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IMAGINARY MONEY AGAINST STICKY RELATIVE PRICES

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Abstract

This paper considers price setting in pure units of account, linked to the means of payment through managed parities. If prices are sticky in the units in which they are set, parity changes may facilitate equilibrium adjustment of relative prices. The paper derives simultaneously the optimal choice of unit of account by each price setter, and the optimal parity policy. The gains from having multiple units of account are computed for a simple calibrated economy. (*JEL* E4, E5, F4)

1 Introduction

The salary is 2,000 livres a year, but I should have to spend six months at Versailles and other six in Paris, or wherever I like. I do not think that I shall accept it, but I have yet to hear the advice of some good friends on the subject. After all, 2,000 livres is not such a big sum. It would be so in German money, I admit, but here it is not. It amounts to 83 louis d'or, 8 livres a year - that is, to 915 florins, 45 kreuzer in our money (a considerable sum, I admit), but here worth only 333 thalers, 2 livres - which is not much. It is frightful how quickly a thaler disappears here.

W.A. Mozart, 1778.

Medieval and early modern Europe was a world of porous monetary borders and sovereigns bent on debasing their coinage. Weights of precious metal and tallies of circulating coins, local and foreign, competed as standards of value.

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In a lesser known twist, there were also units of account without any physical embodiment, defined by legal tender parities to the means of payment. Parities that could be changed at will made the disembodied units of account - which Einaudi (1936) aptly termed ‘imaginary money’ - more than aliases for fixed multiples of the means of payment. Whatever their true historical importance, imaginary monies remain a tantalizing logical possibility.¹

Separation between unit of account and means of payment is also a traditional theme of monetary futurology (Cowen and Kroszner, 1994), and so is the notion of multiple monetary standards within the same region (Cohen, 1998, 1999, 2000). But that literature has focused on sweeping transformations to the means of payment and to the ‘anchoring’ of the price level. Scant thought has been given to a scheme involving more modest changes in the way transactions are settled: to retain under government control a unique standard for the exchange medium, adding imaginary monies as alternative units of account.

A modern day motivation for imaginary monies is the same as for flexible exchange rates: manipulating the parity between the units in which prices are sticky facilitates relative price adjustment and mitigates resource misallocation (Friedman, 1953). The net welfare gains must deduct the calculation burden inherent to multiple units of account.

Just as monetary futurology has been heralding the age of non-territorial, self-organizing networks of users of different means of payment, I consider individual producers who choose among pricing units on grounds other than locational. Firms whose idiosyncratic shocks are highly correlated would like to price together in a separate unit of account, if they could count on parity policy to facilitate their desired relative price adjustment with respect to the rest of the economy. Sectoral links may dominate location as a source of correlation across shocks. It is the self-organizing (and potentially non-territorial) aspect of the scheme, besides the separation between units of account and means of payment, that sets it apart from the conventional problem of optimal currency areas.

This paper fleshes out formally the intuitive case for imaginary monies.² In section 2, a general equilibrium macroeconomic model with sticky prices is augmented to incorporate one imaginary money, and the optimal choice of pricing unit by individual firms is derived simultaneously with the optimal policy towards the imaginary money parity. In section 3, I calibrate the model to quantify the gains on the price misalignment front, to be weighed against one’s

¹Historical references include Bloch (1934), Cipolla (1956, 1982, 1991), Einaudi (1936, 1937), Lane and Mueller (1985), and van Werveke (1934).

²This theme made an incipient appearance in Cowen and Kroszner (1994), who did contemplate the possibility of self-organizing, non-territorial networks of users of multiple units of account. Despite sharing my analogy with OCAs, their initial emphasis was not on the misallocation of production resulting from misaligned prices, but on the risk of contractual obligations to deliver a *given quantity* at predetermined prices - the question of the *invoicing* unit of account (pp. 43-4). Later on, they revert to misallocation costs, and to pricing units chosen according to their correlation with the individual producer’s profit maximizing price, but do so in the context of commodity bundle units of account rather than imaginary monies (p. 94). Their discussion is brief and involves no attempt at modeling or quantification.

best guess of the calculation burden of unit of account duplicity. Section 4 interprets the imaginary money exercise as a stepping stone towards more down-to-earth yet harder problems. Section 5 concludes.

2 An economy with imaginary money

2.1 Basic setup and notation

Consider an economy where prices can be quoted in either one of two units of account. The first is the circulating means of payment, to which I refer as the *real money* ($r\$$). The second, which I call *imaginary money* ($i\$$), is a disembodied unit of account, defined by no more than an officially announced parity X with respect to the real money: $i\$X = r\1 . There is a continuum of differentiated goods indexed by the unit interval. $P(z)$ denotes the price of good z in terms of real money, and $Q(z)$ its price in terms of imaginary money. If good z has its price posted as $i\$Q(z)$, that is understood as willingness to trade the good for $r\$\frac{Q(z)}{X}$.

Each differentiated good is produced by a monopolistically competitive firm, which turns homogeneous labor $h(z)$ into output $c(z)$ according to the simple technology $h(z) = c(z)s(z)$ - increases in $s(z)$ are adverse cost shocks. I allow for a subsidy to employment: the government reimburses the firms for a fraction $(1 - \chi)$ of their payroll, financing that expenditure out of lumpsum taxes. The subsidy can be used to offset distortions due to market power.

There is a representative household with utility function $u(c, h)$, increasing in the CES consumption index:

$$c \equiv \left[\int_0^1 c(z)^{1/\mu} dz \right]^\mu$$

and decreasing in the aggregate amount of work employed in the economy:

$$h \equiv \int_0^1 h(z) dz$$

The household allocates expenditures in order to minimize the cost of each unit of c . As a result, the general price level - the price of the expenditure minimizing bundle - is measured by:

$$P \equiv \left[\int_0^1 P(z)^{\frac{1}{1-\mu}} dz \right]^{1-\mu}$$

$$Q = \left[\int_0^1 Q(z)^{\frac{1}{1-\mu}} dz \right]^{1-\mu}$$

in terms of $r\$$ and $i\$$, respectively ($Q = XP$). The demand for good z , as a function of its real price $p(z) = \frac{P(z)}{P} = \frac{Q(z)}{Q}$, is:

$$c(z) = cp(z)^{\frac{\mu}{1-\mu}} \quad (1)$$

Defining an aggregate index of labor requirements for production:

$$s \equiv \left[\int_0^1 s(z)^{\frac{1}{1-\mu}} dz \right]^{1-\mu}$$

one verifies that:

$$h = cs\delta \quad (2)$$

where:

$$\delta \equiv \int_0^1 \frac{s(z)}{s} p(z)^{\frac{\mu}{1-\mu}} dz$$

can be interpreted as a coefficient of relative price misalignment. δ attains its minimal value of unity when all prices are aligned in proportion to costs: $p(z) = \frac{s(z)}{s}$.

The first best outcome for this economy is the maximum of $u(c, h)$ subject to $h = cs\delta$, requiring $\delta = 1$ and $u_c(c, h) = -u_h(c, h)s$. In the decentralized equilibrium, however, households want $u_c(c, h)w = -u_h(c, h)$, which will be suboptimal unless $w = \frac{1}{s}$.

Suppose for now that firms know $s(z)$ when they set prices. Their profit maximization problem is:

$$\max_{p(z)} [p(z) - \chi s(z)w] cp(z)^{\frac{\mu}{1-\mu}}$$

where w is the real wage rate. This is solved by $p(z) = \mu\chi s(z)w$; integrating over all z , one finds the equilibrium real wage $w = \frac{1}{\mu\chi s}$. There is no price misalignment in this equilibrium, and it will coincide with the first best as long as $\chi = \frac{1}{\mu}$. Absent the employment subsidy, the equilibrium real wage and level of activity would be too low.

The model does not include shocks to preferences. With constant desired mark-ups, they could only affect equilibrium relative prices if marginal costs changed with output, for a given set of cost curves. That channel shuts down if marginal cost curves are horizontal - an assumption, however unrealistic, that I maintain for simplicity.³ The reader should be warned that it tends to overstate

³Blinder et al. (1998) report that most firms perceive their marginal cost curves as either flat or decreasing over the relevant range. They interpret that as supportive of Hall's (1986, 1988) conjecture that marginal costs vary little over the cycle, or even of Ramey's (1991) findings of countercyclical marginal costs. But Blinder et al. admit that industry executives may have confused marginal with average costs in answering to their survey. Evidence of procyclical marginal costs is surveyed in Rotemberg and Woodford (1999), although some procyclicality might be due to aggregate factor scarcity rather than to the shape of each firm's cost curves.

the costs of price misalignment and thus the gains from an imaginary money scheme, if the true marginal cost curves slope upwards.⁴

2.2 Cost shocks and price rigidities

Price misalignment may arise because of nominal rigidities: prices set before costs become known may end up out of line. If some prices are set in real and others in imaginary money, they can be brought closer to alignment, once costs are realized, through manipulation of the imaginary money parity. But pricing decisions, including the choice of unit of account, should anticipate that parity manipulation.

Because consumers costlessly verify all relative prices, there is no reason why firms might *want* to limit deviations of prices from their expected values, and shun pricing units that work to restore price flexibility. Search costs might alter that behavior, as smoother prices help maintain ‘customer goodwill’. The resulting price stickiness would have a welfare enhancing effect of saving on aggregate search costs. Such possibilities are not contemplated in this paper.

I study this problem in a static model. In the runup to the single market period, events unfold as follows: (i) Each firm chooses between $r\text{\$}$ and $i\text{\$}$, and posts a price in the chosen unit; (ii) The values of $s(z)$ realize; (iii) Observing the realized $\{s(z)\}$, and what firms set prices in each unit of account, the government sets the parity X , while it also sets monetary policy instruments (not explicitly modeled) so as to deliver a certain P ; (iv) A fraction α of the firms is randomly selected (as in Calvo, 1983) to post new prices, incorporating all information already revealed. Every firm must satisfy all forthcoming demand at its posted price. In step (i), firms must take into account what they anticipate for $\{s(z)\}$, X and P .⁵

In order to facilitate the analysis, an approximation to the model is taken around the flex-price equilibrium in which all goods share the same $s(z) = \bar{s}$, all prices are equal, and there is no price misalignment. For a generic variable y , \bar{y} shall denote its value at the benchmark equilibrium, and $\hat{y} \equiv \frac{y - \bar{y}}{\bar{y}}$. For every z , $s(z)$ shall have the same probability distribution with mean \bar{s} , so that $E\hat{s}(z) = 0$. Up to a first order approximation, $E\hat{s} = 0$ as well.

From the definition of the coefficient of price misalignment, one can verify that, in a neighborhood of the benchmark equilibrium, and up to a first order approximation, $\hat{\delta} = 0$. In other words, price misalignment is not a first order phenomenon, and can only be studied with higher order approximations to the model. Price misalignment achieved an earlier notoriety in the debate over the costs of inflation (see Fischer, 1981), but at the time the impulse was to dismiss

⁴When sectoral output is below equilibrium, the deadweight loss is overstated because the model assumes that the shortfall could be produced at the low realized marginal cost. When output exceeds equilibrium, the overstatement is due to counting every inframarginal unit as if it had been produced at the high realized marginal cost.

⁵In reality, the government would not observe $\{s(z)\}$ directly. Policy would be less powerful as it would presumably react to indirect signs of cost shocks, such as relative price movements already observed. One could model those with an extra round of partial price adjustment, between the realization of $\{s(z)\}$ and determination of X .

such second order welfare loss as incapable of ‘piling up a heap of Harberger triangles tall enough to fill an Okun gap’ - that is, of making a major difference in the case for disinflation. If the imaginary money scheme involves first order deadweight losses (calculation costs, say, which are not modeled here), then those are certain to trump the welfare losses from price misalignment *whenever cost shocks become small enough*. Yet, the calibrated model of section 3 indicates that the loss from price misalignment - and the gains from imaginary money - need not be negligible.

2.3 Minimizing price misalignment

In its own right, price misalignment is bad for welfare: more work effort h is required to obtain the same amount of the consumption index c , because consumption concentrates on goods that are relatively cheaper to buy, though not so much cheaper to produce. The demanded consumption bundle equates marginal utility ratios to relative prices; if those are not aligned with relative marginal costs, the allocation of consumption is inefficient.

If a social planner, given s , were to choose c, h and δ in order to maximize the household’s utility subject to (2), leaving the allocation of consumption expenditures free to respond to relative prices, then the command optimum ought to minimize δ . But that is not necessarily what a policymaker should do in order to implement the best decentralized equilibrium. Policies that reduce δ impact the equilibrium real wage in one way or another. If the economy operates below its efficient level of activity, then lower δ accompanied by higher w should be welcome. If lower δ comes with lower w , welfare could fall.

Under certain assumptions, a δ -minimizing policy does select the best decentralized equilibrium. Utility could be such that, in keeping with condition $u_c(c, h)w = -u_h(c, h)$, changes in $s\delta$ and w get reflected either in c or in h , *but not both*, cases in which minimization of δ is optimal. In one extreme, minimizing δ is equivalent to minimizing the effort necessary to obtain the *constant* equilibrium level of consumption; likewise in the other extreme, where it maximizes the consumption allowed by the *constant* equilibrium employment.⁶ Among several possible examples, the Cobb-Douglas function $u(c, h) = (c - c^*)^\beta (h^* - h)^{1-\beta}$ approximates that property arbitrarily well with limiting choices of $\beta \in]0, 1[$. As $\beta \rightarrow 0$, all variation concentrates in h , and minimizing δ maximizes equilibrium $u(c, h) \rightarrow h^* - c^*s\delta$. As $\beta \rightarrow 1$, all variation concentrates in c , and minimizing δ again maximizes equilibrium $u(c, h) \rightarrow \frac{h^*}{s\delta} - c^*$.

Without restrictions on the utility function, minimization of δ is guaranteed to be optimal if $\chi = \frac{1}{\mu}$. With that rate of subsidy, the economy would be at the first best equilibrium $\delta = 1$ and $w = \frac{1}{s}$ in the absence of cost surprises. Unexpected cost shocks pull δ away from unity, and that may be more or less mitigated by policy. Whatever δ materializes, though, it is optimal to match it

⁶Another way to put it: if either c or h is fixed, then both are fully determined by $s\delta$, regardless of w , for (2) would not be consistent with only c or h varying with fixed $s\delta$. Thus, policy to reduce δ cannot have welfare side effects through w .

with $w = \frac{1}{s\delta}$. Relatively to the first best equilibrium, w would need to increase just in the same proportion as $s\delta$ falls. The δ -minimizing policy derived below has that property.⁷

Of course, neither set of assumptions is realistic, but they serve to keep the focus on the most direct channel for imaginary money to improve welfare, namely, reducing price misalignment and making the *allocation* of production across goods more efficient. Once minimization of δ is acknowledged as the legitimate policy goal, the developments below need no further recourse to either set of assumptions.

2.4 Optimal pricing

Firms that start by posting an $r\text{\$}$ price $P(z)$ maximize:

$$E \left\{ v \frac{P(z) - \chi s(z) P w}{P} c \left[\frac{P(z)}{P} \right]^{\frac{\mu}{1-\mu}} \right\}$$

where v denotes the household's marginal utility of income in each state of nature. Their prices are:

$$P(z) = \mu \chi \frac{E \left[v c P^{\frac{1}{\mu-1}} s(z) P w \right]}{E \left[v c P^{\frac{1}{\mu-1}} \right]}$$

For firms that start by posting an $i\text{\$}$ price, the latter two equations apply with $Q(z)$ and Q substituted for $P(z)$ and P . Firms selected to set prices after $s(z)$ realizes behave as in the flex-price case: $P(z) = \mu \chi s(z) P w$.

Let Γ be the set of firms that choose to post prices in imaginary money, and γ its measure; let A be the set of firms selected to adjust prices *ex post*, and α its measure. A first order approximation to the pricing rules above yields the following for the *realized* $r\text{\$}$ prices:

$$\widehat{P}(z) = \begin{cases} \widehat{s}(z) + \widehat{P} + \widehat{w} & \text{if } z \in A \\ E\widehat{s}(z) + E\widehat{P} + E\widehat{w} - (\widehat{X} - E\widehat{X}) & \text{if } z \in \Gamma \setminus A \\ E\widehat{s}(z) + E\widehat{P} + E\widehat{w} & \text{if } z \in [0, 1] \setminus (A \cup \Gamma) \end{cases} \quad (3)$$

Integrating (3) over all z and performing some algebraic manipulation, one obtains the following first order relation:

$$\widehat{w} = -\widehat{s} + \frac{1-\alpha}{\alpha} (\widehat{P} - E\widehat{P}) + \gamma \frac{1-\alpha}{\alpha} (\widehat{X} - E\widehat{X}) \quad (4)$$

The equilibrium real wage is thus determined by the aggregate cost shock and the surprises in the $r\text{\$}$ price level and the imaginary money parity. Without surprises in either \widehat{P} or \widehat{X} , the real wage is inversely proportional to the

⁷Up to a first order approximation (since δ displays no first order variation) w must vary in inverse proportion to s , which is verified by (4) and (7).

aggregate cost shock. Consider what would happen if there was still no surprise in \widehat{P} , but the government reacted to the shocks with a surprise *revaluation* of the imaginary money ($\widehat{X} - E\widehat{X} < 0$). All goods in $\Gamma \setminus A$ would become more expensive in terms of $r\$$, while the $r\$$ prices of goods in $[0, 1] \setminus (A \cup \Gamma)$ would not change. To avoid a surprise in \widehat{P} , the $r\$$ prices of the flex-price goods must fall, which is only possible if the equilibrium real wage falls. The larger is the proportion γ of prices buoyed up by the surprise in \widehat{X} , and the smaller is the proportion α of prices that adjust *ex post*, the larger the movement in real wages needs to be. Allowing some upward surprise in \widehat{P} would take pressure off the flex-price goods to compensate for the $r\$$ price increases occurred elsewhere.

2.5 Choice of unit of account

Firms choose between pricing in $r\$$ or in $i\$$ comparing the maximized values of their expected profits under either choice, which yields the following criterion for choosing to price in $i\$$:

$$\left\{ \frac{E \left[vcP^{\frac{\mu}{\mu-1}} s(z)w \right]}{E \left[vcQ^{\frac{\mu}{\mu-1}} s(z)w \right]} \right\}^{\frac{1}{\mu-1}} \left\{ \frac{E \left[vcP^{\frac{1}{\mu-1}} \right]}{E \left[vcQ^{\frac{1}{\mu-1}} \right]} \right\}^{\frac{\mu}{1-\mu}} > 1$$

The strict inequality means that, whenever indifferent between the two units of account in terms of expected profits, firms post prices in $r\$$. That serves to eliminate from Γ any positive measure of price setters with a fleeting allegiance to $i\$$. Here, it is implicitly assumed that pricing in imaginary money carries no inherent disadvantage: the profit function is the same for either choice of unit of account. If there were an arbitrarily small imaginary money handicap in the relationship with customers, then price setters who are otherwise indifferent between $i\$$ and $r\$$ would switch *en masse* to the real money. I choose from the start not to count those in Γ .

I consider a second order approximation to the criterion above, which is easier to manipulate and to interpret:

$$cov \left[\widehat{s}(z) - \widehat{s}, \widehat{X} \right] + \frac{var\widehat{Q} - var\widehat{P}}{2\alpha} + \left(\gamma - \frac{1}{2} \right) \frac{1-\alpha}{\alpha} var\widehat{X} < 0 \quad (5)$$

The covariance term appears for very intuitive reasons. When $\widehat{s}(z) - \widehat{s} > 0$, firm z would like to have its relative price increased, which will happen if it has priced in $i\$$ and $\widehat{X} < 0$. The more likely such parity changes are to produce the firm's desired price changes - the more negative is $cov \left[\widehat{s}(z) - \widehat{s}, \widehat{X} \right]$ - the stronger the incentive to price in $i\$$. Once a price is posted in $i\$$, real profits will depend on $\frac{Q(z)}{Q}$, where the numerator is fixed; likewise for $\frac{P(z)}{P}$, which determines real profits from prices set in $r\$$. Uncertainty about the denominator in these ratios reduces the *expected value* of real profits, as the firm is more likely to be away

from the profit maximizing price.⁸ If Q is more uncertain than P , firms are discouraged from pricing in i §; hence the term in $var\hat{Q} - var\hat{P}$ in (5). The rightmost term in (5) is less transparent, because it stems from the indirect effect of \hat{X} through the equilibrium real wage.

2.6 Monetary policy, real and imaginary

Monetary policy is assumed to control X and P directly. The imaginary money parity is just a number that the policymaker needs to publish. Direct control over P is interpreted as standing in for control of a conventional monetary policy instrument that affects the price level in real money. There will be an optimal choice of both X and P in the sense of minimizing price misalignment. But stability of the purchasing power of the *real* money may be of interest in its own right, constraining the manipulation of P .

It is convenient to work with a second order approximation to δ :

$$\begin{aligned} \hat{\delta} = & \frac{\mu}{\mu-1} \frac{1-\alpha}{\alpha} \left\{ \frac{\alpha}{2} \int_0^1 [\hat{s}(z) - \hat{s}]^2 dz + \frac{1}{2} (\hat{P} - E\hat{P})^2 + \right. \\ & \frac{\alpha\gamma}{2} [1 + \gamma \frac{1-\alpha}{\alpha}] (\hat{X} - E\hat{X})^2 + \gamma (\hat{P} - E\hat{P}) (\hat{X} - E\hat{X}) + \\ & \left. \alpha (\hat{X} - E\hat{X}) \int_{\Gamma} [\hat{s}(z) - \hat{s}] dz \right\} \end{aligned} \quad (6)$$

Minimization of $\hat{\delta}$ with respect to $\hat{P} - E\hat{P}$ and $\hat{X} - E\hat{X}$ produces, up to a first order approximation, the same policy reaction by the government as minimization of the exact δ .

Note first that optimally chosen $\hat{P} - E\hat{P}$ would exactly offset the impact of $\hat{X} - E\hat{X}$ on the equilibrium real wage given by (4):

$$\hat{P} - E\hat{P} = -\gamma (\hat{X} - E\hat{X}) \quad (7)$$

However, when it comes to the purchasing power of the means of payment held by private agents, other considerations besides price misalignment are likely to impinge on the choice of $\hat{P} - E\hat{P}$, namely, the welfare effects of inflation through the holdings of money balances. Without modelling the demand for money, it is not possible to quantify those effects, but their consideration should dampen the desired $\hat{P} - E\hat{P}$ variations. To allow for that possibility, I replace (7) with:

$$\hat{P} - E\hat{P} = -\lambda\gamma (\hat{X} - E\hat{X}) \quad (8)$$

where $\lambda \in [0, 1]$. This nests the extreme cases of a \hat{P} policy intended to minimize price misalignment ($\lambda = 1$) and of an ‘inflation nutter’ ($\lambda = 0$, so that $\hat{P} - E\hat{P} = 0$ no matter what), as well as everything else in between. Note that (8) implies

⁸These deviations have a second order effect on expected profits, which is decisive here, in the absence of first order terms.

$\widehat{Q} - E\widehat{Q} = (1 - \lambda\gamma) (\widehat{X} - E\widehat{X})$. Increases in λ shift the impact of a parity change from the i \$ price level to the r \$ price level. The $\lambda = 1$ partition would be guided by price misalignment considerations alone, being for that reason proportional to the adoption of each unit of account in pricing. That would mean no preference for insulating the purchasing power of r \$ balances held by private agents over that of a disembodied unit of account.

The results for $\lambda = 0$ also apply to another interpretation of the government's behavior: it simply insists in conducting monetary policy as if the imaginary money scheme were not present at all. Such policymakers, despite truly caring about price misalignment ($\lambda < 0$), erroneously stick to $\widehat{P} - E\widehat{P} = 0$, which would indeed be optimal if $\widehat{X} - E\widehat{X} = 0$ but not otherwise - a behavior identical to the inflation nutter's. That might describe what an imaginary money scheme arranged by the private sector would achieve without persuading the government that a reassessment of monetary policy is warranted.

Combined with (8), the first order condition for optimal $\widehat{X} - E\widehat{X}$ is:

$$[\alpha\gamma(1 - \gamma) + (1 - \lambda)\gamma^2] (\widehat{X} - E\widehat{X}) = -\alpha \int_{\Gamma} [\widehat{s}(z) - \widehat{s}] dz \quad (9)$$

while (8) and the criterion (5) for deciding to price in i \$ yields:

$$\Gamma = \left\{ z : cov [\widehat{s}(z) - \widehat{s}, \widehat{X} - E\widehat{X}] < \frac{2\gamma(\alpha + \lambda - 1) - \alpha}{2\alpha} var (\widehat{X} - E\widehat{X}) \right\} \quad (10)$$

An equilibrium is characterized by Γ (with the corresponding γ) and $\widehat{X} - E\widehat{X}$ (contingent on the realization of cost shocks) that satisfy (9) and (10).⁹

In two situations, (9) leaves the parity choice indeterminate. The first is when $\gamma = 0$: since nobody prices in i \$, the choice of parity cannot make any difference for price misalignment. Whether or not that is an equilibrium depends on whether $\Gamma = \emptyset$ is consistent with (10), given an arbitrary choice of parity policy. If policy makes $\widehat{X} - E\widehat{X} = 0$, then indeed $\Gamma = \emptyset$, since no one has any motive to strictly prefer to price in i \$. So, not having any operative imaginary money is always an equilibrium.

The other case of indeterminacy arises when $\gamma = \lambda = 1$ (the integral on the right-hand side of (9) is zero when $\Gamma = [0, 1]$). Even if there was an arbitrary choice of parity policy such that (10) would yield $\Gamma = [0, 1]$, the result would be a fragile equilibrium. If $\gamma = 1$ but λ were instead any less than unity, then (9) would *fully determine* $\widehat{X} - E\widehat{X} = 0$. But parity choice would then be rendered irrelevant for expected profits; with no one strictly preferring to price in i \$, the

⁹ λ need not pertain only to the policymaker's preferences, but may also depend on structural parameters of the model. The policymaker's loss function might be $\widehat{\delta} + \frac{\rho}{2} (\widehat{P} - E\widehat{P})^2$, where $\rho \geq 0$ describes his or her preferences over price misalignment versus disturbances to the r \$ price level. Minimizing that yields (8) and (9) as first order conditions, λ being a function of ρ , α and μ (with range $[0, 1]$). Note that λ would not depend on the *endogenous* parameter γ .

result would need to be $\gamma = 0$ rather than $\gamma = 1$. I disregard both cases of indeterminacy, and focus on equilibria in which imaginary money is present and operative.

3 The gains from imaginary money

3.1 Benchmarks

Consider what would happen if the government turned its back on imaginary money, and kept $\widehat{X} - E\widehat{X} = 0$ always. Regardless of the value of λ , monetary policy would also keep $\widehat{P} - E\widehat{P} = 0$ all the time. Pricing in either unit of account would produce the same expected real profits, so no agent would strictly prefer to price in $i\$$, and the imaginary money would not be used at all. The price misalignment in (6) would reduce to:

$$\widehat{\delta}^* \equiv \frac{\mu}{\mu - 1} \frac{1 - \alpha}{2} \int_0^1 [\widehat{s}(z) - \widehat{s}]^2 dz \quad (11)$$

For a given realization of the cost shocks, $\widehat{\delta}^*$ is a measure of the welfare loss due to price rigidity, unmitigated by imaginary money. It is naturally larger when prices are stickier (α is lower). It is also larger when market power is weaker (μ is lower), because weaker market power is a reflection of higher cross elasticities of substitution, which in turn mean that any given price misalignment causes greater misallocation of production. Misalignment also increases with the dispersion of the idiosyncratic component of cost shocks, as captured by the integral. In particular, cost shocks that hit all industries equally do not matter for price misalignment.

Policies that use the imaginary money can have their impact on price misalignment evaluated by comparing their respective $E\widehat{\delta}$ to the benchmark $E\widehat{\delta}^*$. If $\text{var}[\widehat{s}(z) - \widehat{s}] = \sigma^2$ for all z (henceforth a maintained assumption), then $E\widehat{\delta} = (1 - \theta) E\widehat{\delta}^*$, where:

$$E\widehat{\delta} = \frac{\mu}{\mu - 1} \frac{1 - \alpha}{2} \sigma^2 \quad (12)$$

$$\theta \equiv \alpha \frac{\alpha\gamma(1 - \gamma) + (1 - \lambda^2)\gamma^2}{[\alpha\gamma(1 - \gamma) + (1 - \lambda)\gamma^2]^2} \int_{\Gamma} \int_{\Gamma} \text{corr}[\widehat{s}(z) - \widehat{s}, \widehat{s}(\xi) - \widehat{s}] d\xi dz \quad (13)$$

This θ measures what proportion of the expected price misalignment due to sticky prices gets eliminated by the imaginary money scheme. Because equilibrium γ turns out not to depend on σ or μ , neither does θ .

There are two cases in which θ can be readily turned into a statement about the welfare gains from imaginary money. The first is an economy without monetary frictions, or approaching a cashless limit as in Woodford (1998, 2000).

Aside from minimization of δ , the representative household has no reason to care directly about inflation in $r\$$, and an enlightened government should have $\lambda = 1$. The second case takes as given that $\lambda = 0$: either the government is an inflation nutter, or it does not acknowledge the impact of imaginary money on aggregate conditions. With $\lambda = 0$, the $i\$$ parity affects δ but not the $r\$$ price level. Regardless of the *social* preferences over minimization of δ versus stabilization of P , the reduction of δ captures the full extent of the welfare gains. If, instead, monetary frictions were not negligible but $\lambda > 0$, policy described by (8) and (9) would affect welfare through both δ and the variability of P ; the net welfare effect would depend on the microfoundations of social preferences over these objectives, which I have not modeled.

For the two polar cases, welfare gains depend on θ and on how large the benchmark loss from price misalignment is:

$$\Delta \equiv \theta \frac{E\widehat{\delta}^*}{1 + E\widehat{\delta}^*} \quad (14)$$

Up to first order, Δ measures the proportional increase in consumption allowed by a given work effort, relatively to the benchmark without imaginary money. Unlike θ , it does depend on μ and σ .

3.2 Cosinoid correlations

Calculation of the equilibrium Γ ultimately calls for the correlations among idiosyncratic cost shocks. Bending the continuum of firms into a circle of unit length allows for a symmetric setup: for any two firms, only the distance between them, and not the absolute location of the pair, matters for the correlation between their shocks. A particularly convenient specification for the correlation coefficients is:

$$\text{corr} [\widehat{s}(z) - \widehat{s}, \widehat{s}(\xi) - \widehat{s}] = \cos 2\pi (z - \xi)$$

for all z and ξ on the circle. Because correlations fall monotonically with distance, the equilibrium Γ is always an arc on the circle. But there is no ‘natural’ location for Γ - only its length matters. I fix one endpoint of Γ at zero, and denote the other by γ .¹⁰

Equilibrium γ is found by solving numerically the nonlinear equation:

$$\sin(2\pi\gamma) = \frac{\alpha + 2\gamma(1 - \alpha - \lambda)}{\alpha\gamma(1 - \gamma) + (1 - \lambda)\gamma^2} \frac{1 - \cos 2\pi\gamma}{2\pi}$$

The results derived below are meant as merely illustrative. In less tractable alternatives, correlations determined by distance alone could die away slower

¹⁰The fact that Γ could be located anywhere on the circle does not mean that the partition of the economy between units of account must remain indeterminate. Indeterminacy would indeed occur if the government reacted to any spontaneously arising Γ with policies described by (8) and (9). But it can also pick an arbitrarily located Γ of the right size and commit to a policy guided by substituting that Γ into (8)-(9). Private agents will align themselves accordingly, and there will be no temptation to deviate from the announced policy, which will be optimal *ex post*.

or faster than implied by the cosinoid, which would clearly change the results without necessarily making them more representative of real world economies. There is no way of knowing short of a much greater sacrifice of tractability: to abandon symmetry altogether and embark in a serious effort to match the model to empirical covariance matrices.

3.3 A calibrated economy

In dynamic models with staggered prices, α is calibrated to match the observed duration of prices. Rotemberg and Woodford (1997) calibrate for a mean duration of three quarters, as found by Blinder (1994) and Blinder et al. (1998). Such calibration of a quarterly model implies that only a third of the prices change before one semester of being set, which agrees with the finding of Blinder et al. that 35% of firms adjust prices that often. All that suggests interpreting the length of the period described in my model as one semester, and setting $\alpha = \frac{1}{3}$. Results for lower α 's are also reported: the parity X could change at a higher frequency exactly to offset more of the existing price stickiness.

Table 1 displays the results for γ and θ (both as percentages) for several combinations of α and λ . θ is obviously highest when monetary policy targets price misalignment alone ($\lambda = 1$). The economy is evenly partitioned between r \$ and i \$ price setters, and price misalignment gets reduced by about 40%. In this case, neither γ nor θ depends on α .

Under an inflation nutter ($\lambda = 0$), the i \$ is used by less than a third of firms. Imaginary money is so disadvantaged because its real value bears the entire brunt of changes in X . There is much less reduction in price misalignment: as little as 10% when $\alpha = \frac{1}{5}$. Results follow an intermediate pattern for values of λ between 0 and 1.

Table 2 reports results on Δ (as percentages), for the cashless limit case $\lambda = 1$. I focus on $\mu = 1.15$, as estimated by Rotemberg and Woodford (1997). Recall that higher mark-ups correspond to lower elasticities of substitution, which mitigate the misallocation of production arising from a given degree of price misalignment.

No off-the-shelf numbers existed for σ . Table 3 displays some crude estimates, expressed as percentages and described below. One should regard rows *II* and *III* as conservative estimates of σ , which depend however on certain assumptions and on further estimates of key technology parameters. Row *IV* is still plausible, and row *V* contains outside figures for σ , while those in row *I* are a loose but robust lower bound. I focus on the more conservative σ 's of the second column, which excludes the period of the oil shocks.¹¹

If σ is in the 9-11% range of rows *II* and *III*, gains from imaginary money would be equivalent to 0.8-1.2% of output (with the benchmark $\mu = 1.15$, $\alpha = \frac{1}{3}$). At the 7% lower bound for σ shown in row *I*, there would be gains of 0.5%

¹¹ σ is estimated from yearly data. It is unclear how it should be adjusted to go along with α calibrated for higher frequencies: the adjustment could actually go either way, depending on the serial correlation of the cost shocks within the year, on which I have no information.

of output. Gains would amount to 1.7% of output for $\sigma = 13\%$ (as in row *IV*), rise to 2.2% when $\sigma = 15\%$, and reach 3.8% if σ were the 20% of row *V*.

Numbers of that order are a major achievement for macroeconomic stabilization policy. Unlike tuning a monetary policy rule, however, they come at a cost - the calculation burden of the multiple units of account. Emerson et al. (1992) present estimates of currency *exchange* costs - the costs of transacting in foreign exchange markets - which are *not* the costs that imaginary monies would entail. It is also clear that calculation costs proper - the nuisance of converting prices from one unit of account to another for comparison and settlement - would be more widespread with non-locational imaginary monies than in a world with multiple national currencies, where they apply only to cross-border transactions. There is little hope of putting a value on that nuisance except by introspection.¹²

Corresponding results for $\lambda = 0$ could be easily inferred from tables 1 and 2 (Δ is just proportional to θ): roughly, the welfare gains would range from a fourth to a third of their values under $\lambda = 1$. Gains of the order of 0.3%-0.6% of output (with $\mu = 1.15$, $\alpha = \frac{1}{3}$) would obtain with an imaginary money scheme lacking any cooperation from the monetary authority in terms of moving P around with an eye on δ . Concurrence of $r\text{\$}$ monetary policy is important to reap the full benefits from the scheme - very much so if social preferences call for a low λ . But there is something in imaginary money even under monetary policymakers who behave as if it were not there.

For the sake of comparison, one can compute the optimal command partition of the economy into two groups of price setters, one being mandated to price in $r\text{\$}$, and the other in $i\text{\$}$. Without self-organization, the problem is akin to forming two optimal currency areas, except that they need not follow territorial lines. The enforcement of a non-territorial mandated partition would be impractical - after all, it works in territorial currency areas if monetary functions are not separated, while separation is crucial for imaginary money. What the exercise reveals is how close the decentralized equilibrium is to the command optimum.

Given policy described by (8) and (9), the best command partition of the economy is the one that minimizes θ , as long as price misalignment does represent social welfare - which is true if monetary frictions are negligible and $\lambda = 1$, or else if $\lambda = 0$. Solving that problem numerically, one obtains precisely the same γ , θ and Δ as in the decentralized equilibrium. In either case, the decentralized nature of the imaginary money scheme detracts nothing from what the very best split into two (not necessarily territorial!) currency areas would yield.

¹²In another model-based welfare analysis, Canzoneri and Rogers (1990) find that Europe would gain from monetary union if 'valuation costs' were as high as 0.7% of production costs. The benefits of multiple currencies would be much smaller than suggested here, especially since they take *several* currencies, rather than just two units of account. But the only advantage Canzoneri and Rogers attribute to multiple currencies is the freedom to pursue different seigniorage targets, according to optimal taxation criteria particular to each country - something imaginary monies would not offer. They disregard the effects of sticky prices.

3.4 Estimating σ

The estimates of σ contained in Table 4 are based on the US Manufacturing Industry Database.¹³ It contains yearly cost and output data for the 458 manufacturing industries of the 4-digit SIC.¹⁴

In year t , let $\widehat{MC}_t(z)$ be the percentage deviation of industry z 's nominal marginal cost from its *expected value*, and \widehat{MC}_t their cross-sectional mean. Then, $\widehat{MC}_t(z) = \widehat{s}_t(z) + \widehat{P}_t + \widehat{w}_t$ and $\widehat{MC}_t = \widehat{s}_t + \widehat{P}_t + \widehat{w}_t$. The time series variances of the idiosyncratic cost shock $\widehat{s}_t(z) - \widehat{s}_t$ can be written as:

$$\sigma^2(z) = \text{var} \left[\widehat{MC}_t(z) - \widehat{MC}_t \right]$$

In the model, those are the same for every industry, which is unlikely to hold in the data. Allowing for heteroskedasticity in the model would ruin its symmetry. Instead, I calibrate σ^2 after a cross-sectional weighted average of all $\sigma^2(z)$.

For each industry, I compute a 1959-1996 series $AC_t(z)$ of average costs of intermediate inputs (materials and energy). If the production function is isoelastic in intermediate input use, then $AC_t(z)$ is a constant multiple of the $MC_t(z)$, and $\widehat{AC}_t(z) = \widehat{MC}_t(z)$. But the evidence indicates otherwise: factors are not likely to be that substitutable, and $AC_t(z)$ should vary less than $MC_t(z)$. The correct proxy for the unobservable marginal costs would be:

$$MC_t(z) = \exp \left[\log AC_t(z) + \frac{1 - \epsilon}{\epsilon} \frac{1 - s_M(z)}{s_M(z)} \frac{1 - \mu s_M(z)}{\mu s_M(z)} \log IOR_t(z) \right] \quad (15)$$

where μ is the mark-up, ϵ is the elasticity of substitution between intermediate inputs and primary factors (both assumed common to all industries), $s_M(z)$ is the industry's average share (over time) of intermediate inputs in the value of output, and $IOR_t(z)$ is the ratio of intermediate inputs to output.¹⁵ All $s_M(z)$ and $IOR_t(z)$ are directly computed from the database, and μ is set at 1.15. Besides $\epsilon = 1$, which simply recovers $AC_t(z)$, the first calibration for the elasticity of substitution is $\epsilon = 0.7$, close to the estimate by Rotemberg and Woodford (1996). Their estimate is obtained from 2-digit SIC data, and some of the measured substitution between intermediate inputs and primary factors might be picking up substitution across products that, within the same 2-digit code, happen to be more intensive in one or the other. Within 4-digit industry codes, there should be fewer such opportunities to substitute across products, and one would expect a lower ϵ at that level of disaggregation. Lower estimates would also be expected if, unlike Rotemberg and Woodford, one measured factor

¹³The database and its documentation (Bartelsman and Gray, 1987) can be downloaded from the NBER website.

¹⁴SIC has 459 codes, but the asbestos industry disappeared in mid-sample.

¹⁵The procedure is described by Rotemberg and Woodford (1999, pp. 1064-5), on which this section relies heavily. It assumes constant returns to scale. Here, it is modified in a number of minor ways: to find a proxy for marginal costs rather than for mark-ups; to apply to AC rather than to the labor share in output; to denote the adjustment term as a function of IOR rather than the primary factors to output ratio; and to rely on a log-linear approximation only with respect to IOR , but not with respect to AC itself.

substitution within the year instead of that happening over two year horizons. I also report results for $\epsilon = 0.6, 0.5$ and 0.4 .

To get at the cost *surprise*, I let each industry, guided by Akaike’s Information Criterion, choose an AR specification for $\log MC_t(z) - \log MC_{t-1}(z)$, with any number of lags between 1 and 10, using 1970-1996 as a fixed sample period, and relying on as many data points as necessary from the 1960-1969 pre-sample. The estimated AR’s generate one period ahead forecasts of marginal costs, from 1970 to 1996, which are used to calculate deviations $\widehat{MC}_t(z)$ and their mean $\overline{\widehat{MC}}_t$ (weighted by the industry’s participation in the year’s output). Variances $\sigma^2(z)$ are computed, either over the whole sample (1970-1996) or just over the post-oil shock half-sample (1983-1996), and then averaged into σ^2 with weights given by the participation of each industry in 1983 output.¹⁶ The corresponding σ ’s (expressed as percentages) are reported in Table 4.

Note that estimates of σ based on the 4-digit SIC may still suffer from downward aggregation bias. They average out the variation across differentiated products included in the same 4-digit category, and also all *spatial* variation in cost shocks (for non-tradeables, that variation should also be matched by movements in relative prices).

4 Down to Earth

Disembodied units of account have reappeared a number of times since their medieval occurrences, but never in the spirit of the imaginary monies examined here. The pre-circulation euro had fixed parities with respect to all currencies in EMU. ECU parities were not so irrevocably fixed but it was seldom used in pricing (Bordo and Schwartz, 1989). Indexed units are common under high inflation, but they do not facilitate relative price changes prompted by real shocks - quite to the contrary, they avoid undue relative price changes caused by price staggering.

China had at some point a unique bimetallic standard with a *floating* silver-copper parity. Chen (1975) identified there a fascinating example of ‘currency areas’ self-organized by sector and not by location. Cavallo (2001) speculates about the future of Argentina as a trimonetary economy where firms will choose to price in euros, dollars, or pesos according to sectoral links. Non-territorial self-organization of price setters is present in the two examples, but both differ from the imaginary money scheme in the very real nature of their units of account and in the fact that parities do not deliberately minimize price misalignment.

The imaginary money scheme puts together self-organization of price setters and disembodied units of account linked by parities managed to minimize

¹⁶For $\epsilon < 1$, I truncate the distribution of $\sigma(z)$ to weed out spurious outliers from σ^2 . Lower ϵ should be reflected in less variable *IOR*, or else the adjustment term in (15) would swing wildly. When I apply a lower ϵ across the board, such swings occur in some industries, producing extremely high $\sigma(z)$. Inspection of the cross-sectional distributions of $\sigma(z)$ reveals that discarding the eight highest takes care of the outlier problem in all cases reported here.

price misalignment. Although assembled from familiar pieces, so experimental a monetary design should not be taken literally as a blueprint for reform. More practically, it can be read as stark first approach to problems that are closer at hand but are harder to analyze. Disembodied units of account are in no way central to such problems. The important premises are monetary policymakers concerned about relative price misalignment arising from idiosyncratic real shocks, and some endogenous choice of pricing currency.

Each of these two premises has been separately explored in recent research. Benigno (2001) analyzes a monetary union exposed to (among other things) relative price shocks. Monetary policy attempts to alleviate some of the price misalignment, but it can only do so much when there is a single currency; at any rate, there is no endogenous choice of pricing currency. Devereux and Engel (2001) study the endogenous choice of pricing currency in foreign transactions, obtaining conditions similar to criterion (5). But monetary policies are exogenous, and there are no real shocks to change equilibrium relative prices, only nominal shocks driving relative prices away from equilibrium. If monetary policy goes after relative prices *and* producers have a choice of pricing currency, then the two problems - choice of currency by price setters and the government's choice of policy - are interrelated and must be solved simultaneously.

The imaginary money scheme is the vanilla flavor of that generic exercise: a plain and simple base for a variety of fancier extensions. Such extensions can be tailored to much *less* extravagant setups. One example would restrict imaginary monies to intermediate good sales, thus sparing the consumer from the discomfort of calculating price conversions at retail outlets. Another example would be regional monetary policy coordination with adjustable exchange rates. A freer choice of pricing unit among the coordinating currencies (all real!) might apply only to cross border transactions, or to sectors with a strong foreign trade presence. Participation of each currency in pricing might turn out different from its share in the means of payment. That, in turn, would justify attaching a different weight to stabilizing the real value of each currency. Such considerations are recognized as a relaxed version of those already present in the imaginary money scheme.

5 Conclusion

This paper considers the consequences of allowing producers to choose among different currencies for pricing, and at the same time managing the purchasing power of those currencies in order to minimize resource misallocation due to price stickiness. Producers band together around the available pricing units according to the covariances among their relative price shocks, which may be determined by sectoral links rather than geographical location. The individual choice of pricing currency depends on how the values of the different currencies are managed; their best management in turn depends on who prices in each currency. Individual price setting and policy choice are thus simultaneous problems.

In a static setting, simple conditions make it legitimate for stabilization policy to single-mindedly target price misalignment. First, some other instrument must correct market power distortions, or else stabilization policy must have no bearing on those distortions. Second, monetary frictions must become negligible. If the latter fails, price misalignment still ranks correctly all stabilization policies that do not impinge on the purchasing power of the means of payment (but may affect the purchasing power of disembodied units of account).

When price misalignment is indeed the proper policy target, the decentralized equilibrium replicates the optimal pricing unit assignments. That is important because command assignments are much harder to enforce if pricing units become separable from legal tender currencies - as it must be the case for the split to occur along non-territorial lines. It amounts to saying that OCAs not beholden to geography are theoretically feasible as a product of self-organization, if monetary separation takes hold. Relaxing the locational requirement for membership in a currency 'area' enhances the potential benefits of adjustable parities. On the other hand, the calculation burden of currency multiplicity would go beyond cross-border transactions, being greater than in territorial OCAs.

An admittedly rudimentary model with a single currency and no stretch of calibration puts the deadweight loss from price misalignment at 2.5%-5% of GDP. One more currency could cut the loss by 40%. In the real world, the covariance matrix of relative price shocks might make misalignment losses considerably smaller and also harder to mitigate with multiple currencies. Anyway, the quantitative results warn against an out-of-hand dismissal of such inefficiencies as too small to bother.

Instituting a pure unit of account with a well managed parity produces attractive results even if actual monetary policy presses on with the behavior that was optimal when all prices were sticky in the means of payment. But price misalignment would be reduced to a much greater extent if monetary policy could be enlisted as well, placing the purchasing power of the means of payment at the service of price misalignment mitigation. Otherwise, the use of pure units of account would be discouraged by the higher volatility of their real value, limiting the scope for relative price correction.

Whether gains of any plausible magnitude are enough to compensate for the nuisance of price conversions is a question likely to remain open. A skeptical reader might interpret the numbers reported above as the deadweight losses from our inability to get around that nuisance once we concede defeat in eliminating the root cause of relative price rigidity.

The thought experiment carried out in this paper derives practical interest from potential extensions to more complicated but less far-fetched versions of the problem - as a common thread, the simultaneous choices of pricing currencies by individual producers subject to idiosyncratic shocks, and of monetary policies concerned with price misalignment. Candidate extensions include imaginary monies being used for pricing in a limited class of transactions, where calculation costs are less likely to be prohibitive. They may also involve no imaginary monies at all, but apply to the coordination of monetary policies among real currencies

in a regional integration arrangement, if certain producers are allowed to choose pricing unit according to how much it facilitates their desired relative price adjustment.

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Table 1
 γ and θ

λ	γ			θ		
	$\alpha = \frac{1}{5}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{3}$	$\alpha = \frac{1}{5}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{3}$
0	26.5	28.0	29.9	10.2	11.7	13.8
0.2	28.0	29.5	31.4	13.1	14.8	17.2
0.4	29.9	31.4	33.3	16.9	18.8	21.4
0.6	32.6	34.1	36.0	22.2	24.2	26.7
0.8	37.1	38.5	40.1	29.9	31.6	33.5
1	50	50	50	40.5	40.5	40.5

Table 2
 Δ

α	$\mu \setminus \sigma$	7%	9%	11%	13%	15%	20%
$\frac{1}{5}$	1.10	0.86	1.40	2.05	2.81	3.65	6.07
	1.15	0.60	0.98	1.45	2.00	2.62	4.43
	1.20	0.47	0.77	1.14	1.58	2.08	3.55
$\frac{1}{4}$	1.10	0.80	1.31	1.93	2.64	3.44	5.74
	1.15	0.56	0.92	1.36	1.88	2.46	4.18
	1.20	0.44	0.73	1.07	1.49	1.95	3.35
$\frac{1}{3}$	1.10	0.72	1.17	1.72	2.37	3.09	5.18
	1.15	0.50	0.82	1.22	1.68	2.20	3.76
	1.20	0.39	0.65	0.96	1.33	1.75	3.00

Table 3
 σ

		1970-1996	1983-1996
<i>I</i>	$\epsilon = 1$	8.2	7.0
<i>II</i>	$\epsilon = 0.7$	10.6	9.0
<i>III</i>	$\epsilon = 0.6$	12.4	10.7
<i>IV</i>	$\epsilon = 0.5$	15.4	13.3
<i>V</i>	$\epsilon = 0.4$	21.9	20.2