Affirmative Action in College Admission: long-run implications for efficiency and inequality *

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Abstract

This paper investigates the effects of color-blind affirmative action on welfare, inequality, GDP, investment and intergenerational income correlation. We construct an OLG model with four generations in which parents choose how to invest in their child’s education: which school/college to attend and how much to invest. Motivated by the fact that the main Brazilian universities are going to implement affirmative action in college admission based on income and high school attended, we calibrate the model to match key moments of Brazilian data prior to the implementation to evaluate the potential effects of such policy. We compare two different affirmative action models, the introduction of quotas and the introduction of bonus points on the admission exam. We find that both policies can reduce the intergenerational correlation of earnings, but the final effect on inequality, welfare and GDP depends on the magnitude of the policy. A quota of small magnitude for students in the bottom 20\textsuperscript{th} percentile of the income distribution can increase GDP up to 1.13%.

\textit{JEL: I2, E24, J62.}

\textit{Keywords: Affirmative Action, Intergenerational Mobility, Inequality, Educational Investment.}

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“You do not take a person who, for years, has been hobbled by chains and liberate him, bring him up to the starting line of a race and then say, ‘you are free to compete with all the others,’ and still justly believe that you have been completely fair.

... To this end equal opportunity is essential, but not enough, not enough. Men and women of all races are born with the same range of abilities. But ability is not just the product of birth. Ability is stretched or stunted by the family that you live with, and the neighborhood you live in – by the school you go to and the poverty or the richness of your surroundings. It is the product of a hundred unseen forces playing upon the little infant, the child, and finally the man.”

1 Introduction

Affirmative action (AA) is an important and controversial policy, intended to diminish the disparity in earnings and employment among different social groups. Even though the intentions that support the introduction of AA on college admission process are quite clear, its effects on incentives and economic performance are subject to much debate. On one hand, supporters claim that it levels the playing field: the introduction of AA motivates preferential-treatment students either to exert more effort or to further invest in education as it increases the payoff of costly effort investments for the beneficiaries of the policy. Another argument often evoked by advocates of AA is that larger representation of minorities in high-paying jobs may generate positive role models, influencing future generations and reducing prejudice (Chung 2000). On the other hand, critics of AA argue precisely the opposite, highlighting the possibility of the perverse “patronizing equilibrium” (Coate and Loury 1993, Moro and Norman 2003, Fang and Norman 2006). As a result of the lowered admission standards, AA diminishes incentives for the preferential-treatment students to exert effort in the study process, thus leading to poor economic performance. As discussed by Fryer and Loury (2005), it is theoretically not clear how AA affects incentives.

The general equilibrium effects of AA policies are crucial in order to accurately evaluate the welfare implications as well as implications to inequality, output, earnings persistence between generations, educational investments, etc. Typically, the preferential treatment groups (AA policy beneficiaries) are endogenous. For instance, preferential treatment groups based on type of high school attended (public versus private) or on parents’ income are ultimately endogenously determined by the child’s parents investments at earlier stages of their own lives. Thus, when an AA policy is implemented, parents will take that into account when choosing how to invest in their child’s education. Whenever the preferential treatment group is endogenous, it is imperative to incorporate general equilibrium effects as well as parental educational investment when analyzing the economic consequences of such policy.

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2 There are examples in which the preferential treatment group is not endogenous. Groups based on race or castes are two of such examples.
The main contribution of our paper to the policy debate is in quantifying the impacts of different color-blinded affirmative action policies in economic performance outcomes, such as inequality, intergenerational persistence of earnings, GDP and welfare. We consider two preferential treatment groups: low income students and students from the public school. There are many interesting questions to be answered. (i) What effect does AA have on income inequality: Is AA only beneficial to the already privileged among the preferential-treatment group? (ii) What effect does AA have on the inter-generational persistence of earnings? (iii) What effect does it have on the efficiency of the economy? How does it affect GDP? (iv) What effect does AA have on effort incentives: does it encourage families to invest more or less in education? Does it affect all families (preferential-treatment or not) in the same way? (v) What effect does AA have on human capital accumulation: Does AA policies impact on one generation lead to less constrained educational investment decision for future generations? (vi) How can one compare the different AA policies in terms of the above criteria?

In this paper, we use a four overlapping generation model to investigate the long-term implications of affirmative action on college admission. Hence, our model includes general equilibrium effects and educational investment decisions made by the parents. In our model, each agent lives four periods: young child, old child, young parent and old parent. A straightforward interpretation is that each period corresponds to 16 to 18 years. At each point in time, a family is composed of two members, one parent and one child. There is no credit or saving market between periods. The interpretation is that there is perfect credit market within the 16 years period, however parents cannot save nor borrow against future periods’ income. A child is defined by her innate ability, which is randomly determined at her birth and depends on her parent’s innate ability. All decisions are made at the family unit level. The young parent considers her human capital, her innate ability and her child’s innate ability and has to choose consumption level, how much to invest in the child’s education and whether to educate her child in a public or private school. Public schools receive government funds, but are less efficient to handle parent’s private investments. The government budget is balanced period by period.

The old parent considers her human capital, her innate ability and her child’s acquired and innate abilities and has to choose consumption level, and whether to send the old child to college. College admission is a random process, in which an old child with higher acquired ability have higher probability of admission. The interpretation for such admission process is that there exists an exam, which is a noisy measure of a child’s acquired ability, the child is admitted to the high quality college if and only if she is among the top scorers in the exam. If the old child is not admitted, we consider that the old parent can still invest in the child education by enrolling her in a private college. We calibrate the model to match Brazilian data and use such parameter specification to evaluate different admission policies.

Motivated by a recently approved law in Brazil, we consider affirmative action on admission
policies for two different preferential-treatment groups. (i) public school – an old child that as a young child studied in the public school; and (ii) poor – An old child whose parent has income below a certain threshold.

We find four main results. First, we show that the quotas and extra points policies have the same effects. Quotas or extra point that generates the same increase in the preferential-treatment group admissions’ share have equal effects on equilibrium variables.

Second, for public school students as preferential-treatment group, the policy effects are limited. There is a general equilibrium effect that offsets potential the impact of the policy. The bonus to students from the public school system makes parents enroll their children in the public school (even if it’s inefficient), because it increases the likelihood of college admission.

Third, for low income students as preferential-treatment group, the policy has very different effects depending not only on the magnitude but also on the precision of the target group. If one considers bonus point to the bottom 20% of the income distribution the effects are quite different than if one considers the bottom 60%.

First, the policy monotonically decreases the intergenerational correlation of earnings with maximum decrease of 50%. The final impact in GDP depends on the magnitude of the AA policy. There are two key forces driving output. First, the pool of students admitted changes: for AA policy of small magnitude the pool improves, but if the policy is large the admitted students’ quality decreases. Second, we find that AA policy will increase competition among the student not in the preferential treatment group and will deincentivize their parents’ investments in early education. In equilibrium, there is an over investment in early education, so the AA policy corrects this over investment to some extent. However, if the policy is implement is large scale, parents may end up under investing in early education. When the bonus to low income students is is not too high, then there can be an increase in GDP up to 1.13% when the preferential treatment group are the bottom 20% of income distribution.

The effect on inequality also depends on the magnitude of policy. On one hand, it decreases the intergenerational correlation of earnings, which decreases inequality. On the other hand, it can make the admission process more efficient which increase the dispersion in earnings. For a policy of small magnitude, these two effects are offsetting each other, but the former effect dominates. If the magnitude of the policy is high enough, the admissions becomes more inefficient and the two effects complement each resulting in a significant decrease in inequality (up to 7.56%).

Finally, AA policies can have meaningful welfare implications. The AA policy increases welfare not only by the efficiency channel that increases GDP, but also through a different channel as well. For the low income as the preferential treatment group, if the policy is large (large quotas of bonus points), then many low income students will attend college and these agents will receive a human capital boost. These are low income students, so their marginal utility is large and, as a result, the social welfare gains from these agent offset the welfare loss from the students crowed
out from the admission’s market.

**Related Literature**  Recent set of studies (Hickman 2010, Hickman 2013, Assunção and Ferman 2013) empirically analyze the incentive consequences of AA. Assunção and Ferman (2013) use a difference in differences procedure and find that AA desincentivizes effort for the preferential-treatment students, increasing the achievement gap. Their result depends on the magnitude and type of policy implemented. Hickman uses a semiparametric identification and estimates a structural model that considers admissions as an all-pay-auction. The paper’s main conclusion is that in the absence of AA, there would be significant drop in preferential-treatment college graduation rates and large increase in income inequality.

There exists as well a vast literature analyzing the effects of banning affirmative action in college admissions. Chan and Eyster (2003) analyze it in an exogenous skill distribution setting, while Fryer and Loury (2007) endogenizes it and thus try to understand at which stage of the human capital acquisition process should AA be in place. Finally Epple, Romano, and Sieg (2008) analyze it in a general equilibrium framework, in which colleges compete for students as well. The lesson to be learned is that a ban on AA would sharply diminish the enrollment of preferential-treatment students, but the magnitude of it depends on which stage of human capital accumulation the policy is inserted. Arcidiacono (2005) tries to empirically identify the impact of a ban on affirmative action. The paper conclusion is that, although the impact of such ban would reduce the educational outcome of the preferential-treatment group, the effects on earnings are not clear. Our paper endogenizes the cost of exerting effort, depending mainly on the family income. That is, when facing a credit constraint, the utility cost of investing in education is quite different depending on the family’s wealth. Also, the impact of the AA will depend on how credit constrained the families actually were.

**Affirmative Action in Brazil**  Brazilian educational system is characterized by low quality public schools and high quality free public universities. Even though the vast majority of students in Brazil attend public schools, only a very small fraction of them are admitted to public universities. Most of the spots in public universities are occupied by students who are relatively wealthy and studied in private schools. Indeed, 15% of the public university spots are occupied by students from the top 3% of the income distribution. The admission process is major specific, and there is a vast difference on admission grade cutoffs between majors. In order to be admitted to a public university, the student must be among the top scorers of the admission exam (called *Vestibular*).

If we consider São Paulo University (USP), although 85% of São Paulo high school students

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3University of São Paulo is Brazil’s best university, usually ranked among the top 250 in the world
Table 1: Admissions’ income statistics from FUVEST

This table displays family income admission statistics regarding the 2005 FUVEST exam. The first line indicates the percentage of candidates with family income of less than R$500 a month. The second one, between R$500 and R$1500, and so it follows. The first column reports the fraction of students taking the admission exam, in each income level. The third columns shows the fraction of in fact admitted. The last two column are specifically for medical school. This data considers only FUVEST exam, which is used in the University of São Paulo (USP) admissions.

<table>
<thead>
<tr>
<th>Applicants</th>
<th>Admitted</th>
<th>Applicants (Med School)</th>
<th>Admitted (Med School)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 500</td>
<td>8%</td>
<td>3%</td>
<td>5%</td>
</tr>
<tr>
<td>500 to 1500</td>
<td>31%</td>
<td>17%</td>
<td>19%</td>
</tr>
<tr>
<td>1500 to 3000</td>
<td>23%</td>
<td>25%</td>
<td>23%</td>
</tr>
<tr>
<td>3000 to 5000</td>
<td>17%</td>
<td>22%</td>
<td>19%</td>
</tr>
<tr>
<td>More than 5000</td>
<td>22%</td>
<td>34%</td>
<td>34%</td>
</tr>
</tbody>
</table>

attended public schools, only 39% ⁴ of those who took the admission exam to USP graduated from public schools. Furthermore, only 23% of the students admitted to USP attended public schools, and if one focus on Medicine, the most prestigious and demanded major, that percentage is reduced to only 5%. Table 1 shows the income distribution of USP applicants and admitted students. Given that the admission process is based solely on the exam, the reason why public school students are admitted less often is because of the poor quality of Brazilian public schools. Given that black and brown tend to be poorer than white students, it is no surprise that black and brown students are also vastly underrepresented in public universities.

In August 2012, Brazil’s National Congress approved a law that imposes a quota scheme ⁵ on Federal University admissions, major by major. The law mandates that 50% of students admitted to each major in Federal Universities must have studied in public schools. Furthermore, a fraction of those 50% should be reserved to black and brown students, following the racial distribution in the state the university is located, and finally half of those seats must be assigned to black and brown poor ⁶ students. In 2005, the Federal University of Bahia (UFBA) introduced a similar quota system: The fraction of students from public school increased 50%, from 33.8% to 51%, while in some selected majors it increased more than 400% (e.g. in architecture it increased from 10.7% in 2004 to 43.7% in 2005).

2 Model

We build a four overlapping generations model based on Restuccia and Urrutia (2004). Each agent lives 4 periods: 2 as a child (young child and old child) and 2 as an adult (young parent and old parent). At any point in time, each generation is a continuum of measure 1, and a family unit is composed by one parent and one child. Thus, there are two kinds of family units: one

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⁴ Data from University Foundation for Vestibular (FUVEST), regarding the 2005 admission process.
⁵ Federal law 12.711, August 29th 2012.
⁶ Monthly income less the 1.5 of the minimum wage, which is roughly less than US$500 a month per person.
with a young child and a young parent and another with an old child and an old parent. There is no population growth. Agents are heterogeneous in their innate ability levels, specifically a young child is born with an innate ability level that depends on her parent’s innate ability through an AR(1) process.

The decision is made at the family level. Parents make the consumption and educational investment choices by maximizing the welfare of the family. Hence, at any given time, only young and old parents are making decisions. A particular young parent can be identified by her human capital and her young child’s innate ability. She inelastically supply her human capital services in the labor market and decides how to allocate her labor income between consumption and investment in her young child early education. Besides choosing early education investment and consumption, the young parent also has to decide whether her child will attend public or private early education. On one hand, public school has government subsidy, but has a lower marginal return to early education investment. On the other hand, private school has no government subsidy, but has a higher marginal return.

In the model, early education is interpreted as primary and secondary education. All young child attend early education (no high school drop-out), and, by doing so, they build their own acquired ability which is a combination of their innate ability and the early education investment made by the young parent. Acquired ability directly affects the college admission likelihood and it also matters for future human capital of the child. Acquired ability can be interpreted as knowledge acquired in school, this knowledge directly affects the child’s college admission and it’s related to future labor earning, but it is not purely human capital. Another interpretation for acquired ability is that it’s part of the expected human capital that can be (noisily) measured by an admission’ processes such as exams, SAT scores, recommendations, among others methods. Therefore, our model disentangles acquired ability from human capital: admission’s likelihood is directly related to acquired ability, while labor earnings (wages) are directly defined by the human capital (productivity). This distinction combined with imperfect credit market is the source of inefficiency in equilibrium and it is crucial for affirmative actions policy, since the admission’s process is based on acquired ability rather than solely on expected human capital.

A particular old parent can be identified by her human capital, her old child acquired ability, her old child innate ability and whether her old child has attended private or public school. The old parent chooses how to allocate the family resources, i.e. the labor income from the old parent and the old child when working, between consumption and investment in college education for the old child. The old child can either apply to college or not apply to college. If the old child doesn’t apply to college, then she enters the labor market with her current human capital which is a combination of her innate ability and her acquire ability. If the old child applies to college, then she spends some time applying and will be admitted to the public college with certain probability.

There are two types of colleges in the model: public and private college. Public college is fully
sponsored by the government (tuition-free). However, private and public college have different qualities, i.e. the human capital gain is different when the old child goes to private or public college. As a result of our calibration, public college has a higher quality than the private one; hence, for expositional purposes, we describe the model with the public college being of higher quality. The public college has a fixed measure of spots to be filled in, and, to this end, a noisy signal of the old child acquired ability (admission’s score) is observed by the university and only the top scorers will be admitted. If the old child is not admitted to the public college, she can pay the full tuition cost and attend a private college. The affirmative action will affects the admission’s probability, thus the admission’s probability may depend not only on the acquired ability, but also on the type of affirmative action implemented. Upon admission, the old child can choose between attending public or private college, however, if she’s not admitted, the old child can only attend private college. If the old child does go to college, then she gets a human capital gain from the college education, this gain is different for private and public college.

Each period of the model is interpreted as a 16 to 18 years period, so households face perfect credit market within period: there is only one budget constraint per period. However, there is no credit/savings markets between periods. The economy’s production side is relatively simple, and there is one non-storable good that is produced according to a constant returns to scale technology. There is no capital accumulation and human capital is the only input. Finally, public “tuition-free” education is financed by labor income tax through a budget balanced government.

**Equilibrium Inefficiency and Affirmative Action Mechanism**

The source of inefficiency is that college admission is based on acquired ability, and not directly on expected human capital. Young children will receive early education investments by their parents, and this will result in an acquired ability level when she becomes an old child next period. The admission process is based on the the acquired ability, but the old child future earnings is based on her human capital which is a combination of her acquired ability and her innate ability.

Thus a wealthy parent will optimally invest in her child’s early education not only to maximize future earnings by an increase in acquired ability, but also to increase the probability of being admitted to public college. Acquired ability increases the chance of being admitted and thus reduces the risk of early education investment. As a result, the admission’s market will be highly competitive and low income students will be crowded out from this market, even if they have relatively high expected human capital.

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7This assumption can be relaxed by assuming the the government only finances a fraction of the tuition. Our quantitative and qualitative results are robust to this change.

8In the data (2008 INEP report, tables 7, 9 and 11), there are about 7 times more students applying than public college spots available, while, for private college, the ratio of applicants to private college spot is about 1.3. Moreover, only 5% of the public college spots are unfilled, while 50% of the private college spots are unfilled. Based on this data, we assume that no admission’s score is needed to attend private college.
The equilibrium constrained inefficiency in the model has two sources. First, wealthy parents over invest in early education to increase their children admission’s probability, which increases the admission’s cutoff grade. Second, this over investment in early education will drive the grade point cutoff up, and, as a result, low income children will be crowded out from the admission’s market. This is a pecuniary externality through the admission’s market price: the grade point cutoff.

The affirmative action mitigates this pecuniary externality. AA policies will have two main effects in this context. First, it will make the admission’s market more competitive for the children not in the treatment group (e.g. high income). Second, the treatment group children with the highest acquired ability levels have higher changes of admission in the public college. The policy will substitute the lowest acquire ability children not in the treatment group by the highest acquired ability children in the treatment group. Therefore, the AA policy can potentially improve the pool of students in college. Of course, if the AA policy is too extreme, it can worsen the pool of students as well. We show in the policy evaluation section, the AA policy can increase GDP and welfare; however, if massively implemented, such policy can have have very negative effect on GDP and welfare.

Young Parents Problem   The state variables of a young parent decision problem are given by her human capital and her young child innate ability, \( x_y = (h_y, \pi) \). The young parent considers as given and exogenously determined the wage rate, \( w \), the tax rate implemented by the government, \( \tau \), and the amount spent by the government in early education, \( g \). The young parent chooses to allocate her labor income between consumption \( (c_y) \) and early education investment \( (e) \). Furthermore, she also choose whether the young child will attend public \( (hs = PU) \) or private school \( (hs = PR) \). Given the young child innate ability \( (\pi) \), the young child acquired ability \( (\hat{\pi}) \) will depend on the school attended and on the early education investment according to:

\[
\hat{\pi} = \pi^{\gamma} \hat{e}^{1-\gamma}
\]

where

\[
\hat{e} = \begin{cases} 
\alpha (e + g)^{\kappa} & \text{if } hs = PU \\
 e^{\kappa} & \text{if } hs = PR
\end{cases}
\]

is the the effective early education investment, \( \hat{e} \) is the investment in education received by an young child, depending on whether her parents chose to enroll her in public or private school and on how much did her parents invest in her education.

The parameter \( \gamma \in [0, 1] \) is the innate ability weight in the acquired ability formation.

\( \alpha \in [0, 1] \) represents the public school inefficiency in handling private investments. For low investment, \( e \), the benefit of early education expenditure is higher in the public school. For
higher investments, the private school has higher benefit. $\gamma$ represents the innate ability weight on acquired ability formation. For instance, if $\gamma = 1$, investment in education would have no impact on acquired ability, but, if $\gamma = 0$, acquired ability would depend solely on early education investment.

The young parent budget constraint is given by

$$c_y + e = (1 - \tau) wh_y$$  \hspace{1cm} (2)

where $\tau$ is the labor income tax, $h_y$ is the young parent human capital and $w$ is the wage rate.

Finally, there is a linear relationship between human capital when one is young and when one is old. This captures a life-cycle profile of earnings, however it also implies that the life-cycle profile of earnings is the same across social stratus. The old parent human capital is given by

$$h_o = \xi h_y$$  \hspace{1cm} (3)

where $\xi \geq 1$ is a life cycle parameter.

Formally, the young parent problem is given by

$$V_y(h_y, \pi) = \max_{c_y \geq 0, e \geq 0, h_s \in \{PU, PR\}} u(c_y) + \beta V_o(h_o, \pi, \hat{\pi}, h_s)$$

subject to equations (1), (2) and (3).

In the old parent value function, both the old child acquired ability and the old child innate ability are state variables. The child innate ability appears in the old parent value function for two reasons (i) it affects the benefit of having a college education and (ii) it affects the innate ability distribution of future generations. The acquired ability enters as a state variable, because it affects the college admission’s likelihood. Observe also that whether the young child has studied in the private or in the public school is also a state variable, because it affects the admission’s likelihood under the affirmative action towards public school students. If there were no affirmative action, then that would not be the case as the acquired ability aggregates all the information needed in to college admission.

**Old Parents Decision Problem**  The state variables of an old parent decision problem are given by her human capital, the old child acquired ability, the child innate ability and the school system chosen in the previous period, $x_o = (h_o, \pi, \hat{\pi}, h_s)$. Again, the parent consider as given and exogenously determined both the wage rate, $w$, and the labor income tax rate implemented by the government, $\tau$. The innate ability follows an AR(1) process:

$$\log(\pi') = \rho \log(\pi) + \epsilon, \epsilon \sim N(0, \sigma^2_\pi)$$  \hspace{1cm} (4)
The old parent decision problem can be summarized in four steps. First, the old parent decides if her child will apply to college or not. Second, had she decided to apply, the outcome of the admission process is realized, and the old parent learns if her child was accepted to public college or not, and, if admitted, the old child attends public college. Third, if her child was not admitted to the public college, the old parent can choose between enrolling her child in a private college or not. Fourth, the old parent chooses how much to consume.

In the first step, the old parent decision problem is given by:

$$V_o(h_o, \pi, \hat{\pi}, h_s) = \max \left\{ V_o^{\text{not apply}}(h_o, \pi, \hat{\pi}), V_o^{\text{apply}}(h_o, \pi, \hat{\pi}, \theta) \right\}$$  \hspace{1cm} (5)$$

where $\theta$ is the admission’s outcome, $\theta = 1$ if admitted and $\theta = 0$ if not admitted to public college, $V_o^{\text{not apply}}(h_o, \pi, \hat{\pi})$ is the value of not applying, and

$$V_o^{\text{apply}}(h_o, \pi, \hat{\pi}) = q(\hat{\pi}, \bar{q}, h_s) V_o^{\text{admitted}}(h_o, \pi, \hat{\pi}) + (1 - q(\hat{\pi}, \bar{q}, h_s)) V_o^{\text{not admitted}}(h_o, \pi, \hat{\pi})$$

is the expected value of applying, where $q(\hat{\pi}, \bar{q}, h_s)$ is the probability of being admitted. $\bar{q}$ is the grade point cutoff and will be discussed in details in the next subsection. If the old child if admitted to public college, then she will go to the public college. However, if not admitted, then the old parent will have to choosing between going to the private college and not doing college at all. Next, we will discuss each one of these continuation values.

The value of not applying is given by:

$$V_o^{\text{not apply}}(h_o, \pi, \hat{\pi}) = \max_{c_o \geq 0} u(c_o) + \beta \mathbb{E}_{\pi'} \left[ V_y(h'_y, \pi') \right]$$

s.t.

$$c_o = (1 - \tau) \left( w h_o + w h'_y \right)$$

$$h'_y = \bar{\pi} \psi \hat{\pi}^{1-\psi}$$

where the first constraint is the budget constraint and the second specifies the human capital of the old child when she becomes young parent next period. Human capital is a combination of acquired and innate ability.

If the old child is admitted to college, then the old parent optimization problem is given by:

$$V_o^{\text{admitted}}(h_o, \pi, \hat{\pi}) = \max_{c_o \geq 0} u(c_o) + \beta \mathbb{E}_{\pi'} \left[ V_y(h'_y, \pi') \right]$$

s.t.

$$c_o = (1 - \tau) \left[ w h_o + w h'_y (1 - \bar{\eta} - b) \right]$$

$$h'_y = \bar{\pi} \psi \hat{\pi}^{1-\psi}$$
where again the first constraint is the budget constraint and the second specifies the human capital of the old child when she becomes young parent next period. The parameter $\bar{p} \geq 1$ is the human capital bonus of attending public college.

Conditional on not being admitted to the public college, the old parent chooses between enrolling her child in the private college ($\chi = 1$) or not ($\chi = 0$), and the old parent continuation value is given by:

$$V_{o \text{ admitted}}^{\not \text{ admitted}}(h_o, \pi, \hat{\pi}) = \max_{c_o \geq 0, \chi \in \{0,1\}} u(c_o) + \beta \mathbb{E}_{\pi'} \left[V_y(h'_y, \pi')\right] \quad \text{s.t.}$$

$$c_o + a\chi \bar{\eta} = \left(1 - \tau\right) \left[w h_o + w h'_y \left((1 - \chi) (1 - b) + \chi (1 - \bar{\eta} - b)\right)\right]$$

$$h'_y = \left[(1 - \chi) \pi^\psi \hat{\pi}^{1-\psi} + \chi \bar{p}^\psi \hat{\pi}^{1-\psi}\right]$$

On the budget constraint, $b$ is the time cost of the application process. Also, $a$ represents the cost per unit of time that the old parent incur while the old child is in college, and $\bar{\eta}$ the amount of time it takes to graduate from college. If one is admitted to public college, then one pays no such cost, while on private school one has to pay the full tuition cost.

The impact of college studies on human capital is different, depending on the kind of college attended. On public college, $\bar{p}$ represents how much the student’s human capital will increase after completion, while on private college only a fraction of it, $\underline{p}$.

The multiplicative structure of the human capital implies that the higher the acquired ability and the innate ability, higher is the return of the investment in college if she is admitted. One possible interpretation is that both the acquired ability and innate ability measure how much the student can learn from the material taught at college. Observe that by doing so we are ignoring possible externalities in the education process. The idea that introducing AA for students from public school might lead good students to switch from private to public education, thus increasing the quality of public schools is a common argument used by the advocates of AA. Figer (2010) proposes a model where the ability of a student after college is a function of her peers’ ability. Finally, Velloso (2009) compares the college grades of minority and non-minority students after the introduction of the AA policy and concludes that there exists no significative difference. That is, the students admitted on the AA were able to catch up with the higher acquired ability students during college.

In our model, the admission process and thus the affirmative action policies impact the individual decisions through the probability of public college admission, $q(\hat{\pi}, \bar{q}, hs)$.

9In our calibration $\bar{p} > p$, however there are no assumptions about the relationship between $\bar{p}$ and $p$. The fact that the first is larger than the second (as expected for Brazilian universities) is an interesting result of the calibration.
2.1 Admissions

College admission process is often based on SAT scores, different sorts of admission’s exams, letters of recommendation, essay evaluations, curriculum analysis, etc. In Brazil, admission is based on an exam. To some extent, universities observe a noise measure of student’s ability or knowledge. In our model, when an old child applies to college the university observes a noisy signal \( P \) of her acquired ability. This noisy signal is interpreted as the old child’s points scored in the admission’s evaluation (e.g. exam), and the admission process will be solely based on such score. The old child score in the exam is given by

\[
\log (P) = \log (\hat{\pi}) + \epsilon, \quad \epsilon \sim N \left(0, \sigma_P^2\right)
\]

Taking the exam is costly in terms of time. It takes a fraction of time \( b \) to prepare apply to college. The exam can be taken only once.

The standard admission process is the following. In the economy there is a measure 1 of old child and a measure \( SP \) of college spots. If an old child takes the exam and places herself among the top \( SP \) candidates she is admitted, and, if not, she’s not admitted. Therefore, the probability of admission, \( q(\hat{\pi}, \bar{q}, hs) \), represents the probability of being among the top \( SP \), given my own acquired ability and the distribution of acquired ability among those that will take the exam.

By Law of Large Numbers, there is a unique grade point cutoff \( \bar{q} \) such that every applicant with \( P \geq \bar{q} \) is accepted. In other words, \( \bar{q} \) is such that there is exactly a measure \( SP \) of applicants with grades above \( \bar{q} \), and \( \bar{q} \) is the “price” that clears the admissions’ market. If there is no AA, for a given grade point cutoff \( \bar{q} \) and acquired ability \( \hat{\pi} \), the probability of being accepted in a public college is given by

\[
q(\hat{\pi}, \bar{q}) = Pr\left[ \log (P) \geq \log (\bar{q}) \right]
= Pr\left[ \frac{\varepsilon_p}{\sigma_p} \geq \frac{\log(\bar{q}) - \log(\hat{\pi})}{\sigma_p} \right]
= 1 - \Phi\left( \frac{\log(\bar{q}) - \log(\hat{\pi})}{\sigma_p} \right)
\]

Color blind affirmative action programs will change the probability of admission described above and it may depend on the agent’s income or on the high school type attended. The affirmative policies to be studied here are in two different categories, and can be combined. The first one is the concession of extra points for members of a particular group, while the second one is the implementation of a quota in public college spots for a particular group. The preferential-treatment groups considered are the following: (a) old child that attended public schools; and (b) old child that came from a low income family.
**Extra Points**  For public school attended, the exam grade is given by

$$\log (P) = \begin{cases} 
\log (\hat{\pi}) + \epsilon & \text{if } hs = PR \\
\log (\hat{\pi}) + p_{PU} + \epsilon & \text{if } hs = PU p > 0
\end{cases}$$

and, for low income students, the grades is

$$\log (P) = \begin{cases} 
\log (\hat{\pi}) + \epsilon & \text{if } wh_o > \overline{w} \\
\log (\hat{\pi}) + p_{poor} + \epsilon & \text{if } wh_o \leq \overline{w}
\end{cases}$$

where $p_{PU}$ and $p_{LI}$ are the extra points from the AA policy.

The number of points an old child has scored in the exam depends on her own acquired ability and also on the affirmative action policy in place. The probability of admission, $q(\hat{\pi}, \bar{q}, hs)$, represents the probability of scoring enough points to be among the top $SP$ students, given her own acquired ability, which school system attended, family’s income and the grade point cutoff.

**Quotas**  Another type of affirmative action in the admission process to implement are quotas. This kind of affirmative action has no impact on the number of points a student scores in the admission exam. We still have that $\log (P) = \log (\hat{\pi}) + \epsilon$, $\epsilon \sim N(0, \sigma^2_P)$. The number of points an old child has scored in the exam depends only on her own acquired ability. The probability of admission, $q(\hat{\pi}, \bar{q}, hs)$, represents the probability of scoring enough points to be among the top $SP$ students, given my own acquired ability, my type, parents income and the distribution of acquired ability among the exam takers.

Let’s consider for instance an affirmative action that reserves a certain amount of spots, $SP_p$, for low income children, say parents income are less than $\overline{w}$. For an old child whose parent is poor, in order to be admitted she has to be among the top $SP_p$ low income applicants, while if the family is wealthier she must be among the top $SP_r$ wealthy background children, where $SP_p + SP_r = SP$. The idea here is just as if there were two admission processes, one for rich and one for poor children. The introduction of a quota policy for public education is fairly similar to the above.

In our model, quotas and bonus points have exactly the same effect. The proposition below shows that for every equilibrium with quotas, it’s possible to find an equilibrium with bonus point and no quota that result in the same allocation. Conversely, for every equilibrium with bonus point, it’s possible to find an equilibrium with quota and no bonus point that result in the same allocation.

**Proposition 1.** For every equilibrium with quotas in the admission process, there is an equilibrium with bonus points and no quota that result in the same allocation. The converse the also true, for
every equilibrium with bonus point in the admission process, there is an equilibrium with quotas and no bonus point that result in the same allocation.

Detailed proof in provided in Appendix B

2.2 Production

We assume linear constant returns to scale, i.e. \( F(H) = AH \), and firm’s problem is given by

\[
\max_H Y - wH \\
\text{s.t.} \\
Y = F(H), \quad H \geq 0
\]  

(6)

Let \( H \) be the firm’s policy function.

2.3 Equilibrium

We use the following equilibrium concept to solve the model.

Definition. A Stationary Recursive Competitive Equilibrium is a set of value and policy functions, wage \( w \), government expenditure in early education \( g \), grade point cutoff(s) \( q \), distributions \( \mu_y(h_y, \pi) \) and \( \mu_o(h_o, \pi, \hat{\pi}) \) over the agents’ states such that

- Agents maximize: given \( q \), \( g \) and \( w \), young and old parents optimize and \( H \) solves firm’s problem;
- Markets clear: human capital market clears, output market clears, admissions market clears;
- Government budget is balanced;
- Stationary distribution: \( \mu_y(h_y, \pi) \) and \( \mu_o(h_o, \pi, \hat{\pi}) \) are constant over time.

To solve the market clearing conditions, the human capital market clears, i.e.

\[ H^a = H \]

where \( H^a \) is the aggregate human capital\(^{10}\) and \( H \) is the firm’s demand for human capital.

The resource constraint is given by

\[ Y = C_y + C_o + E + F + \zeta g \]

\(^{10}\)\( H^a \) is the sum of the human capital of all agents working: the old children, the young parents and the old parents. If the child child attends college, she only works a fraction of the period.
where $C_y$ and $C_o$ are the aggregate consumption of the young and old parents. $E$ is the aggregate expenditure in early education, $F$ is the aggregate expenditure in college education (both public and private), and $\zeta g$ is aggregate government expenditure in early education. $\zeta$ is the measure of students in the public school. The government budget being balanced implies that

$$\tau Y = \zeta g + aSP\bar{\eta}.$$ 

Lastly, the admissions market clears, i.e. grade point cutoff(s) will be such that exactly a measure $SP$ of applicant is admitted to public college.

## 3 Calibration

### Table 2: Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
<td>1.5</td>
</tr>
<tr>
<td>Innate ability distribution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>Std of innate ability innovation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Innate ability persistence</td>
<td>0.3</td>
</tr>
<tr>
<td>College</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SP$</td>
<td>Measure of public college spots</td>
<td>0.048</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>Time spent in college (roughly 4 years)</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Noisy of the admission process</td>
<td>0.13</td>
</tr>
<tr>
<td>$b$</td>
<td>Time spent applying to college (roughly 1 year)</td>
<td>0.0625</td>
</tr>
<tr>
<td>$a$</td>
<td>College tuition</td>
<td>2</td>
</tr>
<tr>
<td>Productivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>Life cycle</td>
<td>1.12</td>
</tr>
<tr>
<td>$A$</td>
<td>Marginal productivity</td>
<td>1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Concavity in the early education investment</td>
<td>0.08</td>
</tr>
<tr>
<td>$\overline{p}$</td>
<td>Public college bonus</td>
<td>4</td>
</tr>
<tr>
<td>$p$</td>
<td>Private college bonus</td>
<td>1.23</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Weight of innate ability on young child’s knowledge</td>
<td>0.04</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Weight of innate ability on old child’s human capital</td>
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</tr>
<tr>
<td>Government</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Public school inefficiency</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The numerical solution of the model is described in Appendix A. We assume that agents have
Table 3: Calibration Targets

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\pi$</td>
<td>1.5</td>
<td>Standard Deviation of log earnings</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.3</td>
<td>Inter-generational correlation of earnings (US)</td>
<td>0.40</td>
<td>0.41</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.13</td>
<td>Fraction of applications</td>
<td>0.34</td>
<td>0.31</td>
</tr>
<tr>
<td>$a$</td>
<td>2</td>
<td>Aggregate college expenditure relative to GDP</td>
<td>0.014</td>
<td>0.018</td>
</tr>
<tr>
<td>$\beta$</td>
<td>4</td>
<td>College wage premium</td>
<td>2.73</td>
<td>2.76</td>
</tr>
<tr>
<td>$p$</td>
<td>1.23</td>
<td>Fraction of students in private college</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.08</td>
<td>Early education expenditure relative to GDP</td>
<td>0.060</td>
<td>0.056</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.04</td>
<td>Fraction of low income students in public college</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5</td>
<td>Fraction of low income students in private college</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.9</td>
<td>Fraction in public school</td>
<td>0.88</td>
<td>0.84</td>
</tr>
</tbody>
</table>

CRRA preference, $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ with risk aversion $\sigma$. All parameters values used are in table 2. Some parameters are calibrated directly from the data or from the literature while others have to be calibrated by targeting related moments from the data. The discount factor, risk aversion, life cycle, innate ability persistence are taken from the literature, Restuccia and Urrutia (2004) use the same calibration for these parameters. The discount of 0.52 for 16 years is equivalent to 0.96 per year. The marginal productivity $A$ is normalized to 1.

In the model, each period represents 16 years (one generation), thus we assume that the time spent in college is 4 years ($\eta = 0.25$) and the time spent applying to college 1 year ($b = 1/16$). The tax rate $\tau$ is calibrated from the data, it represents the government expenditure in education (primary, secondary and tertiary) over GDP which is 4% according to the World Bank for 2004.

**College spots** From 2004 PNAD data\textsuperscript{11}, roughly 17% of young adults in the sample were enrolled in college or have already earned a higher education degree. This fraction doesn’t vary much with the age group considered\textsuperscript{12}. From the Brazilian educational census\textsuperscript{13}, in 2004, 28% of total college enrollment is from public institutions of higher education\textsuperscript{14}. Thus, the measure of public spot is calibrated at 4.8% and the measure of private spots is targeted at 12.2%.

The remaining 9 parameters are calibrated to match the following 9 moments of the data simultaneously: (i) standard deviation of log earnings, (ii) intergenerational correlation of earnings, (iii) fraction of applicants, (iv) aggregate college expenditure relative to GDP, (v) college wage

\textsuperscript{11}Pesquisa Nacional por Amostra de Domicílios – Brazilian National Household Survey.
\textsuperscript{12}We considered ages between 18 and 24/26/28/30/32 and the respective fractions were between 16.3% and 17.3%.
\textsuperscript{13}Data statistics were taken from INEP 2008 report.
\textsuperscript{14}In 2004, there were 4,163,733 students enrolled in some institution of higher education out of which 1,178,328 were in public institutions.
premium, (vi) measure of student in private college, (vii) early education expenditure relative to GDP, (viii) fraction of low income students in public college, (ix) fraction in public school. Next, we’ll discuss each of these moments their related parameters in the model and table 3 report all these moments in the model and in data as well as their related parameters.

**Standard deviation of log earnings**  Using the PNAD 2004 data, the standard deviation of log earning is 0.95. In the model the parameter $\sigma_\pi$ affects the innate ability dispersion and therefore the earnings dispersion as well.

**Intergenerational correlation of earnings**  This moment measures the persistence of earning between two generations of the same family. Specifically, it’s the slope coefficient of regressing log earning of the old parent on the log earning of his child as young parent. For the US, the intergenerational correlation of earnings is roughly 0.40 (Solon 1992) using the PSID data. For developing economies, there are no estimates using panel data to the best of our knowledge. Corak (2006, 2010) compute a correlation of 58% for Brazil, but they use a proxy of the old parent’s earnings based on his occupation and education. Corak (2010) compute the persistence of earnings for several different countries, and he finds a huge cross-country variation from 15% in Denmark to 67% in Peru. For our calibration, we are conservative and target the intergenerational correlation of earning at 40%. The parameter $\rho$, which affects the innate ability persistence directly and the earning indirectly, is chosen to match intergenerational correlation of earnings at 40%.

**Fraction of applications**  Based on the Brazilian educational census, the ratio between applicants and college spots is about 7. Since, we have a measure of 4.8% of public college spots, then the measure of applicant should be targeted at roughly 34%. The noisiness in the admission’s exam $\sigma_p$ directly affects this moment. If there is no noise in the admission’s process and the acquired ability is perfectly observed, then only students who are admitted would apply to college. However, as the grade noisiness increases, more students apply to college.

**College wage premium**  College wage premium is the average wage conditional on having a college degree relative to not having a college degree. In 2004, the college wage premium was 2.76 for college graduates between 30 and 48 years old versus having high school degree only. The college wage bonus $\bar{p}$ directly affects this moments as it drives the college wage level up and down accordingly.

**College and early education expenditure relative to GDP**  According to the Brazilian Household Budget Survey (POF)$^{15}$ 2003-2004, the average fraction spent in education was 4.08%

$^{15}$Pesquisa de Orçamento Familiar.
and, from the World Bank data, the government spent 4% of GPD in education. Using the average Brazilian tax revenue rate for the period (15.9%), the total investment in education was roughly 7.44% of total output, 1.4% being invested in higher education and the 6.0% remaining in early education as 18.9% of total education expenditure goes to higher education according to the World Bank 2004 data. We target early education expenditure over GPD at 1.4% by varying the concavity of the education investment function (parameter $\kappa$) and college education expenditure over GPD at 6% by varying the cost of college education (parameter $a$).

Table 4 decomposes the aggregate educational expenditure into private and public investment. This decomposition is not target in our calibration, but the model is fairly close to the data. In the data, government (private) investment in higher education is 0.6% (0.8%) of GDP, and, in the model, government (private) investment is 0.5% (1.3%) of GDP. For early education, government (private) investment is 3.4% (2.6%) of GDP, and, in the model, government (private) investment is 3.5% (2.1%) of GDP.\footnote{The data decomposition provided by INEP/MEC}

**Fraction in public school** Using PNAD data, the fraction of student in public high school was 88% in 2004, the the public school inefficiency parameter $\alpha$ is used to calibrate this moments. If $\alpha = 0$, public school has no return and all young parents would enroll their child in the private school. However, if $\alpha = 1$ then public school is strictly better than the private ones and all young parents would enroll their children in the public schools. Thus, $\alpha$ directly affects the benefit of private versus public school and is closely associated with the fraction of students attending public school.

**Measure of student in private college** If the private college bonus $p$ is low, then only a few students would be willing to go the private college, but as the benefit increases more students would be willing to attend the private college. The quality of the private college is captured by

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
Moments & Data & Model \\
\hline
Total spent in college over GDP & 0.014 & 0.018 \\
Gov. spent in college over GDP & 0.006 & 0.005 \\
Private spent in college over GDP & 0.008 & 0.013 \\
\hline
Total spent in early educ. over GDP & 0.060 & 0.056 \\
Gov. spent in early educ. over GDP & 0.034 & 0.035 \\
Private spent in early educ. over GDP & 0.026 & 0.021 \\
\hline
\end{tabular}
\caption{Expenditure in Education}
\end{table}
the parameters $p$ and it is used to match the measure of students attending private college. As discussed before, a measure of 0.17 students attend college, where 0.048 go to the public college and the remaining 0.122 go to the private college. The measure in the public college is directly calibrated by the parameter $SP$.

**Fraction of low income students in public and private college** We define low income as the bottom 40% of the income distribution, as it roughly represents the fraction of people that can potentially benefit from the affirmative action policy to be implemented in Brazil. Using PNAD data, 7% of the public college students are in the bottom 40% of the income distribution, while only 4% of the private college students have low income. On one hand, public college is tuition free and should have a higher fraction of low income students compared to private college. However, on the other hand, when public college has higher quality than private college, then high income parents can invest more in their child’s early education in order to increase the likelihood of being admitted to the public college. Given this trade-off, public college will have more or less low income students depending on how efficiently high income parent can increase their child’s admission likelihood by investing in early education. This is driven by the innate ability weight in acquired ability function ($\psi$). If the weight $\psi$ is low, then high income parents are able to increase the admission’s likelihood, and, as a result, low income students are crowded out of public college. If the weight $\psi$ is high, then the admission’s process is mostly based on innate ability, and we should observe a higher fraction of low income student in the public college.

The fraction of low income students in the private college will depend on how the human capital depends on the innate ability which is driven by the innate ability weight in human capital function ($\gamma$). Low income students have less resources to invest in early education, thus if human capital depends more on the innate ability, more low income students will be willing to attend private college.

4 Policy evaluation

In this section, we compare the steady-state outcome of the model with and without affirmative action, and we do transition paths from different steady states. We consider bonus points in the admission’s exam for two different preferential-treatment groups: students from the public school system and low income students.

Figure 1 reports the percentage change in Gini, GDP, and intergenerational correlation of earnings, and welfare by varying the bonus point given to the old child applicants in the bottom 20th percentile of the income distribution. The social welfare function considered is simply the sum of the utility of all families. There are many other forms of computing welfare, but this one gives us an idea if agents are better off on average. GDP itself is another measure of welfare, since
there is no unobservable cost to the agents such as effort, however it doesn’t capture redistribution effects. In figure 1, these moments are plotted against the left axis, and the fraction of student in public college that received the bonus points is plotted against the right axis. We vary the bonus points from 0% up to 50%, when all students in the public college are in the preferential-treatment group. The remaining figures obey the same structure, but the preferential-treatment group changes. Figures 2 and figure 3 consider different low income cutoffs: bottom 40th and 60th percentile, respectively. The preferential-treatment group considered in figure 4 are college applicants whose parents are between the 40th and the 60th percentile of the income distribution. Finally, figure 5 report the same set of moments, but the preferential-treatment group consists of the public school students.

Tables 5 and 6 report additional moments for two different treatment groups: child with parents in the bottom 20th percentile of the income distribution (same group as in figure 1) and applicants from the public school (same group as in figure 5). The bonus point considered in the tables are: 2.5%, 5%, 10%, 15% and 30%.

**Bonus points to low income: bottom 20th percentile**  From the figure 1, we can observe that the introduction of affirmative action policy towards low income students can have strong effects on income and inequality. In terms of the intergenerational correlation of earnings, the AA policy has negative impact. For instance, when low income (bottom 20%) students receive a grade bonus of 50%, then this correlation decreases nearly 50%, from 41% to 22%. In the model, agents can invest in early education ($\hat{e}$) which increases the old child’s acquired ability, and as a results, it increases both the human capital and the likelihood of college admission. In the no AA equilibrium, wealthy parents over invest in their child’s education in order to increase the likelihood of their child’s admission. This over investment in early education is optimal as it maximizes the private gains of the families. As a result, the model endogenously generates a persistence in earnings (currently at 41%) higher than the exogenous persistence in the innate ability process (30%). Bonus to low income students makes it more expensive for wealthy agents to so, the grade point cutoff increases and wealthy parent will invest less in their children. All together, there is a decrease in the equilibrium intergenerational correlation of earnings.

The decrease in early education investment can be observed in panel E of table 5. The early education investment made by high income families decreases up to 42% (from 0.12 to 0.07). Low income students have more incentives to invest in early education, however, due to their low income levels, they are not investing in early education. Low income parents don’t have the resources to invest in their child’s education, it’s optimal for them to spend their income in consumption rather than in early education investment (corner solution). The average expenditure in early education of children in public college decreases about 4%, from 1.07 to 1.03. Thus, the AA policy corrects the source inefficiency to some extent by reducing the excessive investment in early education.
The GDP gain can be up to 1.13% when low income students receive a grade bonus of 2.5% (table 5, panel A), but it can be as low as 6% when they receive a bonus grade of 30%. With the AA policy, the top low income students that would under perform (due to lack of early education investment) in the admission’s exam are more likely to be admitted. If the bonus is small (e.g. 2.5%), then the AA policy benefits most the low income students with high expected future human capital; and, as a result, the AA implementation makes the admission process more efficient. When the grade bonus is high, this is no longer the case, and, furthermore, admission’s market becomes more costly for high income families. As a result, high income families decrease investment in early education and the average human capital as well as GDP falls.

Hence, the final impact in GDP depends on the magnitude of bonus point. There can be an increase in efficiency of the admission’s process by admitting better students which potentially increases GDP. In addition, high income families decrease their investments in early education which also affects output. When the bonus point is low, the final effect is positive and GDP increases, however, when the bonus point is high, GDP decreases.

The final effect on inequality also depends on the magnitude of the AA policy. On one hand, the AA decreases the intergenerational correlation of earnings which decreases inequality. On the other hand, AA policy can make the admission process more efficient which by construction increases the earnings dispersion. When the bonus to low income students is high, the former effect dominates and there is a considerable decrease in inequality (up to 7.56%).

Interestingly, welfare increases when the bonus point is low (e.g. 2.5%), then, as the bonus point increases, the welfare decreases. However, if the bonus points is sufficiently high, the welfare increases again. When the bonus point is low (2.5%), the efficiency gain is translated into welfare gain: investment in early education decreases and the admission’s process is more efficient. For a slightly higher bonus points (5% or 10%), the AA policy over compensates the source of inefficiency: early education decreases too much and admission’s process becomes more inefficient. As a result, both GDP and welfare falls. When the bonus point is relatively high (e.g. 30%), then a considerable amount of low income student are admitted to college, including a higher fraction of low acquired ability ones. As discussed before, this will induce an even more inefficient allocation (GDP falls up to 6.2%), but inequality and earnings persistence decrease sharply: 7.56% and 29.52% fall, respectively. However, the higher fraction of low income students in public college will increase their utility considerably. In this scenario, there are much more low income students in college, and, since they are from low income family, the marginal utility of their parents is relatively high. Thus, the welfare gain from these families is relatively high and outweigh the welfare loss from the high income students crowed out from the public college. As a result, high bonus point can increase welfare. For the preferential treatment group considered, it’s welfare improving to implement a bonus of 30% even tough GDP will falls.

This welfare analysis is sensitive to the utility functional form. For CRRA preferences, the
marginal utility goes to infinity as consumption goes to zero. Thus, to some extent, the inada condition on the consumption zero lower bound affects the welfare considerably. Nevertheless, this is an interesting result as output maximizing bonus point differ from the welfare maximizing one.

**Bonus points to low income: other cutoffs**  As the low income cutoff increases (bottom 40\textsuperscript{th} or 60\textsuperscript{th} percentile in figures 2, 3), the policy effects are reduced. GDP and inequality gain/loss are both reduced.

The welfare analysis changes the most. For the bottom 60\textsuperscript{th} percentile of the income distribution as the preferential treatment group (figure 3), there is a welfare loss even when the bonus point is high (e.g. 30\% or 50\%). The reason for this result is that, since the low income cutoff is higher, the bonus point is no longer affecting the students with the lowest income levels among the low income students as it was affecting under the previously discussed income cutoff. Considering the preferential treatment group as students whose parents have income between the 20\textsuperscript{th} and 40\textsuperscript{th} percentiles yields almost the same results as giving bonus points to all student in the bottom 40\textsuperscript{th} percentile (see figure 4). This strongly suggests that the affirmative action policy towards low income students is benefiting the wealthiest among the low income students.

**Bonus points to public school**  If we consider public school students as preferential-treatment group, then the AA effects are limited. Figure 5 and table 6 show that there some effect on income, inequality, welfare, early education investment. When the bonus point is high (e.g. 30\%), then there is a reduction in GDP of 2.18\% and inequality falls by 1.28\%. However, when the bonus point is low, the effects are not as meaningful as the affirmative action for income students: welfare, GDP and inequality all increase slightly. The bonus to students from the public school system makes parents enroll their children in the public school, even though it’s inefficient, because it increases the likelihood of college admission. This is a general equilibrium effect: there is an increase in the number of students attending public school which offsets the potential efficiency gain from the AA policy and therefore has limited effects on equilibrium variables.

In summary, the affirmative action as discussed here can have significant impact on economic variables. The AA policy for low income students can significantly reduce the intergenerational persistence of earnings. If the policy bonus points are high enough, then it decreases inequality at the output expense. In small magnitudes, it can slightly increase welfare, output and inequality.

### 5 Concluding Remarks

We presented a four overlapping generations model in order to evaluate the effects of different color-blind affirmative action on: welfare, inequality, intergenerational correlation of earnings,
GDP, investment in education and human capital accumulation.

We find that the quotas and extra points policies have equivalent effects. We also find that, for public school students as preferential-treatment group, the policy effects are limited, because there is a general equilibrium effect that offsets potential effects of the policy. For low income students as preferential-treatment group, the policy monotonically decreases the intergenerational correlation of earnings with maximum decrease of nearly 50%. The impact on GDP depends on the magnitude of AA policy. However, the GDP gain can be up to 1.1%. The effect on inequality is also depends on the magnitude of AA policy, but there can be a significant reduction in inequality (up to 7.56%). Finally, AA policies can have meaningful welfare implications and can be used as a welfare improving mechanism.

References


**Appendix**

**A Numerical Solution**

**A.1 Steady state equilibrium**

The numerical solution of our model is similar to Restuccia and Urrutia (2004). We use a Markov approximation for the innate ability process as suggested by Tauchen (1986) and define a grid over the state space. For a given grade point cutoff and for a given government expenditure in early education, we solve the value and policy functions numerically. Lastly, we adjust the government expenditure in early education and the grade point cutoff until the government budget is balanced the admission’s market cleared. Formally, we solve the model doing the following steps:

(i) Guess the value function for the old parents, government expenditure in early education and grade point cutoff;

(ii) Solve for the young parent value and policy functions taking as given the old parent value function;

(iii) Solve for the value and policy functions of the old parent taking the young parent value function as given;
(iv) Iterate (ii) and (iii) until convergence;

(v) Guess a distribution over the state space for young parents, and, using the step (iv) young parent policy function, compute the old parent state space distribution. Using the implied distribution and the old parent policy function, compute the young parent state space distribution. Repeat this procedure until convergence.

(vi) Lastly, set the government expenditure in early education and the grade point cutoff to balance the government budget and clear the admission’s market. Using these new values, repeat the previous steps until convergence.

A.2 Transition path

We compute the transition path between the steady state without the AA policy to the steady state with the AA policy. We do this according to the following algorithm:

(i) Guess the number of periods for convergence ($T$).

(ii) Guess the transition path for both government expenditure ($\{g_t\}_{t=1}^T$) and admission’s grade cutoff ($\{q_t\}_{t=1}^T$).

(iii) We assume that in period $T + 1$ the economy will be in the steady state with the AA policy. Given that, we solve the model by backward induction from period $T$ to period 1 taking both the government expenditure and admission’s grade cutoff paths as given.

(iv) Using the policy and value function paths from step (iii), we can compute the evolution of the probability distribution over the state space from period 1 to period $T$, and, we can compute both the admission’s market clearing and the government budget balance conditions.

(v) Adjust government expenditure ($\{g_t\}_{t=1}^T$) and admission’s grade cutoff ($\{q_t\}_{t=1}^T$) paths to clear the admission’s market and the make the government budget balanced.

(vi) Repeat steps (ii) to (v) until convergence of the transition paths.

(vii) Check if the distribution and policy function at period $T$ are the same as in period $T + 1$. If they are not the same, then increase the number of periods for convergence and repeat steps (ii) to (vi) until convergence.

B Proof: Proposition 1

First, let’s take an equilibrium in which a bonus points of $p_{\text{bonus}}$ implemented for a certain treatment group. Let $\bar{q}^*$ and $g^*$ be the grade point cutoff and the government spent in early education
in such equilibrium. Furthermore, let $SP_{pt}$ be the fraction of public college students in the preferential treatment group.

For a given acquired ability level $\hat{\pi}$, a student in the preferential treatment group will be admitted to public college with probability given by

$$q^{\text{bonus}}(\hat{\pi}, \bar{q}^*) = 1 - \Phi \left( \frac{\log (\bar{q}^*) - \log (\hat{\pi}) - \log (p_{\text{bonus}})}{\sigma_P} \right)$$

If not in the treatment group the probability of admission

$$q^{\text{no bonus}}(\hat{\pi}, \bar{q}^*) = 1 - \Phi \left( \frac{\log (\bar{q}^*) - \log (\hat{\pi})}{\sigma_P} \right)$$

To find the equilibrium with quotas, let take the same allocation, but there will be two grade point cutoffs: one for the quota ($q^{\text{quota}}$) and another one for the no quota student ($q^{\text{no quota}}$). These cutoff are defined as

$$\log (q^{\text{quota}}) = \log (\bar{q}^*) - \log (p_{\text{bonus}})$$
$$\log (q^{\text{no quota}}) = \log (\bar{q}^*)$$

Using these cutoffs, the parent’s maximization problem is exactly the same. Then, this allocation along with the government expenditure in early education and the grade cutoffs constitute an equilibrium. Similarly, from an equilibrium with quotas, the grade point cutoff gap between the preferential treatment group and the other students can be used to get the necessary bonus point to construct an equilibrium with bonus point the generates the allocation.
This figure reports the effects of bonus grade to old child whose parents are in the bottom 20\textsuperscript{th} percentile in the income distribution. The effects on GDP, inequality, intergenerational correlation of earnings and social welfare is plotted against left axis, and the fraction of students in the preferential treatment group attending public college is plotted against the right axis. The social welfare considered is simply the sum all agents’ utilities. The bonus point varies from 0\% to 50\%.
Figure 2: Affirmative Action for Low Income: bottom 40th percentile

This figure reports the effects of bonus grade to old child whose parents are in the bottom 40th percentile in the income distribution. The effects on GDP, inequality, intergenerational correlation of earnings and social welfare is plotted against left axis, and the fraction of students in the preferential treatment group attending public college is plotted against the right axis. The social welfare considered is simply the sum all agents' utilities. The bonus point varies from 0% to 50%.

![Figure 2: Affirmative Action for Low Income: bottom 40th percentile](image-url)
Figure 3: Affirmative Action for Low Income: bottom 60\textsuperscript{th} percentile

This figure reports the effects of bonus grade to old child whose parents are in the bottom 60\textsuperscript{th} percentile in the income distribution. The effects on GDP, inequality, intergenerational correlation of earnings and social welfare is plotted against left axis, and the fraction of students in the preferential treatment group attending public college is plotted against the right axis. The social welfare considered is simply the sum all agents' utilities. The bonus point varies from 0\% to 50\%.
Figure 4: Affirmative Action for Low Income: between 20th and 40th percentile

This figure reports the effects of bonus grade to old child whose parents are between 40th and 60th percentile in the income distribution. The effects on GDP, inequality, intergenerational correlation of earnings and social welfare is plotted against left axis, and the fraction of students in the preferential treatment group attending public college is plotted against the right axis. The social welfare considered is simply the sum all agents’ utilities. The bonus point varies from 0% to 50%.
Figure 5: Affirmative Action for Public School

This figure reports the effects of bonus grade to old child that attended public school. The effects on GDP, inequality, intergenerational correlation of earnings and social welfare is plotted against left axis, and the fraction of students in the preferential treatment group attending public college is plotted against the right axis. The social welfare considered is simply the sum all agents’ utilities. The bonus point varies from 0% to 50%.
Table 5: Policy evaluation: extra points for low income (bottom 20%)

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Table 6: Policy evaluation: extra points for high school

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