Pontifícia Universidade Católica do Rio de Janeiro

Departamento de Economia

Monografia de Final de Curso

Comparative Statics of a Cardinal Voting Mechanism

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Orientador: Vinicius Carrasco

06/06/2014

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"Declaro que o presente trabalho é de minha autoria e que não recorri para realizá-lo, a nenhuma forma de ajuda externa, exceto quando autorizado pelo professor tutor."

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“As opiniões expressas neste trabalho são de responsabilidade única e exclusiva do autor.”

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1. Introduction

1.1 Motivation

Ordinal voting systems have been the focus of economists since Kenneth Arrow first rigorously developed Social Choice Theory in his seminal monograph, *Social Choice and Individual Values* (1951). Although ordinal voting systems, such as Majority Rule, have become widespread for their simplicity and effectiveness, MR and other ordinal mechanisms do not account for the different intensities with which distinct individuals value the same issue. Considering all votes equal, insofar as preferences are concerned, has a troubling corollary: it is not possible to identify what the social optimum is, nor can we ever be sure that it will be implemented.

In order to achieve the social optimum, it is necessary to consider not the choice that is valued by most, but simply the choice that is most valued (by all). In upholding society's desire of equality, it is unfair to treat ardent and apathetic voters alike. It is impossible, however, to rely on a simple "truth-telling" mechanism, where heterogeneous individuals are asked their preference intensity over some range. Such a mechanism would clearly not be strategy-proof, since any non-indifferent voter would have an incentive to overstate the intensity of his preference. In this case, the results obtained by Majority Rule would be replicated.

A strategy-proof mechanism that aggregates preferences with different values must satisfy a set of properties, including compatibility and participation constraints. Moreover, so that one may satisfy the principles of equality and fairness held by modern democratic societies, no money transfers should be allowed to compensate for utility transfers. Although VCG mechanisms have been adapted to voting systems, such as in Tideman and Tullock (1976), and could appropriately capture voting intensity, they do not satisfy the fairness condition set above and are thus inapplicable to our problem.

In a setting where agents vote on N issues throughout time, the mechanism developed herein – Budget Voting (BV) – attempts to comply with these constraints by granting all agents an initial budget with K votes that is never replenished. Since any vote "spent" on a given issue decreases the vote-budget, agents can no longer overstate their preferences without consequence.

Budget Voting is by no means an optimal mechanism insofar as ex-ante and ex-post efficiency are concerned, but it does improve upon Majority Rule and some other voting mechanisms in particular settings. BV also provides us with a clear, implementable rule to determine preference intensity that carries no political stigma and does not rely on monetary transfers.

1.2 Related Literature

Our mechanism pertains to a set of models that have multidimensional settings with non-homogeneous preferences and non-transferable utility, since strategy-proof mechanisms that aggregate preference intensities, that are deemed fair and that are able to create Pareto improvements are contingent on these characteristics. Three papers which develop such models and closely resemble ours are Casella (2005), Jackson and Sonnenschein (2007) and Hortala-Vallve (2011).

Casella (2005) proposes a system of Storable Votes, where throughout time agents can choose to vote on an issue or abstain. When opting for the latter the agent "saves" a vote, which can be used to decide on other issues throughout time. Formally, Casella proposes a system in which I agents meet throughout T periods to take binary decisions – either maintaining the status quo or changing it in some manner. In every time period, each individual has his preference indexed by a parameter $v_{it} \leq |1|$ chosen from a continuous distribution *F(v)* symmetric around zero, and upon observing it, decides if to cast a vote on the the issue at hand. Whenever the vote is not used, it is "stored", and thus agents are able to accumulate votes that may be used on those matters that they regard as most important.

Our mechanism differs slightly from Casella's, since it does not allow individuals to develop an inventory of votes (it instead gives them a fixed budget that they can use throughout their "voting lives"). Furthermore, we initially assume only that agents' preference distribution has median zero, such that the probability of any agent being for or against any issue ex-ante is always the same. Nonetheless, BV shares Storable Votes' restriction to binary decisions and both mechanisms explore decisions over a time-dimension; agents do not know their full preference profile at the time of voting.

Jackson and Sonnenschein (2007) demonstrate that in a setting where agents are required to take several decisions (simultaneously or over time), linking these decisions by restricting agents from declaring preference intensities that do not reflect their preference distribution leads to Pareto improvements. This occurs because, as the number of decisions made increases, agents match their voting profiles with the frequency of preferences generated by their prior distribution. Truth telling, therefore, becomes a dominant strategy.

Our mechanism differs greatly from Jackson's and Sonnenschein's, since it does not require that the prior distribution of preferences be known. Moreover, whereas they create a rule that rations preference intensity declarations based on their frequency, addressing settings with a large number of issues (where issues $N \rightarrow \infty$), BV focuses on a simpler rule that does not rely on the law of large numbers. Since each of these two mechanisms is designed to deal with a distinct facet of a common problem, it is impractical to compare their efficiency.

Hortala-Vallve (2011) develops a system of "Qualitative Voting", where in a setting with a closed agenda each agent is supplied with a budget to vote on a number of issues that must be accepted or dismissed. Since all votes are cast simultaneously on all issues and by all agents, an individual disperses of his whole budget on the issues that he feels are relatively more important. Formally, in a setting with I voters and N binary decisions, QV compels agents to cast votes from their budget in a way that maximizes their expected utility. This mechanism Pareto dominates Majority Rule in its setting.

Qualitative Voting is descriptively and practically quite similar to our mechanism, differing only temporally; Budget Voting is essentially a dynamic extension of QV. Nonetheless, the implications on agents' actions of not knowing the prior distribution of preferences are not trivial, and the consequences of this dimensional extension are the object of this monograph.

2. Model

2.1 Setting

In this model, a *game* is a setting where I agents approve or refute N changes over T periods, with any one decision occurring exclusively over one period. Agents are each provided with a budget at the beginning of the game containing K votes, and at the end of each turn, their remaining budget is defined as $K_{in} = K - \sum_{t=1}^{n-1} k_{it}\,, n \in N$.

At the beginning of each period, all agents privately observe their preference profile θ_{it} drawn from a commonly known class of probability distributions with median zero and finite average. Furthermore, all agents observe their constant and commonly known discount rate $\delta \in (0,1]$ before deciding simultaneously how many discrete votes $k_{it} \in K_{in}$ to allocate, if any, for or against a given change. All agents are assumed to be rational utility maximizers who are risk-neutral across time.

If an agent values change positively, each vote cast by him will be tallied as $\{+1\}$ on a vote counter, while votes from agents who value change negatively will be tallied as {-1} on that same vote counter. After the end of each period, the value on the vote counter is observed and the choice *c* made over an issue assumes one of two values – $c \in \{0,1\}$ – where zero maintains the status quo, and one enacts change. In case of a tie, Majority Rule is used as tiebreak criteria and, if a tie is still at hand, a coin-toss is then used to determine whether change is enacted or not. Formally:

$$c = \begin{cases} 1, & if \ \sum_{i=1}^{I} k_{it} > 0 \\ 0, & if \ \sum_{i=1}^{I} k_{it} < 0 \end{cases}$$

Agent *i*'s utility in each period *t* can be represented by the function $U(c, \theta_{it}) = cu(\theta_{it})$, where $u(\cdot)$ is a well-behaved utility function that is constant over time and strictly increasing in θ, with $u(0) = 0$. If agent $i \in \{1, \ldots, I\}$ has a preference profile $\theta_{it} > 0$, he would prefer that *c* = 1 at period *t* and that change be enacted, while if $\theta_{it} < 0$, he would prefer *c* = 0 and that the status quo be maintained. Since agent *i* can only choose how many votes to cast, he can only affect *p(c)*, which we may interpret as the probability of change. Therefore, agent *i*' constrained maximization problem is:

$$\max_{k_t^i \,\forall\, t \in T} p(k_{i1}, k_{-i1}) \max\{u(\theta_{it}), 0\} + \big(1 - p(k_{i1}, k_{-i1})\big) \min\{u(\theta_{it}), 0\}$$
$$+ \sum_{t=2}^{T} p(k_{it}, k_{-it}) \max\{E[u(\theta_{it})], 0\}\, \delta^{t-1}$$
$$+ \sum_{t=2}^{T} \big(1 - p(k_{it}, k_{-it})\big) \min\{E[u(\theta_{it})], 0\} \delta^{t-1} \ s.t. \sum_{t=1}^{T} k_{it} = K$$

At the beginning of each new turn *t*, every agent *i* renews his maximization problem with his new preference profile θ_{it}, and decides how many votes to allocate on the issue at hand by comparing the utility of voting on present change *vis-à-vis* to the utility of storing votes for the future. An agent will vote on an issue if the marginal expected utility of an extra vote on that issue is larger than the marginal expected utility of saving said vote. Notice that this occurs only over *T – 1* periods, as in the last period – upon observing their preference profiles – all agents will dispense of all of their remaining votes.

2.2 Comparative Statics

To understand how Budget Voting works, it is pertinent to analyze how the mechanism functions in its simplest setting and how it reacts to small changes in its parameters. Consider, for the purpose of concreteness, an anecdotal situation in which a couple – *i* and *j* – distraught by its inability to resolve disagreements concerning the household, decides to implement Budget Voting. Suppose that they are currently concerned with whether or not to change their drapes, and assume initially that they wish to implement BV only through two periods with a budget endowment of two votes. Assume, initially, that agent *i* values change positively in *t = 1*, and that the expected utility of change is positive. Under these circumstances, the ex-ante expected utility in *t =1* under each possible voting pattern is:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{3}{4}p_{j1}(0) + p_{j1}(1) + p_{j1}(2)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + p_{j1}(2)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)$$

where $p_{jt}(x)$ is the probability that agent *j* will cast *x* votes in period *t*. Since we are only considering a two-period vote, $p_{j1}(x)$ is sufficient information to understand agent *j*'s voting strategy through all periods. Notice that we are not concerned with whether agent *j* votes for or against change, but only with the number of votes he uses.

It is simple to compare these three functions and find the players' optimal strategies. Notice that $E[u(\theta_{i2})]\delta > u(\theta_{i1}) \rightarrow E[U_{i1}(0)] > E[U_{i1}(1)] > E[U_{i1}(2)]$ and, similarly, $E[u(\theta_{i2})]\delta < u(\theta_{i1}) \rightarrow E[U_{i1}(0)] < E[U_{i1}(1)] < E[U_{i1}(2)]$. This demonstrates that when BV is applied under this specific set of assumptions, the allocation of agent *i*'s votes is independent of agent *j*'s voting pattern. Additionally, it is shown that under these restrictive hypotheses $E[U_{i1}(1)]$ is a weakly dominated strategy and is therefore unlikely to be played. It is natural to consider that an analogous situation would arise if $u(\theta_{i1}) < 0$, and the expected utility of change were negative. In this case, the ex-ante utilities would be:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{4}p_{j1}(0)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)$$

and, as expected, all that matters for voting decisions to be made is the relative value of current change *vis-à-vis* the expected utility of future change. We now have $E[u(\theta_{i2})]\delta > u(\theta_{i1}) \rightarrow E[U_{i1}(0)] < E[U_{i1}(1)] < E[U_{i1}(2)]$ while, on the other hand, $E[u(\theta_{i2})]\delta < u(\theta_{i1}) \rightarrow E[U_{i1}(0)] > E[U_{i1}(1)] > E[U_{i1}(2)]$. Consequently, whenever the utility of current and future change have the same sign – that is, both are seen as concomitantly good or bad – frontier solutions will ensue and the results of Majority Rule will be replicated. This will surprisingly also be the case if the utility of current change and the expected utility of future change have different signs. The ex-ante expected utilities when $u(\theta_{i1}) < 0$ and $E[u(\theta_{i2})] > 0$ are:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{3}{4}p_{j1}(0) + p_{j1}(1) + p_{j1}(2)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + p_{j1}(2)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)$$

while, when $u(\theta_{i1}) > 0$ and $E[u(\theta_{i2})] < 0$, they are:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{4}p_{j1}(0)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)$$

By solving for the equations above and comparing the results to those found previously, we find that in a simple, two-person, two-vote setting, BV always replicates MR, since: $|E[u(\theta_{i2})]|\delta > |u(\theta_{i1})| \rightarrow E[U_{i1}(0)] > E[U_{i1}(1)] > E[U_{i1}(2)]$ and, similarly, $|E[u(\theta_{i2})]|\delta < |u(\theta_{i1})| \rightarrow E[U_{i1}(0)] < E[U_{i1}(1)] < E[U_{i1}(2)]$. As is quite clear, this means that in its base scenario Budget Voting fully captures preference intensity and agents vote truthfully, since they always have a positive probability of affecting the result.

To further investigate Budget Voting, it is logical to look at how the mechanism's results change in extensions of the simple scenario studied above. Firstly, let us look at what happens when we increase agents' budgets to contain three votes, instead of the initial two, and when current and expected future change are valued positively. In this two-period, two-person, three-vote setting, the ex-ante expected utilities are:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{3}{4}p_{j1}(0) + p_{j1}(1) + p_{j1}(2) + p_{j1}(3)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + p_{j1}(2) + p_{j1}(3)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{3}{4}p_{j1}(2) + p_{j1}(3)\right)$$

$$E[U_{i1}(3)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + p_{j1}(2) + \frac{3}{4}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{3}{4}p_{j1}(3)\right)$$

whereas, when current change is valued negatively, yet the expected utility of future change is still positive, we shall have:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{3}{4}p_{j1}(0) + p_{j1}(1) + p_{j1}(2) + p_{j1}(3)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + p_{j1}(2) + p_{j1}(3)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{3}{4}p_{j1}(2) + p_{j1}(3)\right)$$

$$E[U_{i1}(3)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{3}{4}p_{j1}(3)\right)$$

When both current and expected future change are negative:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{4}p_{j1}(0)\right)$$
$$E[U_{i1}(1)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)$$
$$E[U_{i1}(2)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)$$
$$E[U_{i1}(3)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{4}p_{j1}(3)\right)$$

and finally, when current change is positive and expected future change is valued negatively:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{4}p_{j1}(0)\right)$$
$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)$$
$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2) + \frac{1}{2}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)$$
$$E[U_{i1}(3)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + p_{j1}(2) + \frac{3}{4}p_{j1}(3)\right) + E[u(\theta_{i2})]\delta\left(\frac{1}{2}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2) + \frac{1}{4}p_{j1}(3)\right)$$

Once having compared the expected utilities above in each of the four possible variants of a two-person, two-period, three-vote setting, it becomes quite clear that the additional votes available in agents' budgets to do not alter the results found previously. In this scenario agents' votes depend only upon the relation between the absolute value of the current utility of change and the discounted expected utility of future change. Once again, we shall have the rather simple result: $|E[u(\theta_{i2})]|\delta > |u(\theta_{i1})| \rightarrow E[U_{i1}(0)] > E[U_{i1}(1)] > E[U_{i1}(2)] > E[U_{i1}(3)]$ and $|E[u(\theta_{i2})]|\delta < |u(\theta_{i1})| \rightarrow E[U_{i1}(0)] < E[U_{i1}(1)] < E[U_{i1}(2)] < E[U_{i1}(3)]$. In fact, no matter the amount by which we increase agents' budgets in a two-person, two-period scenario, BV replicates the results of Majority Rule.

Although strategic voting does not occur in the base scenarios analyzed above, it may happen in Budget Voting when the environment becomes more complex. When a new agent enters the game, for example, the likelihood of any given vote being pivotal is reduced, and the cost of frontier solutions – in which all votes are cast in a single period – increases. Consider a scenario in which we have three agents, two periods and two votes. Once again, assume that agent *i* values change positively in the future. In this case, agent *i*'s ex-ante expected utility for each of his possible choices when the utility of current change is positive is:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{3}{4}\sum_{x=y}^{2} p_{k1}(x)p_{j1}(y)\,p_{k1}(0) + \frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x} p_{k1}(x)p_{j1}(y)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(1)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + p_{j1}(2)\right) + p_{k1}(2)\left(\frac{3}{4}p_{j1}(0) + p_{j1}(1) + p_{j1}(2)\right)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{k1}(0)\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(1)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right) + p_{k1}(2)\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + p_{j1}(2)\right)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{k1}(0)\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right) + p_{k1}(1)\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(\frac{3}{4}\sum_{x=y}^{2} p_{k1}(x)p_{j1}(y)\,p_{k1}(0) + \frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x} p_{k1}(x)p_{j1}(y)\right)$$

In this particular setting, the results of BV are:

$$E[U_{i1}(0)] > E[U_{i1}(1)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(2)\left(\frac{1}{4}p_{j1}(1)\right)\right)$$

$$< E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)\right)$$

$$E[U_{i1}(1)] > E[U_{i1}(2)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)\right)+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)\right)\right)$$
$$< E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$

$E[U_{i1}(0)] > E[U_{i1}(2)] \rightarrow$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{2}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)\right)$$
$$< E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$

By comparing each of the possible voting patterns above it becomes clear that Budget Voting may induce strategic voting. A player may choose to vote on a given issue (current or future) not because it is what he most values, but because he believes the likelihood of his vote being pivotal is sufficiently high for the issue at hand. Since preference intensity is no longer the only factor to which agents look before deciding how to vote, part (or all) of the welfare gains of Budget Voting over MR may be extinguished as the possibility of strategic voting rises.

Although an increase in the number of players in our model introduces the possibility of strategic voting, it also causes players to have greater incentives to spread out their votes: $E[U_{i1}(1)]$ is no longer a weakly dominated strategy. In fact, as the number of agents in the environment increases, "spreading the vote" becomes increasingly attractive (albeit not necessarily monotonically, since tie-break rules for even and odd numbers of players are applied somewhat differently. This result is upheld regardless of how present and expected future change are valued. Let us look first at the ex-ante expected utility when $u(\theta_{i1}) < 0; E[u(\theta_{i2})] < 0$:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{1}{4}\sum_{x=y}^{2} p_{k1}(x)p_{j1}(y)\,p_{k1}(0) + \frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x} p_{k1}(x)p_{j1}(y)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right) + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)\right)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(2)\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(\frac{1}{4}\sum_{x=y}^{2} p_{k1}(x)p_{j1}(y)\,p_{k1}(0) + \frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x} p_{k1}(x)p_{j1}(y)\right)$$

The results are:

$$E[U_{i1}(0)] > E[U_{i1}(1)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(2)\left(\frac{1}{4}p_{j1}(1)\right)\right)$$

$$> E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)\right)$$

$$E[U_{i1}(1)] > E[U_{i1}(2)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)\right) + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)\right)\right)$$

$$> E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left. + p_{k1}(2)\left(\frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right)$$

$$E[U_{i1}(0)] > E[U_{i1}(2)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{2}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)\right)$$
$$>E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$

Quite clearly, we have the familiar result that when maintaining the status quo is preferred both in the present and in the future, the result is – in absolute value – the same as that when change is preferred in both periods. Let us now analyze what happens when stability is preferred in the present, but is expected to be negative in other periods:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{1}{4}\sum_{x=y}^{2}p_{k1}(x)p_{j1}(y)\,p_{k1}(0)+\frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x}p_{k1}(x)p_{j1}(y)\right)$$
$$+E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{3}{4}p_{j1}(0)+\frac{3}{4}p_{j1}(1)+\frac{3}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(1)\left(\frac{3}{4}p_{j1}(0)+\frac{3}{4}p_{j1}(1)+p_{j1}(2)\right)+p_{k1}(2)\left(\frac{3}{4}p_{j1}(0)+p_{j1}(1)+p_{j1}(2)\right)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{2}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{2}p_{j1}(0)+\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$
$$+E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{3}{4}p_{j1}(0)+\frac{3}{4}p_{j1}(1)+\frac{1}{2}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(1)\left(\frac{3}{4}p_{j1}(0)+\frac{3}{4}p_{j1}(1)+\frac{3}{4}p_{j1}(2)\right)+p_{k1}(2)\left(\frac{1}{2}p_{j1}(0)+\frac{3}{4}p_{j1}(1)+p_{j1}(2)\right)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$
$$+E[u(\theta_{i2})]\delta\left(\frac{3}{4}\sum_{x=y}^{2}p_{k1}(x)p_{j1}(y)\,p_{k1}(0)+\frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x}p_{k1}(x)p_{j1}(y)\right)$$

The results are:

$$E[U_{i1}(0)] > E[U_{i1}(1)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(2)\left(\frac{1}{4}p_{j1}(1)\right)\right)$$

$$> -E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left.+\,p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)\right)$$

$$E[U_{i1}(1)] > E[U_{i1}(2)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)\right)+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)\right)\right)$$

$$> -E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left.+\,p_{k1}(2)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$

$$E[U_{i1}(0)] > E[U_{i1}(2)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{2}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left.+\,p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)\right)$$

$$> -E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(2)\right)\right.$$

$$\left.+\,p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$

And finally, when $u(\theta_{i1}) > 0; E[u(\theta_{i2})] < 0$, the ex-ante expected utilities are:

$$E[U_{i1}(0)] = u(\theta_{i1})\left(\frac{3}{4}\sum_{x=y}^{2}p_{k1}(x)p_{j1}(y)\,p_{k1}(0)+\frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x}p_{k1}(x)p_{j1}(y)\right)$$

$$+\,E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right.$$

$$\left.+\,p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)\right)\right)$$

$$E[U_{i1}(1)] = u(\theta_{i1})\left(p_{k1}(0)\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right.$$
$$\left. + p_{k1}(2)\left(\frac{1}{2}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right)$$
$$+ E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)\right.$$
$$\left. + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(2)\left(\frac{1}{2}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)\right)$$

$$E[U_{i1}(2)] = u(\theta_{i1})\left(p_{k1}(0)\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right) + p_{k1}(1)\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right.$$
$$\left. + p_{k1}(2)\left(\frac{3}{4}p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)\right)$$
$$+ E[u(\theta_{i2})]\delta\left(\frac{1}{4}\sum_{x=y}^{2} p_{k1}(x)p_{j1}(y)\, p_{k1}(0) + \frac{1}{2}\sum_{x=0}^{2}\sum_{y\neq x} p_{k1}(x)p_{j1}(y)\right)$$

Whose results are:

$$E[U_{i1}(0)] > E[U_{i1}(1)] \rightarrow$$
$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(2)\left(\frac{1}{4}p_{j1}(1)\right)\right)$$
$$< -E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left. + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(1)\right)\right)$$

$$E[U_{i1}(1)] > E[U_{i1}(2)] \rightarrow$$
$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)\right) + p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)\right)\right)$$
$$< -E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)\right) + p_{k1}(1)\left(\frac{1}{4}p_{j1}(0) + \frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left. + p_{k1}(2)\left(\frac{1}{4}p_{j1}(1) + \frac{1}{4}p_{j1}(2)\right)\right)$$

$$E[U_{i1}(0)] > E[U_{i1}(2)] \rightarrow$$

$$u(\theta_{i1})\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{2}p_{j1}(0)+\frac{1}{4}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{4}p_{j1}(1)\right)\right)$$
$$<-E[u(\theta_{i2})]\delta\left(p_{k1}(0)\left(\frac{1}{4}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)+p_{k1}(1)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(2)\right)\right.$$
$$\left.+p_{k1}(2)\left(\frac{1}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{4}p_{j1}(2)\right)\right)$$

Alas, it is quite clear that regardless of how current and future change are valued, in a three-person, two-period setting, Budget Voting sets incentives for strategic voting and reduces the odds of agents pooling all of their votes in one single period. Moreover, it is interesting to consider that in this setting, if agents were to be risk-averse throughout time, they would be more likely to cast all of their votes on one single period.

The final development of the base scenario that we will study is that of a temporal extension. When Budget Voting is applied to more than two periods, agents are faced with a higher opportunity cost when casting a vote, and are therefore more likely to store votes for the future. It is also interesting to note that within a three-period setting, players will be required to solve two optimization problems (although we are primarily concerned with the one that occurs in $t = 1$). From now on, let us always consider that the expected utility of change is positive. In a two-person, three-period, two-vote scenario in which $u(\theta_{i1}) > 0$, the ex-ante expected utility for each possible voting pattern is:

$$E[U_{i1}(0,0)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0)+\frac{1}{2}p_{j1}(1)+\frac{1}{2}p_{j1}(2)\right)$$
$$+E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0)+\frac{1}{2}p_{j2}(1)+\frac{1}{2}p_{j2}(2)\right)\right.$$
$$\left.+p_{j1}(1)\left(\frac{3}{4}p_{j2}(0)+\frac{1}{2}p_{j2}(1)\right)+p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$
$$+E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0)+p_{j2}(1)+p_{j2}(2)\right)+p_{j1}(1)\left(p_{j2}(0)+p_{j2}(1)\right)\right.$$
$$\left.+p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$E[U_{i1}(0{,}1)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\Bigg(p_{j1}(0)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right)$$

$$+ p_{j1}(2)\left(p_{j2}(0)\right)\Bigg)$$

$$+ E[u(\theta_{i2})]\delta^2\Bigg(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1) + p_{j2}(2)\right) + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + p_{j2}(1)\right)$$

$$+ p_{j1}(2)\left(p_{j2}(0)\right)\Bigg)$$

$$E[U_{i1}(0{,}2)] = u(\theta_{i1})\left(\frac{3}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\Bigg(p_{j1}(0)\left(p_{j2}(0) + p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + p_{j2}(1)\right)$$

$$+ p_{j1}(2)\left(p_{j2}(0)\right)\Bigg)$$

$$+ E[u(\theta_{i2})]\delta^2\Bigg(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right)$$

$$+ p_{j1}(1)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\Bigg)$$

$$E[U_{i1}(1{,}0)] = u(\theta_{i1})\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\Bigg(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right)$$

$$+ p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\Bigg)$$

$$+ E[u(\theta_{i2})]\delta^2\Bigg(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1) + p_{j2}(2)\right) + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + p_{j2}(1)\right)$$

$$+ p_{j1}(2)\left(p_{j2}(0)\right)\Bigg)$$

$$E[U_{i1}(1,1)] = u(\theta_{i1})\left(p_{j1}(0) + \frac{3}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$E[U_{i1}(2,0)] = u(\theta_{i1})\left(p_{j1}(0) + p_{j1}(1) + \frac{3}{4}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}(1) + \frac{1}{2}p_{j2}(2)\right) + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

Clearly, as with when we have more than two agents, an extension over more than two periods gives players incentives to vote on more than one issue. Observe that votes may be cast in the third period, even though the utility of casting votes then is always inferior to that of casting votes in the second period (since the expected utility of change is the same, and the discount factor $\delta \in (0,1)$). Let us now look at the ex-ante expected utilities of agent *i* when the utility of current change is negative:

$$E[U_{i1}(0,0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0) + p_{j2}(1) + p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$E[U_{i1}(0,1)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1) + p_{j2}(2)\right) + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$E[U_{i1}(0,2)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(0) + \frac{1}{2}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(p_{j2}(0) + p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$E[U_{i1}(1,0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1) + p_{j2}(2)\right) + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$E[U_{i1}(1,1)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(1) + \frac{1}{2}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1) + \frac{1}{2}p_{j2}(2)\right) + p_{j1}(1)\left(p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$E[U_{i1}(2,0)] = u(\theta_{i1})\left(\frac{1}{4}p_{j1}(2)\right)$$

$$+ E[u(\theta_{i2})]\delta\left(p_{j1}(0)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}(1) + \frac{1}{2}p_{j2}(2)\right) + p_{j1}(1)\left(\frac{3}{4}p_{j2}(0) + \frac{1}{2}p_{j2}(1)\right)\right.$$

$$\left. + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

$$+ E[u(\theta_{i2})]\delta^2\left(p_{j1}(0)\left(\frac{1}{2}p_{j2}(0) + \frac{1}{2}p_{j2}(1) + \frac{3}{4}p_{j2}(2)\right)\right.$$

$$\left. + p_{j1}(1)\left(\frac{1}{2}p_{j2}(0) + \frac{3}{4}p_{j2}(1)\right) + p_{j1}(2)\left(\frac{3}{4}p_{j2}(0)\right)\right)$$

Once again, the results that originate from a temporal extension demonstrate that Budget Voting is susceptible to “interior” solutions, in which votes are cast on more than one issue. This does not occur due to a diminishing probability of being pivotal – as when we increase the number of players in the game – but due to the rising opportunity cost of casting a vote on any given period, and the effects of this are not necessarily negative. Clearly, as the number of periods increases, agents will have greater incentives to store votes. Therefore, as they progress through the game, they become more propense to spend their votes at each given turn. This culminates – in the next-to-last period – in sincere voting and frontier solutions (if there are only two agents) or in one of the prior investigated extensions of the mechanism.

3. Conclusion

This monograph has sought to understand a very simple voting mechanism that seeks to capture preference intensity for groups of people that meet repeatedly over time. Agents are given an initial budget of votes to be used along a certain period throughout which numerous decisions with binary outcomes are made. This results in voters casting a greater portion of their budget on issues whose outcomes are more strongly felt by them, while also producing incentives for voters not to overstate their preferences. Since agents now have the possibility of transferring votes from periods of weak preference intensity to periods of strong preference intensity, ex-ante welfare should increase.

Although agents are always more likely to vote on issues that are felt as being relatively more important to them than others, as the number of voters and periods increases, incentives to spread votes through different issues rise. While an increase in the number of voting periods increases the opportunity cost of votes – which is not a negative trait in and of itself – the increase in the number of voters makes players' votes less likely to be pivotal, and henceforth augments the probability of strategic voting. It is suggested, both by our analysis as that of other papers concerning cardinal voting mechanisms, that the likelihood of strategic voting is diminished as the number of voting periods increases and as the variance of preferences is held to a certain limit.

However simple the description of Budget Voting may be, the mechanism is quite prone to becoming extremely complicated as the number of agents and periods increases, and a practical issue surges as to whether individuals would be able to find the equilibrium strategies that would allow it to increase welfare over ordinal voting mechanisms (especially Majority Rule). It is thus the object of the author to pursue the practical and theoretical efficiency gains of BV over other mechanisms in the future, especially seeing as Budget Voting provides us with a simple, implementable voting mechanism that seems likely to increase general welfare.

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