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Stability of General Equilibria in Labor-managed Economies: a non-tatonnement approach

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1. Introduction

A labor-managed economy (LME) is distinguished by three operating rules: (1) firms are managed by their members, who also comprise the firm's labor force; (2) a firm's members are residual income claimants, thus their collective income is the firm's profit; and (3) firms and consumers operate autonomously, with interactions among agents occurring through free markets.

In this paper we define a non-tatonnement process for this type of economy, and show that it converges to the set of general competitive equilibria of the economy.

The original concern that instability might be a pervasive problem for a LME is due to Ward's initial analysis [11, 12] of the labor-managed firm (LMF). Ward assumed that a LMF is required to evenly distribute profits among all of its members (making the LMF a perfectly egalitarian cooperative), and proved that if all members are strict income maximizers, then the LMF has a negatively-sloped output-supply function. This result is guaranteed if labor is the only variable input for the LMF, and is likely when there are several variable inputs.

This result has been interpreted to mean that, without significant entry and exit of firms into and out of a given market, there would be inefficient (destabilizing) responses to price changes in a LME. Two categories of challenge to the Ward result have emerged to date. One has been to question the presumed motivation, hence behaviour, of the LMF. The other has been to develop a (static) characterization of general equilibrium for the LME which yields Pareto efficiency. Both of these deserve brief mention.

Vanek [10, pp. 56-7] argued first that Ward ignored the social (or collective) nature of the LMF. As a social unit, the LMF should be viewed as having a fixed membership, which rules out the possibility of a perversely-sloped supply curve. Similarly, Steinherr and Thisse [9] argue that if members of a LMF are risk averse and that dismissal of workers is random, then supply curves again become normal (the merit of these arguments is, of course, subject to debate and remains to be resolved). A second line of criticism of Ward's analysis of the LMF focuses on the presumption of strictly egalitarian membership. Meade [7], for example, claims that it would be sensible to require that the addition of a new member, or the dismissal of an old one (as a response to changing market conditions), be subject to mutual consent. This could be achieved by assigning differential membership shares to members of the LMF based on their date of joining, and allowing dismissed members to retain some claim to the profits of the firm after their departure. This inegalitarian cooperative will also display normal supply curves. Finally, Bonin [1] has demonstrated that Meade's requirement of mutual consent can be exactly satisfied by side payments based on the existence of an extra-firm alternative wage. A member entering a firm would pay a fee, and a dismissed member would receive compensation, exactly equal to the difference between the firm's going wage and the alternate wage.

The other resolution of the perverse-supply behaviour of the LMF has been to study the LME from a static general equilibrium point of view. Vanek's [10] was the first such analysis, and he divided the problem into two parts. At the microeconomic, or structural, level, Vanek begins with an assumption that whenever there is a group of unemployed workers, they will form a new firm producing the most profitable (on a per-worker basis) output possible. This directly generates full employment at equilibrium. Similarly, whenever one LMF yields a higher payment to its worker than does another LMF, the second will switch markets (or naturally lose members to the more profitable firm). As a result, in equilibrium, there will be full employment and all (homogeneous) workers will receive identical wages across firms. It is then shown that such an equilibrium (under standard environmental conditions) is equivalent to a Walrasian equilibrium. Hence, it is Pareto optimal.

Vanek does address the problem of stability directly, but only at the macroeconomic level, where all outputs are aggregated into a single homogeneous good produced by a single national firm. Using a standard, static notion of equilibrium, Vanek shows that the LME equilibrium might be unstable, but claims that this is not too likely.

While Vanek's general equilibrium analysis of the LME is insightful, it is not technically rigorous. A more formal, but conceptually similar, framework is provided by the work of Dreze [4] and Ichiishi [6] who treat the LME as a production-coalition economy where workers can freely form into coalitions which thus become firms. Dreze, in particular, views the explanation of endogenous firm formation as an important task of his analysis. As with Vanek, the assumption of free coalition formation and dissolution amounts to an assumption of free entry and exit of firms which guarantees both full employment and equality of workers' incomes in equilibrium. In the production-coalition approach, classical assumptions on technologies, preferences, and the environment suffice to guarantee the existence of a competitive equilibrium for the LME. Further, with natural definitions of sustainability, it can be shown, for a given classical environment, that the sets of Pareto optima, sustainable competitive equilibria for the LME, and sustainable Walrasian equilibria, are all identical. This last result, in particular, provides a very positive appraisal of the performance of an LME. Ward [12] accurately anticipated, but did not prove, this result in his earlier work.

Two fundamental problems remain, however, in the literature on LMEs. First, the existence of a Pareto optimal equilibrium for an LME is of little consequence unless it can be shown that the LME institutions actually allow such an equilibrium to be attained (cf. [8, p. 276]).

Second, there exists no proof of existence of equilibrium with a fixed number of firms. Our results show that at least for some preferences and endowments such equilibria exist, but this is not an "existence" proof in the traditional sense. Indeed, the existence of at least one equilibrium for *each* possible set of initial endowments and preferences is not a necessary (or sufficient) condition for non-tatonnement stability, although it is a necessary condition for tatonnement stability. This is because, in a non-tatonnement process, the endowments of agents are not fixed. In the remainder of the introduction we provide a rationale for the specific methods we employ in the rest of the paper.

First, as stated above, all existing work on LME equilibrium deals with a long-run situation in that free entry and exist are presumed. Further, the assumption that entry and exit are free implicitly presumes a perfectly functioning capital market. In the LME, financial capital does exist and is employed by workers to purchase capital goods and other inputs. Explicitly, however, financial capital is devoid of equity and managerial attributes. Because of this, capital *markets* do not exist in the LME. Rather, financial capital is more like a bond market. Further, the institutional rule, used in Yugoslavia, that a firm must maintain the value of its (physical) capital automatically leads to some immobility of capital.

All of these factors would seem to argue for *not assuming* perfectly functioning capital markets in an LME, even if capital goods used as inputs are traded on perfectly competitive markets. In particular, the presumption that workers can freely form and dissolve firms seems implausible. Our analysis below breaks with standard practice by assuming a fixed number of firms. The stability results obtained are thus applicable to short run analysis.

The dynamic process employed below is of the non-tatonnement variety. Specifically, trade agreements and price adjustments are governed by a particular type of process where (1) consumers can make trades and commitments to trade which are binding, (2) firms make binding commitments to receive inputs and deliver outputs (but do *not* produce until the process terminates), (3) firms and consumers (as workers) make binding agreements on membership changes at mutually advantageous terms, and (4) markets satisfy a Hahn-process assumption that all net excess demands for a given good (except possibly labor) are on one side of the market.

The process unfolds outside of real time. There is no production or consumption until equilibrium is reached. Firms and households trade in commitments to deliver commodities in equilibrium. In each period of time households and firms face a price vector for goods, and think that those prices will not change thereafter. Households consider the possibility of changing their supplies of labor, if necessary through trading in membership rights. Under these conditions, households maximize utility, and firms maximize profit per worker, and supplies and demands are determined. Then there is trade in commodities and membership rights. If a worker leaves a labor-managed firm, he receives a payments equal to his of the firm's profit, less his opportunity wage. If a worker joins a labor-managed firm, he pays a fee that depends on the amount of labor he will supply. Next, the prices of goods, the expected wages of firms and the prices of membership rights change. The whole process is repeated until equilibrium is reached. Then households deliver inputs (including labor) to firms according to their households deliver inputs (including labor) to firms according to their commitments, firms produce outputs so as to be able to satisfy their commitments, and finally there is consumption.

Our use of a non-tatonnement process, allowing for exchange and binding commitments, but not production, or consumption prior to equilibrium attainment, necessitates some elaboration of the concept of membership rights. When a worker joins a firm, this amounts to a contract or commitment (1) to work a certain number of hours for that firm, (2) to participate in the management of that firm, and (3) to share in the profits (or losses) of that firm. Should the worker later quit, or be dismissed, prior to the attainment of equilibrium, then no work has actually been performed, and the worker should receive no labor compensation. The worker has been a member of the firm for some time, however, and has a claim to the profits accrued during his tenure with the firm. In our model the worker does not work until equilibrium is reached. Therefore, the profits that accrue during his tenure are all “speculative”, due to transactions.

This leads naturally to a notion of discriminatory membership rights in a firm based not only on the duration of membership, but the exact times at which a worker enters and leaves a firm, similar in nature to Meade’s Inegalitarian Cooperative. In fact, the rule we use below is Bonin’s (naive) capitalized value of Meade’s inegalitarian membership shares. This procedure works as follows. There is an opportunity wage, w_c, available to all workers. If a worker enters a firm j at time t with the expected per-member profit of $w_j(t)$, then the worker pays an entrance fee of $w_j(t) - w_c(t)$. Alternatively, the worker can simply agree to forego collecting this portion of his share of the firm’s profits. Upon paying this fee, the worker has full membership rights in the firm. If the worker later (at time s) leaves the firm, he is paid compensation in the amount $[w_j(s) - w_c(s)]$. Thus a worker who joins firm j at time t and leaves at time s earns $\left[w_j(s) - w_j(t)\right] - [w_c(t) - w_c(t)]$ for his tenure in the firm, but receives no compensation for labor provided (which equals zero in any case). The amount $[w_j(s) - w_j(t)]$, of course, is exactly equal to the change in per-member profit accrued by the worker during his membership, while $[w_c(s) - w_c(t)]$ is the decrease in the value of membership rights due to changes in the opportunity wage w_c.

A comment on or modelling technique may be appropriate here. We assume that there is a continuum of workers, but just a finite number of firms. This means, of course, that there are “many” workers per firm. But the reason for modelling the household sector as a continuum is that in this way the discontinuities that would arise when workers would be switching jobs can be smoothed out. In the theory of competitive equilibrium these discontinuities are avoided by allowing workers to be simultaneously employed by several firms, but to permit this would be inappropriate in the context of our model. The assumption that there is a continuum of workers is thus of a purely technical nature.

The structure of the paper is as follows. In Section II the model is defined and discussed in its static components, while Section III contains the assumptions that formally define the process, and again a discussion of these assumptions. The quasi-stability theorem is stated in Section IV, but its proof is relegated to the Appendix. We give some heuristics on the proof and argue that equilibria are Pareto efficient. Finally, in Section V we mention some possible alternative institutional specifications for the model which are discussed with regards to the Yugoslav reality. We find that these institutional variations would necessarily be associated with unstable processes.

II. The Model

There are $(n+1)$ commodities, indexed by $h = 0, 1, 2, \ldots, n$. Commodity 0 is time, and commodity 1 is money and the numeraire. The price of commodity h is denoted by p^h, $h = 1, 2, \ldots, n$.

There is a continuum of households, indexed by $i \, \varepsilon \, I \equiv [0,1]$. We denote the Lebesgue measure on [0,1] by n. Household i has a twice differentiable, monotonic, strictly quasiconcave utility function: $U_i\colon R^{n+1} \to R$.

For $h = 0, 1, 2, \ldots, n$, the actual stock of commodity h owned by household i is $\bar{x}_i^h$, and the corresponding desired stock is x_i^h.

The actual commitment of firm j to deliver commodity h is denoted by $\bar{y}_j^h$, if $1 \leq h \leq n$, and the amount of labor used by the firm is denoted by $\bar{y}_j^0$. The desired commitment of firm j to deliver commodity h is denoted by y_j^h, and firm j's desired input of labor is denoted by y_j^0.

We are modelling an economy in which each firm uses "many" workers, and this is reflected in the assumption that there is only a finite number of firms. Firm j has a production set Y_j, which is given by

$$Y_j = \left[y \, \varepsilon \, R^{n+1} \mid y^0 \geq 0, \Phi_j(y) \leq 0 \right]$$

where $\Phi_j\colon R^{n+1} \to R$ is twice continuously differentiable. We make the following assumptions on the technologies of firms:

(F1) $y_j \, \varepsilon \, Y_j \Longrightarrow y_j^1 \leq 0$ $\quad (j = 1, 2, \ldots, m)$

(F2) Y_j is a closed and convex subset of R^{n+1} $\quad (j = 1, 2, \ldots, m)$

(F3) $Y_j \cap (-Y_j) = \{0\}$ $\quad (j = 1, 2, \ldots, m)$

(F4) $Y_j \supset -R_+^{n+1}$ $\quad (j = 1, 2, \ldots, m)$

(F5) For all $j \, \varepsilon \, \{1, 2, \ldots, m\}$, if $\{y_k\}$ is a sequence of net output vectors, such that

$$y_k \, \varepsilon \, Y_j \qquad \forall \, k \, \varepsilon \, Z_+$$

$$\lim_{k \to \infty} y_k^0 = 0$$

then there exists some $h \, \varepsilon \, \{2, 3, \ldots, n\}$ such that

$$\lim_{k\to\infty} \frac{\partial \Phi_j\,(y_k)/\partial y_k^0}{\partial \Phi_j\,(y_k)/\partial y_k^h} = -\infty$$

If $\{y_k\}$ is a sequence of net output vectors such that

$$y_k\ \varepsilon\ Y_j \qquad\qquad \forall\ k\ \varepsilon\ Z_+$$
$$\lim_{k\to\infty} y_k^0 = +\infty$$

Then, for $n = 1, 2, \dots, n$

$$\lim_{k\to\infty} \frac{\partial \Phi_j\,(y_k)/\partial y_k^0}{\partial \Phi_j\,(y_k)/\partial y_k^h} = 0$$

Assumption (F1) states that money is not produced. Assumptions (F2), (F3) and (F4) are standard in the literature. Assumption (F3) states that production processes are irreversible, and (F4) allows for free disposal. Assumption (F5) is a type of Inada condition. It states that the marginal productivity of labor in the production of some good becomes infinite as the firm's input of labor converges to zero, and it is needed to avoid the possibility of $y = 0$ being the wage-maximizing net output vector for firm j given a strictly positive price vector. It also states that if $\{y_k\}_k\ \varepsilon\ Z_+$ is a sequence of feasible net output vectors, such that $\| y_k \| \to \infty$ then all marginal productivities of factors have to converge to zero along the sequence.

The amount of labor supplied by household i to firm j is denoted by $\bar{\ell}_i^j$. We assume that, at a certain point in time, a household can be a member of at most one labor-managed firm, and that there is a uniform bound to the endowments of time of households. That is, if we define the endowment of time of household i to be x_i^{0e}, then, with an appropriate normalization, $x_i^{0e} \leq 1\ \forall\ i\ \varepsilon\ I$.

The following relations also hold

(2.1) $\quad 0 \leq \bar{\ell}_i^j \leq x_i^{0e} \leq 1 \qquad (i\ \varepsilon\ I, j = 1, 2, \dots, m)$

$\qquad \bar{\ell}_i^j\ \bar{\ell}_i^k = 0 \qquad (j, k\ \varepsilon\ \{1, 2, \dots, m\}, j \neq k)$

It follows from our previous definitions that

$$\bar{x}_i^0 = x_i^{0e} - \sum_{j=1}^{m} \bar{\ell}_i^j \qquad (i\ \varepsilon\ I)$$

(2.2) $\quad \bar{y}_j^0 = \int_I \bar{\ell}_i^j\, d\mu \qquad (j = 1, 2, \dots, m)$

We define Γ to be the set of all square integrable-functions $\Psi: I \to R$, and

$$p = (1, p^2, \dots, p^n)'$$

$$\bar{\ell}_i = \sum_{j=1}^{m} \bar{\ell}_i^j = x_i^{0e} - \bar{x}_i^0 \qquad (i \,\varepsilon\, I)$$

$$\bar{\ell}^j = \{\bar{\ell}_i^j\}_{i\,\varepsilon\, I} \;\; (\varepsilon\, \Gamma) \qquad (j = 1, 2, \dots, m)$$

$$\bar{\ell} = (\bar{\ell}^1, \bar{\ell}^2, \dots, \bar{\ell}^m)'$$

$$\bar{x}_i = (\bar{x}_i^1, \bar{x}_i^2, \dots, \bar{x}_i^n)' \qquad (i \,\varepsilon\, I)$$

$$x_i = (x_i^1, x_i^2, \dots, x_i^n)' \qquad (i \,\varepsilon\, I)$$

$$\bar{x}^h = \{\bar{x}_i^h\}_{i\,\varepsilon\, I} \;\; (\varepsilon\, \Gamma) \qquad (h = 0, 1, \dots, n)$$

$$\bar{x} = (\bar{x}^1, \bar{x}^2, \dots, \bar{x}^n)'$$

$$\bar{y}_j = (\bar{y}_j^1, \bar{y}_j^2, \dots, \bar{y}_j^n)' \qquad (j = 1, 2, \dots, m)$$

(2.3) $$\bar{y} = \left((\bar{y}_1^0, \bar{y}_1'), (\bar{y}_2^0, \bar{y}_2'), \dots, (\bar{y}_m^0, \bar{y}_m')\right)'$$

$$y_j = (y_j^1, y_j^2, \dots, y_j^n)' \qquad (j = 1, 2, \dots, m)$$

$$\bar{x}^h = \int_I \bar{x}_i^h d\mu \qquad (h = 0, 1, \dots, n)$$

$$X^h = \int_I X_i^h d\mu \qquad (h = 0, 1, \dots, n)$$

$$\bar{Y}^h = \sum_{j=1}^{m} \bar{y}_j^h \qquad (h = 0, 1, \dots, n)$$

$$Y^h = \sum_{j=1}^{m} y_j^h \qquad (h = 0, 1, \dots, n)$$

$$Z^h = (X^h - \bar{X}^h) - (Y^h - \bar{Y}^h) \qquad (h = 0, 1, \dots, n)$$

A firm can accumulate profits in two different ways. It can buy or sell commitments to deliver the commodities indexed by $h = 2, 3, \dots, n$, or it can buy or sell membership rights, at the price of u_j. We assume that the market for membership rights works well enough so that, at each moment of time, there exists some w_c such that

$$w_1 - u_1 = w_2 - u_2 = \cdots = w_m - u_m = w_c$$

Analytically, the actual profits of firm j at time t are given by

(2.4) $\bar{\pi}_j(t) = \bar{\pi}_j(0) + \int_o^t \left[p'(\tau)\dot{\bar{y}}_t(\tau) + u_j(\tau)\dot{\bar{y}}_j^0(\tau)\right]d\tau$

The actual wage paid by the firm satisfies

$$\bar{w}_j = \frac{\bar{\pi}_j}{\bar{y}_j^0}$$

that is, all the members of a labor-managed firm receive an equal share of its profits.

The objective of the firm is maximize profit per worker, so the firm chooses (y_j^0, y_j) to be the solution of max w_j subject to

(2.5) $w_j = \dfrac{p'(y_j - \bar{y}_j) + \bar{\pi}_j + u_j(y_j^0 - \bar{y}_j^0)}{y_j^0}$

$y_j^0 > 0$

$(y_j^0, y_j)\ \varepsilon\ Y_j$

Equation (2.5) can be solved to give

(2.6) $w_j = \dfrac{p'(y_j - \bar{y}_j) + \bar{\pi}_j + w_c(\bar{y}_j^0 - y_j^0)}{\bar{y}_j^0}$

Assumption (F5) implies that this problem has a solution with $y_j^0 > 0$. Given this, it follows from (2.6) and (F5) that if, along a trajectory, there is a positive lower bound for the price vector, then, along that trajectory, there is a positive lower bound to y_j^0. Also, it is clear from (2.6) that the firm's maximization problem is equivalent to the problem of a firm that can hire labor at the fixed wage w_c and does not consider changing its membership.

The expected wage of firm j, w_j, is the optimal value of the problem above. We define

$$\bar{w} = (\bar{w}_1, \bar{w}_2, \dots, \bar{w}_m)'$$
$$w = (w_1, w_2, \dots, w_m)'$$
$$\bar{\pi} = (\bar{\pi}_1, \bar{\pi}_2, \dots, \bar{\pi}_m)'$$

We assume that, for all $t \geq 0$, if firm j does not expect to make additional profits, then it does not want to change its vector of net outputs. That is, for all $t \geq 0$ and for $j = 1, 2, \dots, m$,

(2.7) $w_j = \bar{w}_j \Longrightarrow y_j = \bar{y}_j, y_j^0 = \bar{y}_j^0$

Also, a firm's stock of labor is never greater than its desired stock of labor. That is, a firm can lay-off workers or reduce their numbers of hours worked at will. Analytically this is expressed by $y_j^0 \geq \bar{y}_j^0$.

Workers cannot be forced to work more than their desired number of hours. Analytically, this is expressed by $x_i^0 \leq \bar{x}_i^0$.

We define w_i, the wage available to the i[th] household, as $w_i = w_j$ if $\bar{\ell}_i^j > 0$ and $w_i = w_c$ if $\bar{\ell}_i = 0$. We are thus assuming that the household always believes in the existence of available jobs.

The actual wage received by household i is denoted by $\bar{w}_i$ and given by

$$\bar{w}_i = \bar{w}_j \text{ if } \bar{\ell}_i^j > 0$$
$$\bar{w}_i = 0 \text{ if } \bar{\ell}_i = 0$$

A household that changes the amount of labor that it provides to a labor-managed firm has to trade in membership rights. This is true whether the household is being admitted as a member, is leaving a firm, or is just changing the amount of labor provided to its employer. This kind of trading is discussed in Meade [7] and Bonin [l].

The household's optimization problem is

$$\max U_i\,(x_i^0, x_i)$$

subject to

(2.8) $$p'(x_i - \bar{x}_i) + w_i(x_i^0 - \bar{x}_i^0) \leq (w_i - \bar{w}_i)\bar{\ell}_i + (w_i - w_c)(x_i^0 - \bar{x}_i^0) \qquad 0 \leq x_i^0,\ 0 \leq x_i$$

The vector of x_i^0, x_i of desired stocks of the household is chosen in the set of solutions to this problem.

The budget constraint of the household can be rewritten as

(2.9) $$p'^{x_i} + w_c x_i^0 \leq \omega_i \equiv p'^{\bar{x}_i} + w_c \bar{x}_i^0 + (w_i - \bar{w}_i)\bar{\ell}_i$$

It is clear from (2.9) that the price of leisure for the household is w_c and not w_i. The expected wealth of household i, ω_i is defined in (2.9). We also define eu, the actual wealth of household i, as

$$\bar{\omega}_i = p'\bar{x}_i + w_c\bar{x}_i^0 - \bar{w}_i\bar{\ell}_i$$

The interpretation of the budget constraint (2.8) is that' the value of the excess demands of household i cannot exceed the additional wages and resources from trades in membership rights that it expects to receive.

We denote by $V_i(p, w_c, \omega_i)$ the indirect utility function of household i and by λ_i the Lagrange multiplier associated with its budget constraint. That is, λ_i is the marginal utility of wealth for household i.

At this point one must notice that the behaviour of both households and firms is naive. That is, in each period of time agents think that equilibrium will obtain at the given prices, after trade. In most non-tatonnement models agents display such naive behaviour, the most important exception being the work of Fisher [5]. Considerable insights on the role of expectations in stability theory are obtained in [5], where it is shown that a necessary condition for the instability of a competitive economy is the appearance of unforeseen opportunities that can be exploited by agents. However, Fisher does not supply a connection between the actual events that take place in the economy and changes in expectations. In the present model we follow a tradition in non-tatonnement stability theory by specifying such a connection in the form of static expectations. Admittedly, a more reasonable rule of expectations formation would be preferable. In the present paper we advance the positive theory of the stability of labor- managed economies to a point comparable to the one reached in the theory of the stability of competitive economies (see [5]). That is, positive results are obtained under the assumption that agents are naive, but not under more realistic rules of expectation formation. It remains to be seen whether, as for competitive economies, a necessary condition for the instability of labor-managed economies is the appearance of unforeseen opportunities.

Let $S = \mathbb{R}^{(n+1)(m+1)+3m} x \Gamma^{m+n}$ and let S^* be the dual space of S. Define S' as the subset of S consisting of all $(p, w_c, w, \bar{w}, \bar{\pi}, \bar{y}, \bar{\ell}, \bar{x})$ such that $p > 0$, $w_c > 0$, $w > 0$, and

$$0 \leq \bar{\ell}_i^j \qquad (i \ \varepsilon \ I, j = 1, 2, \dots, m)$$

(2.10) $$\bar{\ell}_i \leqq x_i^{0e} \qquad (i \ \varepsilon \ I)$$

$$\bar{x}_i \geq 0 \qquad (i \ \varepsilon \ I)$$

$$\left(\bar{y}_j^0, \bar{y}_j\right) \ \varepsilon \ Y_j \qquad (j = 1, 2, \dots, m)$$

In the next section we define our process as a dynamical system on the set S'. We endow S with a product topology, by giving to $\mathbb{R}^{(n+1)(m+1)+3m}$ the standard topology and identifying Γ with $L^2(I)$. The set $S' \subset S$ is given the subspace topology.

Given a vector $\hat{s} \ \varepsilon \ S'$, the economy defined by $\hat{s}$ is the set of all $s \ \varepsilon \ S'$ such that

(2.11) $$\int_I (\bar{x}_i^h - \hat{\bar{x}}_i^h) d\mu = \sum_{j=1}^m (\bar{y}_j^h - \hat{\bar{y}}_j^h) \qquad (h = 1, 2, 3, \dots, m)$$

$$\int_I (\bar{x}_i^0 - \hat{\bar{x}}_i^0) d\mu = \int_I \sum_{j=1}^m \left(\hat{\bar{\ell}}_i^j - \bar{\ell}_i^j\right) d\mu$$

The equations (2.11) express the fact that the economy is closed. We say that firm j is in equilibrium if $w_j = \bar{w}_j$. As assumed above, in this case, $y_j = \bar{y}_j$ and $y_j^0 = \bar{y}_j^0$. We say that household i is in equilibrium if $x_i^h \leq \bar{x}_i^h$ for $h = 0, 1, 2, \dots, n$. A vector $s \ \varepsilon \ S'$ is said to be an equilibrium for the economy defined by $s \ \varepsilon \ S'$ if s belongs to the economy defined by $\hat{s}$ and, at s, all firms and almost all households are in equilibrium. The set of all equilibria for the economy defined by s is denoted by $E(s)$.

III. The Dynamic Process

Formally, the dynamic process consists of a dynamical system $\Phi: S' \text{x } \mathbb{R}_+ \to S'$, satisfying the following assumptions:

i) For all $\tilde{s}\ \varepsilon\ S'$ and all $t \geq 0$, $\Phi(\tilde{s}, t)$ is in the economy defined by $\tilde{s}$. That is, the equations (2.11) hold with $\tilde{s} = \hat{s}$ and $s = \Phi(\tilde{s}, t)$.

ii) For any $\hat{s}\ \varepsilon\ S'$, there exists $\delta(s) > 0$ such that, for all $t \geq\ 0$,

$$\bar{y}_j^0 \geq \delta(s) \qquad (j = 1, 2, \dots, m)$$

$$\overline{w}_j(t) \geq 0 \qquad (j = 1, 2, \dots, m)$$

iii) Given $s\ \varepsilon\ S'$ let $s(t)\ \equiv \Phi(s, t)$. Then the right time derivatives exist.

$$\dot{p}(t), \dot{\bar{y}}_j^0(t) \qquad (j = 1, 2, \dots, m)$$

$$\dot{\bar{y}}(t), \dot{w}_i(t) \qquad (i\ \varepsilon\ I)$$

$$\dot{\overline{w}}_i(t) \qquad (i\ \varepsilon\ I)$$

$$\dot{w}_c(t), \dot{\bar{x}}_i^0(t) \quad (i\ \varepsilon\ I)$$

$$\dot{\bar{x}}_i(t)^2$$

$$\dot{p}^h Z^H \geq 0 \qquad (h = 2, 3, \dots, H)$$

With the inequality holding strictly if either $Z^h > 0$ or $Z^h < 0$ and $p^h > 0$, and $\dot{w}_c Z^0 \geq 0$ with the restrict inequality if either $Z^0 > 0$ or $Z^0 < 0$ and $w_c > 0$.

iv) Given $s\ \varepsilon\ S'$, for all $t \geq 0$ and $h = 0, 2, 3, \dots, n$,

$$x_i^h \neq \bar{x}_i^h \geq 0 < \left(x_i^h - \bar{x}_i^h\right) Z^h \qquad (i\ \varepsilon\ I)$$

$$y_j^h \neq \bar{y}_j^h \geq 0 > \left(y_j^h - \bar{y}_j^h\right) Z^h \qquad (j = 1, 2, \dots, m)$$

We define $S'' \subset S'$ as the subset of S' where iv is satisfied.

v) For any $s\ \varepsilon\ S$, there exist $\underline{p}(s)$, $\bar{p}(s)$, $\underline{w_c}(s)$ and $\overline{w}_c(s)$ satisfying

$$0 < \underline{p}(s) < \bar{p}(s)$$

$$0 < \underline{w_c}(s) < \overline{w}_c(s)$$

and such that, for all $\hat{s}$ in the economy defined by s,

$$\hat{p}^h < \underline{p}^h(s) \geq Z^h(\hat{s}) > 0$$

$$\hat{p}^h > \bar{p}^h(s) \geq Z^h(\hat{s}) < 0$$

$$\widehat{w} < \underline{w_c}(s) \geq Z^0(\hat{s}) < 0$$

$\hat{w} > \underline{w}_c(s) \geq Z^0(\hat{s}) > 0$

vi) For any $s \, \varepsilon \, S'$ the integral

$$V(s) = \int_I V_i(p, w_c, \omega_i) d\mu$$

exists.

vii) Given $s \, \varepsilon \, E'$ and the trajectory $\Phi(s,t)$, for any $t \geq 0$, let $C(t) \subset I$ be the set of households whose employment situation is changing at time t. Then, $\mu[C(t)] = 0, \forall \, t \geq 0$.

viii) For any $s \, \varepsilon \, S'$ and all $t \geq 0$,

(3.1) $\dot{\bar{\omega}}_i = \dot{p}'\bar{x}_i + w_i\dot{\bar{x}}_i^0 + \dot{w}_c\bar{x}_i^0 \quad i \not\varepsilon \, C(t))$

(3.2) $\dot{\omega}_i = \dot{p}'\bar{x}_i + \dot{w}_i\bar{\ell}_i + \dot{w}_c\bar{x}_i^0 \quad (i \, \varepsilon \, I)$

ix) A vector $s \, \varepsilon S'$ is an equilibrium for the economy defined by s itself if and only if's is a rest point of Φ.

We now discuss these assumptions.

Assumption i states that the economy is closed, except for the fact that labor can be exchanged for wages (money) abroad. This assumption could be relaxed by allowing trading in other commodities with fixed prices.

In our model households are indifferent between working in different labor-managed firms. Therefore, the first relation in Assumption ii is both necessary and plausible. Indeed, there is no reason why all members of a firm would not eventually leave, but there is also no reason why they should leave. The second inequality imposes non-negativity of wages, which is a weak "No Bankruptcy" assumption. One can imagine that some firms may go bankrupt, and there are two possibilities: either all firms go bankrupt, and the economy reverts to pure exchange, or some firms survive. In the latter case we redefine the process to include only surviving firms.

Assumption iii States that prices move in the directions given by the corresponding excess demands, if possible without violating the non-negativity constraints. This applies also to the "true" wage rate w_c.

Assumption iv is a Hahn Process assumption. Except for money, there are agents with unsatisfied demands in at most one side of a market. The Hahn Process assumption is the driving force behind our stability proof. On this, see the discussion in Section IV. This assumption is frequently made in works on the stability theory of competitive economies (see [5]) and the reasoning that usually justify it in that context apply equally well to labour-managed economies, as far as commodities are concerned.

We have assumed that, for all i, $x_i^0 \leq \bar{x}_i^0$ and for all j, $x_j^0 \leq \bar{x}_j^0$. This means that, as far as the labour “market” is concerned, the only possible violations of assumption iv are of the type in which there are some firms with positive excess demands for labour and some households with negative excess demands for labour. It is clear that this situation can be transformed into one in which only one side of the market is unsatisfied by a sequence of bilateral trades in which at least one of the agents involved on the trade has an expected gain. For example, a firm with a positive excess demand for labour may hire an unemployed worker or a worker who wants to work more hours may leave a firm, knowing that there are other firms that have positive excess demands for labour and therefore would want to employ him for the desired number of hours. Given this, it is reasonable to extend Assumption iv to the “market” for labour. Such a sequence of trades would not exist in general in the absence of the assumptions $x_i^0 \leq \bar{x}_i^0$ and $x_j^0 \leq \bar{x}_j^0$. For example, if all firms are in equilibrium, one worker in firm j wants to work more hours, and one worker in firm k wants to work less hours, then these workers might not want to leave their respective firms, since for each one of them the possibility of finding another job after leaving depends on whether the other also leaves his firm or not. The existence of trades would then require a higher degree of communication between agents, and the Hahn Process assumption would be implausible.

Assumption v is also important for the proof of stability. Clearly, its role is to ensure the boundedness of prices. It can be derived from mild assumptions on the preferences of households.

Given that the utility functions of households are normalized, Assumption vi simply states that for any $s \; \varepsilon \; S'$ the function $i \longrightarrow V_i(p, w_c, \omega_i)$ is measurable. It allows us to construct a function $V + W : S' \longrightarrow \mathbb{R}$ that behaves like a Lyapounov function, but is not necessarily continuous.

Although Assumption vii is not explicitly used below, without it we could not reasonably expect the variables of the system to change continuously. The possibility of having Assumption vii and thereby eliminating aggregate discontinuities is precisely the reason why we model a labor-managed economy with a continuum of households. Without it there would be a discontinuity on x whenever a set of positive measure of workers would change their employment situations. This is because although such a change does not affect the expected wealth of a household, it does affect its actual wealth, and therefore its money holdings. It goes without saying that Assumption vii) does not imply that the memberships of firms are constant or almost constant. In any finite period of time infinitesimal changes integrate to finite values.

viii is a “No Swindling” assumption (cf. [5, p. 54]). Equation (3.1) is equivalent to

$$p'\dot{x}_i = (w_i - w_c)\dot{x}_i^0 + \dot{\bar{w}}_i\bar{\ell}_i + \bar{w}_i\dot{\bar{\ell}}_i \qquad (i \not\varepsilon \; C(t))$$

which States that for a household $i \; \varepsilon \; C(t)$, the net value of purchases of commodities in a given period (time excepted) has to equal the sum of the additional revenues from trading in membership rights and wages that it receives in that period. For $i \not\varepsilon \; C(t)$, (3.2) follows easily from (2.9) and (3.1).

It states that the expected wealth of a household $i \not\varepsilon C(t)$ is not affected by its trades, but only by price and wage changes. For i£ C (t) we should expect the same to be true. That is, a household should not be able to profit (or loss) from switching jobs. Therefore (3.2) must also hold for $i\ \varepsilon\ C(t)$, with $\dot{w}_i$ and $\bar{\ell}_i$ denoting the values that these variables assume after the switch.

Finally, Assumption ix simply states the identity of the sets of equilibria of the dynamic process and of the labor-managed economy. We could have stated it as a result, which would easily follow from iv and some trivial assumptions, but to postulate it simplifies matters.

IV. Quasi-Stability

The dynamic process defined in Section III is quasi-stable in the weak topology of S. More precisely, for any set of initial conditions $s\ \varepsilon\ S''$ the vector of state variables $s(t) \equiv \Phi(s,t)$ converges weakly to the set of equilibria defined by s. By weak convergence we mean convergence in the weak topology of S. Here we only State the theorem, and make some remarks about it. The proof can be found in the Appendix.

Theorem 1: For any $\hat{s}\ \varepsilon\ S''$ the trajectory $\Phi(\hat{s},t)$ converges weakly to the set of equilibria for the economy defined by $\hat{s}$. More precisely, if $\{t_k\}_k$ is an increasing, unbounded sequence, then the sequence $\{\Phi(s,t_k)\}_k$ has a subsequence $\{\Phi(t_{k_\alpha})\}_\alpha$ such that, for some $\tilde{s}\ \varepsilon\ E(\tilde{s})$,

$$\lim_{\alpha\to\infty} < \Phi\left(s,t_{k_\alpha}\right) - \tilde{s}, s^* \geq 0 \qquad\qquad \forall\ s^*\ \varepsilon\ S^*$$

Moreover, the wealth $w_i(s,t_k)$ converge to $\widetilde{w}_i$ for all $i\ \varepsilon\ I$, and, for $h = 0,1,\dots,n$, the actual stocks $\bar{x}^{-h}(s,t_{k_\alpha})$ converge in measure to $\tilde{\bar{x}}^h$, that is, for all $\varepsilon > 0$

$$\lim_{\alpha\to\infty} \mu\left\{i\ \varepsilon\ I \parallel \bar{x}_i^h\left(s,t_{k_\alpha}\right) - \tilde{\bar{x}}_i^h\right| > \varepsilon\right\} = 0 \qquad\qquad (h = 0,1,\dots,n)$$

The crucial Assumption for the proof of the Theorem is iv, the Hahn Process assumption. As usual, it is used to show that since all agents are in the same side of a market, all are hurt by the price movement of that market. Notice that this is true for our type of LME, as well as for competitive economies. If we are talking about households this is obvious, since the objective function of the household is the same in the two types of economies. According to equation (2.6) above, in our model the firm behaves as if it were trying to maximize profits with a fixed membership and facing a fixed wage w_c. Then the objective function of the labor-managed firm is identical to that of a competitive firm, and the usual reasoning involving the Hahn Process assumption (cf. [5, pp. 31-2]) applies here.

Of course, the point of all this is building a Lyapounov function. We run into a technical difficulty here because, since there is an infinite number of workers, the "sum" of their utilities

$$V(s) = \int_I V_i(s)d\mu$$

need not be a continuous function of s, even though each $V_i(s)$ is continuous. We model an economy with an infinite number of workers to avoid another kind of discontinuity, which would occur when a worker switched jobs. It turns out that we get more than additional complexity by doing so. We can show that, if the function W is defined by

$$W(s) = \sum_{j=1}^{m} w_j(s)$$

then $V + W$ has the essential properties of a Lyapounov function. That is, $V + W$ is: i) decreasing outside the set of equilibria, and ii) constant on the ω-límit set of a trajectory. Notice that ii) follows from i) if a continuous function is being considered.

The weak topology of S is used to guarantee that any bounded sequence has a convergent subsequence. This, together with the properties of $V + W$ discussed above ensures quasi-stability.

We repeat that our result is independent of the truth value of an "existence theorem", since we have a non-tatonnement model, where endowments are not fixed.

It is natural to ask whether or not it is possible to give any characterization of equilibria. In particular, given the absence of an explicit domestic labor market, such issues as full employment and efficiency are not foregone conclusions. Because there is not a real labor market in the LME, there is no common wage rate. Rather there are m payment rates: one w_j for each of the m firms. This fact is somewhat misleading however. From the consumer's point of view, w_j is a profit share, if the consumer belongs to firm j, but is the relevant wage rate regardless of the individual's employment status. If consumer i belongs to firm j, then an increase in labor supplied to j increases i's income at the rate w_j, but this is partially offset by the payment of the fee $u_j = (w_j - w_c)$. This leaves the net return to increasing hours worked of w_c. The same holds true if the consumer decreases hours worked, joins or leaves a firm, or changes the hours worked in the foreign sector.

Similarly, w_c represents the true cost to any firm of expanding employment. Thus, w_c is effectively a uniform wage rate for labor. This wage rate is common to all firms and to all workers.

In essence, then, the differences among the w_j's represent pure wealth effects, and are not true differences in wage rates. This guarantees that the LME economy here modelled entails $n + 1$ markets with cominon prices on each market.

During the whole process, and, a fortiori, in equilibrium, full employment is maintained. This is because the foreign sector absorbs any amount of labor that is not employed domestically.

Our assumptions on technology, preferences and (non-labor) markets are all quite standard. Given that w_c becomes an implicit wage rate for all firms and all consumers, it follows that an LME equilibrium is isomorphic to a standard Walrasian equilibrium. Of course, such an equilibrium must be Pareto optimal.

V. Institutional Variations

At present there is no firmly-entrenched theory of the LME. Rather, there is substantial variation of opinion as to what the objective function of the LKF actually is or should be, how capital markets work or should be structured, and so on. Further, there is the question as to how exactly the theory should correspond to (the evolving) institutional reality of Yugoslavia. Above, we have refrained from appealing to the Yugoslav case too extensively. In this final section we offer a few comments on how our model compares with the Yugoslav economy and how our results might be affected by variations in how the LME is modelled.

The 'classical' analysis of LME's does not entail our assumption of an implicit market for membership rights. In this standard case, maximizing profit per member guarantees a negatively sloped output supply curve when labor is the only variable input, and this perverse supply curve is a strong possibility even when there are multiple variable inputs.

In our model, this institutional specification would alter the firm's objective function to one of maximizing

$$w_j(\tau) = \frac{p'(\tau)\left[y_j(\tau) - \bar{y}_j(\tau)\right] + \bar{\pi}_j(\tau)}{y_j^0(\tau)}$$

The fact that the resulting optimal choice of $y_j(t)$ might respond perversely to $p'(t)$ does not inherently cause any problems for our stability analysis. Rather, a severe problem is created by the inclusion of the $[\bar{\pi}_j(t)]$ term. If, at any time, t, the accumulated profit of the firm exceeds the value of its contracts (i.e., $\bar{\pi}_j(t) > p'(t)\bar{y}_j(t)$), then the firm can generate infinite per-worker profits by buying-out its commitments (*viz.*, setting $y_j(t) = 0$) and reducing $y_j^0(t)$ toward zero, and distributing $(\bar{\pi}_j(t) - p'(t)\bar{y}_j(t))$ among its non-existing members.

If it could be guaranteed that $\bar{\pi}_j - p'\bar{y}_j$ never became positive, then stability of the LME could be demonstrated. This would be true regardless of the direction of the changes in y_j and y_j^0 in response to changes in p. Hence, the potential Marshallian instability of the Labor-Managed firm turns out not to be a cause of non-tatonnement instability in a general equilibrium setting.

A final noteworthy observation regarding the possible benefit to workers of liquidating their company is that this problem is explicitly dealt with in the Yugoslav constitution. Specifically, worker management is curtailed when it comes to a shut-down decision. First, firms are required by law to maintain the value of their enterprises. Second, if a firm is liquidated, the proceeds revert to the State.

These restrictions can be incorporated in our model in two ways. First, the requirement that a firm's value not be depleted can be expressed as a requirement that, for at least one component (capital) of y_j, $y_j^h, \dot{\bar{y}}_j^h \leq 0$. Together with an initial condition $\bar{y}_j^h(0) < 0$, this guarantees that $\bar{y}_j(t) =$

0 is in fact infeasible.

Even more explicitly, the prohibition on workers closing down their firm cart be translated as a requirement that $\bar{y}_j^0$ have a lower bound.

The final possible institutional variation taken directly from Yugoslav experience is that workers not be charged for entering a firm but that they would be eligible for compensation upon their departure, especially if this is involuntary (e.g., due to dismissal by co-workers). This procedure is argued for on two grounds – fairness and incentives to undertake investment at the expense of current income. This procedure would, however, create problems in terms of the analysis of the preceding sections. The reason is simply that workers would be able to generate income, without bounds, by repeatedly joining and leaving firms, receiving compensation for each departure. Obviously, this practice would be individually rational and would foster considerable instability.

The rule requiring entering (departing) members of a firm to make payment (receive compensation) equivalent to the difference between that firm's going wage and the opportunity wage transforms the firm's optimization problem into one of maximizing wages (per capita profits) for the existing membership. Since, by definition, the existing membership is fixed, this problem is equivalent to maximizing total profits, treating new membership as an input 'purchased' at the market wage rate, w_c. This can be interpreted as implying that, in equilibrium, membership will be such that the marginal revenue product of labor equals the implicit wage rate, w_c.

There is, however, no guarantee that at equilibrium actual wages paid will equal w_c. Nor is there anv reason why wages would be common across firms. Further, within a firm, the net payment to members will vary according to the time at which they joined the firm. Charter members of the firm will receive the wage $\bar{w}_j^\star = \bar{\pi}_j^\star / y_j^{0\star}$. A member who joined the firm at time $t > 0$ will receive the wage $\bar{w}_j^\star$, but will have paid an entrance fee of $(w_j(t) - w_c)$, leaving a net payment of $w_c + [\bar{w}_j^\star - w_j(t)]$.

Consider now what happens when a worker switches firms. First, a worker might want to move from firm j to firm k if this led to an expected rise in w_i. But the worker who transfers receives compensation $(w_k - w_j)$ from firm j and pays compensation $(w_k - w_c)$ to firm k. In net, worker i pays a fee $(w_k - w_j)$ to transfer from k to j. This fee, of course, is exactly equal to the expected wage increase from the transfer. Similarly, if $w_k < w_j$, there is no gain.

It would be possible to object to our compensation rules on the ground that they create universal indifference among workers as to their place of employment. As an alternative, Meade [7] and Bonin [2] suggest compensations that are within a range such that both the worker who enters or exits a firm, and all other members of a firm are made strictly better off by the move. For example, a firm j taking on new members would charge an entry fee of $(w_j - w_c) - \varepsilon$ for some $0 < \varepsilon < (w_j - w_c)$

that would render expansion beneficial to both existing and new members. Similarly, a contracting firm would pay departing members compensation of $(w_j - w_c) + \delta$ where $\delta > 0$. It would further be possible to make δ payable only to members who leave the firm involuntarily (and in fact to charge δ to members who quit).

The terms ε and δ would be variables depending, for example, on the differences $(w_j - w_c)$ and $(y_j^0 - \bar{y}_j^0)$. Presumably, ε and δ would become null in equilibrium. Again such rules would create incentives for workers to keep switching jobs indefinitely.

Appendix

Lemma 1: Given $s \ \varepsilon \, S''$ and with $s(t) = \Phi(s,t)$ for any $t \ > \ 0$,

(A.1) $$\dot{w}_j(t) \leq 0 \qquad (j = 1, 2, \dots, m)$$

(A.2) $$\dot{V}_i(t) \leq 0 \qquad (i \, \varepsilon \, I)$$

Moreover, if firm j is not in equilibrium at time t, then the corresponding inequality in (A.l) holds strictly, and if household i is not in equilibrium at time t, then the corresponding inequality in (A.2) holds strictly.

Proof: We differentiate (2.6), using the Envelope Theorem, to get

(A.3) $$\dot{w}_j = \frac{\dot{p}'\left(y_j - \bar{y}_j\right) - p'\dot{\bar{y}}_j + \dot{\bar{\pi}}_j - u_j\dot{\bar{y}}_j^0 + \dot{w}_c(\bar{y}_j^0 - y_j^0)}{\bar{y}_j^0}$$

From (2.4), equation (A.3) simplifies to

(A.4) $$\dot{w}_j = \frac{\dot{p}'(y_j - \bar{y}_j)}{\bar{y}_j^0} + \frac{\dot{w}_c(\bar{y}_j^0 - y_j^0)}{\bar{y}_j^0}$$

and (A.1) and the related assertions in the statement of the Lemma follow from Assumptions ii, iii, and iv.

To prove (A.2), we first write

$$\dot{V}_i = \frac{\partial V_i}{\partial p}\dot{p} + \frac{\partial V_i}{\partial \omega_i}\dot{\omega}_i + \frac{\partial V_i}{\partial w_c}\dot{w}_c$$

or, using equation (3.2),

(A.5) $$\dot{V}_i = \lambda_i[\dot{w}_i\bar{\ell}_i - \dot{p}'(x_i - \bar{x}_i) - \dot{w}_c(x_i^0 - \bar{x}_i^0)]$$

The term $\dot{w}_i\bar{\ell}_i$ in the right-hand side of (A.5) is always no positive, from (A.1). Therefore, Assumptions ii and iii imply (A.2) and the related assertions about households in disequilibrium. This completes the proof of the Lemma.

Lemma 2: For any $s \, \varepsilon \, S$" the trajectory $\Phi(s, \mathbb{R}_+)$ is bounded.

Proof: Assumption v clearly implies that p and w_c are bounded, and that for large t, $p(t) \leq \bar{p}$ and $w_c(t) \leq \bar{w}_c$. From Lemma 1, each w is no increasing in time. Also, $w_j \geq \overline{w}_j$. Then w_j and $\overline{w}_j$ are bounded from above. From Assumption ii, $\overline{w}_j$ is bounded from below, so the same is true of w_j. Assumptions (F2), (F3), and (F4) imply that unbounded outputs can only be produced by the use of unbounded inputs (cf. Debreu [2]). Then the equations (2.11) implies that $\bar{y}_j^h$ is bounded along a trajectory for $h \, \varepsilon \, \{0, 2, 3, \dots, n\}$. Since money is not produced, the same is trivially true for $h \ = \ 1$. The boundedness of $\bar{\ell}$ follows from (2.1). It remains to be proved that $\bar{x}$ is bounded.

For any $i \, \varepsilon \, I$, $\dot{V}_i(t) \leq 0$ for all $t \geq \ 0$, from Lemma 1. Then, for all $i \, \varepsilon \, I$ and all $t \geq \ 0$, $V_i(t) \leq V_i(0)$.

By revealed preference, we have

$$p'(t)\bar{x}_i(t) + w_c(t)\bar{x}_i^0(t) \leq p'(t)x_i(0) + w_c(t)x_i^0(0)$$

or

(A.7) $$p'(t)\bar{x}_i(t) \leq p'(t)x_i(0) - w_c(t)[\bar{x}_i^0(0) - x_i^0(t)]$$

Since $0 \leq x_i^0 \leq 1$ for all $i \, \varepsilon \, I$, (A.7) implies

$$\underline{p}'\bar{x}_i(t) \leq \bar{p}'x_i(0) + \bar{w}_c$$

and therefore, for $h = 1, 2, \ldots, n$, and sufficiently large t we have $\bar{x}_i^h(t) \leq M$, with

$$M = \frac{\bar{p}'x_i(0) + \bar{w}_c}{\min\left(1, \underline{p}^2, \ldots, \underline{p}^n\right)}$$

Proof of Theorem 1: The space $L^2(I)$ is self-dual and separable, and therefore S also has these properties. It follows that bounded sequences in S have weakly convergent subsequence (cf. [3], p. 62), and we conclude that the set of limit points of the trajectory starting at $\hat{s}$ is nonempty. It remains to prove that this set is contained in $E(\hat{s})$. This would follow immediately from the existence of a Lyapounov function, and $V + W$, where W and V are as defined in Lemma 1, would seem to be a natural candidate. However, $V + W$ need not be continuous in the weak topology of S', and we must follow a slightly longer line of reasoning.

The properties of a Lyapounov function that are crucial for a proof of quasi-stability are that it must be decreasing outside of the set of equilibria, and constant on the set of limit points of a trajectory. Given the first property, continuity implies the second. From Lemma 1, $V + W$ is decreasing outside of the set of equilibria of the process defined in Section III.

However, as stated above, $V + W$ need not be continuous. Nevertheless, we show below that $V + W$ is constant on the set of limit points of a trajectory, and thus displays all the properties that are needed to prove our quasi-stability theorem.

Clearly, W is continuous in S', with either the strong or weak topology. Now, consider an increasing sequence $\{t_k\}_{k \, \varepsilon \, Z_+}$ and a vector $\tilde{s} \, \varepsilon \, S'$ such that $\lim_{k\to\infty} t_k = \infty$ and, in the weak topology of S,

(A.8) $$\lim_{k\to\infty} \Phi(s, t_k) = \tilde{s}$$

It must also be the case that, for all $i \, \varepsilon \, I$,

(A.9) $$\lim_{k\to\infty} \omega_i(t_k) = \widetilde{\omega}_i$$

To see this, notice that, for all $i \, \varepsilon \, I$, we have

$$\lim_{k\to\infty} V_i[p(s, t_k), w_c(s, t_k), \omega_i(s, t_k)] = \inf\{V_i[p(s, t), w_c(s, t), \omega_i(s, t)] | t \geq 0\}$$

since V_i is monotonic nonincreasing in t and bounded from below. Now suppose that, for some i, (A.9) does not hold. From Lemma 2, $\omega_i(s, \mathbb{R}_+)$ is a bounded set, and hence there are ω_i^1, ω_i^2 and subsequence $\{t_{k_\alpha}\}$ and $\{t_{k_\beta}\}$ such that

(A.10) $$\lim_{\alpha\to\infty} \omega_i\left(s, t_{k_\alpha}\right) = w_i^1 < w_i^2 = \lim_{\beta\to\infty}(s, t_{k_\beta})$$

From nonsatiation,

$$V_i(\tilde{p}, \widetilde{w}_c, \omega_i^1) < V_i(\tilde{p}, \widetilde{w}_c, \omega_i^2)$$

and hence, since V_i is continuous, (A.8) and (A.10) imply that there exists some N such that, if $\alpha > N$ and $\beta > N$, then

$$V_i[p(s, t_{k_\alpha}), w_c(s, t_{k_\alpha}), \omega_i(s, t_{k_\alpha})] < V_i\left[p\left(s, t_{k_\beta}\right), w_c\left(s, t_{k_\beta}\right), \omega_i\left(s, t_{k_\beta}\right)\right]$$

This contradicts the monotonicity of V_i in t, and therefore (A. 8) must hold for all $i\ \varepsilon\ I$.

Now, from (A.8), and the fact that (A.9) holds for all $i\ \varepsilon\ I$, we have

(A.11) $$V_i(\tilde{p}, \widetilde{w}_c, \widetilde{\omega}_i) = \lim_{k\to\infty} V_i[p(t_k), w_c(t_k), \omega_i(t_k)]$$

for all $i\ \varepsilon\ I$. Also, for all $i\ \varepsilon\ I$,

(A.12) $$V_i[p(t_k), w_c(t_k), \omega_i(t_k)] \le V_i[p(0), w_c(0), \omega_i(0)] \qquad (k\ \varepsilon\ \mathbb{Z}_+)$$

hence we can apply the Lebesgue bounded convergence theorem to conclude that

$$\int_I V_i(\tilde{p}, \widetilde{w}_c, \omega_i)\, d\mu = \lim_{k\to\infty} \int_I V_i[p(t_k), w_c(t_k), \omega_i(t_k)] d\mu = \inf\{\int_I V_i[p(t), w_c(t), \omega_i(t)] d\mu | t \ge 0\}$$

and the function V is constant on the ω-limit set of the trajectory associated with s. Since W is continuous W is also constant on that ω-limit set, and so is $V + W$.

We conclude that $\Phi(\hat{s}, t)$ converges weakly to $E(\hat{s})$.

Now, for each $i\ \varepsilon\ I$, $h\ \varepsilon\ \{0, 1, \ldots, n\}$ and $k\ \varepsilon\ \mathbb{Z}_+$ define

$$E_k^{h+}(\varepsilon) = \{i\ \varepsilon\ I | \bar{x}_i^h(s, t_k) > \tilde{\bar{x}}_i^h + \varepsilon\}$$

$$E_k^{h-}(\varepsilon) = \{i\ \varepsilon\ I | \bar{x}_i^h(s, t_k) < \tilde{\bar{x}}_i^h - \varepsilon\}$$

$$E_k(\varepsilon) = E_k^+ \cup E_k^- = \{i\ \varepsilon\ I | \bar{x}_i^h(s, t_k) - \tilde{\bar{x}}_i^h > \varepsilon\}$$

From (A.8), (A.9), and Assumption (v) it follows that

$$\lim_{k\to\infty} x_i^h(s, t_k) = \tilde{\bar{x}}_i^h \qquad (\forall\ i\ \varepsilon\ I, h = 0, 1, \ldots, n)$$

and therefore, for $h = 0, 2, \ldots, n$,

(A.13) $$\lim_k \sup \min\left\{\mu\left[E_k^{h+}(\varepsilon)\right], \mu\left[E_k^{h-}(\varepsilon)\right]\right\} = 0$$

For suppose not. Then there exists some $\eta > 0$ and an increasing sequence $\{k_\infty\} \to \infty$ such that

$$\mu\left[E_{k_\propto}^{h+}(\varepsilon)\right] > \eta < \mu\left[E_{k_\propto}^{h-}(\varepsilon)\right]$$

For all $\alpha\ \varepsilon\ \mathbb{Z}_+$. Now, if $|x_i^h(s, t_{k_\alpha} - \tilde{\bar{x}}_i^h| \le \varepsilon/2$, then

(A.14) $$i\ \varepsilon\ E_{k_\alpha}^{h+}(\varepsilon) \Rightarrow x_i^h\left(s, t_{k_\alpha}\right) < \bar{x}_i^h(s, t_{k_\alpha})$$

$$i \; \varepsilon \; E_{k_\alpha}^{h-}(\varepsilon) \Rightarrow x_i^h(s, t_{k_\alpha}) > \bar{x}_i^h(s, t_{k_\alpha})$$

For large α,

(A.15) $$\mu\{i \; \varepsilon \; I \big| x_i^h(s, t_{k_\alpha}) - \bar{\bar{x}}_i^h \big| \leq \varepsilon/2\} > (1-\eta)$$

since $x^h(s, t_k)$ converges pointwise to $\bar{\bar{x}}^h$ and pointwise convergence implies convergence in measure. From (A.14) and (A.15) it follows immediately that, for large α,

$$\mu\{i \; \varepsilon \; I \big| x_i^h(s, t_{k_\alpha}) > \bar{x}_i^h(s, t_{k_\alpha})\} > 0 < \mu\{i \; \varepsilon \; I | x_i^h(s, t_{k_\alpha}) < \bar{x}_i^h(s, t_{k_\alpha})\}$$

which is a contradiction of the Hahn Process assumption (iv). Hence, (A.13) holds.

Now, suppose that there exists an increasing sequence $\{k_\alpha\} \to \infty$ such that, say, $\mu\left[E_{k_\alpha}{}^{h+}(\varepsilon)\right] > \delta$ for all $\alpha \; \varepsilon \; \mathbb{Z}_+$, and some $\delta > 0$.

From (A.13),

(A.16) $$\lim_{\alpha \to \infty} \mu\left[E_{k_\alpha}^{h-}(\varepsilon)\right] = 0$$

By the same reasoning, if $\varepsilon' < \varepsilon$ and

$$\mu\left[E_{k_{\alpha_\beta}}^{h-}(\varepsilon')\right] > \delta' > 0$$

for some subsequence of $\{k_\alpha\}$ then

$$\lim_{\beta \to \infty} \mu\left[E_{k_{\alpha_\beta}}^{h+}(\varepsilon')\right] = 0$$

and hence, since $\varepsilon' < \varepsilon$,

$$\lim_{\beta \to \infty} \mu\left[E_{k_{\alpha_\beta}}^{h+}(\varepsilon)\right] = 0$$

a contradiction. It follows that

(A.17) $$\lim_{\alpha \to \infty} \mu\left[E_{k_\alpha}^{h-}(\varepsilon')\right] = 0 \qquad \forall \; \varepsilon' \leq \; \varepsilon$$

Take $\varepsilon' \leq \; \varepsilon$. Then, from the boundedness of $\bar{x}$, there exists some $M \; > \; 0$ such that

(A.18) $$\int_I \left[\bar{x}_i^h(s, t_{k_\alpha}) - \bar{\bar{x}}_i^h\right] d\mu \geq \varepsilon\mu\left[E_{k_\alpha}^{h+}(\varepsilon)\right] - M\mu\left[E_{k_\alpha}^{h-}(\varepsilon')\right] - \varepsilon'\{\left[1 - \mu E_{k_\alpha}^{h+}(\varepsilon)\right] - \mu\left[E_{k_\alpha}^{h-}(\varepsilon')\right]\}$$

The weak convergence of $\bar{x}^h(s, t_{k_\alpha})$ implies that the left side of (A.18) converges to zero. Then, from (A.17),

$$0 \geq \varepsilon\mu\left[E_{k_\alpha}^{h+}(\varepsilon)\right] - \varepsilon'$$

or

$$\mu\left[E_{k_\alpha}^{h+}(\varepsilon)\right] \leq \frac{\varepsilon'}{\varepsilon}$$

Since ε'/ε can be made arbitrarily small, we conclude that

$$\lim_{\alpha \to \infty} \mu\left[E_k^{h+}(\varepsilon)\right] = 0$$

and obtain a contradiction. Similarly, $\mu[E_k^{h-}(\varepsilon)]$ must converge to zero, and it follows that the actual stocks $\bar{x}^h(s, t_k)$ converge in measure to $\tilde{\bar{x}}^h(s)$, for $h = 0, 2, \ldots, n$.

It remains only to show convergence in measure for $h = 1$. From (2.9), and the fact that $\widetilde{w}_i = \widetilde{\overline{w}}_i$, we can write

$$\bar{x}_i^1 - \tilde{\bar{x}}_i^1 = \omega_i - \widetilde{\omega}_i - \left[\sum_{h=2}^{n}(p^h \bar{x}_i^h - \tilde{p}^h \tilde{\bar{x}}_i^h) + (w_c \bar{x}_i^0 - \widetilde{w}_c \tilde{\bar{x}}_i^0) + (w_i - \overline{w}_i)\bar{\ell}_i\right]$$

Boundedness then implies that there exists some $M > 0$ such that

$$\left|\bar{x}_i^1 - \tilde{\bar{x}}_i^1\right| \leq |\omega_i - \overline{\omega}_i| + |w_i - \overline{w}_i| + M\left[\sum_{\substack{h=0 \\ h\neq 1}}^{n}\left|\bar{x}_i^h - \tilde{\bar{x}}_i^h\right| + \sum_{h=2}^{n}|p^h - \bar{p}^h| + |w_c - \widetilde{w}_c|\right]$$

from which the convergence in measure of $\bar{x}_i^1(t_k)$ to $\tilde{\bar{x}}_i^1$ follows easily from previous results.

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