages). In this case, there are no revaluation effects and the price

¹(14) page 14

²With long-term debt, jumps in the nominal interest rate also affects the real value government debt.

level may still be fiscally determinate (as long as fiscal policy is active and monetary policy is passive). (16) briefly describes this situation³.

An interesting property of this equilibrium is that the revaluation effects, stressed by the fiscal theory, do not occur and, nevertheless, its main predictions do not change. For instance, in the event of a negative fiscal shock, in this equilibrium, agents raise consumption today and accept a lower future return on government real bonds. This endogenous response reduces the government cost of financing, reestablishing the fiscal solvency, without any kind of revaluation mechanism.

The existence of this equilibrium challenges the interpretation of the fiscal theory as one in which the price level, as an stock price, has its value determined by a valuation equation, which relates the expected discounted future stream of dividends (in the case of the government debt, surpluses) and the quantity of stocks (or nominal bonds) - see (6). The price level is still determined by the "intertemporal fiscal equilibrium condition", but the analogy with the stock markets loses its appeal.

In section 4 we develop a model where these three sources of fiscal financing are available and in section 6.1.2, using the estimated model, we return to this discussion analyzing, the role of each one of them.

³In section B (Appendix) we display the dynamic responses of some variables of interest to a monetary and a fiscal shock, in a textbook New-Keynesian model, with indexed debt and surpluses, under a fiscal Regime.

3 Simplified Model

In this section, we present a model of a frictionless endowment economy, where 100% of government bonds are real (or perfectly indexed to inflation) and the fiscal policy has some non-indexed components. A simple way of modeling this hybrid fiscal policy rule (with indexed and non-indexed components) is:

$$s_t = \left(\tau_t + \frac{T_t}{P_t} \right) - \left(z_t + \frac{Z_t}{P_t} \right)$$

Where s_t is the real value of surplus; τ_t and z_t are the indexed components of revenue and expenditure; T_t and Z_t are the non-indexed components. As in (10), the nominal components are permanently eroded by inflation. In the more general case (presented in the next section), the fiscal policy may be indexed to past inflation rates and, therefore, have their values affected by inflation, only temporarily.

Under this structure, inflation directly affects the real value of surpluses. The sign of this effect, however, depends on the difference between T_t and Z_t . In the particular case where $T_t = Z_t$, this effect is null. For simplicity, consider the case where z_t , T_t and Z_t are constant:

$$s_t = \left(\tau + \frac{T}{P_t} \right) - \left(z + \frac{Z}{P_t} \right) \quad (3-1)$$

In this simplified model, the Government has only real debt. Its flow budget constraint is:

$$b_t = R_{t-1}^r b_{t-1} - s_t \quad (3-2)$$

Where R_{t-1}^r is the bond's real return, b_t is value of debt and s_t is the real value of the surplus, in period t .

The fiscal authority sets a rule for the indexed component of the revenue and other three constants (The nominal component of revenue and transfers, plus the real component of transfers):

Where, τ and b are the steady-state levels of τ_t and b_t . ϵ_t^t represents an exogenous fiscal policy shock.

The representative agent receives an endowment y every period and solve:

$$\max_{\{c_t, b_t\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \beta^{t+j} \frac{C_{t+j}^{(1-\sigma)}}{1-\sigma} \quad (3-3)$$

$$\text{s.t. } C_t + b_t + s_t \leq y + R_{t-1}^r b_{t-1} \quad (3-4)$$

The monetary authority controls the interest rate of a nominal bonds, which satisfies a non-arbitrage relation with the real bonds, and follows a standard Taylor rule:

$$R_t = \frac{1}{\beta} \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \epsilon_t^M \quad (3-5)$$

The equilibrium in this model is defined as an initial debt stock b_{-1} , a sequence of prices $\{P_t, R_t^r, R_t\}$ and allocations $\{C_t, b_t, \tau_t, z, T, Z\}$ satisfying the individual optimality condition and the market clearing conditions. The proposition that follows establishes the conditions for the existence of a unique bounded equilibrium.

Proposition: *If the nominal component of revenues has a different value than the nominal component of expenditure, ($Z \neq T$) then, as in (1), there are two disjoint regions of the parametric space with a locally unique bounded equilibrium:*

Region 1 (Monetary Regime): $|\phi_\pi| > 1$ and $\left| \frac{1}{\beta} - \phi_b \right| < 1$

Region 2 (Fiscal Regime): $|\phi_\pi| < 1$ and $\left| \frac{1}{\beta} - \phi_b \right| > 1$

Proof in the Appendix (Section A).

The first thing worth noting here is that, although the government does not have nominal bonds, there is the possibility of a fiscally dominant regime (Regime F). Therefore, in this case, the Fiscal Theory of the Price Level (FTPL) operates through the "revaluation" of the government's net transfers. This point is informally made in (17).

There are interesting theoretical features of the equilibrium in regime F. As stressed in the literature of the FTPL, under this regime, the price level is determined by the government's intertemporal budget constraint (or "government debt valuation equation", (6)). In this case, a negative fiscal shock ($\Delta s < 0$) results in a higher inflation, which revalues real value of the government debt. The same happens here, if the non-indexed expenditure is larger than the non-indexed revenue ($T < Z$). In this case, a negative fiscal shock is "financed" by the erosion of the real value of government expenditure. This fiscal effect of inflation also resembles, in a stylized fashion, the "Bacha effect".

To understand this result, it is worth considering the government debt valuation equation:

$$b_{t-1} = \sum_{j=0}^{\infty} \beta^j E_t \left\{ \tilde{\tau}_{t+j} - \frac{Z_{t+j} - T_{t+j}}{P_{t+j}} \right\}$$

Where $\tilde{\tau}_t \equiv \tau_t - z_t$ and $V_t \equiv Z_t - T_t$. Rearranging the terms, the equation can be rewritten as:

$$b_{t-1} + \frac{\sum_{j=0}^{\infty} Q_t(t+j) E_t[V_{t+j}]}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \tilde{\tau}_{t+j}$$

For simplicity, consider the case of a pegged nominal interest rate $\phi_\pi = 0$. Also, since $E_t[V_{t+j}] = V$, in this example, this equation can be even more simplified:

$$b_{t-1} + \frac{V}{1-\beta} \frac{1}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \tilde{\tau}_{t+j}$$

If ($T < Z$), then $V > 0$ and, therefore, the government net transfers works exactly as a nominal debt "cushion".

In this model, since the only source of fiscal financing is the "erosion" of the real value of the fiscal variables, an awkward prediction occurs when the revenue of the government is less indexed than the expenditure ($T > Z$). In this case, as inflation reduces the real value of surpluses, a negative fiscal shock generates deflation. If however, the government has a sufficiently large stock of nominal debt, then the negative fiscal shock generates a positive response of inflation (as predicted by standard models), which, through the "asymmetry" ($T > Z$), reinforces the fiscal shock, generating even higher inflation rates¹.

3.1

Stepping on a rake

The effects of a monetary policy shock also changes under this circumstances. A standard result in the FTPL literature is the rise of inflation following a monetary tightening. In contrast with that prediction, here, if ($T < Z$), the monetary policy shock has a negative impact over

¹If revenue is less indexed than expenditure ($T > Z$), for sufficiently large values of nominal debt, the inflation response for a negative fiscal shock ($\Delta s < 0$) is positive. However, starting from the case where 100% of debt is Nominal, there is a discontinuity on the inflation response as debt gets more indexed: If a small fraction of debt gets indexed, the response is amplified, but if a large fraction gets indexed, than the response inverses and the shocks generates deflation. In this model, the discontinuity occurs when $B_{t-1} = (Z - T)/(1 - \beta)$ - the nominal value of debt equals the present value of the difference between the nominal components of transfers and revenues

inflation in the short run. In the long-run, however, prices are higher than it was before the shock. Figure 3.1 displays the dynamic responses of some variables of interest to a monetary policy shock that takes the form of a positive realization of $\epsilon_t^M = 1^2$.

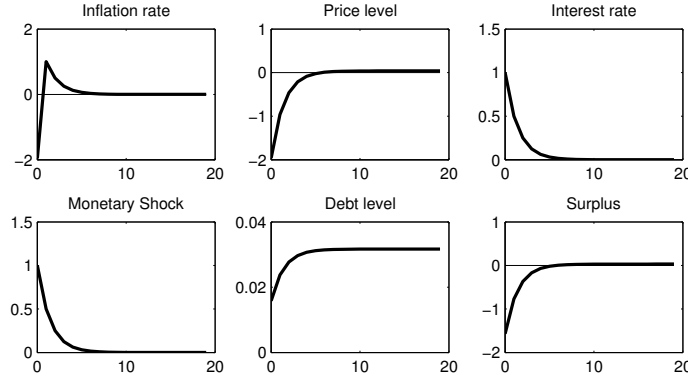


Figure 3.1: Monetary Policy Shock - Frictionless endowment economy with real debt and non-indexed surpluses ($T < Z$)

This result is also easily understood through the debt valuation equation:

$$b_{t-1} + \frac{\sum_{j=0}^{\infty} Q_t(t+j) E_t[V_{t+j}]}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \tilde{r}_{t+j}$$

Where $Q_t(t+j)$ is equal to the inverse of the nominal bond return from period t to $t+j$. Now the result can be clearly described: the interest rate rise have a negative effect on bond prices ($Q_t(t+j)$) reducing the present value of the nominal government expenditures and, hence, generating a negative wealth effect on private agents. The effect on price level can be rationalized as following: the negative wealth effect reduces demand for goods, which, in a flex-price economy, translates, immediately, in a reduction of the price level. Also, the price response is proportional to the change in the present value of nominal expenditures ($\sum_{j=0}^{\infty} Q_t(t+j) E_t[V_{t+j}]$). Finally, since the real value of net transfers is permanently eroded, the debt level changes to a higher steady-state level.

The same result occurs in an economy with long-term nominal debt and indexed surpluses. (12) and (13) analyse this case. In (12) words:

After initially dropping, the inflation rate rises back above its steady state level by as much as it initially fell, and the rise is more sustained than the initial drop. This is the “stepping on a rake”

²The log-linearized equations and the parameters calibration are described in the Appendix (Sections A and A.2)

phenomenon: Apparently effective measures to reduce inflation come back, after a delay, to produce precisely the opposite of the desired effect.

4 Model

In this section, we propose a model where the government has a more complete debt structure and the indexation rule takes different forms. We incorporate this fiscal side in a medium-scale DSGE model based on (18), which, in turn, is based on (19), (20) and (21).

4.1 Government Debt and Budget Constraint

The government issues one-period indexed bonds and nominal bonds of all maturities. For simplicity, we assume that the maturity structure declines at a constant rate ρ , as in (5). Hence, the government flow budget constraint is:

$$P_t b_{t-1}^r + B_{t-1}^m \left[1 - \sum_{j=1}^{\infty} Q_t(t+j) \rho^j \right] = P_t s_t + P_t Q_t^r b_t^r + B_t^m \sum_{j=1}^{\infty} Q_t(t+j) \rho^{j-1} \quad (4-1)$$

Where $\rho^{j-1} B_t^m$ and $Q_t(t+j)$ are, respectively, the quantity and the price of a nominal bond that matures in $t+j$. b_t^r and Q_t^r are, respectively, the quantity and price of the indexed bond.

I also assume that government supply of the real bonds is a constant fraction of the total supply of bonds ($b_t^r + B_t^m/P_t$).

$$b_t^r = \alpha \frac{B_t^m}{P_t} \quad (4-2)$$

The real value of the government surplus has three components:

$$s_t = T_t - g_t - z_t$$

$$T_t, g_t \geq 0, \text{ and } z_t \in \mathbb{R}$$

Where T_t is its revenues from distortionary taxes, g_t is the real value of its purchases and z_t is the real value of its net transfers.

4.2

Tax Revenue

The tax revenue is:

$$T_t = \tau_t(R_t^k K_t + W_t L_t) \quad (4-3)$$

K_t and R_t^k represents, respectively, aggregate effective capital and its return. W_t and L_t represents, respectively, the real wage and the labor supply. Note that the same tax rate τ_t is applied to both the labor income and wage income. This specification is based on (21).

The tax rate for capital and labor respond to the lagged debt-to-output ratio ($\hat{d}_{t-1}^m \equiv \frac{B_{t-1}^m/P_{t-1}}{Y_{t-1}}$) according to the following rule:

$$\tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau) \gamma_\tau \hat{d}_{t-1}^m + \epsilon_t^\tau, \epsilon_t^\tau \sim N(0, \sigma_\tau^2) \quad (4-4)$$

4.3

Government Purchases and Transfers

A general structure for the indexation of fiscal instruments can be represented as an infinite sum of terms involving past values of the price level:

$$\begin{aligned} Z_t &= z_{1,t} P_t + z_{2,t} P_{t-1} + z_{3,t} P_{t-2} + \dots \\ &= \sum_{j=1}^{j=\infty} z_{j,t} P_{t-j+1} \end{aligned}$$

The usual indexation structure of most monetary models is the particular form whereby $z_{j,t} = 0$ for all $j > 1$. In this case, $Z_t = z_{1,t} P_t$ (is perfectly indexed to the price level). This general form for modeling the indexation structure is completely characterized by the terms $z_{j,t}$, which are functions potentially involving any other real variable from the system.

In order to estimate this structure, we impose two sets of restrictions, defining, therefore, two different specifications. The first one sets $z_{j,t} = 0$ for all j , with the exception of $j = 1$ and $j = t$. The resulting model is very similar to the case analyzed in the last section. The second one imposes an geometrically decaying structure. They are described in detail in the following sections.

4.3.1

First Specification

As in the simple model described in the last section, we assume that the government net transfers have two different forms of indexation rules. The

first one captures the standard type of indexation of the literature - the case whereby the fiscal rule is inflation-contingent. The second one captures the most dramatic case whereby there are no indexation at all. Finally, there is an exogenous component that represents unexpected transfers shocks.

$$z_t = \left(z^+ + \frac{Z^+}{P_t} \right) - \left(z^- + \frac{Z^-}{P_t} \right) + \eta_t^z$$

$$\eta_t^z = \rho_z \eta_{t-1}^z + \epsilon_t^z, \quad \epsilon_t^z \sim N(0, \sigma_z^2)$$

The term $\left(z^- + \frac{Z^-}{P_t} \right)$ can be interpreted as lump-sum taxes. Defining $\equiv (z^+ - z^-)$ and $V_{2,t} \equiv (Z_t^+ - Z_t^-)$, the real value of transfers can be written as:

$$z_t = \left(v_{1,t} + \frac{V_{2,t}}{P_t} \right) + \eta_t^z$$

As in last section's model, $V_{2,t}$ can be either negative or positive. This implies that inflation can have a positive or a negative impact on the real value of transfers and, therefore, on surpluses. For simplicity, we assume that $V_{2,t}$ and $v_{1,t}$ are constants. The log-linearized equation for the net transfers is¹:

$$\hat{z}_t = - \underbrace{\left(\frac{V_2/P}{z} \right)}_{\phi_z} \hat{p}_t + \eta_t^z \quad (4-5)$$

The distribution of the parameter $\phi_z \in [-1, 1]$ is estimated and used to gauge, first, the sign of the effect of inflation on net transfers and, second, the fraction of the net transfers that are not indexed to the price level.

The government purchases have the same indexation structure:

$$G_t = G_{1,t} + G_2 + P_t \eta_t^g$$

$$G_{1,t} = g_{1,t} P_t$$

$$\eta_t^g = \rho_g \eta_{t-1}^g + \epsilon_t^g, \quad \epsilon_t^g \sim N(0, \sigma_g^2)$$

Its real value is, therefore:

$$g_t = g_{1,t} + \frac{G_2}{P_t} + \eta_t^g$$

I assume that the indexed component ($g_{1,t}$) respond to the lagged debt-

¹The log deviations of a variable X_t from its steady state X is defined as \hat{x} .

to-output ratio ($\hat{d}_{t-1}^m \equiv \frac{B_{t-1}^m/P_{t-1}}{Y_{t-1}}$) according to the following rule:

$$g_{1,t} = - \left[1 - \left(\frac{G_2/P}{g} \right) \right]^{-1} \gamma_g \hat{d}_{t-1}^m$$

The term $\left[1 - \left(\frac{G_2/P}{g} \right) \right]^{-1}$ multiplying the reaction parameter γ_g is included to make sure the reaction function does not depend on the indexation structure.

The log-linearized equation for the government purchases is:

$$\hat{g}_t = -\gamma_g \hat{d}_{t-1}^m - \underbrace{\left(\frac{G_2/P}{z} \right)}_{\phi_g} \hat{p}_t + \eta_t^g \quad (4-6)$$

4.3.2

(Second Specification) Geometric Structure

In this second specification we allow the fiscal variables to be indexed to past inflation values and impose a geometric decaying weight to the indexation structure:

$$Z_t = z(1 - \theta_z) \left[P_t + \theta_z P_{t-1} + \theta_z^2 P_{t-2} + \dots \right] + P_t \eta_t^z$$

$$\eta_t^z = \rho_z \eta_{t-1}^z + \epsilon_t^z, \quad \epsilon_t^z \sim N(0, \sigma_z^2)$$

Here, z is equal to the steady-state level of government real net transfers and $\theta_z \in [0, 1]$ is the parameter that defines the indexation structure. Note that the two polar cases of the first specification are also contemplated here: if $\theta_z = 0$, then $Z_t = zP_t + \eta_t^z P_t$ is perfectly indexed to inflation. Also, if $\theta_z = 1$, then $Z_t = P_{t-1}z + \eta_t^z$, which is exactly the case where there is no indexation at all². Moreover, under this specification, for all values of θ_z different from 1, inflation affects the real value of transfers only temporarily, as opposed to the first specification, where the effect is permanent.

As an example, Figure 4.1 left panel depicts how weights are distributed into the past price levels and the right panel represents the cumulative indexation. The blue bars represent the case whereby $\theta_z = 0.5$ and the red bars when $\theta_z = 0.75$. When $\theta_z = 0.5$, after one period, 75% of the total transfers

²The two specifications are equivalent if $\theta_z = 1$ and $\phi_z = 1$ (no indexation at all); and also if $\theta_z = 0$ and $\phi_z = 0$ (perfect indexation)

have already had their nominal values readjusted whereas when $\theta_z = 0.75$ it takes 4 periods for the same fraction of the transfers to be readjusted.

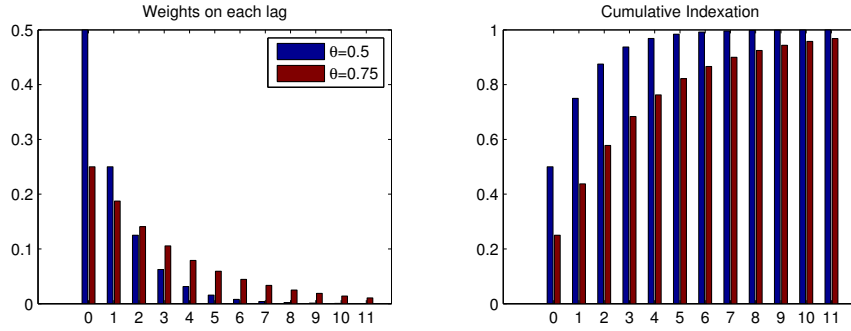


Figure 4.1: Geometric Indexation Structure

The government purchases have the same indexation structure, but it also respond to the debt level:

$$G_t = g(1 - \theta_g) \left[P_t + \theta_g P_{t-1} + \theta_g^2 P_{t-2} + \dots \right] + P_t (-\gamma_g d_{t-1}^m + \eta_t^g)$$

$$\eta_t^g = \rho_g \eta_{t-1}^g + \epsilon_t^g, \quad \epsilon_t^g \sim N(0, \sigma_g^2)$$

To simulate/estimate the model we write the geometric structure in a recursive way, using auxiliary variables v_g and v_z . The log-linearized equations for the real value of transfers and purchases are:

$$\hat{z}_t = v_{z,t} + \eta_t^z \quad (4-7)$$

$$v_{z,t} = \theta_z (v_{z,t-1} - \pi_t) \quad (4-8)$$

$$\hat{g}_t = v_{g,t} - \gamma_g \hat{d}_{t-1}^m + \eta_t^g \quad (4-9)$$

$$v_{g,t} = \theta_g (v_{g,t-1} - \pi_t) \quad (4-10)$$

4.4

Monetary Policy

Regarding the monetary policy, we assume a simple Taylor rule of the form:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) (\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t) + \epsilon_t^r, \quad \epsilon_t^r \sim N(0, \sigma^r) \quad (4-11)$$

The rest of the model is quite standard and it is based on (18). We describe the private sector of the model in the next subsection.

4.5

Households and Firms

Firms

The production in this economy is made by a final goods producer and an intermediate good producer. The perfectly competitive final goods producer uses a continuum of intermediate goods $y_t(i)$, according to the constant return-to-scale technology as in (22):

$$\left[\int_0^1 y_t(i)^{1/(1+\eta_t^p)} di \right]^{(1+\eta_t^p)} \geq Y_t, \quad (4-12)$$

where η_t^p is the exogenous time-varying markup to the intermediate goods' price.

The final goods producer chooses Y_t and $y_t(i)$ to maximize profit subject to the technology. The demand for $y_t(i)$ is given by:

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-(1+\eta_t^p)/\eta_t^p} \quad (4-13)$$

Where $(1 + \eta_t^p)/\eta_t^p$ is the elasticity of substitution between the intermediate goods.

The intermediate goods producers are monopolistic competitors in their product market. Firm i produces by a Cobb–Douglas technology:

$$y_t(i) = A_t^{1-\alpha} k_t^\alpha(i) L_t(i)^{1-\alpha}. \quad (4-14)$$

Where A_t is a permanent shock to technology and its growth rate, $a_t = \ln A_t - \ln A_{t-1}$, follows a stationary AR(1) process:

$$a_t = (1 - \rho_a)\gamma + \rho_a a_{t-1} + \epsilon_t^a, \quad \epsilon_t^a \sim N(0, \sigma_a^2) \quad (4-15)$$

Where γ is the steady-state growth rate.

The model has a price stickiness a la (23). The intermediate firms have a probability of $(1 - \omega_p)$ each period to reset their prices. Those cannot do so index their prices to past inflation according to the rule:

$$p_t(i) = p_{t-1}(i) \pi_{t-1}^{\chi^p} \pi^{1-\chi^p}. \quad (4-16)$$

Labor packers

A labor packer, in a perfectly competitive market, purchases a continuum of differentiated labor inputs $L_t(j)$ from the households and gather them to produce a composite labor service L_t by the technology:

$$L_t = \left[\int_0^1 L_t(i)^{1/(1+\eta_t^w)} dj \right]^{(1+\eta_t^w)},$$

where η_t^w denotes a time-varying exogenous markup to wages. Its demand function is:

$$L_t(i) = L_t \left(\frac{W_t(j)}{W_t} \right)^{-(1+\eta_t^w)/\eta_t^w} \quad (4-17)$$

Where $W_t(j)$ is the wage receive from the labor packer by the household j and W_t is the wage for the composite labor service paid by intermediate firms.

Households

Each household maximizes its discounted utility subject to its budget constraint:

$$E_t \sum_{s=0}^{\infty} \beta^s u_{t+s}^b \left[\ln(C_{t+s} - \theta C_{t+s-1}) - \frac{\phi L_{t+s}(j)^{1+\nu}}{1+\nu} \right] \quad (4-18)$$

where $\beta \in (0, 1)$ is the discount factor, $\theta \in (0, 1)$ is external habit formation, $\nu \geq 0$ is the inverse of the Frisch labor elasticity and φ is the disutility weight on labor. Each household owns one unit of specialized labor $L(j)$ and sets its wage in that market. u_t^b represents a stochastic component of the preference and it follows a AR(1):

$$\ln u_t^b = (1 - \rho^b) \ln u^b + \rho^b \ln u_{t-1}^b + \epsilon_t^b, \quad \epsilon_t^b \sim N(0, \sigma_b^2) \quad (4-19)$$

The household flow budget constraint is:

$$\begin{aligned} P_t b_{t-1}^r + B_{t-1}^m \left[1 - \sum_{j=1}^{\infty} Q_t(t+j) \rho^j \right] + D_t + \\ (1 - \tau_t) W_t(j) L_t(j) + (1 - \tau_t) R_t^k \nu_t \bar{K}_{t-1} - \psi(\nu_t) \bar{K}_{t-1} = \\ C_t + I_t + P_t s_t + P_t Q_t^r b_t^r + B_t^m \sum_{j=1}^{\infty} Q_t(t+j) \rho^{j-1} \end{aligned}$$

Each household owns an equal share of all intermediate firms and receives the share D_t of intermediate firms profits. Also, the households control both the size of the capital stock and its utilization rate ν_t . Effective capital, $k_t = \nu_t \bar{k}_{t-1}$ is rented to firms at the rate R_t^k . The cost of capital utilization is $\psi(\nu_t)$ per unit of physical capital. In the steady state, $\nu = 1$ and $\psi(1) = 0$. Define a parameter $\psi \in [0, 1)$ such that $\psi''(1)/\psi'(1) \equiv \psi/1 - \psi$. Then, the law of motion for private capital is:

$$\bar{k}_t = (1 - \delta) \bar{k}_{t-1} + u_t^i \left[1 - s \left(\frac{i_t}{i_{t-1}} \right) \right] i_t \quad (4-20)$$

Where $s(i_t/i_{t-1})$ is the adjustment cost as in (20) and (19). By assumption, $s(\gamma) = s'(\gamma) = 0$ and $s''(\gamma) = s > 0$. The term u_t^i captures exogenous variations in the efficiency with which investment can be transformed into

physical capital, as in (24). It follows an AR(1) process:

$$\ln u_t^i = (1 - \rho^i) \ln u^i + \rho^i \ln u_{t-1}^i + \epsilon_t^i, \quad \epsilon_t^i \sim N(0, \sigma_i^2) \quad (4-21)$$

In each period a fraction $1 - \omega_w$ households are allowed to re-optimize their wage rate by maximizing:

$$E_t \left[\sum_{s=0}^{\infty} (\beta \omega_w)^s u_{t+s}^b U(C_{t+s|t}, L_{t+s|t}) \right] \quad (4-22)$$

Subject to their budget constraint (Eq.??) and the labor demand function (Eq. 4-17). Those who cannot reset their wages, adjust their values according to the following rule:

$$W_t(j) = W_{t-1}(j) (\pi_{t-1} e^{a_{t-1}})^{\chi^w} + (\pi e^\gamma)^{(1-\chi^w)} \quad (4-23)$$

5 Estimation Methodology

In the estimation, we use quarterly data for three samples: 1955Q1–1966Q4, 1967Q1–1979Q2 and 1984Q1–2007Q4 - the same data used in (18). (25), in a regime switching model, find evidence of changes in policy parameters between these three periods, motivating the cutoffs. The period of 1967Q1–1979Q2, known as the “Great Inflation”, is characterized by fast economic growth and high inflation. It ended with the appointment of Paul Volcker as chairman of the FED board, in August 1979. The third period, known as the “Great Moderation” is characterized by low inflation and low volatility in macroeconomic aggregates.

I use nine time series in the estimation: consumption, investment, wages, government spending, tax revenue, government debt, hours worked, inflation and interest rate:

$$\begin{bmatrix}
 dlCons_t \\
 dlInv_t \\
 dlWage_t \\
 dlGovSpend_t \\
 dlTaxRev_t \\
 dlGovDebt_t \\
 lHous_t \\
 lInfl_t \\
 lFedFunds_t
 \end{bmatrix}
 =
 \begin{bmatrix}
 100\gamma \\
 100\gamma \\
 100\gamma \\
 100\gamma \\
 100\gamma \\
 100\gamma \\
 0 \\
 0 \\
 0
 \end{bmatrix}
 +
 \begin{bmatrix}
 \hat{c}_t - \hat{c}_{t-1} + \hat{a}_t \\
 \hat{i}_t - \hat{i}_{t-1} + \hat{a}_t \\
 \hat{w}_t - \hat{w}_{t-1} + \hat{a}_t \\
 \hat{g}_t - \hat{g}_{t-1} + \hat{a}_t \\
 \hat{t}_t - \hat{t}_{t-1} + \hat{a}_t \\
 \hat{b}_t - \hat{b}_{t-1} + \hat{a}_t \\
 \hat{L}_t \\
 \hat{\pi}_t \\
 \hat{R}_t
 \end{bmatrix}$$

Where l and dl represents 100 times the log and the log difference of each variable. The small letters represent the transformed quantity of a level variable: \hat{b}_t is the transformed real debt level, \hat{a}_t is the percentage deviation of technology growth rate from the steady-state growth rate γ . In the appendix (Section C.3) we provide a detailed description of the data.

Using the software Dynare, and based on (26) and (27), first, we use the minimization routine *csmi_nwel.m* (by Prof Christopher Sims) to search for a local minimum of the negative log posterior function. This value is used

to initialize the estimation. The posterior distribution is estimated using the random walk Metropolis-Hastings algorithm. We perform, for each estimation, two chains of 2 million draws of the posterior distribution. We discard the first half of the draws at the end of the estimation, leaving us with a final sample size of 1 million. Diagnostic tests implemented in Dynare are performed to ensure convergence of the MCMC chains.

Using (28) methodology, we find that all the estimated parameters are locally identified. Some of the parameters, however, are calibrated. As in (18), the discount factor β is set to 0.99. The capital income share α is set to 0.3, which implies a labor share of 0.7. The quarterly depreciation rate for capital, δ , is set to 0.025, implying the annual depreciation rate of 10%. η^w and η^p are set to 0.14, so the steady-state markups in the product and labor markets are 14%. We use the mean values of data from 1955Q1 to 2007Q4 to calibrate the fiscal variables. The tax rate is set to 0.185, the ratio of government spending to output is set to 0.104 and the ratio of government debt to annual output is set to 0.348. Since the Treasury Inflation-Protected Securities (or TIPS) were introduced only in 1997¹, the parameter λ (share of indexed debt on total debt) is set to zero. The parameter ρ is calibrated, so the average maturity of nominal debt is equal to 43 months. Also, when computing these statistics from the data, output is defined as the sum of consumption, investment, and government consumption and investment, consistent with the output definition in the model.

For each sample, we estimate the model with the two different specifications for the indexation structure. Therefore, there are a total of six estimations.

Regarding the monetary and fiscal policy parameters (ϕ_π , ϕ_y , γ_τ and γ_g), in all estimations, we set our prior for these policy parameters with a distribution centered in Regime M. As in (20), for ϕ_π , we set a normal distribution with mean 1.5 and standard deviation of 0.25; for ϕ_y , a normal distribution with mean 0.25 and standard deviation of 0.05. As in (18), for γ_τ and γ_g we set a normal distribution with means, respectively, of 0.15 and 0.05, and standard deviation of 0.05.

The table with the complete set of priors and posterior estimates is in the Appendix (Section C.5).

¹https://www.treasurydirect.gov/indiv/research/indepth/tips/res_tips.htm

6

Estimation Results

Regarding the posterior distribution, the mean estimates for the standard parameters are close to the estimates of (20) - See appendix. The posterior distribution for the fiscal parameters are also close to the estimation in (18).

6.1

(1st Specification) - Estimation Results and Counterfactuals

6.1.1

Evidence of Bacha Effect

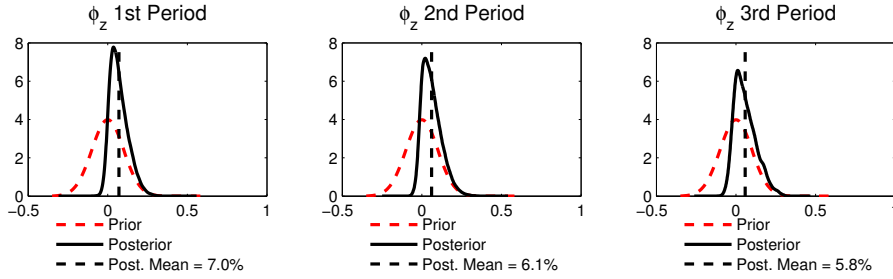
As discussed in the model's section, the government net transfer is modeled as:

$$z_t = \left(z_t^+ + \frac{Z^+}{P_t} \right) - \left(z_t^- + \frac{Z^-}{P_t} \right) + \eta_t^z$$

The parameter $\phi_z \equiv \frac{(Z^+ - Z^-)/P}{z}$ is a measure of the non-indexed fraction of the government net transfers. If it is positive, than an inflation rise has a negative impact on the real value of transfers. In the model, this happens when the size of the government nominal transfers is greater than the size of the government nominal lump-sum taxes ($Z^+ > Z^-$).

I set a Normal distribution with zero mean and standard deviation $\sigma = 0.1$ as the prior for this parameter. *A priori*, therefore, there is equal probability of having a negative or a positive sign. As shown in Figure 6.1 The posterior distribution for this parameter have mean in the positive region, in all three periods of the sample (1955Q1–1966Q4, 1967Q1–1979Q2 and 1984Q1–2007Q4). Moreover, since the revenue from distortive taxes is perfectly indexed, $\phi_z > 0$ implies that inflation has a positive impact over the real surpluses (the “Bacha effect”).

When interpreting this results, it is important to note that, in the estimation, we use data for government purchases, revenue and debt level. As a result, the variable \hat{z}_t is linked to the residual of government flow budget constraint. In this case, what we call as “net transfers” actually captures anything that affects the debt level other than revenue and expenditure.

Figure 6.1: Prior and Posterior Distributions of ϕ_z

In order to analyze how this phenomenon affects the predictions of the model, first, we impose the Fiscal regime, setting $\phi_\pi = \phi_y = \gamma_g = \gamma_\tau = 0$. This implies that the nominal interest rate is constant and the fiscal policy does not respond to the debt level. Then, we set the rest of the parameters on the mean of their posteriors (for the period of 1955Q1–1966Q4¹) and compare the dynamic responses of some variables of interest to a monetary and fiscal policy shocks, under different values of ϕ_z and ϕ_g (“Indexation Parameters”). The table summarizing the estimation results is on the appendix (section C.5).

Figure 6.2 displays the impulse response functions (IRFs) of some variables of interest to a monetary policy shock that takes the form of positive realization of ϵ_t^r of size 0.25. The red dashed lines represent the IRFs when the “Indexation Parameters” are set on their mean values and the black lines represents the IRFs when they are set to zero (perfect indexation).

When there is imperfect indexation the immediate inflation response to the shock is negative, whereas when indexation is perfect it is positive. This negative response in the short-run gets larger the less indexed is the fiscal instruments and the longer is the nominal debt maturity. The cumulative inflation response, measured by the price level response, is 0.20 percentage points higher when indexation is perfect. Moreover, when indexation is imperfect the debt level steady-state changes to a higher level (99% higher than the initial level). This happens because the net transfers real value is permanently eroded by the higher inflation.

¹The main points made here do not depend on the period choice.

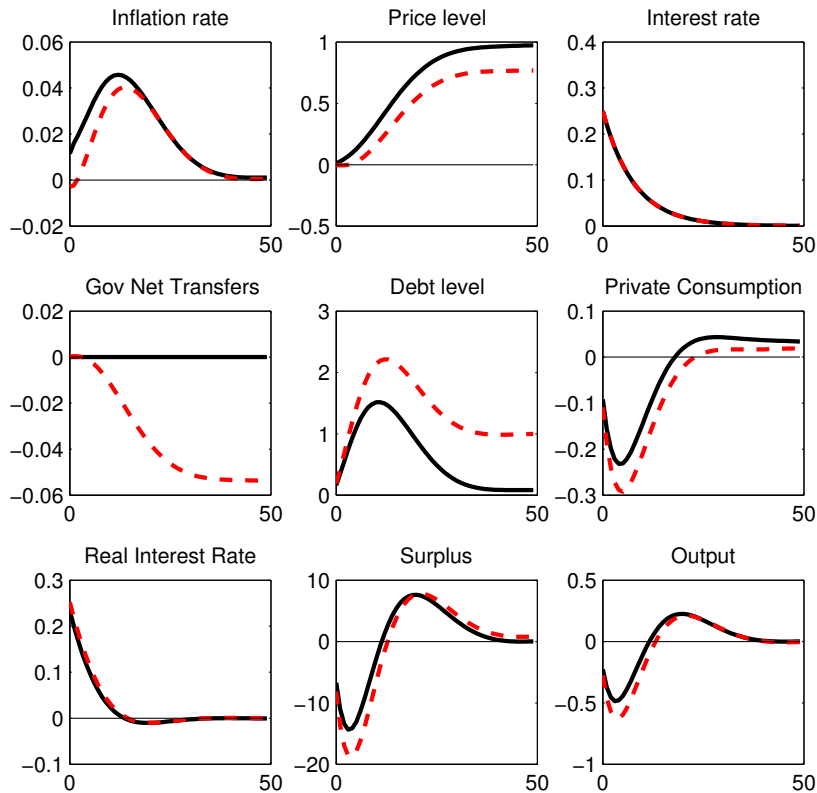


Figure 6.2: IRF Monetary Shock - Red (Dashed): $\phi_z = 7.0\%$, $\phi_g = 1.7\%$; Black: $\phi_z = \phi_g = 0$

Figure 6.3 illustrates the IRFs of fiscal shock (an unexpected increase in net transfers). First, as expected, the inflationary result of the fiscal shock is lower when transfers are not totally indexed. As discussed in section 3, this happens because when part of the transfers are no indexed, inflation erodes not only the real value of debt but also the real value of transfers and purchases. Thus, less inflation is needed to reestablish the “fiscal intertemporal equilibrium condition”. Second, when part of the transfers are not indexed, the inflation generated in the shock takes the real value of the transfers and purchases to a lower steady-state level. Consequently, debt and surpluses move to a higher steady-state level. This would not happen if the transfers were indexed to the past inflation.

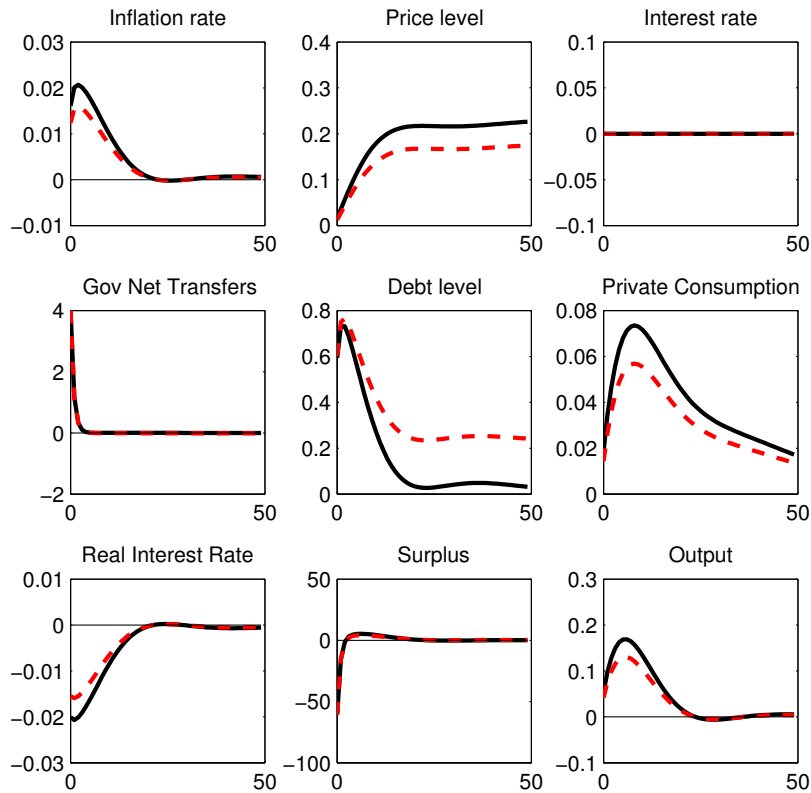


Figure 6.3: IRF Fiscal Shock - Red (Dashed): $\phi_z = 7.0\%$, $\phi_g = 1.7\%$; Black: $\phi_z = \phi_g = 0$

6.1.2 The role of each channel

In order to assess the role of each one of the channels operating in the FTPL, we compare the cumulative inflation following a negative fiscal shock, when different combinations of those channels are “available”. Table 6.1 summarize the results.

To be available, each source of fiscal financing requires an specific condition on the model: for the debt to be used as a cushion, it must have some nominal part; the erosion of the real value of expenditures requires imperfect indexation; finally, for the real discount rate to respond to variations on government transfers, there must be some kind of nominal rigidity. The left side of the table indicates if these conditions are met (those market with “X”) and, therefore, if the related channels are operating. The right side indicates both the cumulative inflation and how much higher it is in relation to the benchmark (the case where all three are operating - the first line of the table). For instance, the value 17.70% represents the cumulative inflation in the case where all government debt is real, but there are still imperfect indexation

($\phi_z = 7.0\%$, $\phi_g = 1.7\%$) and nominal stickiness (the parameters associated to these frictions are calibrated using the mean of their posteriors).

	Nominal Debt	Imperfect Indexation	Nominal Rigidity	Cumulative Inflation (Price Level)	Difference from the Benchmark (bps)
1	X	X	X	17.45%	0
2		X	X	17.70%	0.25
3	X		X	22.62%	5.2
4	X	X		39.6%	22.2
5	X			87.04%	69.6
6		X		72.49%	55.0
7			X	23.04%	5.6
8				No equilibrium	

Table 6.1: Inflationary effect of a fiscal shock ("X" indicates the presence of a feature)

The first thing worth noting here is that, in an economy with nominal rigidity, the role of the nominal debt "cushion" is strikingly diminished. In a flexible-price economy with indexed surpluses, as in most FTPL literature² the cumulative inflation response tend to infinity as the share of nominal debt goes to zero. However, when prices and nominal wages do not freely adjust, the change in that response is much lower. In fact, in our estimated model, when comparing the case where debt is totally nominal with the case where it is totally real, the cumulative inflation barely changes: it gets only 0.25 basis points higher, when surplus indexation is imperfect (lines 1 and 2 of Table 6.1), and 0.42 when surpluses are perfectly indexed (lines 3 and 7). This happens because in the sticky price model, the consumption is also affected by the fiscal shock, reducing the real interest rates and, therefore, the government real interest payments. From the fiscal perspective, the endogenous response of the real rates reduces the real interest expenditure and, hence, the "size" of the fiscal shock and its inflationary effect.

In the appendix (Section B) we show that this result also holds in a textbook New-Keynesian model with a moderate degree of price stickiness.

Figure 6.4 displays the IRFs of the model with and without nominal debt (both with nominal rigidities). It makes clear that is not only the inflation response that barely changes: the same applies to all other variables of interest. However, as we reduce the degree of nominal rigidity in the economy, the role of nominal debt as a source of fiscal financing increases. In a flex-price economy, comparing the case in which debt is nominal and surpluses are imperfectly indexed (line 4) with the case in which the only source of finance is the erosion

²As examples: (1), (2) and (6)

of the real value of expenditures (line 6), we see that the inflation response is much higher (32.89 percentage points) in the latter.

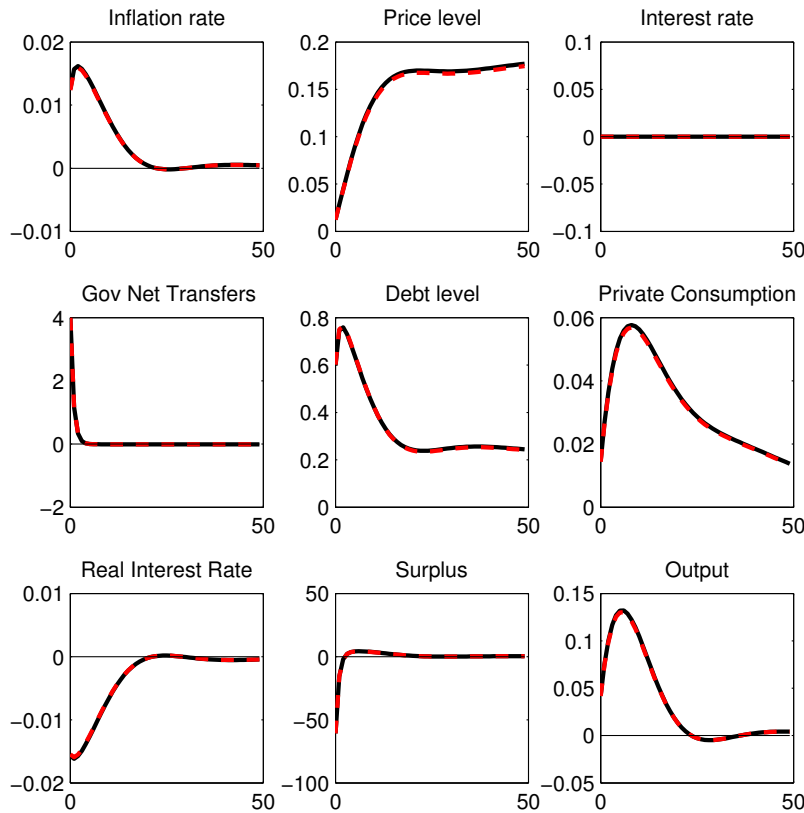


Figure 6.4: IRF Fiscal Shock - Black: All debt is real; Red (Dashed): All debt is nominal

A direct way of assessing the role of the “erosion of the real value of expenditures” as a source of fiscal financing is to compare the inflation response when this channel is available to the response when it is not. In the case where prices are rigid and the government has nominal debt, we compare lines 1 and 3. The difference of inflation attributed to this source is (22.62%-17.45%) 5.17% percentage points. In the case where prices are rigid but debt is real, we compare lines 2 and 7. In this case the difference of inflation attributed to this source is (23.04%-17.7%) 5.34% percentage points.

6.2

Estimation Results (2nd Specification)

In this specification the parameters of interest are θ_z and θ_g associated to the log-linearized equations for the real value of transfers and purchases:

$$\hat{z}_t = v_{z,t} + \eta_t^z \tag{6-1}$$

$$v_{z,t} = \theta_z(v_{z,t-1} - \pi_t) \tag{6-2}$$

$$\hat{g}_t = v_{g,t} - \gamma_g \hat{a}_{t-1}^m + \eta_t^g \tag{6-3}$$

$$v_{g,t} = \theta_g(v_{g,t-1} - \pi_t) \tag{6-4}$$

We assume as priors a beta distribution with mean 0.5 and standard deviation of 0.1. Figures 6.5 and 6.6 depict the estimation results.

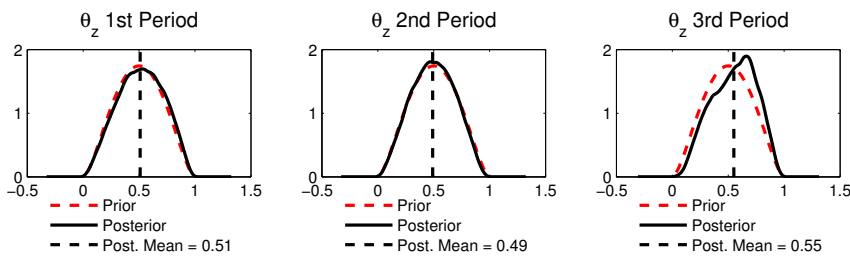


Figure 6.5: Prior and Posterior Distributions of θ_z

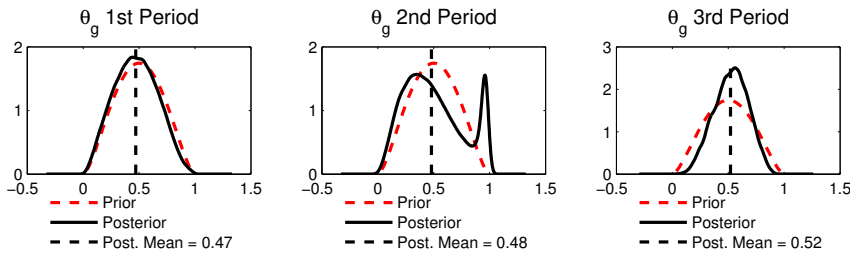


Figure 6.6: Prior and Posterior Distributions of θ_g

Taken together, these estimates reveals that, in spite of being locally identified (under (28) methodology), data seems to add little information on their values, since the posterior estimates are almost identical to their prior distributions. In this case, there is little we can tell about their values and, therefore, on the counterfactuals exercises we use arbitrary values.

As in Section 6.1, where we analyzed the results under the first specification, here we set all the parameters, with exception of those related to monetary and fiscal policies, on the mean of their posteriors. Also, we choose different values of θ_g and θ_z , to assess how would economy behave under different indexation structures.

Figure 6.7 displays the impulse response functions (IRFs) of some variables of interest to a monetary policy shock that takes the form of positive realization of ϵ_t^r of size 0.25. The red dashed lines and the blue dash-dotted lines represents the IRFs when the fiscal policy is imperfectly indexed, with parameters, respectively $\theta_g = \theta_z = 0.75$ and $\theta_g = \theta_z = 0.95$. The black lines represents the IRFs when they are set to zero (perfect indexation).

Qualitatively, the results are very similar to those found under the first specification: when surpluses are not perfectly indexed (i) inflation rates are lower, (ii) debt level grows more and (iii) output and private consumption have a larger variation. A relevant difference, however, is that, under this specification, variable such as the debt level, surpluses, government net transfers and private consumption do not move to a different steady-state after the shock. This happens because, now, the real value of purchases and transfers are eroded only temporarily, whereas under the first specification the effect is permanent. Furthermore, when that imperfection is more extreme (the case where $\theta_z = \theta_g = 0.95$), the inflation response is negative in the short-run.

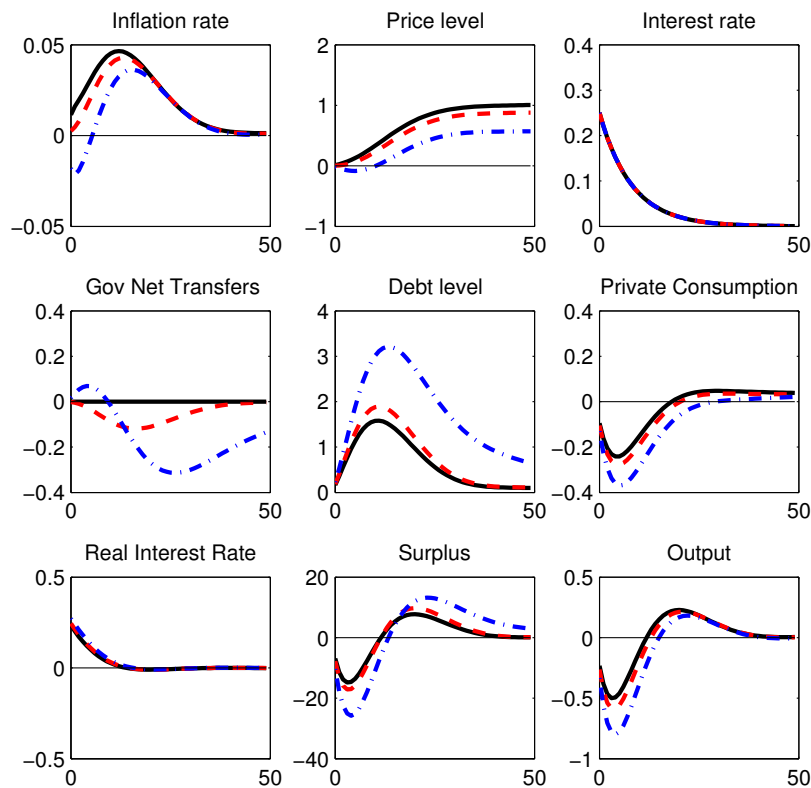


Figure 6.7: IRFs Monetary Shock - Blue (Dashed-dotted): $\theta_z = \theta_g = 0.95$ Red (Dashed): $\theta_z = \theta_g = 0.75$; Black: $\theta_z = \theta_g = 0$

Figure 6.8 displays the IRFs to an expansionary fiscal shock that takes the form of positive realization of ϵ_t^z of size 4. Again, there are three different values for the indexation parameters: 0.95, 0.75 and zero. The results are also qualitatively similar to those under the first specifications, with the exception that here only the price level changes to a different steady-state. Also, when the fiscal variables have a very weak indexation ($\theta_z = \theta_g = 0.95$), the cumulative inflation response is reduced, virtually, to a half of what it would be under perfect indexation.

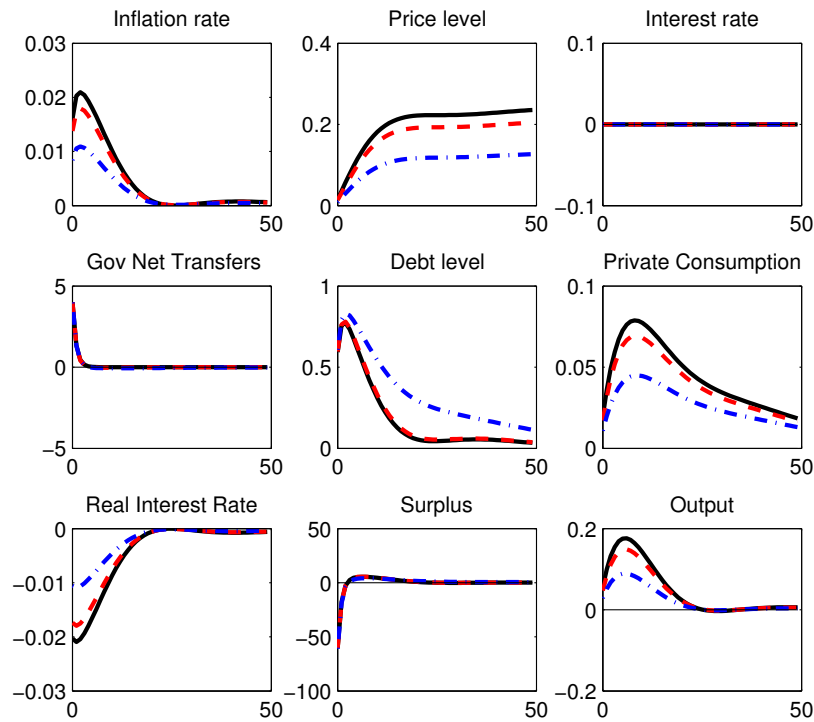


Figure 6.8: IRFs Fiscal Shock - Blue (Dashed-dotted): $\theta_z = \theta_g = 0.95$ Red (Dashed): $\theta_z = \theta_g = 0.75$; Black: $\theta_z = \theta_g = 0$

7 Conclusion

In this article, we propose a model that accounts for different forms of indexation rules for the government purchases and transfers. We show that the FTPL may operate through the erosion of the real value of expenditures, other than the revaluation of the government debt. That implies theoretical changes on model dynamics. Through the estimation of the model, we evaluate the relevance of these changes for the U.S. economy.

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A Proof of the Proposition

We log-linearize the equations of the model under the steady-state, organize it in a linear system and apply Blanchard (1980) solution conditions. In the following equations $i_t = \ln(R_t)$; $\rho = \ln(\beta^{-1})$. π_t and b_t are, respectively, the log deviation of inflation and real debt from their steady-state.

Euler equation implies, for every period, that:

$$R_t^r = \beta^{-1} \quad (\text{A-1})$$

Government flow budget constraint:

$$b_t = \beta^{-1}b_{t-1} - \frac{1-\beta}{\beta}s_t \quad (\text{A-2})$$

Taylor rule:

$$i_t = \rho + \phi_\pi \pi_t + \epsilon_t^M \quad (\text{A-3})$$

Fisher relation:

$$i_t = \rho + E_t \pi_{t+1} \quad (\text{A-4})$$

Surplus:

$$\frac{1-\beta}{\beta}s_t = \phi_b b_{t-1} + \phi_p p_t - \epsilon_t^F \quad (\text{A-5})$$

Where $\phi_p = \frac{(Z-T)/P}{b}$ measures the how relevant is the the nominal component of the surplus.

The log-linearized system of these equations can be represented as:

$$E_t \begin{pmatrix} b_t \\ \pi_{t+1} \\ p_t \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{\beta} - \phi_b\right) & -\phi_p & -\phi_p \\ 0 & \phi_\pi & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ \pi_t \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^F \\ \epsilon_t^M \\ 0 \end{pmatrix} \quad (\text{A-6})$$

The eigenvalues of the matrix are:

$$\begin{aligned} \lambda_1 &= 1 \\ \lambda_2 &= \frac{1}{\beta} - \phi_b \\ \lambda_3 &= \phi_\pi \end{aligned}$$

Blanchard (1980) solution conditions for existence and uniqueness of a bounded solution are satisfied if:

$$|\phi_\pi| > 1 \text{ and } \left| \frac{1}{\beta} - \phi_b \right| < 1 \quad (\text{Monetary Regime}) \text{ or}$$

$$|\phi_\pi| < 1 \text{ and } \left| \frac{1}{\beta} - \phi_b \right| > 1 \quad (\text{Fiscal Regime}).$$

Note, finally, that the condition of $\phi_p \neq 0$ is also necessary, since if $\phi_p = 0$ and $\left| \frac{1}{\beta} - \phi_b \right| > 1$ the debt level has an explosive path, which violates the transversality condition.

$$b_t = \left(\frac{1}{\beta} - \phi_b \right) b_{t-1} - \epsilon_t^F \implies E_t b_{t+1} > b_t, \quad \forall t$$

A.1

Solution for the Price Level under Fiscal Regime

The solution for the price level in the Fiscal Regime can be obtained through the government flow budget condition:

$$b_t = \left(\frac{1}{\beta} - \phi_b \right) b_{t-1} - \phi_p p_t + \epsilon_t^F$$

Defining $\left(\frac{1}{\beta} - \phi_b \right)^{-1} \equiv \gamma$ and $-\phi_p \equiv \phi$:

$$b_{t-1} = \gamma b_t + \gamma \phi p_t + \gamma \epsilon_t^F$$

Taking expectations and substituting forward:

$$b_{t-1} = \gamma \epsilon_t^F + \gamma \phi p_t + \gamma^2 \phi E_t p_{t+1} + \gamma^3 \phi E_t p_{t+2} + \dots$$

Now, from Taylor rule and Fisher relation:

$$\mathbb{E}_t(p_{t+1} - p_t) = \phi_\pi(p_t - p_{t-1}) + \epsilon_t^M$$

Which can be rewritten as a second order difference equation for the price level:

$$\mathbb{E}_t(p_{t+1}) = (1 + \phi_\pi)p_t - \phi_\pi p_{t-1} + \epsilon_t^M$$

Rewriting in matrix form:

$$\underbrace{\begin{bmatrix} \mathbb{E}_t p_{t+1} \\ p_t \end{bmatrix}}_{E_t Z_{t+1}} = \underbrace{\begin{bmatrix} (1 + \phi_\pi) & -\phi_\pi \\ 1 & 0 \end{bmatrix}}_A \underbrace{\begin{bmatrix} p_t \\ p_{t-1} \end{bmatrix}}_{Z_t} + \underbrace{\begin{bmatrix} \epsilon_t^M \\ 0 \end{bmatrix}}_{V_t}$$

So we have:

$$b_{t-1} = \gamma \epsilon_t^F + \phi \gamma \begin{bmatrix} 1 & 0 \end{bmatrix} Z_t + \gamma E_t \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \phi \gamma Z_{t+1} + \gamma b_{t+1} \right]$$

if $|\phi_\pi\gamma| < 1$, then both eigenvalues of (γA) are, in absolute value, lower than 1. So:

$$b_{-1} = \gamma\epsilon_t^F + \phi\gamma \begin{bmatrix} 1 & 0 \end{bmatrix} Z_t + \phi\gamma^2 \begin{bmatrix} 1 & 0 \end{bmatrix} \underbrace{\left[I + \gamma A + \gamma^2 A^2 + \dots \right]}_{=[I-\gamma A]^{-1}} E_t Z_{t+1} + \underbrace{\lim_{j \rightarrow \infty} \mathbb{E}_t \gamma^{j+1} b_j}_{=0, \text{ if } |\gamma| < 1}$$

$$\left[I - \gamma \begin{bmatrix} (1 + \phi_\pi) & -\phi_\pi \\ 1 & 0 \end{bmatrix} \right]^{-1} \equiv B$$

$$b_{-1} = \gamma\epsilon_t^F + \phi\gamma \begin{bmatrix} 1 & 0 \end{bmatrix} Z_t + \phi\gamma^2 \begin{bmatrix} 1 & 0 \end{bmatrix} B E_t Z_{t+1}$$

Assuming $p_{-1} = 0$:

$$p_t = b_{t-1} \frac{(1-\gamma)(1-\phi_\pi\gamma)}{\phi\gamma[(1-\gamma)(1-\gamma\phi_\pi) + \gamma]} - \epsilon_t^M \frac{\gamma}{\phi\gamma[(1-\gamma)(1-\gamma\phi_\pi) + \gamma]} - \epsilon_t^F \frac{(1-\gamma)(1-\phi_\pi\gamma)}{\phi[(1-\gamma)(1-\gamma\phi_\pi) + \gamma]}$$

Note that, the condition $|\gamma| < 1$ is equivalent to $\left| \frac{1}{\beta} - \phi_b \right| > 1$.

A.2 Calibration

The calibration used to generate the IRFs of a monetary policy shock (displayed in Figure ??) is shown on table below. The parameter $\phi_p = \frac{(Z-T)/P}{b}$ is equal to $\frac{(Z-T)/P}{s} \frac{s}{b} = \frac{(Z-T)/P}{s} \frac{(1-\beta)}{\beta}$. So assuming that $T = 0$ (all revenue is indexed), $\frac{z+Z/P}{s} = 4$ (transfers are 4 times higher than surpluses) and that 20% of the transfers are not indexed, than $\phi_p = 0.008$.

Parameter	Value
β	0.99
σ	1
ϕ_π	0
ϕ_b	0
ϕ_p	0.008

Table A.1: Calibration - Frictionless, Endowment Economy Model

B FTPL in a textbook New-Keynesian Model - Nominal vs Real debt

We simulate an standard New-Keynesian model described in gali2015monetary, including a simple fiscal policy rule for surpluses. The log-linearized equations are:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \quad (\text{B-1})$$

$$y_t = E_t y_{t+1} - \sigma^{-1} r_t \quad (\text{B-2})$$

$$i_t = \phi_\pi \pi_t + \eta_t^m \quad (\text{B-3})$$

$$r_t = i_t - E_t \pi_{t+1} \quad (\text{B-4})$$

$$b_{t-1} - (1 - \lambda)\pi_t = (1 - \beta)s_t + \beta \left[\lambda(b_t - r_t) + (1 - \lambda)(b_t - i_t) \right] \quad (\text{B-5})$$

$$s_t = \gamma b_{t-1} - (z/s)z_t \quad (\text{B-6})$$

$$z_t = \eta_t^z \quad (\text{B-7})$$

$$\eta_t^z = \rho \eta_{t-1}^z + \epsilon_t^z \quad (\text{B-8})$$

The calibration is:

Parameter	Value
β	0.99
σ	1
α	1/3
θ	2/3
ϕ	1
ϵ	6
ϕ_π	0
γ	0
ρ	0.5
λ	{0,1}
(z/s)	4

Table B.1: Calibration NK Model

As in gali2015monetary, the parameter $\kappa \equiv \lambda_2(\sigma + ((\phi + \alpha)/(1 - \alpha)))$, where $\lambda_2 \equiv (((1 - \theta)(1 - \beta * \theta))/\theta)((1 - \alpha)/(1 - \alpha + \alpha\epsilon))$.

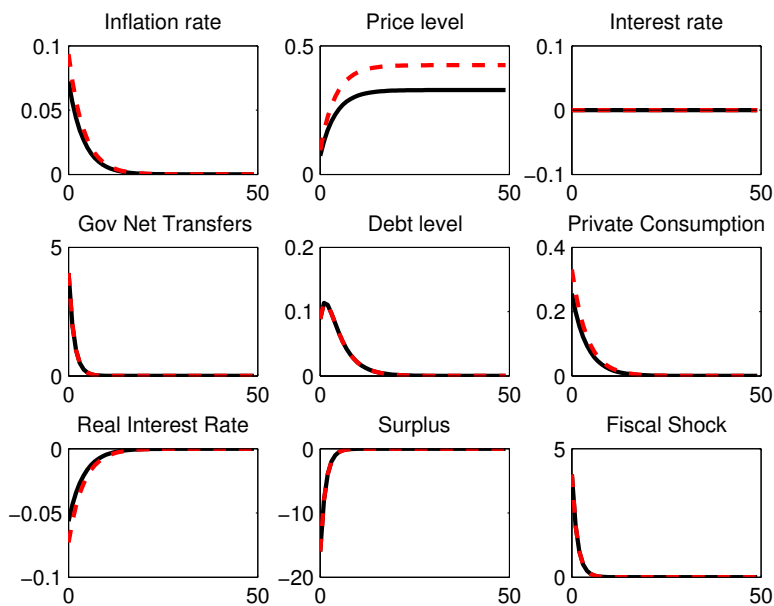


Figure B.1: IRF Fiscal Shock - Red (Dashed): $\lambda = 1$ (Real debt); Black: $\lambda = 0$ (Nominal debt)

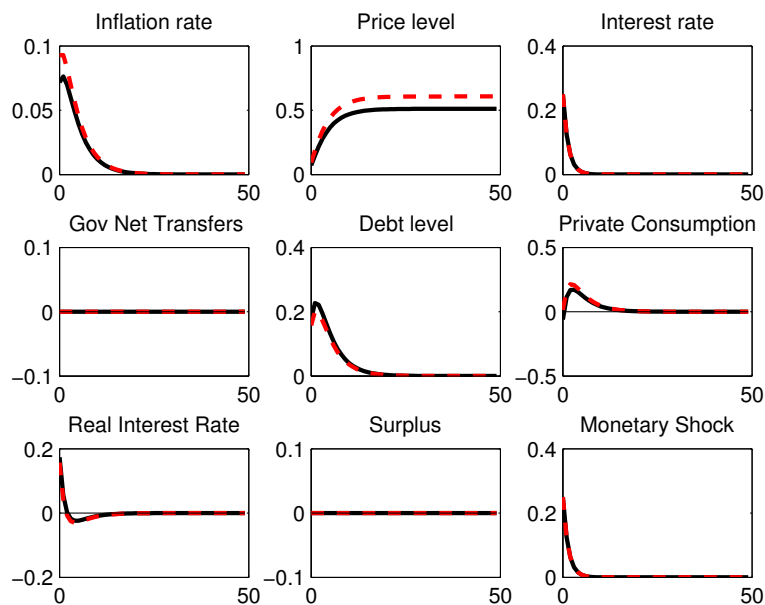


Figure B.2: IRF Monetary Shock - Red (Dashed): $\lambda = 1$ (Real debt); Black: $\lambda = 0$ (Nominal debt)

C

Model Equations, Data Description and Results

C.1

Model Equations (Log-Linearized)

Here we present all the equations used to simulate the model:

Production function:

$$\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \quad (\text{C-1})$$

Capital-Labor ratio:

$$\hat{R}_t^k - \hat{w}_t = \hat{L}_t - \hat{k}_t \quad (\text{C-2})$$

Marginal Cost:

$$\hat{m}c_t = \alpha \hat{R}_t^k + (1 - \alpha) \hat{w}_t \quad (\text{C-3})$$

Household FOC for consumption:

$$\hat{\lambda}_t = \hat{R}_t + E_t[\hat{\lambda}_{t+1}] - E_t[\hat{\pi}_{t+1}] - E_t[\hat{a}_{t+1}] \quad (\text{C-4})$$

Euler Equation:

$$\hat{\lambda}_t = \hat{R}_t + E_t[\hat{\lambda}_{t+1}] - E_t[\hat{\pi}_{t+1}] - E_t[\hat{a}_{t+1}] \quad (\text{C-5})$$

Household FOC for capacity utilization:

$$\hat{R}_t^k - \frac{\tau}{1 - \tau} \hat{\tau}_t = \frac{\psi}{1 - \psi} \hat{v}_t \quad (\text{C-6})$$

Household FOC for capital:

$$\hat{q}_t = E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{a}_{t+1} + \beta e^{-\gamma} (1 - \tau) R^k E_t \hat{R}_{t+1}^k - \beta e^{-\gamma} \tau R^k E_t \hat{\tau}_{t+1} + \beta e^{-\gamma} (1 - \delta) q E_t \hat{q}_{t+1} \quad (\text{C-7})$$

Household FOC for investment:

$$(1 + \beta) \hat{i}_t - \frac{1}{s e^{2\gamma}} \hat{q}_t + \hat{a}_t - E_t \hat{i}_{t+1} - E_t \hat{a}_{t+1} = \hat{i}_{t-1} \quad (\text{C-8})$$

Effective capital:

$$\hat{k}_t = \hat{v}_t + \hat{k}_{t-1} - \hat{a}_t \quad (\text{C-9})$$

Law of motion for capital:

$$\hat{k}_t = (1 - \delta)e^{-\gamma}(\hat{k}_{t-1} - \hat{a}_t) + [1 - (1 - \delta)e^{-\gamma}](\hat{u}_t^i + \hat{i}_t) \quad (\text{C-10})$$

Phillips curve:

$$\pi_t = \frac{(1 - \beta\omega^p)(1 - \omega^p)}{\omega^p(1 + \beta\chi_p)}\hat{m}c_t + \frac{\chi_p}{1 + \beta\chi_p}\pi_{t-1} + \frac{\beta}{1 + \beta\chi_p}E_t\{\pi_{t+1}\} + \hat{\eta}_t^* \quad (\text{C-11})$$

Where $\hat{\eta}_t^* = \frac{(1 - \beta\omega^p)(1 - \omega^p)}{\omega^p(1 + \beta\chi_p)}\hat{\eta}_t$ is the rescaled shock.

Aggregate resource constraint:

$$\hat{y}_t = (c/y)\hat{c}_t + (i/y)\hat{i}_t + (g/y)\hat{g}_t + \psi'(1)(k/y)\hat{v}_t \quad (\text{C-12})$$

Government budget constraint:

$$\hat{b}_{t-1}^m - (1 - \lambda)\hat{\pi}_t - a_t = (1 - \beta)\hat{s}_t + \beta \left\{ \lambda \left[\hat{b}_t^m - \hat{r}_t \right] + (1 - \lambda) \left[\hat{b}_t^m + (1 - \rho)\hat{P}_t^m \right] \right\} \quad (\text{C-13})$$

Where $\lambda = \frac{b^r}{b^r + \frac{(1 + \rho P^m)B^m}{P}}$ is the fraction of the real debt in the total debt. Note also that $\frac{s}{b^r + \frac{(1 + \rho P^m)B^m}{P}} = (1 - \beta)$ is the quotient between the real surplus and the real value of debt.

Long-term bond portfolio price:

$$\hat{P}_t^m = \beta_t \hat{P}_{t+1}^m - \hat{R}_t; \quad (\text{C-14})$$

Government surplus:

$$\hat{s}_t = (T/s)\hat{T}_t - (g/s)\hat{g}_t - (z/s)\hat{z}_t \quad (\text{C-15})$$

Transfers rule (1st specification):

$$\hat{z}_t = - \underbrace{\left(\frac{V_2/P}{z} \right)}_{\phi_z} \hat{p}_t + \eta_t^z \quad (\text{C-16})$$

Transfers rule (2nd specification):

$$\hat{z}_t = v_{z,t} + \eta_t^z \quad (\text{C-17})$$

$$v_{z,t} = \theta_z(v_{z,t-1} - \pi_t) \quad (\text{C-18})$$

Government purchases rule (1st specification):

$$\hat{g}_t = -\gamma_g \hat{a}_{t-1}^m - \underbrace{\left(\frac{G_2/P}{z} \right)}_{\phi_g} \hat{p}_t + \eta_t^g \quad (\text{C-19})$$

Government purchases rule (2nd specification):

$$\hat{g}_t = v_{g,t} - \gamma_g \hat{d}_{t-1}^m + \eta_t^g \quad (\text{C-20})$$

$$v_{g,t} = \theta_g (v_{g,t-1} - \pi_t) \quad (\text{C-21})$$

Tax-rate rule:

$$\tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau) \gamma_\tau \hat{d}_{t-1}^m + \epsilon_t^\tau, \epsilon_t^\tau \sim N(0, \sigma^\tau) \quad (\text{C-22})$$

Monetary-policy rule:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) (\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t) + \epsilon_t^r, \quad \epsilon_t^r \sim N(0, \sigma^r) \quad (\text{C-23})$$

C.2

Equation for the government budget constraint

Government Budget Constraint:

$$b_{t-1}^r + \frac{B_{t-1}^m (1 + \rho P_t^m) P_{t-1}}{P_{t-1} P_t} = s_t + Q_t^r b_t^r + \frac{B_t^m P_t^m}{P_t}$$

Defining $\tilde{x}_t \equiv X_t/A_t$ and, specifically $\tilde{b}_t^m \equiv \frac{B_t^m}{A_t P_t}$:

$$\tilde{b}_{t-1}^r \frac{A_{t-1}}{A_t} + \tilde{b}_{t-1}^m (1 + \rho P_t^m) \frac{P_{t-1}}{P_t} \frac{A_{t-1}}{A_t} = \tilde{s}_t + Q_t^r \tilde{b}_t^r + \tilde{b}_{t-1}^m P_t^m$$

Log-Linearized:

$$\hat{b}_{t-1}^m - (1 - \lambda) \hat{\pi}_t - a_t = (1 - \beta) \hat{s}_t + \beta \left\{ \lambda \left[\hat{b}_t^m - \hat{r}_t \right] + (1 - \lambda) \left[\hat{b}_t^m + (1 - \rho) \hat{P}_t^m \right] \right\}$$

Where $\lambda = \frac{b^r}{b^r + \frac{(1 + \rho P^m) B^m}{P}}$ is the fraction of the real debt in the total debt. Note also that $\frac{s}{b^r + \frac{(1 + \rho P^m) B^m}{P}} = (1 - \beta)$ is the quotient between the real surplus and the real value of debt.

C.3

Data Description

As noted in the text, we use exactly the same data as traum2011monetary, with the same transformations. All the data used in the estimation is from the National Income and Product Accounts Tables, by the Bureau of Economic Analysis. All data in levels are nominal values. Nominal

data are converted to real values by the price deflator for GDP (Table 1.1.4, line 1).

Consumption

Consumption, C , is defined as the sum of personal consumption expenditures on nondurable goods (Table 1.1.5, line 5) and services (Table 1.1.5, line 6).

Investment

Investment, I , is defined as the sum of personal consumption expenditures on durable goods (Table 1.1.5, line 4) and gross private domestic investment (Table 1.1.5, line 7).

Tax revenue

Tax revenue, T , is defined as the sum of federal personal current tax (Table 3.2, line 3), federal taxes on corporate income (Table 3.2, line 7), and federal contributions for social insurance (Table 3.2, line 11).

Government spending

Government spending, G , is defined as federal government consumption expenditure and investment (Table 1.1.5, line 22).

Government debt

Government debt, B , is the market value of privately held gross federal debt, published by the Federal Reserve Bank of Dallas. The quarterly data are the monthly data at the beginning of each quarter.

Hours worked

Hours worked are constructed from the following variables:

H : the index for nonfarm business, all persons, average weekly hours duration, 1992=100, seasonally adjusted (from the Department of Labor).

Emp : civilian employment for 16 years and over, measured in thousands, seasonally adjusted (from the Department of Labor, Bureau of Labor Statistics, CE16OV). The series is transformed into an index where 1992Q3=100. Hours worked are then defined as

$$N = \frac{H * Emp}{100}$$

Wage rate

The wage rate is defined as the index for hourly compensation for nonfarm business, all persons, 1992 = 100, seasonally adjusted (from the U.S. Department of Labor).

Inflation

The gross inflation rate is defined using the price deflator for GDP (Table 1.1.4, line 1).

Interest rate

The nominal interest rate is defined as the average of daily figures of the federal funds rate (from the Board of Governors of the Federal Reserve System).

C.4

Definition of observables

The observable variable X is defined by making the following transformation to variable x :

$$X = \ln \left(\frac{x}{Popindex} \right) * 100$$

where Popindex is the index of Pop, constructed such that 1992Q3 = 1; Pop the civilian noninstitutional population in thousands, ages 16 years and over, seasonally adjusted (from the Bureau of Labor Statistics), LNS10000000; x = consumption, investment, hours worked, government spending, tax revenues. The real wage rate is defined in the same way, except that it is not divided by the total population.

C.5

Estimation Results

Parameters	Prior Mean	Prior Dist	Prior Stdv	1955Q1–1966Q4			1967Q1–1979Q2			1984Q1–2007Q4		
				Post. Mean	0.05	0.95	Post. Mean	0.05	0.95	Post. Mean	0.05	0.95
γ	0.005	norm	0.0005	0.005	0.004	0.006	0.005	0.004	0.006	0.005	0.005	0.006
ν	2	gamma	0.75	2.59	1.26	3.86	2.22	1.03	3.37	2.85	1.50	4.23
θ	0.5	norm	0.2	0.92	0.89	0.97	0.94	0.90	0.98	0.85	0.80	0.90
ψ	0.5	norm	0.2	0.33	0.03	0.59	0.32	0.09	0.55	0.43	0.22	0.64
s	4	gamma	1	3.83	2.40	5.21	4.23	2.71	5.68	5.50	3.93	7.05
ω_p	0.66	beta	0.1	0.87	0.79	0.94	0.70	0.63	0.77	0.94	0.91	0.98
ω_w	0.66	beta	0.1	0.73	0.63	0.82	0.73	0.64	0.82	0.86	0.80	0.92
χ_p	0.5	beta	0.152	0.27	0.08	0.45	0.45	0.26	0.64	0.26	0.07	0.45
χ_w	0.5	beta	0.152	0.46	0.24	0.69	0.55	0.33	0.78	0.39	0.17	0.60
ϕ_x	1.5	norm	0.25	1.72	1.37	2.09	1.63	1.28	2.00	1.89	1.52	2.30
ϕ_y	0.125	norm	0.05	0.14	0.06	0.22	0.16	0.09	0.23	0.14	0.08	0.21
γ_g	0.05	norm	0.05	0.02	-0.03	0.07	0.00	-0.03	0.04	-0.01	-0.03	0.02
γ_r	0.15	norm	0.05	0.12	0.05	0.19	0.11	0.05	0.18	0.05	0.04	0.06
ϕ_g	0	norm	0.1	0.02	-0.14	0.17	0.10	-0.05	0.25	0.07	-0.09	0.22
ϕ_z	0	norm	0.1	0.07	-0.02	0.16	0.06	-0.03	0.16	0.06	-0.04	0.17
ρ_a	0.7	beta	0.2	0.54	0.38	0.70	0.37	0.19	0.54	0.48	0.36	0.61
ρ_b	0.7	beta	0.2	0.33	0.11	0.54	0.41	0.19	0.63	0.93	0.87	0.98
ρ_r	0.7	beta	0.2	0.88	0.85	0.92	0.91	0.87	0.95	0.87	0.84	0.90
ρ_i	0.7	beta	0.2	0.67	0.45	0.90	0.65	0.34	0.95	0.87	0.77	0.96
ρ_p	0.7	beta	0.2	0.70	0.48	0.92	0.28	0.08	0.46	0.54	0.26	0.79
ρ_w	0.7	beta	0.2	0.40	0.17	0.63	0.43	0.16	0.70	0.26	0.11	0.41
ρ_g	0.7	beta	0.2	0.95	0.88	1.00	0.99	0.97	1.00	0.99	0.99	1.00
ρ_t	0.7	beta	0.2	0.96	0.94	0.99	0.88	0.75	0.99	0.90	0.83	0.97
ρ_z	0.7	beta	0.2	0.29	0.10	0.47	0.43	0.24	0.62	0.29	0.13	0.43
σ_a	0.1	invg	2	1.14	0.94	1.33	0.97	0.81	1.13	0.77	0.68	0.87
σ_g	0.1	invg	2	2.24	1.86	2.61	2.00	1.66	2.34	1.64	1.44	1.84
σ_m	0.1	invg	2	0.22	0.18	0.26	0.18	0.14	0.21	0.14	0.12	0.15
σ_z	0.1	invg	2	1.03	0.69	1.36	1.03	0.69	1.39	0.34	0.26	0.41
σ_b	0.1	invg	2	5.50	1.19	9.41	5.88	1.30	10.30	0.31	0.10	0.52
σ_p	0.1	invg	2	0.08	0.04	0.11	0.20	0.15	0.25	0.08	0.05	0.11
σ_w	0.1	invg	2	0.14	0.10	0.19	0.19	0.12	0.26	0.25	0.20	0.31
σ_t	0.1	invg	2	2.43	2.03	2.82	3.48	2.92	4.03	2.38	2.11	2.67
σ_z	0.1	invg	2	41.63	34.39	48.87	66.82	56.07	77.78	50.58	44.95	55.91

Table C.1: Estimation Results - First Specification

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Parameters	Prior Mean	Prior Dist	Prior Stdv	1955Q1–1966Q4			1967Q1–1979Q2			1984Q1–2007Q4		
				Post. Mean	0.05	0.95	Post. Mean	0.05	0.95	Post. Mean	0.05	0.95
γ	0.005	norm	0.0005	0.005	0.004	0.006	0.005	0.004	0.006	0.005	0.004	0.006
ν	2	gamma	0.75	2.58	1.29	3.89	2.17	0.96	3.31	3.25	1.68	4.79
θ	0.5	norm	0.2	0.92	0.87	0.97	0.93	0.89	0.97	0.84	0.79	0.90
ψ	0.5	norm	0.2	0.37	0.08	0.64	0.43	0.18	0.67	0.43	0.22	0.64
s	4	gamma	1	3.81	2.41	5.18	4.12	2.71	5.54	5.38	4.03	6.67
ω_p	0.66	beta	0.1	0.86	0.79	0.93	0.68	0.61	0.75	0.95	0.92	0.97
ω_w	0.66	beta	0.1	0.73	0.64	0.82	0.73	0.64	0.83	0.86	0.81	0.92
χ_p	0.5	beta	0.152	0.27	0.08	0.45	0.49	0.30	0.69	0.22	0.06	0.38
χ_w	0.5	beta	0.152	0.46	0.23	0.68	0.59	0.37	0.83	0.35	0.16	0.55
ϕ_z	1.5	norm	0.25	1.64	1.21	2.07	1.57	1.19	1.92	1.79	1.40	2.19
ϕ_y	0.125	norm	0.05	0.15	0.06	0.23	0.16	0.09	0.23	0.16	0.11	0.21
γ_g	0.05	norm	0.05	0.03	-0.02	0.08	0.00	-0.03	0.04	-0.01	-0.03	0.02
γ_r	0.15	norm	0.05	0.11	0.04	0.19	0.11	0.04	0.17	0.04	0.02	0.05
ρ_a	0	norm	0.1	0.54	0.38	0.71	0.38	0.20	0.56	0.49	0.38	0.60
ρ_b	0	norm	0.1	0.35	0.11	0.57	0.42	0.21	0.64	0.93	0.88	0.98
ρ_r	0.7	beta	0.2	0.88	0.84	0.92	0.91	0.87	0.94	0.87	0.84	0.90
ρ_i	0.7	beta	0.2	0.69	0.48	0.91	0.62	0.34	0.92	0.86	0.77	0.94
ρ_p	0.7	beta	0.2	0.72	0.52	0.94	0.27	0.09	0.46	0.58	0.37	0.79
ρ_w	0.7	beta	0.2	0.40	0.17	0.64	0.44	0.16	0.70	0.26	0.11	0.41
ρ_g	0.7	beta	0.2	0.96	0.90	1.00	0.99	0.97	1.00	0.99	0.99	1.00
ρ_t	0.7	beta	0.2	0.95	0.91	0.99	0.85	0.72	0.97	0.90	0.84	0.96
ρ_z	0.7	beta	0.2	0.33	0.13	0.53	0.58	0.40	0.76	0.24	0.10	0.37
θ_z	0.7	beta	0.2	0.51	0.19	0.85	0.49	0.18	0.82	0.55	0.23	0.86
θ_g	0.7	beta	0.2	0.47	0.15	0.78	0.48	0.17	0.98	0.52	0.27	0.77
σ_a	0.1	invg	2	1.14	0.94	1.33	0.99	0.82	1.14	0.78	0.68	0.88
σ_g	0.1	invg	2	2.26	1.88	2.63	2.07	1.71	2.40	1.65	1.45	1.84
σ_m	0.1	invg	2	0.22	0.18	0.26	0.18	0.14	0.21	0.14	0.12	0.15
σ_i	0.1	invg	2	1.02	0.69	1.33	1.06	0.71	1.39	0.35	0.28	0.42
σ_b	0.1	invg	2	5.05	1.36	8.66	5.16	1.30	8.70	0.29	0.11	0.48
σ_p	0.1	invg	2	0.07	0.04	0.11	0.20	0.16	0.25	0.07	0.05	0.10
σ_w	0.1	invg	2	0.14	0.09	0.19	0.19	0.12	0.26	0.25	0.20	0.31
σ_t	0.1	invg	2	2.43	2.03	2.84	3.46	2.86	3.99	2.39	2.11	2.66
σ_z	0.1	invg	2	42.84	35.29	49.86	70.73	58.72	82.20	50.22	43.44	56.71

Table C.2: Estimation Results - Second Specification