CAN A WELL-FITTED EQUILIBRIUM ASSET PRICING MODEL PRODUCE MEAN REVERSION?

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SUMÁRIO

Num artigo recente, Cechetti, Lam e Mark (1990a) tiveram por objetivo mostrar que a correlação serial negativa no retorno de longo prazo das ações eram consistentes com um modelo de equilíbrio de determinação de preços de ativos. Neste artigo, mostramos que este resultado se baseia num erro de especificação do modelo de Markov para mudanças de regime do processo de dotação e numa hipótese forte sobre o conhecimento do estado futuro da economia pelo agente representativo. Uma vez que a especificação correta do modelo de Markov é escolhida para o processo de dotação e a hipótese usual sobre o conhecimento do agente é restabelecida, o modelo não produz a reversão à média na magnitude encontrada nos dados. Além do mais, a débil reversão à média produzida pelo modelo é atribuída ao viés causado pelo tamanho pequeno da amostra.

ABSTRACT

In a recent paper, Cechetti, Lam and Mark (1990a) intended to demonstrate that negative serial correlation in long horizon stock returns was consistent with an equilibrium model of asset pricing. In this paper, we show that their result relies on a misspecified Markov swiching model for the endowment process and on a strong assumption about the knowledge of the future state of the economy by the representative agent. Once the proper Markov specification is chosen for the endowment process and the normal assumption is made about the agent's knowledge, the model does not produce mean reversion in the magnitude detected in the data. Furthermore, the small amount of mean reversion produced by the model is only due to small sample bias.

The negative serial correlation detected in long horizon stock returns by Poterba and Summers (1988) and Fama and French (1988) could be interpreted as evidence in favor of the inefficiency of financial markets, or predictability of returns. Stock prices take irrational long swings away from their fundamental value. However, Fama and French (1988) propose also an efficient market or rational pricing explanation based on equilibrium expected returns. If shocks to expected returns are uncorrelated with shocks to rational forecasts of dividends, a shock to expected returns must be compensated by an opposite movement in the current price. If moreover expected returns are highly correlated but mean reverting, a shock to expected returns has no long term effect on expected prices. Mean reversion in prices is thus implied by mean reversion in expected returns. This is why we observe a negative correlation between the return for holding a stock from k periods before the shock occurred to the time of the shock, and the return for holding a stock from the time of the shock to k periods ahead. To support the efficient market explanation, one needs therefore an equilibrium model that can produce mean reverting expected returns. This is precisely the task undertaken with apparent success by Cechetti, Lam and Mark (1990)(hereafter CLM). In a Lucas exchange economy with infinitely many identical agents, they show that consumption smoothing motives with a moderate degree of risk aversion can produce negative autocorrelation in real returns of the magnitude observed in the data. Compared with similar exercises aimed at explaining the equity premium puzzle for example, the strength of their demonstration seems to come from the fact that the parameters of the endowment process are estimated by maximum likelihood instead of being calibrated to match a few moments of the data. More precisely, they specify for the endowment process a Markov switching (MS) model characterized by two states, one of low growth and one of normal growth, and estimate by maximum likelihood the parameters of this model for three historical series on real consumption, GNP and dividend growth rates. The justification for using these three series comes from the fact that the Lucas model does not provide a way to select any particular series among the three since it imposes an equality between consumption, dividends, and output.

In this paper, we show that if instead of imposing one particular MS model, one specifies a larger class of MS models and lets each historical series of endowment decide on the best model according to various testing procedures, the chosen specification is a two-state Markov switching model with one mean and two variances. The measures of mean reversion (variance ratios and regression coefficients at various lags) implied by this specification for the endowment process are much less supportive of the equilibrium model, leaving the small sample bias as the main source of explanation for the generated negative autocorrelation in returns.

Furthermore, we show that the results obtained by CLM are very sensitive to the specific dating chosen for the Markov variable which represents the state of the economy in the endowment equation. This dating gives the agents foreknowledge of the state of the economy next period. Choosing a more natural dating while maintaining the remaining features of their specification produces positive autocorrelation in returns.

To assess the empirical validity of such an equilibrium asset pricing model, there are two possible ways to proceed. One can start by fitting the data on the endowment process with the best specification within a class of models and generate return series using the theoretical formulas implied by the model. The evaluation then consists in comparing the simulated series to the observed series. This comparison can however be made in many dimensions, mean reversion chosen in this paper being only one of them. Another way to proceed will be to use the theoretical return formulas and the observed return series to estimate by maximum likelihood the parameters of the endowment process and compare them with actual data on consumption. This is the approach taken in a more general model by Bonomo and Garcia (1990).[1] CLM follow a third approach which is less satisfactory from the point of view of assessing the model. Although they estimate the parameters of the endowment process by maximum likelihood, their specification choice is geared at matching as closely as possible a particular set of

[1] In that paper, we model the endowment process as a joint bivariate Markov process and derive the return formulas following the same method as in this paper. The assessment of the model is done along the two possible ways outlined in the text. First, we compare, for both real and excess returns, the unconditional first and second moments, the variance ratios and univariate regression coefficients, and the regression statistics of returns on the dividend-price ratio produced by the model to the actuals. Second, we estimate using the return formulas the parameters of the endowment that will rationalize the observed returns.

statistics on returns, that is mean reversion statistics. But by doing so, their demonstration reduces to showing that there exists one equilibrium model that can produce mean reversion, without showing that it is fully consistent with the observed endowment data.

In Section I, we present the equilibrium asset pricing model and derive closed-form solutions for the returns. In Section II, we estimate by maximum likelihood various MS models for the endowment process and select the best specification for each of the three series of consumption, GNP and dividends. In Section III, we use the maximum likelihood estimates of the parameters obtained in Section II and the theoretical return formulas derived in Section I to generate the small sample and large sample empirical distributions of the variance ratios and regression coefficients at various lags for four specifications of the endowment process which differ by the number of different means and variances and by the timing of the Markov variable. We then compare the respective implications of each specification for mean reversion. We interpret the results in Section IV and conclude in Section V.

I. The Model

Our assumptions, apart from those related to the specification of the endowment process, are standard in the literature.

An infinitely-lived representative agent maximizes her intertemporal vN-M utility function over her lifetime and receives each period an endowment of a nonstorable good. Assuming additive time separability and constant discounting of the utility function, the utility at time t, V_t, can be written as:

$$V_t = E_t \sum_{t=0}^{\infty} \beta^j U(C_{t+j}) \tag{1}$$

where E_t denotes expectation conditional on information available at time t, U an atemporal utility function and C_t the agent's consumption at time t. We assume that U is the power utility function with constant relative risk aversion γ:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

The first-order condition for an interior maximum implies that the price of an asset l at time t (p_t^l) is equal to the expected product at time t of the asset payoff at time t+1 (x_{t+1}^l) and the marginal rate of substitution between t and t+1 ($m_{t,t+1}$), that is:

$$P_t^l = E_t(m_{t,t+1}\, x_{t+1}^l)$$

Using our assumptions about the utility function stated in (1) and (2), this condition can be rewritten as:

$$P_t^1 = \beta E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} x_{t+1}^1\right)$$

An equity is defined as an asset which gives the right to the endowment stream in the economy. Since the unique good in the economy is nonstorable, the dividend D is identical to consumption. Therefore, the equity price at time t satisfies:

$$P_t^e = \beta E_t\left[\left(\frac{D_{t+1}}{D_t}\right)^{-\gamma}\left(P_{t+1}^e + D_{t+1}\right)\right]$$

Iterating the above equation gives:

$$P_t^e = D_t^{\gamma}\sum_{j=1}^{\infty}\beta^j E_t D_{t+j}^{1-\gamma} \tag{3}$$

We postulate that the logarithm of the endowment process follows a random walk where both the mean and the variance change according to a Markov state variable S_t which takes values 0,1,...,K-1 (in the case of K states). The Markov process $\{S_t\}$ has the following transition probability matrix π:

$$\pi = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0[K-1]} \\ p_{10} & p_{11} & \cdots & p_{1[K-1]} \\ \vdots & \vdots & \vdots & \vdots \\ p_{[K-1]0} & p_{[K-1]1} & \cdots & p_{[K-1][K-1]} \end{bmatrix}$$

The endowment process can then be written as:

$$d_t - d_{t-1} = \alpha_0 + \alpha_1 S_{1,t} + \ldots + \alpha_{K-1} S_{K-1,t} + (\omega_0 + \omega_1 S_{1,t} + \ldots + \omega_{K-1} S_{K-1,t})\epsilon_t \tag{4}$$

where $S_{i,t}$ is a function of the state of the economy, S_t, taking value 1 whenever $S_t = i$ and 0 otherwise; d_t is $\ln D_t$; ϵ_t is a N(0,1) error term. Then, in state i, the mean and standard deviation of the growth rate of the endowment will be given by $(\alpha_0+\alpha_i, \omega_0+\omega_i)$.

Given the process defined by (4) and the transition probability matrix π, we can find closed form solutions for the asset prices and derive formulas for returns.

Iterating n times equation (4) in exponential form, we obtain:

$$D_{t+n}^{1-\gamma}=D_t^{1-\gamma}\exp\left((1-\gamma)\sum_{j=1}^{n}\alpha_0+\alpha_1 S_{1\,t+j}+\dots+\alpha_{K-1}S_{K-1,\,t+j}+(\omega_0+\omega_1 S_{1\,t+j}+\dots+\omega_K S_{K-1\,t+j})\,e_{t+j}\right)$$

Taking the conditional expectation of both sides with respect to the information set at time t and using the independence of the sequences (S_m) and (ϵ_m) results in:

$$E_t D_{t+n}^{1-\gamma}=D_t^{1-\gamma}E_t\exp(\mu_0 n+\mu_1 i_{1\,t,n}+\dots+\mu_{K-1}i_{K-1\,t,n}) \tag{5}$$

where:

$$\mu_0=(1-\gamma)\,\alpha_0+\frac{(1-\gamma)^2}{2}\omega_0^2$$

$$\mu_j=(1-\gamma)\,\alpha_j+\frac{(1-\gamma)^2}{2}(2\omega_0\omega_j+\omega_j^2) \qquad j=1\dots K-1$$

$$i_{j\,t,n}=\sum_{h=1}^{n}S_{j\,t+h} \qquad j=1\dots K-1$$

The expectation term on the right hand side of (5) can be written in matrix form :

$$E_t\exp(\mu_0 n+\mu_1 i_{1\,t,n}+\dots+\mu_{k-1}i_{k-1\,t,n})=I_{K,t}A^{n}1$$

where $I_{K,t}$ is a 1xK row vector with 1 in the column corresponding to the state at time t and zeros in the other columns, $\mathbf{1}$ is a Kx1 column vector of ones, and A is defined as follows:

$$A=e^{\mu_0}\begin{bmatrix} p_{00} & p_{01}e^{\mu_1} & \cdots & p_{0K-1}e^{\mu_{K-1}} \\ p_{10} & p_{11}e^{\mu_1} & \cdots & p_{1K-1}e^{\mu_{K-1}} \\ \vdots & \vdots & \vdots & \vdots \\ p_{K-10} & p_{K-11}e^{\mu_1} & \cdots & p_{K-1K-1}e^{\mu_{K-1}} \end{bmatrix}$$

So, expression (5) can be substituted in the equity price equation (3) to obtain the following formula:

$$P_t^e=D_t\rho(S_t) \tag{6}$$

where:

$$\rho(S_t)=I_{K,t}[(I-\beta A)^{-1}-I]\,1 \tag{7}$$

When K=2, it is easy to see that ρ becomes:

$$\rho(S)=\frac{1-\beta e^{\mu_0}(p_{00}+p_{11}-1)\,e^{\mu_1(1-S)}}{\Delta}-1 \tag{8}$$

where:

$$\Delta=\beta^2(p_{00}+p_{11}-1)\,e^{2\mu_0+\mu_1}-\beta e^{\mu_0}(p_{11}e^{\mu_1}+p_{00})+1$$

Defining the one period equity return, R_t, as

$$R_t = \frac{P_{t+1}+D_{t+1}}{P_t}$$

we can use (6) to arrive at the following return formula:[2]

$$R_t = \frac{\rho(S_{t+1})+1}{\rho(S_t)} \exp(\alpha_0+\alpha_1 S_{t+1}+(\omega_0+\omega_1 S_{t+1})\epsilon_{t+1}) \tag{9}$$

The natural informational assumption is to provide the agent with the knowledge of the state of the economy she is facing, that is S_t (or equivalently $S_{1,t}, S_{2,t}, \ldots, S_{K-1,t}$) is known at time t. An alternative assumption is to provide the agent with the foreknowledge of the state of the economy one period ahead, that is S_{t+1} is known at time t, as CLM do. An equivalent way of stating this assumption is to maintain that the agent at time t knows at most S_t but to make $d_t - d_{t-1}$ a function of S_{t-1} instead of S_t. In that case, the endowment equation becomes:

$$d_t - d_{t-1} = \alpha_0+\alpha_1 S_{1,t-1}+\ldots+\alpha_{K-1}S_{K-1,t-1}+(\omega_0+\omega_1 S_{1,t-1}+\ldots+\omega_{K-1}S_{K-1,t-1})\epsilon_t \tag{10}$$

The price and price dividend ratio formulas can be derived in a similar way in this case. It is easy to see that instead of (5) we should have:

$$E_t D_{t+n}^{1-\gamma} = D_t^{1-\gamma} E_t \exp(\mu_0 n+\mu_1 i_{1\ t-1,n}+\cdots+\mu_{K-1} i_{K-1\ t-1,n}) \tag{11}$$

The matrix formula for the expectation term in the right hand side of (11) is:

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[2] When K=2, $S_{1,t}$ and S_t are identical.

$$I_{K,t}BA^{n-1}1$$

where the matrix B is defined as follows:

$$B=e^{\mu_0}\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & e^{\mu_1} & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & e^{\mu_{k-1}}\end{bmatrix}$$

Using (11) in (3) results in the following formula for the price dividend ratio:

$$\rho^L(S_t)=\beta I_{K,t}B(I-\beta A)^{-1}1 \tag{12}$$

When K=2, the expression for the price dividend ratio is:

$$\rho^L(S)=\frac{\beta e^{\mu_0+\mu_1 S}-\beta^2 e^{2\mu_0+\mu_1}(p_{00}+p_{11}-1)}{\Delta} \tag{13}$$

The return formula becomes:

$$R_t^L=\frac{\rho(S_{t+1})+1}{\rho(S_t)}\exp(\alpha_0+\alpha_1 S_t+(\omega_0+\omega_1 S_t)e_{t+1}) \tag{14}$$

II. Estimation of the Endowment Process

Since we specify a general Markov switching model with K states for the endowment process, the logical first step is to check if a two-state model fits better than a one-state model, i.e. the random walk model. CLM showed that a two-mean, one variance

Markov model fits the data better than a random walk[3]. We will therefore skip that step in our gradual testing procedure.

The estimation and testing strategy is to start with the estimation of the two-state MS models, select the best specification within that class, and then test for the presence of a third state keeping as the null hypothesis the best specification in the two-state class. The procedure stops when there is no evidence for the presence of a higher number of states.

The upper panel of Table 1 presents the maximum likelihood estimates of three two-state models for each of the series of consumption, dividend , and GNP growth rates. Taking consumption as an example, the first model (C1) is the two-state MS model with two means (α_0, $\alpha_0+\alpha_1$) and one variance (ω_0^2). The second model (C2) refers to the one-mean (α_0) and two-variance (ω_0^2, $(\omega_0+\omega_1)^2$) model. Finally, C3 is the two-state encompassing model with two means (α_0, $\alpha_0+\alpha_1$) and two variances (ω_0^2, $(\omega_0+\omega_1)^2$). To test the constraint $\omega_1=0$, one can use a likelihood ratio test of C1 against C3 since C1 is nested in C3. The X^2 p-value line indicates a value of 0.011 for the null of C1 against the alternative C3, which means that C1 can be rejected in favor of C3 almost at the 1% level of confidence.

The two other sets of columns for dividends and GNP confirm this evidence against the constraint $\omega_1=0$. We could stop the exercise here for the two-state specification and decide that the (C3, D3, G3) models are the preferred ones. However, for reasons

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[3] They also compare their two-mean one variance MS model to an AR(1) or AR(2) specification in terms of forecasting power. For GNP and consumption, there is no clear winner, but for dividends the MS model performs much better. A case is therefore made for the MS model given its analytical tractability.

of parsimony, given the unsignificance and close-to-zero value of α_1, we can reestimate the model under the constraint $\alpha_1=0$. The results are presented in the C2, D2 and G2 columns. The X^2 p-value line confirms that C2 (resp. D2, G2) cannot be rejected in favor of C3 (resp. D3, G3) at the 84% (resp. 80%, 97%) level. We will therefore retain C2 (resp. D2, G2) as our null hypothesis against the three-state formulation, the logical next step in our testing procedure.

Although the two-state specification is nested in the three-state specification, there are many parameter combinations of the three-state model which provide a two-state representation. A two-state specification can be obtained by setting α_2 and ω_2 to zero, or by setting the corresponding transition probabilities to the third state to zero. Note also that in the latter case the parameters α_2 and ω_2 have no role and could take any values[4]. Because of this identification problem, the classical hypothesis testing procedures (LR, LM, Wald) used for nested models fail to apply to test for the number of states[5].

To illustrate the problems involved, we will choose as a null hypothesis a linear model in growth rates with one mean (α_0) and one variance (ω_0^2) and as an alternative a two-state MS model (adding the parameters α_1, ω_1, p_{00} and p_{11} to the null specification). To start with the LM test, we note that the first-

[4] In the former case, the probabilities that link the third state to the other states could take any of infinitely many values without affecting the two-state representation, i.e. any of the infinitely many combinations of probability values that correspond to the same unconditional (steady state) probabilities for the first and second states.

[5] As indicated in Hamilton (1989) p. 377, footnote 12.

order conditions with respect to α_1, ω_1, p_{00} and p_{11} are identically zero at the constrained MLE if we choose $\alpha_1=0$, $\omega_1=0$, $p_{00}=1$ and $p_{11}=0$ as our specific version of the null[6]. Therefore, one can never reject the null of a single state even when the two-state model is the truth. For the LR and Wald tests, one can always estimate the two-state model, but two sorts of problems arise if the single state model is true. First, attempting to estimate the two-state model might prove unsuccessful, or convergence might be very slow. Second, the regularity conditions (identification and rank conditions) are violated. Under the null of a single state ($\alpha_0=\alpha_1$, $\omega_0=\omega_1$),the probability parameters p_{00} and p_{11} are unindentified, since any value between 0 and 1 will leave the likelihood value unchanged. Therefore the scores with respect to p_{00} and p_{11} will be identically zero and the asymptotic information matrix will be singular. No valid asymptotic test can therefore be constructed under the null of a single state.

To overcome these problems, we will use various tests described in Appendix A[7], i.e. the Davies (1987) bound test, the Gallant (1977) test and the Davidson and MacKinnon (1981) J-test. The first test has been proposed by Boldin (1989) in the Markov switching model context, while the second has been suggested by Gallant (1977) for non-linear models where a similar problem applies. The third test is the well-known test for non-nested

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6 Notice also that, as mentioned above, there are infinitely many combinations which represent the null hypothesis.

7 A description of additional tests that could be useful in guiding the choice of the number of states is found in Garcia and Perron (1990).

models. The results for the tests of the best two-state MS model (C2, D2 G2) against a three-state MS model are shown in the bottom panel of Table 1. For the consumption series, convergence could not be achieved with the three-state model, an indication, as mentioned earlier, that the series is best characterized by two states in variance. This is confirmed by the Gallant test and the J-test. For the dividend and GNP series, the Davies quick rule[8] and the J-test favor the two-state specification, but the Gallant test is not as clear, especially for dividends.[9] Given our current ignorance about the respective powers of these tests, we are inclined to accept the two states in variance specification.

The estimation procedure gives as a by-product the probabilities of being in state 0 or 1 at time t given the information available at time t.[10] These probabilities are shown for each of the series in Figure 1. For the consumption series, the variance switches from the high state to the low state in 1950 and stays there almost until the end of the sample. For the GNP series, the high variance period extends mainly from 1930 to 1949. For the dividend series the changes are more frequent, but the same low variance is exhibited after 1950. Given the correspondence between the endowment and the stock prices in the Lucas model, this characterization of the endowment process is in line with the

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[8] See Appendix A for a definition of the quick rule for the Davies bound test.

[9] A more complete Markov specification for the dividend and GNP series should include some autoregressive parameters but no closed-form solution can be found for the stock price when autocorrelation parameters are present.

[10] See Hamilton (1989) for a detailed explanation of these so-called filter probabilities.

previous evidence [Officer (1973), Schwert (1989)] of a much higher variance in stock prices in the 1930s[11]. In the next section, we will analyze the effects of the various characterizations of the endowment process on the generated measures of mean reversion.

III. Implications of Different Specifications for Negative Autocorrelation in Real Returns

In this section, we will compare the negative autocorrelation patterns in returns generated by the equilibrium model for four different specifications of the endowment process: one mean, two variances (1M2V) for both S_t and S_{t-1}, two means, one variance (2M1V) for both S_t and S_{t-1}. We use as measures of negative autocorrelation both the variance ratios ($VR= Var(R_{t,t+m})/mVar(R_t)$, for m equal 2 to 10) and the multiperiod returns regression coefficients (regression of R_{t+m} on R_{t-m}, for m equal 1 to 10).

Tables 2 and 4 report the results of the Monte Carlo experiment for the variance ratios and the regression coefficients respectively. The Monte Carlo distributions for these statistics are generated in the following way. Given a randomly drawn vector of N(0,1) errors ϵ_{t+1} and a randomly drawn vector of S_t according to the transition probabilities estimated in Section II, we generate series of returns according to formulas (9) or (14) for R_t and R_t^L respectively, with the estimates obtained in Section II for the α

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11 Equation (6) shows that when we stay in the same state for a period of time, the growth rate of P^e mimics the growth rate of D. Then a higher variance for the endowment growth will translate into a higher variance for the stock price growth.

and ω parameters of the consumption growth series.[12] We replicate the procedure 10,000 times and compute each time the variance ratio and the regression coefficient. We therefore obtain the respective distributions of variance ratios and regression coefficients at various lags.

For the length of the series, we choose 116 observations (the number of observations for the actual returns) to generate the small sample distributions, and 1,160 observations for the large sample ones. We report the medians of the distributions as well as the percentage of the distribution below the actuals. For the small sample results, this percentage is to be interpreted as a p-value for the hypothesis that the actuals are produced by the model. The closer it is to the 50% line, the more support for the hypothesis. The large sample statistics are produced to evaluate whether the model is capable of generating some negative autocorrelation even in large samples. In other words, we want to assess the magnitude of the small sample bias present in the small sample results.

Table 2 shows that the combination of the 2M1V, S_{t-1} specification (the one chosen by CLM) with a concave utility function (γ=1.7) is the only one to generate negative autocorrelation of the magnitude exhibited by the actual data: for all other combinations the variance ratios are in general substantially larger.

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[12]Results obtained with the GNP and dividend series estimates are not reported because of space considerations. The conclusions reached are very similar in nature.

The large sample results in the bottom part of the table reinforce the conclusions one can infer from the top part: only the 2M1V, S_{t-1} specification combined with a concave utility function generates large sample variance ratios substantially smaller than one. In other words, the variance ratio values smaller than unity obtained from the other models are due to small sample bias. As shown in table 3, the population variance ratios (see Appendix B for their derivation) for the three other specifications are greater than one when a coefficient of relative risk aversion of 1.7 is assumed.

The figures in tables 2 and 3 reveal the importance for the CLM results of both the 2M1V specification and the lagged state (S_{t-1}) structure. If the MS specification that best fits the data, i.e. the 1M2V specification, is substituted for the 2M1V one while the lagged state structure is maintained, a lower small sample bias will remain as the only source of a slight negative autocorrelation. If we otherwise change the lag structure to recover the usual one while maintaining the 2M1V specification an even more striking result is obtained when the utility function is concave: the variance ratio is substantially greater than one and increasing with the return horizon. If the endowment model which best fits the data and the natural lag structure is chosen, the results are similar to the ones with the lagged structure: unlike the results obtained using the 2M1V specification, the 1M2V specification results are robust to the choice of the lag structure.

returns.

When one state is persistent and the other is not, as in the 2M1V specification, the pattern of autocorrelation depends on two factors: the magnitude of the change in the price dividend ratio when changing states (since ρ(0) and ρ(1) appear in the return equation in that case), and the timing of the Markov variable in the exponent term in the return equations (9) and (14).

For the first factor, it is easily seen from equations (8) and (13) that the difference between the price dividend ratios in the two states is given by:

$$\rho(0)-\rho(1)=\frac{\beta(p_{00}+p_{11}-1)}{\Delta}[\exp(\mu_0)-\exp(\mu_0+\mu_1)] \qquad \textbf{(15)}$$

$$\rho^L(0)-\rho^L(1)=\frac{\beta}{\Delta}[\exp(\mu_0)-\exp(\mu_0+\mu_1)] \qquad \textbf{(16)}$$

when the endowment equation is (4) or (10) respectively.

In this latter specification, it is also assumed that the representative agent knows S_t at time t-1. This fact explains the absence in (16) of the term ($p_{00}+p_{11}-1$) which appears in (15). This term reflects the uncertainty faced by the agent regarding the state of endowment growth in the next period. Given the estimated values for the transition probabilities in the 2M1V specification (p_{11}=0.5265, p_{00}=0.9760), one can see that the difference between ρ(0) and ρ(1) is about halved when the S_t specification is chosen instead of the S_{t-1} retained by CLM. Also, in both cases, ρ(0) is less than ρ(1) for the specific values chosen or estimated for the

The results of Table 4 for the regression coefficients are analogous to the results obtained for the variance ratios[13], that is, the 2M1V,S_{t-1} specification is the only one to generate coefficients that are negative enough to approach the actual data. What are the special features in this specification that explain these results?

IV. Interpretation of the Results

To understand how the characteristics of the Markov switching model for the endowment process affect the theoretical autocorrelation of returns, one has to look at equations (9) and (14) which define the equilibrium returns. The return is seen to depend on the Markov state in two adjacent periods and on the realization of the i.i.d. term ϵ in the second period. If both states are persistent (high p_{00} and p_{11}), the state observed in period t is likely to remain unchanged and the successive returns are likely to differ only through ϵ_{t+1}. This tends to generate positive autocorrelation in long-horizon returns. Conversely, when both states show little persistence (low p_{00} and p_{11}), the observed state is likely to change, producing negative autocorrelation in returns. The fact that the estimated p_{00} and p_{11} are both very high for the 1M2V model is at least part of the explanation for the slightly positive value found for the population autocorrelation of

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[13] The regression coefficients (b) and the variance ratios (V) are linked by the following formula: $b(k) = V(2k)/V(k) - 1$

parameters.[14]

To see the influence of the second factor, the timing of the Markov variable, one can look again at the return equations. In the explanation that follows, we neglect the effect of the i.i.d. term ϵ. Suppose the economy has been in state 0 for some periods. When there is a change from state 0 to state 1, equations (8) and (13) will read as follows:

$$R_{t+1}=\frac{\rho(1)+1}{\rho(0)}\exp(\alpha_0+\alpha_1)$$

$$R_{t+1}=\frac{\rho(1)+1}{\rho(0)}\exp(\alpha_0)$$

Then both $\rho(1)$ and $\rho^L(1)$ appear in the numerator and $\rho(0)$ and $\rho^L(0)$ in the denominator in both specifications and, since $\rho(1)$ (resp. $\rho^L(1)$) is greater than $\rho(0)$ (resp.($\rho^L(0)$)), the part of the equation which multiplies the exponential term will increase in both cases (but more for the lagged specification for the reasons mentioned above). For the exponential term however, only α_0 will appear in the lagged specification, driving the returns much higher than in the normal specification where α_1 (high in absolute value and negative) will also be present. For the next period, if we stay in state 1 the return tends to be lower than in state 0 (since α_1 is appearing in the exponential term in both cases and $\rho(1)$ (resp. $\rho^L(1)$) is greater than $\rho(0)$ (resp. $\rho^L(0)$), but there is still a good chance of returning to state 0, since state 1 is not very

[14] In their discussion of the sign of $\rho^L(0)-\rho^L(1)$ on page 407, CLM state that it is always negative when the coefficient of relative risk aversion, γ, is greater or equal to 1, without mentioning that the validity of this statement depends on the parameter values. The statement is not true for example if the bad state is slightly more persistent than the good state.

persistent (p_{11}=0.5265). In this case, the return equations will be given by:

$$R_{t+1}=\frac{\rho(0)+1}{\rho(1)}\exp(\alpha_0)$$

$$R^L_{t+1}=\frac{\rho^L(0)+1}{\rho^L(1)}\exp(\alpha_0+\alpha_1)$$

For the lagged specification the return will be very low because $\rho^L(1)$ will now appear in the denominator, $\rho^L(0)$ in the numerator and α_1 in the exponential term. This sequence of positive and negative spikes in returns is what generates the negative autocorrelation pattern associated with the 2M1V lagged specification. For the non-lagged specification the exponential term will smooth out both positive and negative spikes (because α_1 will be present when the other term is high but not when it is low), resulting in the suppression of the negative autocorrelation effect.

For the 1M2V specification, the timing of the Markov variable does not alter substantially the results, since p_{00} and p_{11} are high - making the difference $\rho(0)-\rho(1)$ almost identical in both lag structures - and α_1 is 0.

The mean reversion effect in equilibrium asset pricing models based on a regime-switching endowment can be produced by two kinds of dynamics. A high variance state of some persistence increases risk in returns and implies that a higher expected return is required compared with a low variance state. Switching from a high variance state to a low variance state can produce mean reversion in asset prices. The main reason why the two-variance, one-mean estimated model cannot produce mean reversion is that the states are too persistent. The other kind of dynamics is generated by the intertemporal substitution of the agent when faced with alternating high and low growth rates in her endowment. When in addition the agent knows the state of the economy in the next period, she can adjust more radically her consumption in response. The facts that the mean growth rates in the estimated model are very far apart and that this foreknowledge assumption is made are the two reasons of the success of the CLM model in producing mean reversion.

An additional feature of the results remains to be explained. Table 3 exhibits a monotonic relationship between the population variance ratios and the return horizon. To understand this fact, it must be remembered that the longer the horizon, the more numerous are the changes in states, hence the larger is the measure of autocorrelation.

V. Conclusion

In this paper, we showed that, among various Markov specifications for the endowment process, mean reversion of the magnitude detected in the data was only obtained by combining an arbitrary assumption about the timing of the Markov variable S_t with a misspecification of the process. This was the particular combination chosen by CLM on the grounds of being on the tradeoff frontier between a model that completely matches the data and one that is tractable. The timing of the state is justified by the fact that in actual economies future nominal dividend payments are announced in advance. In a long-term setting with annual observations as it is the case here, such a justification is not very convincing. Moreover, this specification is used to model the consumption and GNP series as well, for which obviously the same justification does not apply. This dating of the Markov variable coupled with the fact that the agents in the economy are assumed to know also the state at time t has, as we saw, a very important bearing on the results. We showed that when the natural timing of the unobservable Markov variable is matched with the best tractable specification in this class of models, the amount of negative autocorrelation generated is substantially smaller than what is found in the data. Moreover, the small sample bias stands as its only explanation.

Furthermore, it must be emphasized that none of the models generates negative autocorrelation in excess returns, contrary to what is apparent in actual data, as shown in Table 5[15].

Finally, as we mentioned in the introduction, it is not enough to show that one particular equilibrium model can generate mean reversion. To gain acceptance for this model, one will have to show that it is also capable of reproducing both the endowment process features and the other characteristics of the return series such as the unconditional moments or the regression coefficients on

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[15] The formulas for the theoretical variance ratios and regression coefficients are derived in Appendix B.

other financial variables. In this paper, we applied our effort to best characterizing the endowment series and the model did not stand the test. It remains as an open question whether a more elaborate model for the endowment process can reproduce, in a representative agent economy setting, the characteristics of the return series.

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Appendix A

1. Davies Bound Test

The procedure proposed by Davies applies when a vector γ of dimension say q is present only under the alternative hypothesis. Define the likekihood ratio statistic as a function of γ:

$$LR(\gamma)=2(\ln L_1(\gamma_g)-\ln L_0^*) \tag{A.1}$$

where $L_1(\gamma_g)$ denotes the likekihood value of the objective function evaluated at γ_g (a given value for γ) under the alternative hypothesis, and L_0^* the maximized value obtained under the null hypothesis (where γ is not present). Let γ^* be the argmax of $L_1(\gamma)$ and M be the maximum of $LR(\gamma) \equiv 2(\ln L_1^* - \ln L_0^*)$. Davies derives the following upper bound for the significance of M:

$$Pr[LR(\gamma)>M]=Pr[\chi_q^2>M]+VM^{\frac{q-1}{2}}\exp^{-\frac{M}{2}}\frac{2^{\frac{-q}{2}}}{\Gamma(\frac{q}{2})}$$

where $\Gamma(.)$ denotes the gamma function and V is defined as:

$$V=\int_{\gamma_l}^{\gamma_u}\left|\frac{\partial LR(\gamma)^{\frac{1}{2}}}{\partial\gamma}\right|d\gamma \tag{A.2}$$
$$=\left|LR(\gamma_1)^{\frac{1}{2}}-LR(\gamma_l)^{\frac{1}{2}}\right|+\left|LR(\gamma_2)^{\frac{1}{2}}-LR(\gamma_1)^{\frac{1}{2}}\right|+\cdots+\left|LR(\gamma_u)^{\frac{1}{2}}-LR(\gamma_n)^{\frac{1}{2}}\right|$$

where $\gamma_1, \gamma_2, \ldots, \gamma_n$ are the turning points of $LR(\gamma)$. A quick rule is obtained upon making the assumption that the likelihood ratio has a single peak. In that case V reduces to $2M^{1/2}$. Our testing procedure uses this quick rule and estimates the model under the alternative hypothesis to obtain L_1^* (and therefore M and V) to calculate the significance level. Another procedure, which avoids estimating the model under the alternative, would consider a fine grid of values for the vector γ, compute V by substituting these values into formula (A.2), and find the maximized value $L_1(\gamma^*)$ over the grid (with the associated vector γ^*) to compute M.

2. Gallant's Test Procedure

Consider the following models under the null and alternative hypotheses:

$$H_0: y_t = g(x_t,\psi) + e_t$$
$$H_1: y_t = g(x_t,\psi) + \tau\, d(x_t,\varphi) + e_t$$

The basic idea of the test is straightforward. Let z_t be a given vector of variables which do not depend on unknown parameters. If τ_0, the true value of τ, is equal to 0, the least squares estimator of δ in the following regression:

$$y_t = g(x_t, \psi) + z_t' \delta + e_t \tag{A.3}$$

is estimating the null vector. Let $\beta \equiv (\alpha_0, \alpha_1, \alpha_2, \omega_0, \omega_1, \omega_2, p_{i,j}$ (i,j =1,2,3)) be the vector of parameters in the three-state model (in the two-state model the vector is defined similarly without α_2, ω_2, and $p_{i,j}$ (i,j =3)). The Gallant procedure applied to determining the number of states in a Markov switching model follows four steps:

i) For a given set of values for β (say m) indexed by i, calculate the fitted values $\hat{y}_i$ for the model with the larger number of states.

ii) If the matrix $Y \equiv (y_1, \ldots, y_m)$ is too big, extract a few principal components, say d, (or the first few vectors of the orthogonal matrix in a singular value decomposition of Y).

iii) Add these principal components (call them z_t, a vector of dimension d) to the model with the lower number of states, i.e. estimate (A.3) where the function $g(x_t, \psi)$ represents the model with the lower number of states.

iv) Compute the following residual sums of squares:

$$H_1: \quad \tilde{\sigma}^2 = \sum_{t=1}^{T} (y_t - g(x_t, \tilde{\phi}) - \tilde{\delta} z_t)^2$$

$$H_0: \quad \hat{\sigma}^2 = \sum_{t=1}^{T} (y_t - g(x_t, \hat{\phi}))^2$$

The likelihood ratio test, with size α, rejects the null hypothesis if:

$$T \equiv \frac{\tilde{\sigma}^2}{\hat{\sigma}^2} > 1 + \frac{d\, F_\alpha}{(T-d-\mu)}$$

where μ is the number of parameters estimated under the null hypothesis, d is the dimension of the vector z_t and F_α denotes the α percentage point of a $F(d, T-\mu-d)$ distributed random variable.

Appendix B: Derivation of the Population Variance Ratios and Regression Coefficients

Formulas B.2-B.9 for equity returns(**R**) and safe asset returns($\mathbf{R^f}$) first and second moments are easily derived from the equity return formula (2) in the text and the folowing formula for the risk-free asset:

$$R_t^f(S_t) = \frac{\phi(S_t)}{\beta}\exp\left(-\gamma\alpha_0 - \frac{\gamma^2\omega_0^2}{2}\right) \qquad \textbf{(B.1)}$$

where

$$\phi(r) = \frac{1}{(q+(1-q)w)(1-r)+(1-p+pw)r}$$

and

$$w=\exp\left(\gamma\alpha_1+\gamma^2(\omega_0\omega_1+\frac{\omega_1^2}{2})\right)$$

Moment Formulas:

$$ER_t=\exp\left(\alpha_0+\frac{\omega_0^2}{2}\right)\sum_{\substack{r=0,1\\s=0,1}}\pi(r+1)\,C(r+1,s+1)\,\lambda_m(r,s) \qquad \textbf{(B.2)}$$

where:

$$\lambda_m(r,s) = \frac{(\rho(s)+1)^2}{\rho(r)^2}\exp\left(2\left(\alpha_1+2\omega_0\omega_1+\omega_1^2\right)s\right)$$

$$\pi=\left[\frac{1-p}{2-p-q},\frac{1-q}{2-p-q}\right]$$

$$C=\begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}$$

$$E(R_t^2)=\exp\left(2\left(\alpha_0+\omega_0^2\right)\right)\sum_{\substack{r=0,1\\s=0,1}}\pi(r+1)\,C(r+1,s+1)\,\lambda_{m2}(r,s) \qquad \textbf{(B.3)}$$

where:

$$\lambda_{m2}(r,s)=\frac{(\rho(s)+1)^2}{\rho(r)^2}\exp\left(2\left(\alpha_1+2\omega_0\omega_1+\omega_1^2\right)s\right)$$

$$E(R_tR_{t-k})=\exp\left(2\alpha_0+\omega_0^2\right)\sum_{\substack{r=0,1\\s=0,1\\u=0,1}}\pi(r+1)\,C(r+1,s+1)\,C^{k-1}(s+1,u+1)\,C(u+1,v+1)\,\lambda_{m2c}(r,s,u,v)$$

(B.4

where:

$$\lambda_{m2c}(r,s,u,v)=\frac{(\rho(s)+1)(\rho(v)+1)}{\rho(r)\rho(u)}\exp\left(\left(\alpha_1+\omega_0\omega_1+\frac{\omega_1^2}{2}\right)(v+s)\right)$$

$$ER_t^f=\frac{\exp\left(-\gamma\alpha_0-\frac{\gamma^2\omega_0^2}{2}\right)}{\beta}\sum_{r=0,1}\pi(r+1)\phi(r) \qquad \textbf{(B.5)}$$

$$E(R_t^{f^2})=\frac{\exp\left(-2\gamma\alpha_0-\gamma^2\omega_0^2\right)}{\beta^2}\sum_{r=0,1}\pi(r+1)\phi_m^2(r) \qquad \textbf{(B.6)}$$

$$E(R_t^fR_{t-k}^f)=\frac{\exp\left(2\gamma^{\alpha_0}-\gamma^2\omega_0^2\right)}{\beta^2}\sum_{\substack{r=0,1\\s=0,1}}\pi(r+1)\,C^k(r+1,s+1)\,\phi(r)\,\phi(s) \qquad \textbf{(B.7)}$$

$$E(R_{t-k}^fR_t)=\frac{\exp\left(\alpha_0(1-\gamma)+\frac{\omega_0^2}{2}(1-\gamma^2)\right)}{\beta}\sum_{\substack{r=0,1\\s=0,1\\u=0,1}}\pi(r+1)\,C^k(r+1,s+1)\,C(s+1,u+1)\,\theta(r,s,u)$$

$k=0,1,2,\ldots,10$

(B.8)

where:

$$\theta(r,s,u)=\frac{(\rho(u)+1)\phi(r)}{\rho(s)}\exp\left(\left(\alpha_1+\omega_0\omega_1+\frac{\omega^2}{2}\right)u\right)$$

$$E(R_{t-k}R_t^f)=\frac{\exp\left(\alpha_0(1-\gamma)+\frac{\omega_0^2}{2}(1-\gamma^2)\right)}{\beta}\sum_{\substack{r=0,1\\ s=0,1\\ u=0,1}}\pi(r+1)\,C(r+1,s+1)\,C^{k-1}(s+1,u+1)\,\theta(u,r,s) \qquad k=1,2,\dots,10$$

(B.9)

Using (B.2) to (B.9) it is straightforward to find the correlation coefficients (ρ) at various lags for the equity returns and the excess returns. Then, the theoretical values for the k-period variance ratio (V) and regression coefficient (b) can be obtained by the application of the following formulas:

$$V(k)=1+\frac{2}{k}\sum_{j=1}^{k-1}(k-j)\rho_j$$

$$b(k)=\frac{\sum_{j=1}^{k}(j\rho_j+(k-j)\rho_{k+j})}{k+2\sum_{j=1}^{k-1}(k-j)\rho_j}$$

For the specification which uses S_{t-1} in the endowment equation the theoretical values for the variance ratios and regression coefficients can be found in a similar way.

TABLE 1-Maximum Likelihood Estimates of Two-State Models and Test Results

	Consumption			Dividends			GNP		
	C1	C2	C3	D1	D2	D3	G1	G2	G3
α_0	0.0236 (6.44)	0.0200 (6.89)	0.0205 (5.90)	0.0171 (1.53)	-.00008 (-0.01)	0.0010 (0.14)	0.0240 (5.78)	0.0179 (5.58)	0.0180 (5.54)
α_1	-0.0899 (-4.57)	---	-0.0020 (-0.27)	-0.370 (-6.56)	---	-0.0093 (-0.43)	-0.1756 (-7.19)	---	-0.0013 (-0.05)
ω_0	0.0329 (12.04)	0.0176 (6.32)	0.0177 (6.49)	0.1050 (13.69)	0.0429 (7.69)	0.0428 (7.83)	0.0431 (15.05)	0.0310 (11.70)	0.0310 (12.04)
ω_1	---	0.0289 (5.90)	0.0288 (5.89)	---	0.1400 (6.87)	0.1404 (7.16)	---	0.0781 (4.29)	0.0781 (4.22)
p_{11}	0.5265 (1.97)	0.9849 (59.27)	0.9848 (59.44)	0.1753 (0.82)	0.8430 (8.40)	0.8367 (8.52)	0.5089 (2.32)	0.9305 (15.03)	0.9305 (15.75)
p_{00}	0.9760 (44.11)	0.9610 (28.23)	0.9613 (28.74)	0.9508 (40.52)	0.8289 (9.66)	0.8238 (9.53)	0.9821 (93.93)	0.9858 (66.08)	0.9858 (77.31)
L	269.96	276.37	276.41	179.67	191.744	191.81	294.41	311.366	311.367
χ^2 p-value	0.011	0.84	---	0.000	0.80	---	0.000	0.97	---

Note: Asymptotic t-ratios in parentheses.

Tests of Two States (C2, D2 or G2) against Three States

	C2	D2	G2
Davies Test	---	0.878	1.0
Gallant Test	0.562	0.029	0.114
J-Test (Davidson and MacKinnon) (Standard error in parentheses)	0.040 (0.069)	-0.097 (0.359)	-0.52 (0.570)

Notes: .For the Davies test, we report a p-value for the null hypothesis of a two-state model. It could not be calculated for the consumption model since convergence could not be achieved with three states.
.The Gallant test is an F-test, for which we report also a p-value for the null of a two-state model. It has been performed by using in the two-state model the estimated growth rate based on Ml estimates of the three-state model for both dividends and GNP where convergence was obtained. For consumption, we gave a series of values to the three-state model parameters and calculated the corresponding consumption growth rate. We formed a matrix Y with these calculated growth rates and extracted the first vector of the orthogonal matrix obtained from a Singular Value Decomposition of Y.
.The J-test is a t-test on $\hat{\alpha}$ in the model: $y = (1-\alpha)\, f(\beta) + \alpha g + u$, where $f(\beta)$ stands in our case for the model with the lower number of states and g for the estimated growth rate with the higher number of states, calculated as indicated for the Gallant test for the respective series.

TABLE 2-Median of Distribution of Variance Ratios of Returns for Models Calibrated to Consumption

k	Actual	$\gamma=0$ S_{t-1} 2M1V	$\gamma=0$ S_{t-1} 1M2V	$\gamma=0$ S_t 2M1V	$\gamma=0$ S_t 1M2V	$\gamma=1.7$ S_{t-1} 2M1V	$\gamma=1.7$ S_{t-1} 1M2V	$\gamma=1.7$ S_t 2M1V	$\gamma=1.7$ S_t 1M2V
					T = 116				
2	1.0137	0.9853 (0.59)	0.9932 (0.58)	0.9863 (0.58)	0.9918 (0.59)	0.9502 (0.68)	0.9930 (0.58)	1.1162 (0.21)	0.9921 (0.58)
3	0.8664	0.9609 (0.31)	0.9773 (0.23)	0.9602 (0.32)	0.9774 (0.23)	0.8951 (0.44)	0.9775 (0.23)	1.1739 (0.06)	0.9775 (0.23)
4	0.8351	0.9354 (0.33)	0.9636 (0.24)	0.9335 (0.33)	0.9657 (0.24)	0.8524 (0.47)	0.9635 (0.24)	1.1992 (0.06)	0.9653 (0.24)
5	0.7978	0.9115 (0.32)	0.9483 (0.23)	0.9118 (0.32)	0.9543 (0.23)	0.8175 (0.47)	0.9490 (0.24)	1.2115 (0.07)	0.9537 (0.23)
6	0.7459	0.8898 (0.28)	0.9330 (0.20)	0.8905 (0.28)	0.9368 (0.20)	0.7898 (0.43)	0.9340 (0.20)	1.2182 (0.06)	0.9383 (0.20)
7	0.7259	0.8713 (0.29)	0.9187 (0.22)	0.8728 (0.29)	0.9209 (0.21)	0.7685 (0.44)	0.9195 (0.22)	1.2167 (0.06)	0.9215 (0.21)
8	0.7363	0.8560 (0.33)	0.9043 (0.26)	0.8558 (0.34)	0.99048 (0.26)	0.7490 (0.48)	0.9041 (0.26)	1.2065 (0.06)	0.9041 (0.26)
9	0.7102	0.8405 (0.33)	0.8901 (0.26)	0.8405 (0.33)	0.8894 (0.25)	0.7317 (0.47)	0.8908 (0.26)	1.1954 (0.09)	0.8891 (0.26)
10	0.7242	0.8260 (0.37)	0.8724 (0.31)	0.8254 (0.37)	0.8732 (0.31)	0.7156 (0.51)	0.8735 (0.31)	1.1870 (0.11)	0.8732 (0.31)
					T = 1160				
2	1.0137	1.000 (0.63)	0.9999 (0.66)	0.9983 (0.63)	0.9989 (0.68)	0.9554 (0.90)	1.0000 (0.67)	1.1453 (0.00)	0.9992 (0.68)
3	0.8664	0.9977 (0.02)	0.9974 (0.00)	0.9961 (0.02)	0.9975 (0.00)	0.9228 (0.20)	0.9971 (0.00)	1.2408 (0.00)	0.9977 (0.00)
4	0.8351	0.9968 (0.02)	0.9954 (0.00)	0.9930 (0.02)	0.9962 (0.00)	0.9001 (0.21)	0.9948 (0.00)	1.3049 (0.00)	0.9964 (0.00)
5	0.7978	0.9938 (0.01)	0.9936 (0.00)	0.9903 (0.01)	0.9950 (0.00)	0.8830 (0.17)	0.9923 (0.00)	1.3500 (0.00)	0.9951 (0.00)
6	0.7459	0.9905 (0.00)	0.9915 (0.00)	0.9874 (0.00)	0.9939 (0.00)	0.8709 (0.09)	0.9906 (0.00)	1.3821 (0.00)	0.9943 (0.00)
7	0.7259	0.9889 (0.00)	0.9902 (0.00)	0.9854 (0.00)	0.9920 (0.00)	0.8607 (0.09)	0.9891 (0.00)	1.4041 (0.00)	0.9924 (0.00)
8	0.7363	0.9863 (0.01)	0.9892 (0.00)	0.9838 (0.00)	0.9907 (0.00)	0.8531 (0.13)	0.9877 (0.00)	1.4210 (0.00)	0.9907 (0.00)
9	0.7102	0.9839 (0.01)	0.9875 (0.00)	0.9810 (0.00)	0.9892 (0.00)	0.8462 (0.11)	0.9865 (0.00)	1.4343 (0.00)	0.9893 (0.00)
10	0.7242	0.9823 (0.01)	0.9866 (0.00)	0.9795 (0.01)	0.9876 (0.00)	0.8401 (0.15)	0.9854 (0.00)	1.4439 (0.00)	0.9878 (0.00)

Notes:
- .The figures between parentheses give the percentage of Monte Carlo distribution below the actual value.
- .All models we refer to in this table are two-state Markov switching models.
- .2M1V stands for the model with two (state) means and one (state) variance.
- .1M2V stands for the model with two (state) variances and one (state) mean.
- .S_t stands for the model where the Markov variable at time t enters the endowment equation at time t.
- .S_{t-1} stands for the model where the Markov variable at time t-1 enters the endowment equation at time t.
- .The numbers for the two-mean-one-variance(2M1V) lagged(S_{t-1}) Markov switching model differ slightly from CLM results becau of data revisions at the end of the sample.

TABLE 3-Population Variance Ratios for Models Calibrated to Consumption					
k	Actual	S_{t-1} 2M1V	S_{t-1} 1M2V	S_t 2M1V	S_t 1M2V
2	1.0137	0.9524	1.0000	1.1481	1.0000
3	0.8664	0.9206	1.0001	1.2471	1.0001
4	0.8351	0.8987	1.0002	1.3153	1.0002
5	0.7978	0.8831	1.0002	1.3638	1.0002
6	0.7459	0.8716	1.0002	1.3992	1.0002
7	0.7259	0.8630	1.0003	1.4259	1.0003
8	0.7363	0.8564	1.0003	1.4465	1.0003
9	0.7102	0.8511	1.0004	1.4627	1.0004
10	0.7242	0.8469	1.0004	1.4759	1.0004

Note: The values are calculated for γ=1.7.

TABLE 4-Median of Monte Carlo Distribution and Population Value
of Regression Coefficients of Returns for Models Calibrated to Consumption

k	2M1V γ=0 T=116	2M1V γ=1.7 T=116	2M1V γ=0 T=1160	2M1V γ=1.7 T=1160	2M1V γ=1.7 Population	1M2V γ=0 T=116	1M2V γ=1.7 T=116	1M2V γ=0 T=1160	1M2V γ=1.7 T=1160	1M2V γ=1.7 Population
					S_{t-1}					
1	-0.0145 (0.59)	-0.0492 (0.68)	-0.0001 (.64)	-0.0445 (0.91)	-0.0475	-0.0070 (0.58)	-0.0070 (0.58)	-0.0002 (0.68)	-0.0000 (0.68)	0.0000
2	-0.0397 (0.18)	-0.0899 (0.30)	-0.0033 (0.00)	-0.0568 (0.01)	-0.0564	-0.0219 (0.12)	-0.0217 (0.13)	-0.0027 (0.00)	-0.0034 (0.00)	0.0001
3	-0.0574 (0.38)	-0.1026 (0.49)	-0.0061 (0.03)	-0.0563 (0.18)	-0.0531	-0.0345 (0.32)	-0.0346 (0.32)	-0.0041 (0.01)	-0.0047 (0.02)	0.0001
4	-0.0678 (0.46)	-0.1072 (0.56)	-0.0076 (0.09)	-0.0522 (0.28)	-0.0470	-0.0482 (0.42)	-0.0480 (0.42)	-0.0055 (0.07)	-0.0061 (0.07)	0.0002
5	-0.0776 (0.33)	-0.1113 (0.40)	-0.0084 (0.01)	-0.0478 (0.03)	-0.0409	-0.0648 (0.31)	-0.0640 (0.31)	-0.0066 (0.01)	-0.0066 (0.01)	0.0002
6	-0.0868 (0.40)	-0.1156 (0.46)	-0.0101 (0.02)	-0.0433 (0.06)	-0.0357	-0.0780 (0.39)	-0.0775 (0.39)	-0.0088 (0.02)	-0.0081 (0.02)	0.0002
7	-0.0994 (0.38)	-0.1234 (0.42)	-0.0115 (0.01)	-0.0393 (0.03)	-0.0314	-0.0918 (0.37)	-0.0927 (0.37)	-0.0095 (0.02)	-0.0094 (0.01)	0.0003
8	-0.1119 (0.32)	-0.1349 (0.36)	-0.0116 (0.00)	-0.0367 (0.00)	-0.0279	-0.1047 (0.33)	-0.1041 (0.33)	-0.0112 (0.00)	-0.0102 (0.00)	0.0003
9	-0.1241 (0.29)	-0.1449 (0.32)	-0.0125 (0.00)	-0.0346 (0.00)	-0.0251	-0.1179 (0.29)	-0.1172 (0.29)	-0.0124 (.00)	-0.0121 (0.00)	0.0003
10	-0.1368 (0.21)	-0.1563 (0.22)	-0.0140 (0.00)	-0.0345 (0.00)	-0.0227	-0.1304 (0.22)	-0.1303 (0.22)	-0.0139 (0.00)	-0.0129 (0.00)	0.0003
					S_t					
1	-0.0141 (0.58)	0.1163 (0.22)	-0.0016 (0.64)	0.1453 (0.00)	0.1481	-0.0081 (0.59)	-0.0080 (0.59)	-0.0011 (0.69)	-0.0009 (0.69)	0.0001
2	-0.0417 (0.18)	0.0879 (0.04)	-0.0044 (0.00)	0.1401 (0.00)	0.1456	-0.0020 (0.12)	-0.0205 (0.12)	-0.0024 (0.00)	-0.0024 (0.00)	0.0001
3	-0.0556 (0.36)	0.0524 (0.26)	-0.0071 (0.03)	0.1145 (0.00)	0.1219	-0.0375 (0.32)	-0.0370 (0.32)	-0.0039 (0.01)	-0.0034 (0.01)	0.0001
4	-0.0657 (0.46)	0.0207 (0.27)	-0.0094 (0.09)	0.0899 (0.00)	0.0997	-0.0481 (0.42)	-0.0482 (0.42)	-0.0042 (0.07)	-0.0042 (0.07)	0.0002
5	-0.0757 (0.33)	-0.0073 (0.21)	-0.0107 (0.01)	0.0711 (0.00)	0.0822	-0.0645 (0.32)	-0.0638 (0.32)	-0.0059 (0.01)	-0.0055 (0.01)	0.0002
6	-0.0886 (0.41)	-0.0309 (0.30)	-0.0110 (0.02)	0.0571 (0.00)	0.0690	-0.0812 (0.40)	-0.0823 (0.40)	0.0073 (0.02)	-0.0069 (0.02)	0.0002
7	-0.1017 (0.38)	-0.0526 (0.31)	-0.0109 (0.01)	0.0462 (0.00)	0.0590	-0.0967 (0.38)	-0.0955 (0.38)	-0.0083 (0.01)	-0.0083 (0.01)	0.0003
8	-0.1166 (0.32)	-0.0750 (0.27)	-0.0121 (0.00)	0.0374 (0.00)	0.0513	-0.1114 (0.33)	-0.1114 (0.33)	-0.0098 (0.00)	-0.0095 (0.00)	0.0003
9	-0.1280 (0.29)	-0.0949 (0.25)	-0.0128 (0.00)	0.0313 (0.00)	0.0453	-0.1247 (0.30)	-0.1238 (0.30)	-0.0111 (0.00)	-0.0107 (0.00)	0.0003
10	-0.1398 (0.21)	-0.1126 (0.18)	-0.0141 (0.00)	0.0254 (0.00)	0.0405	-0.1418 (0.22)	-0.1414 (0.22)	-0.0128 (0.00)	-0.0127 (0.00)	0.0003

Notes:
- .The figures between parentheses give the percentage of Monte Carlo distribution below the actual value.
- .All models we refer to in this table are two-state Markov switching models.
- .2M1V stands for the model with two (state) means and one (state) variance.
- .1M2V stands for the model with two (state) variances and one (state) mean.
- .S_t stands for the model where the Markov variable at time t enters the endowment equation at time t.
- .S_{t-1} stands for the model where the Markov variable at time t-1 enters the endowment equation at time t.
- .The numbers for the two-mean-one-variance(2M1V) lagged(S_{t-1}) Markov model differ slightly from CLM results because of data revisions at the end of the sample.

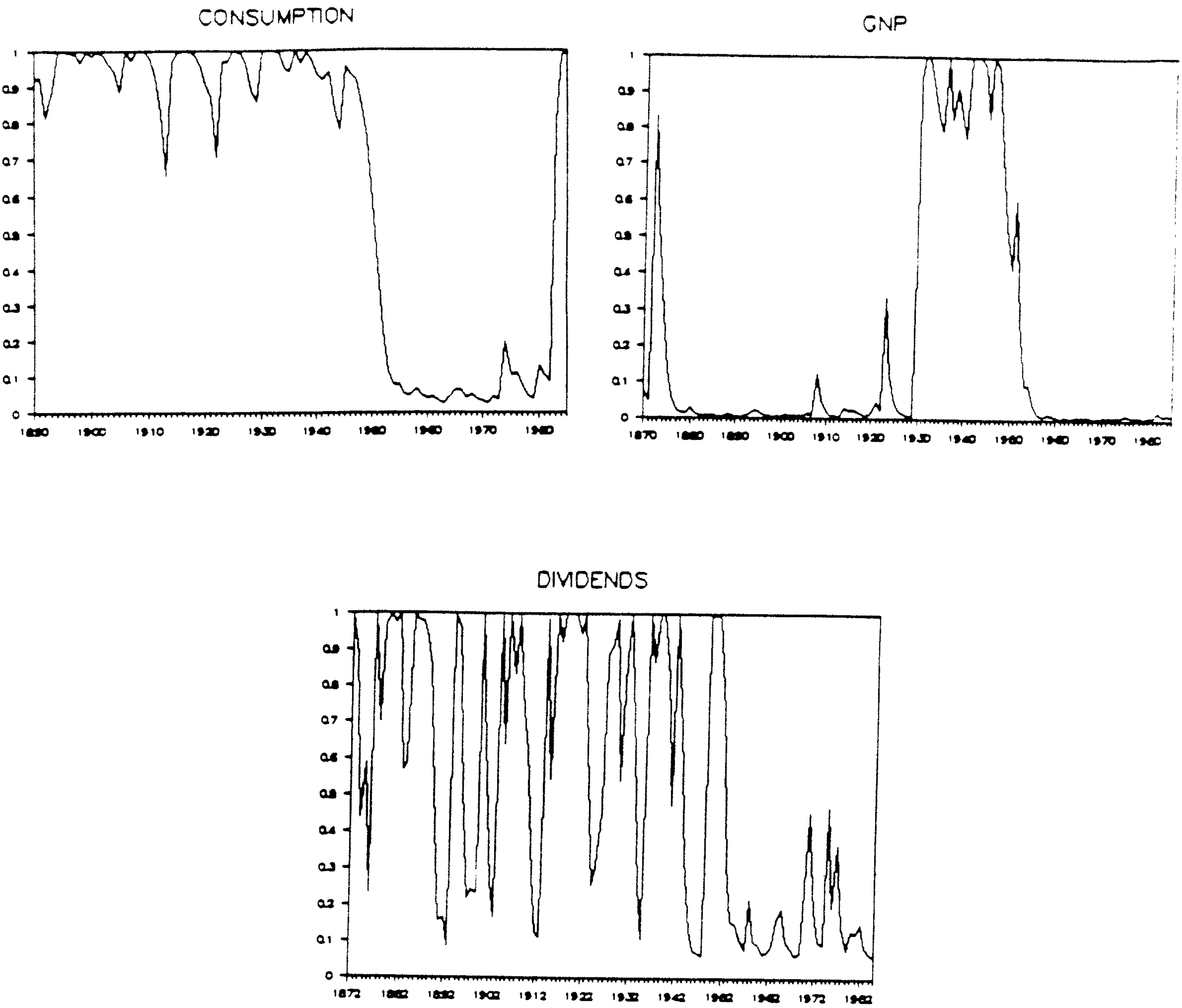
k	Actual	2M1V	1M2V	2M1V	1M2V
		S_{t-1}		S_t	
2	1.0492	0.9993	1.0013	0.9994	1.0160
3	0.9300	0.9989	1.0026	0.9990	1.0311
4	0.9121	0.9986	1.0038	0.9987	1.0454
5	0.8639	0.9983	1.0050	0.9985	1.0589
6	0.7848	0.9982	1.0062	0.9983	1.0717
7	0.7246	0.9980	1.0073	0.9982	1.0837
8	0.7123	0.9980	1.0083	0.9981	1.0952
9	0.7038	0.9979	1.0094	0.9981	1.1060
10	0.7148	0.9978	1.0104	0.9980	1.1162

Table 5-Population Values for the Excess Returns Variance Ratios

Note: The values are calculated for γ=1.7.

FIGURE 1

Probabilities of a High Variance State



Note: These probabilities are a sub-product of the nonlinear filter used to estimate the parameters (for full details, see Hamilton (1989)).