DEPARTAMENTO DE ECONOMIA
PUC-RIO

TEXTO PARA DISCUSSÃO
N°. 449

NON-MONOTONE INSURANCE CONTRACTS AND THEIR EMPIRICAL CONSEQUENCES

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DEZEMBRO 2001

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December 4, 2001

Abstract

The goal of this paper is to show the possibility of a non-monotone relation between coverage and risk which has been considered in the literature of insurance models since the work of Rothschild and Stiglitz (1976). We present an insurance model where the insured agents have heterogeneity in risk aversion and in lenience (a prevention cost parameter). Risk aversion is described by a continuous parameter which is correlated with lenience and, for the sake of simplicity, we assume perfect correlation. In the case of positive correlation, the more risk averse agent has higher cost of prevention leading to a higher demand for coverage. Equivalently, the single crossing property (SCP) is valid and implies a positive correlation between coverage and risk in equilibrium. On the other hand, if the correlation between risk aversion and lenience is negative, not only may the SCP be broken, but also the monotonicity of contracts, i.e., the prediction that high (low) risk averse types choose full (partial) insurance. In both cases riskiness is monotonic in risk aversion, but in the last case there are some coverage levels associated with two different risks (low and high), which implies that the *ex-ante* (with respect to the risk aversion distribution) correlation between coverage and riskiness may have every sign (even though the *ex-post* correlation is always positive). Moreover, using another instrument (a proxy for riskiness), we give a testable implication to disentangle single crossing and non single crossing under an *ex-post* zero correlation result: the monotonicity of coverage as a function of riskiness. Since by controlling for risk aversion (no asymmetric information), coverage is a monotone function of riskiness, this also gives a test for asymmetric information. Finally, we relate this theoretical results to empirical tests in the recent literature, specially the Dionne, Gouriéroux and Vanasse (2001) work. In particular, they found an empirical evidence that seems to be compatible with asymmetric information and non single

*This research is supported by CNPq of Brazil and Faperj of Rio de Janeiro. For their helpful comments, we thank Daniel Ferreira, Piero Gottardi, Richard Kihlstrom, Walter Novaes and Bernard Salanié. A preliminary version of this paper was presented in PUC-Rio, in DELTA and in Lausanne at the 56th European Meeting of the Econometric Society, 2001.

crossing in our framework. More generally, we build a hidden information model showing how omitted variables (asymmetric information) can bias the sign of the correlation of equilibrium variables conditioning on all observable variables. We show that this may be the case when the omitted variables have a non-monotonic relation with the observable ones. Moreover, because this non-monotonic relation is deeply related with the failure of the SCP in one-dimensional screening problems, the existing literature on asymmetric information does not capture this feature. Hence, our main result is to point out the importance of the SCP in testing predictions of the hidden information models.

1 Introduction

The goal of this paper is to show the possibility of a non-monotone relation between coverage and risk which has been considered in the literature of insurance models since the work of Rothschild and Stiglitz (1976). We propose an insurance model with adverse selection and moral hazard. The insured agents are heterogeneous with respect to risk aversion and lenience (precaution cost parameter). We borrow the Holmstrom and Milgrom (1987) framework of providing incentives for precaution for an agent with constant risk aversion. Differently from the standard literature, we assume that there exists a correlation between risk aversion and lenience. To simplify the model, we study only perfect correlation, which gives two stylized cases. In the positive correlation one, more risk averse agents have higher marginal cost of precaution. Thus, these two heterogeneities go in the same direction, i.e., the more risk averse agents buy more insurance and then they have less cautious behavior which reinforces the necessity of buying insurance. Therefore, one might expect to see positive correlation between coverage and risk. This is precisely the Rothschild and Stiglitz prediction and it is strongly related with the SCP.

In the negative correlation case, more risk averse agents have lower marginal cost of precaution. Thus, depending on the variance of the wealth (which determines the inference power of cautious behavior), those two heterogeneities can act in opposite directions: one effect can dominate the other, breaking down the SCP and the monotonicity property. More specifically, in a monopolistic market, the insuree's marginal utility of reducing coverage is decomposed into two effects: one is the marginal benefit, the prevention effort, which decreases with risk aversion (due to the negative correlation assumption) and the other is the marginal cost of risk premium, which increases with risk aversion.[1] For instance, if the variance is low, the informativeness about precaution is more precise, diminishing the moral hazard (or the cost of risk premium effect) and making the adverse selection dominant. Thus, the optimal contract will reflect more the screening feature which is driven by the first effect and the coverage will be non-increasing in risk aversion. On the other hand, if the variance is high, the wealth realization is less informative about precautious behavior and

[1] We provide a specific framework: cost of precaution with homogeneity of degree zero in effort and risk aversion, which has exactly this feature.

the screening is more costly. Thus, the second effect predominates and leads to non-decreasing coverages in risk aversion. For intermediate variances, there is a combination of these two effects: coverage is non-decreasing for low risk averse types and non-increasing for high ones.[2]

We also carry out the analysis in the competitive case. We use the perfect Bayesian equilibrium concept (signaling game) to sustain the actuarially fair constraint on the contract (zero profit). If the equilibrium involves full revelation, since there is no rent extraction by the insurance company (zero profit), the distortion is only provoked by the moral hazard and self-selection constraints. If the equilibrium involves pooling, the trade-off between distortion and rent extraction also appears as a consequence of cross subsidies. We show that for intermediate levels of variance, the quasi separable equilibrium[3] has discrete pooling: low and high risk averse insurees choosing the same coverage and having the same marginal rate of substitution between coverage and premium.

Puelz and Snow (1994) was the pioneering reference in testing asymmetric information on insurance market. Their results were consistent with the presence of asymmetric information. However, as pointed out by Dionne *et al.* (2001), these results may be spurious because of omitted non-linear effects in the regressions, for instance. Chiappori and Salanié (2000) also discuss the problems of Puelz and Snow test and propose a more robust test for asymmetric information using a data base from French insurance contracts. They tested the validity of positive correlation between coverage and riskiness of the contract conditional on all observable variables. They measured *ex-post* riskiness as the occurrence of an accident during one year and reduced the class of contracts to two: partial and full insurance. Their conclusion was that controlling for all observable variables, they could not reject that this correlation would be different from zero. Therefore, if the Rothschild and Stiglitz (1976) monotonicity property (MP): *"contracts with more comprehensive coverage are chosen by agents with higher expected accident costs"* were valid for that data set, this would lead to the conclusion that there is no binding adverse selection (no asymmetric information). Most of the models predict a positive correlation between insurance contract coverage and risk, implying that this claim is quite robust. Seemingly, it does not require the SCP and it remains true when moral hazard or multidimensional screening are introduced (see Chiappori and Chassagnon (1997) and Villeneuve (1996), for instance). Indeed, Chiappori *et al.* (2001) show that in competitive insurance markets *ex-post* positive correlation is extremely general for the case of two outcomes: loss/no loss.

Another recent and similar test is Dionne *et al.* (2001), which uses data from a large private insurer in Quebec. They show that when non-linearities of the

[2]Moreover, since the SCP does not hold, we have to adopt the Araujo and Moreira (2000) approach to solve the problem. In particular, the monotonicity is not the only necessary condition for implementability and new conditions are necessary (and sufficient) for either implementability or optimality.

[3]Since the SCP does not hold, there is no full separable equilibrium. We introduce the notion of quasi separable equilibrium, which involves low degree of pooling and high coverage. See section 5 for the details.

risk classification variables are taken into account, the conditional independence of coverage and riskiness cannot be rejected again. However, as we will argue next their findings may be compatible with asymmetric information and the failure of the SCP.

Our first main implication is that the equilibrium contract, in the monopolistic or competitive cases, might not be a monotonic function of risk aversion anymore, opening the possibility for *ex-ante* zero correlation between coverage and risk (even though the *ex-post* correlation is positive, which is in conformity with Chiappori *et al.* (2001) result).[4] Indeed, we provide a numerical example confirming this. In other words, the task of this paper is to present a robust framework with adverse selection and moral hazard where every sign of correlation between coverage and risk is consistent with asymmetric information. The second main implication is to propose empirical consequences for the existence of single crossing and the presence of asymmetric information in itself. More precisely, under *ex-post* zero correlation between coverage and riskiness, the monotonicity between coverage and riskiness can be tested as the null hypothesis for the absence of asymmetric information. If it is not rejected, then there is no evidence of asymmetric information because this is perfectly consistent with pure moral hazard equilibrium. Otherwise, the SCP does not hold and asymmetric information is present. The corollary of this reasoning is our third main and general result: if the SCP fails, another instrument is necessary for controlling the effect of the omitted variable on the observable ones (otherwise, the correlation between them may be bias).

In a preliminary analysis of the data, Dionne *et al.* (2001) provides a figure that shows the relation between risk classes and observable deductible choices (contract coverage), which we remake in section 5.1. The remarkable feature of this relation is the non-monotonicity under *ex-post* zero correlation between coverage and riskiness, which is completely compatible with asymmetric information and the failure of the SCP in our framework. In other words, the results of Dionne *et al.* (2001) can be interpreted as an evidence of non single crossing following the model developed in this paper. In the extensions and conclusions of the paper we also give two possible applications of our results for labor market models.

In sum, we build a hidden information model showing how omitted variables (asymmetric information) can bias the sign of the correlation of equilibrium variables conditioning on all observable variables. In one-dimensional screening problems with two instruments the SCP guarantees a monotonic relation of the omitted variable with the instruments. However, we argue that if the SCP is violated, then this relation may be not monotonic. The immediate consequence is that in former case no proxy for hidden information variable is needed, but in the last case absence of such proxy may bias the correlation results of the

[4]The *ex-ante* correlation means that we are calculating it with respect to the prior distribution of risk aversion in the population. In particular, in a non-monotonic equilibrium coverage, there may exist two levels of risk (low and high) associated with a given coverage. On the other hand, the *ex-ante* correlation means that for a given coverage we take the average risk with respect to the types that pools in such coverage.

instruments. Moreover, because this non-monotonic relation is deeply related with the failure of the SCP in one-dimensional screening problems, the existing literature on asymmetric information does not capture this feature. Therefore, our main result is to show the importance of the SCP in testing predictions of the hidden information models.

In the following section we present the monopolistic insurance model. The perfect correlation between adverse selection parameters is treated in Section 3. The fourth section extends the analysis to the competitive case. Section 5 presents our main zero correlation result, proposes the empirical test for the presence of single crossing and asymmetric information and provides the possible empirical evidence of non single crossing. The last section gives the extensions and concluding remarks.

2 The Monopolistic Insurance Market

We present an insurance model with moral hazard and adverse selection. We depart from the usual models by introducing two heterogeneities in order to link the *ex-ante* adverse selection with the *ex-post* moral hazard. More precisely, the insuree has constant absolute risk aversion θ and lenience (cost of precaution heterogeneity) η, which are adverse selection parameters.

Suppose that the wealth ω is normally distributed with mean e and variance σ^2. The variable e is the prevention effort controlled by the insuree.[5] The cost of prevention is a function of e and η: $C(e,\eta)$. We assume the standard properties for C:[6] $\partial_e C > 0$ and $\partial_{ee} C > 0$. The insuree's preference depends on the wealth and effort and it is represented by the following Von Neumann-Morgenstern utility function:

$$-\exp[-\theta\omega - C(e,\eta)].$$

The insurance company is risk-neutral.

The stages of the model are the following: (1) the insurance company chooses the menu of contracts (indexed by the insuree's parameters); (2) the insuree (self) selects his contract; (3) the insuree decides his level of prevention; (4) finally, the state of nature is realized and the contracts are enforced:

[5] In the context of Holmstrom and Milgrom (1987), this model can be thought as a continuous-time one in which the insuree's wealth is a diffusion process whose trend, e, he can control: $d\omega_t = edt + \sigma dW_t$, where $\{W_t\}_{t\in[0,1]}$ is a Brownian motion. A possible interpretation for this process is that its drift is a sufficient statistic for the account number of loss/no loss that the insuree has along the total period of insurance in a continuous time setup. More precisely, we can approximate this diffusion process by a binomial tree which has in each node two outcomes: loss and no loss. So, the drift is a good proxy for riskiness, which is the ex-post probability of accident in the model of two outcomes (see Schmidt (1992)). In this case, Holmstrom and Milgrom (1987) show that the optimal insurance contract for the whole period must give the agent a bonus that depends linearly on the number of periods in which the wealth increased, and ω (final wealth) is a sufficient statistics for this number.

[6] From now on we will use the following notation for derivatives: $\partial_e C$ is the partial derivative of C with respect to e and so on; $\dot{e}(\theta)$ is the derivative of e with respect to θ.

Contracts offered	Announcement of type	Choice of effort	State realized Contracts enforced
------- |	------- |	------- |	------- |
ex-ante	interim 1	interim 2	ex-post

In the following subsections we study the pure moral hazard and moral hazard with adverse selection cases. In the first case, the insurance company can verify the insuree's parameters and in the second it cannot.

2.1 Pure moral hazard

An important benchmark is when the insurance company can control for risk aversion and lenience, i.e., there is no adverse selection and the insurance problem is a pure moral hazard. This is also equivalent to solving the last two stages of the model.

From Holmstrom and Milgrom (1987) the optimal contract with moral hazard must only be linear: $\alpha\omega + \beta$, where α is called the power and β is the constant part. In terms of the certainty equivalent of the agent (insuree) and the principal (insurance company), their expected utilities in the space of linear contracts (α, β) can be equivalently represented by,[7] respectively,

$$
\begin{aligned}
\overline{v}(\alpha, \beta, e, \theta, \eta) &= \alpha e + \beta - C(e, \eta) - \frac{\sigma^2}{2}\theta\alpha^2 \\
\overline{u}(\alpha, \beta, e, \theta, \eta) &= e - [\alpha e + \beta] = e - C(e, \eta) - \frac{\sigma^2}{2}\theta\alpha^2 - \mathcal{V}^{(\alpha,\beta,e)}(\theta, \eta)
\end{aligned}
$$

where e is the prevention effort and $\mathcal{V}^{(\alpha,\beta,e)}(\theta, \eta) = \overline{v}(\alpha, \beta, e, \theta, \eta)$ is the informational rent of the insuree with type (θ, η) who chooses contract (α, β) and effort e.

There is a single insurance company solving the following program:

$$
\begin{gathered}
\max_{(\alpha,\beta,e)} \overline{u}(\alpha, \beta, e, \theta, \eta) \\
\text{s.t. } e \in \operatorname*{argmax}_{\widehat{e}} \overline{v}(\alpha, \beta, \widehat{e}, \theta, \eta) \quad (IC) \\
\overline{v}(\alpha, \beta, e, \theta, \eta) \geq \overline{v}(1, 0, e, \theta, \eta) \quad (IR)
\end{gathered}
$$

where the first constraint is the incentive compatibility of effort and the second is the participation constraint that guarantees at least the agent's opportunity cost (the utility in the null contract $\alpha = 1$ and $\beta = 0$).

[7] We are using the well known identity:

$$E[\exp(\omega)] = \exp[E(\omega) - .5V(\omega)]$$

where ω is normally distributed, E is the expectation and V the variance operators.

Given a contract (α, β), the optimal effort is determined by the first order condition of the IC constraint, i.e., the equalization of its marginal benefit and cost:

$$\alpha = \partial_e C(e, \eta) \qquad (1)$$

Let $e = e(\alpha, \eta)$ be the effort induced for a type (θ, η) by the contract (α, β), i.e., the implicit solution of equation (1) (observe that, by concavity, (1) is necessary and sufficient for optimality).

Taking into account equation (1), the insuree's indirect utility function

$$V(\alpha, \beta, \theta, \eta) = v(\alpha, \theta, \eta) + \beta$$

is quasi-linear, where $v(\alpha, \theta, \eta) = \alpha e(\alpha, \eta) - C(e(\alpha, \eta), \eta) - \frac{\sigma^2}{2}\theta\alpha^2$ and the insurance company's expected profit is then

$$U(\alpha, \beta, \theta, \eta) = e(\alpha, \eta) - C(e(\alpha, \eta), \eta) - \frac{\sigma^2}{2}\theta\alpha^2 - \mathcal{V}^{(\alpha,\beta)}(\theta, \eta)$$

i.e., it is the expected social surplus less the informational rent of the insuree (where we drop the dependence on e because it is now a direct function of (α, η)).

Therefore, the insurance company's program reduces to:

$$\begin{gathered} \max_{(\alpha,\beta)} U(\alpha, \beta, \theta, \eta) \\ \text{s.t. } V(\alpha, \beta, \theta, \eta) \ \geq w_0(\theta, \eta) \quad (IR) \end{gathered}$$

where

$$w_0(\theta, \eta) = V(1, 0, \theta, \eta)$$

is the opportunity cost of the agent with type (θ, η). This gives the following well-known characterization of the optimal contract (see Holmstrom and Milgrom (1987)):

$$\begin{aligned} \alpha(\theta, \eta) &= \partial_e C(e(\alpha, \eta), \eta) = \frac{1}{1 + \sigma^2\theta\partial_{ee}C(e(\alpha, \eta), \eta)} \\ \beta(\theta, \eta) &= w_0(\theta, \eta) - v(\alpha(\theta, \eta), \eta) \end{aligned}$$

where the last equation is the binding IR constraint, i.e., $\mathcal{V}^{(\alpha,\beta)}(\theta, \eta) \equiv w_0(\theta, \eta)$ (given the verifiability of the parameters).

In order to relate the decision variables of our model to the standard insurance ones, let us define the following variables that have a one to one correspondence to the decision variables previously defined:

Definition 1 *Given a contract (α, β) faced by an agent with coefficient of risk aversion θ and induced prevention effort $e(\alpha, \eta)$, we define*

$$\begin{aligned} co &:= 1 - \alpha \\ \pi &:= 1 - e(\alpha, \eta) \\ p &:= w_0 - \beta \end{aligned}$$

as his coverage, riskiness, lenience η and premium in (α, β), respectively.

Observe that *co* measures the standard deviation of the proportion of wealth to which the insuree is not exposed (or equivalently, the one that the insurance company faces):

$$(Var[\alpha\omega + \beta - \omega])^{1/2} = 1 - \alpha = co$$

In Rothschild and Stiglitz (1976), risk is the probability of the bad state of nature, which coincides with the consumer's type. In our case, the state space is infinite and the type is the insuree's risk aversion. However, in the Holmstrom and Milgrom (1987) framework (with continuous time) the optimal prevention effort, e, is constant over time (because of the non-wealth effect of the CARA utility functions) and, therefore, $\pi = 1 - e$ is a non-ambiguous proxy for riskiness and lenience. It is the reciprocal measure of precautious behavior, i.e., it is the (absolute[8]) decrease in effort of prevention due to adverse selection. The premium is the difference between the certainty equivalent of initial wealth and the reimbursement.

2.2 Moral hazard with adverse selection

Now the insurance company cannot verify the parameters of their insurees. Taking into account equation (1) again, the problem now reduces to a two-dimensional screening program.

A direct mechanism (or contract) is a pair of functions that maps the announcement of types to allocations: $(\alpha(\theta, \eta), \beta(\theta, \eta))_{(\theta, \eta)}$. From standard arguments from the literature of mechanism design,[9] the insurance company program is equivalent to maximizing its expected profit subject to the truth-telling constraint (incentive compatibility) and the participation constraint:

$$\begin{aligned} & \max_{(\alpha,\beta)} E\{U(\alpha(\cdot,\cdot), \beta(\cdot,\cdot), \cdot, \cdot)\} \\ \text{s.t. } & (\theta, \eta) \in \operatorname*{argmax}_{(\widehat{\theta}, \widehat{\eta})} V(\alpha(\widehat{\theta}, \widehat{\eta}), \beta(\widehat{\theta}, \widehat{\eta}), \theta, \eta) \quad (IC) \\ & V(\alpha(\theta, \eta), \beta(\theta, \eta), \theta, \eta) \ \geq w_0(\theta, \eta) \qquad (IR) \end{aligned}$$

where the expectation is taken with respect to the prior distribution of types.

This is usually a very demanding and difficult problem to treat in general (see Armstrong and Rochet (1999) for instance). In the next section we will make a simplifying assumption: that θ and η are perfectly correlated. This will allow us to reduce the screening dimension to one and to give a full characterization of the solution. Moreover, we will deal with two extreme cases: positive and negative correlation. As we shall see, these cases are very related with the

[8] One may also consider a relative measure like the decrease in effort compared to the first best situation.

[9] According to the Revelation Principle, we can restrict ourselves to the direct and truthful mechanisms.

validity or not of the SCP, which will be behind the monotonicity of optimal contracts with respect to the one-dimensional adverse selection parameter (say the risk aversion).

3 Perfect Correlation between Risk Aversion and Lenience

Assume that θ and η are random variables that are perfectly correlated, i.e., $\eta = a\theta + b$, where a and b are constants. This reduces the adverse selection to one parameter, say θ, with cumulative distribution F on $[1, 2]$ and density $f > 0$. Therefore, from now on we will drop the dependence through η and substitute θ directly in its place. In particular, we assume that the cost of prevention depends directly on θ.

Now a contract is a pair of functions $(\alpha, \beta) : [1, 2] \to \Re^2$ and the monopolistic insurance company solves the following program:

$$\begin{array}{l} \max\limits_{(\alpha,\beta)} \int_1^2 U(\alpha(\theta), \beta(\theta), \theta) f(\theta) d\theta \\ \text{s.t. } \mathcal{V}^{(\alpha,\beta)}(\theta) \geq \mathcal{V}^{(\alpha,\beta)}(\widehat{\theta}|\theta) \quad (IC) \\ \qquad \mathcal{V}^{(\alpha,\beta)}(\theta) \geq w_0(\theta) \quad (IR) \end{array}$$

where $\mathcal{V}^{(\alpha,\beta)}(\widehat{\theta}|\theta) = V(\alpha(\widehat{\theta}), \beta(\widehat{\theta}), \theta)$ and $\mathcal{V}^{(\alpha,\beta)}(\theta) = \mathcal{V}^{(\alpha,\beta)}(\theta|\theta)$ is the rent function of the type θ agent.

From equation (1) and the envelope theorem, the first order condition of the IC constraint is equivalent to

$$\frac{d}{d\theta}\mathcal{V}^{(\alpha,\beta)}(\theta) = \partial_\theta v(\alpha(\theta), \theta). \quad (2)$$

The next two subsections separate the analysis into two cases: the positive correlation (single crossing) one where the marginal cost of prevention (lenience) is increasing with risk aversion and the negative correlation (non-single crossing) one where the relation is reversed. In this last case, we focus on a particular situation: the degree prevention cost with homogeneity of degree zero.

3.1 The positive correlation case

In this case, more risk averse agents have higher marginal prevention cost. The immediate consequence is that this agent will buy more insurance in equilibrium. Formally:

Assumption A1: $\partial_{\theta e} C(e, \theta) > 0$.

We have the following immediate consequence:

Proposition 2 *Assume that A1 holds. A contract $(\alpha(\theta), \beta(\theta))$ satisfies the IC constraint (i.e., it is implementable) if and only if it satisfies equation (2) and $\alpha(\cdot)$ is non-increasing.*

The proof is in Appendix A. The intuition for this result is that the marginal rate of substitution between coverage and premium is non-decreasing in risk aversion ($\partial_{\alpha\theta}v(\alpha,\theta) < 0$), which implies that the more risk averse agent gets a lower powered incentive scheme, i.e., higher coverage. In other words, there is a positive correlation between coverage and risk aversion. Moreover, using (1), the optimal prevention $e(\cdot)$ for a given implementable power $\alpha(\cdot)$ satisfies

$$\dot{e}(\theta) = \frac{\dot{\alpha}(\theta) - \partial_{\theta e}C(e,\theta)}{\partial_{ee}C(e,\theta)}$$

and then $e(\cdot)$ is also non-increasing. Thus, proposition 2 gives the well known Rothschild and Stiglitz MP: positive correlation between coverage (co) and riskiness (π). In the next subsection, we will analyze the case where MP may not hold.

3.2 The negative correlation case

Assume that the disutility of the precaution effort is homogeneous of degree 0 in (e,θ):

Assumption A2: $C(e,\theta) = c\left(\frac{e}{\theta}\right)$.

Assumption A2 means that there exists a negative correlation between risk aversion and lenience. In particular, an ex-ante high (low) risk averse agent will be more (less) diligent ex-post, decreasing (increasing) his ex-ante marginal utility for high coverage. As a consequence, we will see that for some cases the adverse selection reduced form of this model does not have the SCP, which may imply that the optimal insurance contract is non-monotonic.

Araujo and Moreira (2000) develop a framework where it is possible to characterize the optimal contracts in situations like these. We will apply this approach here to characterize the optimal insurance contract. The main consequence is the existence of a framework where zero correlation and binding asymmetric information are compatible.

First let us characterize the optimal contract with pure moral hazard (the proof is in Appendix A).

Proposition 3 *Suppose that A2 holds and $c' > 0$, $c'' > 0$ and $c''' \geq 0$. Then the optimal coverage and riskiness in the pure moral hazard case are decreasing functions of risk aversion.*

This proposition is a comparative statics and means that the marginal cost of prevention effect dominates risk aversion in the risk sharing rule. Hence, the efficient risk sharing in the pure moral hazard case under assumption A2 is to induce more prevention effort the higher the risk aversion is (since prevention is less costly for these types) by giving more power (or less coverage). This leads to a positive correlation between coverage and riskiness.

Let us move to the case of moral hazard with adverse selection. An important issue here is the validity of the SCP, which means the constant sign of $\partial_{\alpha\theta}v(\alpha,\theta)$. Using standard arguments (see Guesnerie and Laffont (1984)), the second order condition of the IC constraint implies that α is non-decreasing (increasing) when $\partial_{\alpha\theta} v(\alpha, \theta) > (<) \ 0$. When this sign does not change, the SCP holds and the first and second order conditions of the IC constraint are sufficient for implementability (see proposition 2).

In the following subsection we present the example of quadratic cost of prevention with uniform distribution of risk aversion in order to characterize the optimal contract explicitly.

3.3 Quadratic cost of prevention

Assume A2 with quadratic cost of prevention:

$$c(e) = \frac{e^2}{2}.$$

Given a contract (α, β), the optimal prevention effort is $e(\alpha, \theta) = \theta^2 \alpha$ and the pure moral hazard prevention power is:

$$\alpha^{pm}(\theta) = \frac{1}{1 + \sigma^2/\theta}.$$

Under moral hazard and adverse selection the marginal insuree's rent with respect to power and risk aversion is

$$\partial_{\alpha\theta} v(\alpha, \theta) = 2\theta\alpha - \sigma^2 \alpha$$

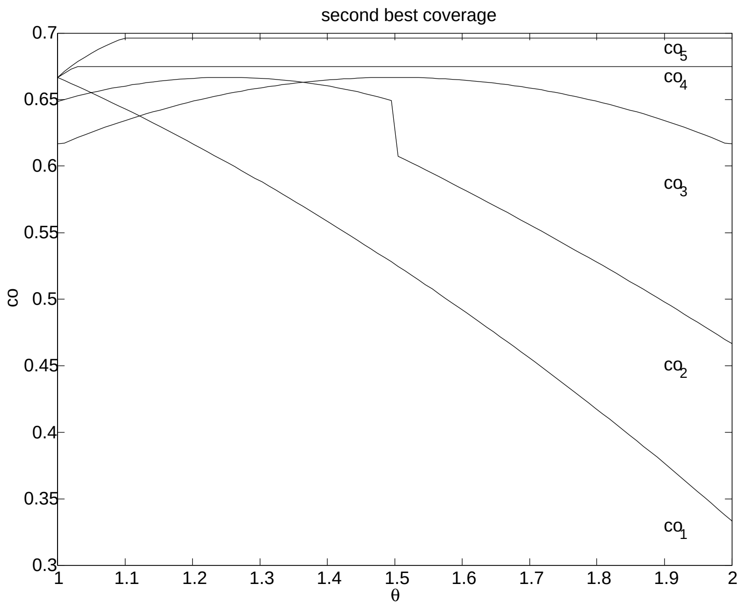
(see Appendix A). The first term, $2\theta\alpha$, is the marginal benefit in reducing coverage and raising risk aversion (due to A2) and the second term, $\sigma^2\alpha$, is the marginal cost of risk premium. Depending on what term predominates, the sign may be positive or negative. Formally, $\partial_{\alpha\theta} v(\alpha, \theta) \gtrless 0$ if and only if $\theta \gtrless \frac{\sigma^2}{2}$. Thus there are two extreme situations where the SCP holds: $\sigma^2 \leq 2$ or $\sigma^2 \geq 4$.

If the variance is low ($\sigma^2 \leq 2$), the implementable coverages are non-increasing functions of risk aversion. The intuition is that the observation of ω is very informative about the prevention effort taken by the insuree for low variances. The insurance contract reflects more the screening aspect of the incentive scheme, which is driven by the fact that the more risk averse type has lower marginal cost of prevention (assumption A2). In Appendix B we show that the insuree's rent function is decreasing in θ and the IR constraint is binding for an interval of high risk averse types (possibly degenerated).

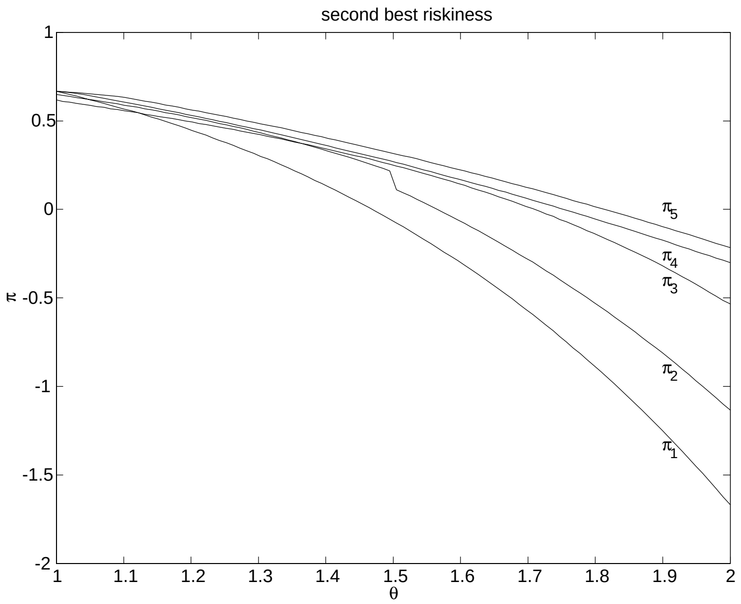
If the variance is high ($\sigma^2 \geq 4$), the implementable coverages are non-decreasing functions of θ. The intuition is the same as in the single crossing case because the cost of the risk premium increases with the variance and, for high variance levels, the total marginal cost of risk premium increases with risk aversion. Therefore, when the variance is high, ω is less informative about the insuree's prevention effort, also making the screening costly (which, in particular, implies much pooling). In this case the insuree's rent is non-decreasing with θ. In both cases, the coverage is increasing with σ^2.

For intermediate values of variance, $\frac{\sigma^2}{2} \in (1,2)$, the SCP does not hold, and then an implementable coverage still has to satisfy the up and down stream incentives: it must be non-decreasing (respectively increasing) for $\theta \in [1, \frac{\sigma^2}{2}]$ (respectively $\theta \in [\frac{\sigma^2}{2}, 2]$). However, there is also an extra necessary condition for implementability: if two types, θ and $\widehat{\theta}$, choose the same coverage $co = 1-\alpha$, then $\partial_\alpha v(\alpha,\theta) = \partial_\alpha v(\alpha,\widehat{\theta})$ or $\widehat{\theta} = \sigma^2 - \theta$.[10] We interpret this condition as a cross stream incentive because if it was not satisfied, a low (high) risk averse type would mimic a high (low) type by slightly moving his coverage and prevention.

In Appendix B we give a complete analysis of the optimal contract. In particular, low and high risk types choose the same coverage (pooling) allowing zero correlation. The following two figures give the optimal coverage $(co_1, ..., co_5)$ and riskiness $(\pi_1, ..., \pi_5)$ for $\sigma^2 = 2$, 2.5, 3, 3.5 and 4, respectively.



[10] Observe that $\widehat{\theta}$ is not a function of α, which greatly simplifies the analysis. For the more general case, the regions of positive and negative sign of $\partial_{\alpha\theta} v$ are separated by the curve $\partial_{\alpha\theta} v = 0$. See Araujo and Moreira (2000) for more details.



4 The Competitive Insurance Market

Jullien *et al.* (2000) also showed that under monopoly provision of insurance, the zero correlation result is possible. However, in their paper the prevention effort is not monotonic in risk aversion and in ours prevention effort always increases with risk aversion.[11] We showed that if there is a negative correlation between risk aversion and lenience, coverage may not be monotonic in risk aversion, even though riskiness is decreasing in risk aversion.[12] However, all these models predict positive correlation between coverage and risk aversion and there are two types (the exception is Chiappori *et al.* (2001)) and two outcomes models.

Now we extend the analysis to the competitive insurance markets. There are $I > 1$ insurance companies and each proposes a contract $(\alpha(\theta), \beta(\theta))$ that specifies a power (or coverage $co(\theta)$) and a fixed reimbursement (or a premium $p(\theta)$) for each report θ by the insuree. The incentive compatibility and participation constraints remain the same. Moreover, given an equilibrium contract $(\alpha(\theta), \beta(\theta))$, the optimal prevention response of the agent with type θ is also the same: $e(\theta|\theta)$, where

$$e(\widehat{\theta}|\theta) = e(\alpha(\widehat{\theta}), \theta)$$

[11] In a previous paper, Jullien *et al.* (1999) claim that this monotonicity is the general case.

[12] De Meza and Webb (2000) is rather in the same spirit as ours: the low risk averse agent is risk neutral, he does not make any effort, and more risk-averse agents make more effort. The model is competitive, but there is a public policy that imposes an administrative cost of insurance. Moreover, because of non-monetary cost of effort, the SCP does not hold, which may imply pooling equilibrium and support zero correlation between coverage and risk.

is the prevention effort of θ when he announces to be type $\widehat{\theta}$. The main difference from monopoly is that we impose the actuarially fair or balancedness constraint on the equilibrium contract because of the competitiveness between insurance firms. To support this actuarially fair constraint we will adopt the perfect Bayesian equilibrium concept (signaling game):

Definition 4 *A perfect Bayesian equilibrium (PBE) for the insurance model is a profile of strategies* $\{c(\theta) = (\alpha(\theta), \beta(\theta)), e(\cdot|\theta))\}_{\theta\in[1,2]}$ *and ex-post beliefs* $\mu(\cdot|c)$ *such that the following conditions are satisfied:*

1. Zero expected profit or actuarially fair constraint (best response of the insurance companies):

$$\beta(\theta) = (1 - \alpha(\theta)) \int e(\theta|\widetilde{\theta}) d\mu(\widetilde{\theta}|c(\theta))$$

2. Maximization of the expected total surplus (best response of the insurees):

$$\begin{aligned} \theta &\in arg \max_{\widehat{\theta}\in[1,2]} \beta(\widehat{\theta}) + v(\alpha(\widehat{\theta}), \theta) \\ s.t. &: \quad \beta(\theta) + v(\alpha(\theta), \theta) \geq w_0(\theta) \end{aligned}$$

3. Consistency of beliefs: $\mu(\theta|c)$ *is the Bayesian updating given the best responses 1. and 2., i.e., it is the probability a posteriori of* θ *given* c.

In Appendix C we characterize possible features of the equilibrium contract: separating and continuous or discrete pooling. For an illustration, let us analyze the case of assumption A2 with quadratic cost and uniform distribution. The separating part of the equilibrium is characterized by the following ordinary differential equation (ODE), which is equivalent to the first order condition of the problem in 2.:

$$\frac{\dot{\alpha}}{\alpha} = \frac{2}{h(\alpha, \theta)}$$

where $h(\alpha, \theta) = \frac{\sigma^2 \alpha}{1-\alpha} - \theta$.

Observe that the curve implicitly defined by $h(\alpha, \theta) = 0$ is precisely the pure moral hazard power α^{pm} and the region in the space $\theta \times \alpha$ where $h(\alpha, \theta) > (<)$ 0 is given by points (θ, α) above (below) the curve α^{pm}. If the variance is low (high), i.e., $\sigma^2 \leq 2$ (≥ 4), the SCP holds and this means that the separating equilibrium is implementable if and only if its power is non-decreasing (increasing). Therefore, the solution of the ODE will be a separating equilibrium whether its initial (final) condition at type 1 (2) is above (below) $\alpha^{pm}(1)$ $(\alpha^{pm}(2))$. Moreover, the restriction in the maximization problem in 2. is the IR constraint. By Appendix B, it is easy to see that for the quadratic cost it is only necessary to check at extreme values 1 or 2, which is satisfied for every initial (or final) coverage value in the interval $[0, 1]$.

For intermediate variances $(2 < \sigma^2 < 4)$, there is no full separation because the SCP does not hold. However, the necessary conditions for incentive compatibility derived in the monopolistic case are also valid here: α should be non-decreasing (increasing) in the region $\partial_{\alpha\theta}v > (<)\ 0$ and if θ and $\widehat{\theta}$ are pooling in the same contract α (where it is not flat), then $\partial_{\alpha}v(\alpha,\theta) = \partial_{\alpha}v(\alpha,\widehat{\theta})$. In the case of assumption A2 with quadratic cost this implies that $\widehat{\theta} = \sigma^2 - \theta$ and the posteriori Bayesian updating for θ given α is $\lambda = 1/2$. Thus, the first order condition in problem 2. gives the following ODE, which characterizes the discrete pooling part:

$$\frac{\dot{\alpha}}{\alpha} = \frac{2}{\sigma^2 - \theta + \frac{\sigma^2}{2\theta-\sigma^2}h(\alpha,\theta)}.$$

Since the equilibrium must be U-shaped (for the reasons just explained above), analyzing the sign of this ODE, $\alpha(\sigma^2/2)$ must be greater than $\alpha^{pm}(\sigma^2/2) = 1/3$. Conversely, if α is only U-shaped or U-shaped plus a monotonic part (non-increasing if it is the first part or non-decreasing if it is the last part) satisfying both ODEs in each case, then α will be an equilibrium.[13] However, for high intermediate variances ($3 \leq \sigma^2 \leq 4$), there is no such equilibrium since the power would be increasing for low risk aversion because the separating part of the equilibrium would be in the region above the curve α^{pm}, i.e., the region where incentives are overpowered with respect to the pure moral hazard, which would violate the incentive compatibility. For these cases we can only have equilibria with high degrees of pooling (continuous pooling).

Since the concept of PBE leads to an indeterminacy of equilibria, the important issue now is their selection. Wilson (1977), Rothschild and Stiglitz (1979) and Spence (1979) have started a debate about the equilibrium concept in the context of adverse selection. Riley (1979) proposed a concept of reactive equilibrium that rules out all but a single equilibrium: the separating equilibrium, in the continuous type setup. Although it is an ad hoc definition, it has been mostly used in the literature. In the next subsection we provide a criterion that selects a single equilibrium: the quasi separable one, since total separability is not possible when the SCP does not hold. This concept tries to generalize the same features of the reactive equilibrium: separability and Pareto optimality.

4.1 Refinement of equilibria

The next definition provides the selection criterion that we will use. It is based on two principles: the highest degree of separation and coverage.[14] More precisely:

[13] A proof of this fact can be adapted from Araujo and Moreira (2000). Observe that it is not a straightforward result because we have to show that the necessary local second order conditions (monotonicity) and the equalization of the marginal rate of substitution between coverage and premium across pooling types guaranteed by the Bayesian updating are sufficient conditions for incentive compatibility.

[14] One interesting paper that applies this definition of reactive equilibrium is Cresta and Laffont (1987). They discuss the value of information in the classical setup of the Riley (1979) model. The definition that follows is very much inspired in their work

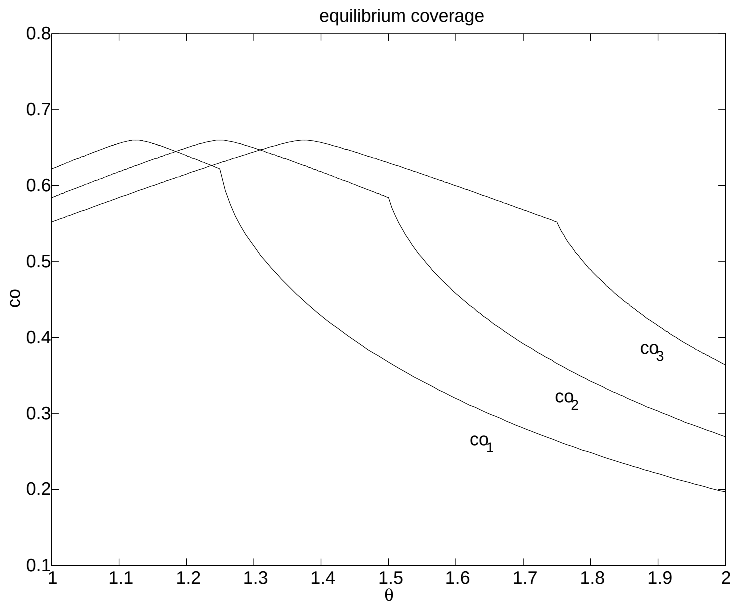
Definition 5 *A PBE is quasi separable if the following conditions are satisfied:*

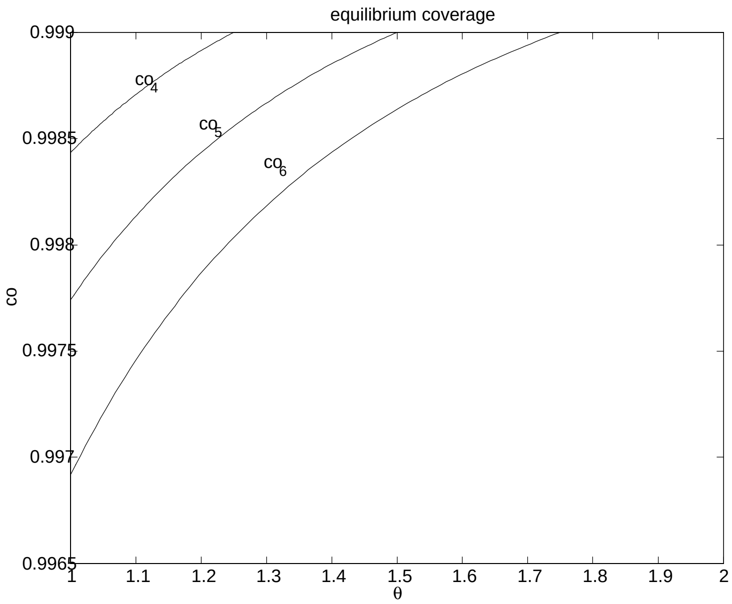
1. Given a type in a pooling part, there is a different one such that pools with him and their marginal rate of substitution between coverage and premium are the same.

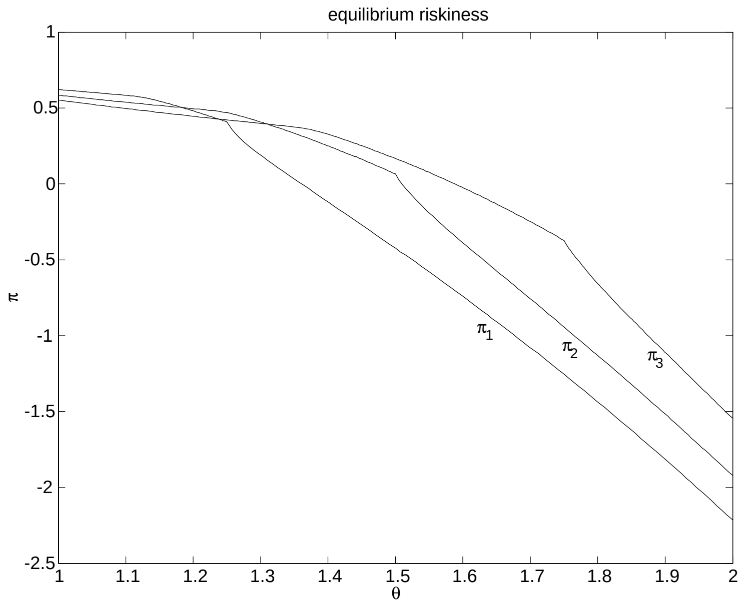
2. There is no other PBE satisfying condition 1. giving no less coverage for every type and with at least one type getting higher coverage.

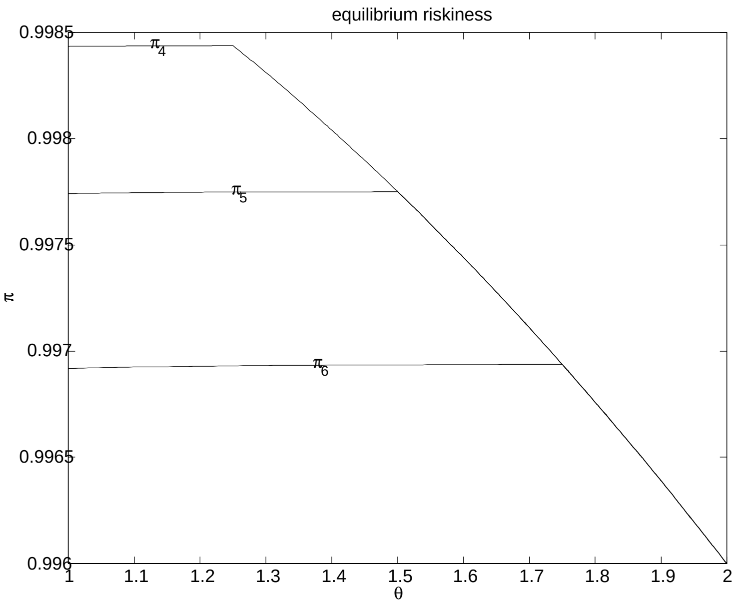
The first condition is intuitive and says that if there is pooling, then the marginal rate of substitution between coverage and premium must be equalized pairwise. This property has to do with the highest degree of separability. In particular, only separable or discrete pooling parts or continuous pooling that is a degenerated U-shaped part are possible. The second property gives the boundary condition and determines uniquely the equilibrium. It gives a Pareto improvement criterion of selection: coverage.

Under assumption A2 with quadratic cost and uniform distribution, the following figures show the quasi separable equilibrium coverage $(co_1, ..., co_6)$ and riskiness $(\pi_1, ..., \pi_6)$ for the $\sigma^2 = 2.25$, 2.5, 2.75, 3.25, 3.5 and 3.75, respectively.




equilibrium coverage
co
θ
co4
co5
co6
0.999
0.9985
0.998
0.9975
0.997
0.9965
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


equilibrium riskiness
π
θ
π1
π2
π3
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2



For $0 < \sigma^2 \leq 2$, the SCP holds and the quasi equilibrium is the full separating one with the highest coverage, i.e., with initial condition $co(1) = 1 - \alpha^{pm}(1)$. For $2 < \sigma^2 < 3$, the equilibrium consists of two parts: a discrete pooling part in the interval $[1, \sigma^2 - 1]$ plus a separating part in the interval $[\sigma^2 - 1, 2]$ without discontinuity and with the highest coverage: $co(\sigma^2/2) = 1 - \alpha^{pm}(\sigma^2/2) = 2/3$. (See the discussion after definition 4).

For $3 \leq \sigma^2 < 4$, only low risk averse types are screening out, i.e., it is separating in the interval $[1, \sigma^2 - 2]$ with finial condition $co(\sigma^2 - 2) = 1$ and constant equal to 1 in the interval $[\sigma^2 - 2, 2]$. Finally, for $\sigma^2 \geq 4$, again the SCP holds and the quasi equilibrium is separable with final condition $co(2) = 1$. These are the Pareto dominating members of the class of equilibria and they are the ones with the largest possible initial conditions.[15]

Observe that we assumed uniform distribution in the interval $(1, 2)$. However, we can deal with other kinds of distributions. For instance, consider the following family of power probability distribution for risk aversion:

$$f_n(\theta) \cong (\overline{\theta} - \theta)^n$$

where n is non negative real number and f_n is a density function with support in an interval $[\underline{\theta}, \overline{\theta}]$. The interesting feature of these distributions is that when n is large they put more weight in the low risk averse types, which reinforces the negative relation between coverage and riskiness.[16]

[15] Observe that the ODEs are not Lipschitizian at those initial conditions. However, by continuity, the quasi separable equilibrium can be defined as the limit of a sequence of equilibria with initial conditions that converge to the respective one.

[16] This will be important for strengthening the results that we will present in the next section. For instance, in Appendix C we build an example using this family of distributions where only non-degenerated discrete pooling is present.

5 Testable Implications

In this section we present our main results. They provide the links between the validity or not of the SCP and the positive correlation or not between coverage and riskiness. Also, they provide a testable consequence of our model that allows us to disentangle the absence of adverse selection from non single crossing under the zero correlation result. First we need the following:

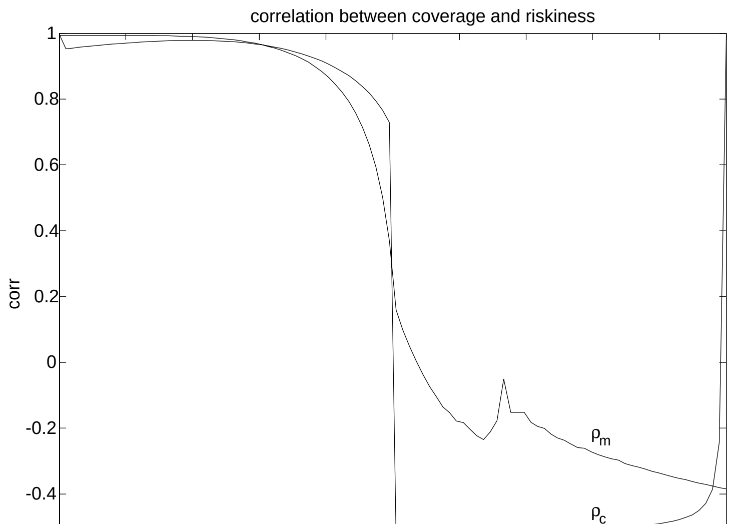
Definition 6 *The (ex-ante) correlation between coverage and risk is defined by*

$$\rho = corr\,(co(\cdot), \pi(\cdot))$$

where the "corr" means the correlation with respect to the distribution of θ.

Proposition 7 *Assume A2 with quadratic cost. If risk aversion is uniformly distributed, then in the monopoly or competitive markets ρ assumes negative and positive values, depending on σ^2.*

Proof. *The proof is an immediate consequence of the following figure. It gives the computation of ρ as a function of the variance in the interval $(2, 4)$ for the monopolistic and competitive cases, where the label m (c) means monopoly (competition).* ■



This proposition is our first main result. In the presence of moral hazard and adverse selection, if there is a negative correlation between risk aversion and lenience, the SCP might not hold, leading to (possibly) *ex-ante* zero correlation between coverage and riskiness. From this perspective, the zero correlation result is consistent with asymmetric information and non single crossing. In

Appendix C we give another example for the competitive case involving only non-degenerate discrete pooling using the power distribution family presented before (see the end of the last section).

Under monopoly provision of insurance, our result is similar to Jullien *et al.* (2000). However, since coverage is always a monotonic function of risk aversion, the only way to obtain every sign of the correlation between riskiness and coverage is when riskiness is not a monotonic function of risk aversion.

It is also important to observe that even though in the competitive case every sign of the correlation between coverage and riskiness may be possible, there is no contradiction with Chiappori *et al.* (2001). The reason is that our concept of correlation is *ex-ante* and their is *ex-post* (see footnote 4). Indeed, in conformity with them, if we compute the *ex-post* correlation, we will obtain a positive value.

The following proposition gives the testable implication of our model and it is straightforward from the previous discussion. Essentially it says that asymmetric information and zero correlation between coverage and riskiness are compatible if coverage is a non-monotonic function of riskiness.

Proposition 8 *Assume negative correlation between risk aversion and lenience (i.e., A2).*

(i) Under symmetric information, there is a positive relation between coverage and riskiness and their ex-post correlation must be zero.

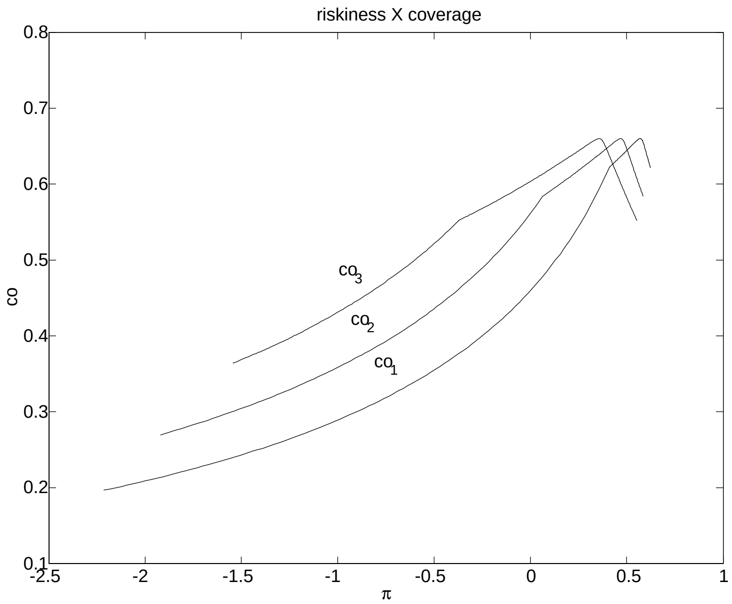
(ii) Under asymmetric information, if riskiness is decreasing with risk aversion and coverage is a non monotonic function of riskiness, then the SCP does not hold. In this case all signs of the ex-ante correlation between coverage and riskiness may be possible.

(iii) Under ex-post zero correlation between coverage and riskiness, there exist two possibilities: no binding asymmetric information if coverage is monotonic in riskiness or asymmetric information consistent with non single crossing if coverage is non monotonic in riskiness.

An equivalent way to interpret this proposition is that if there is symmetric information about risk aversion, then the equilibrium is the pure moral hazard one, which leads to an increasing relation between coverage and riskiness (positive correlation) - see proposition 3. Moreover, controlling for all observable variables, the *ex-post* correlation between coverage and riskiness must be zero. Also, under asymmetric information, it gives a necessary condition for single crossing: coverage must be a monotonic function of riskiness. Finally, under *ex-post* zero correlation between coverage and riskiness, the monotonicity of the relation between coverage and riskiness gives a test for asymmetric information.

Remark 9 *An immediate and important consequence of the previous proposition is that if the SCP is violated, another instrument may be necessary for controlling the effect of omitted variables on observable ones.*

For instance, under A2 with quadratic cost and uniform distribution for risk aversion, the equilibrium relationship between riskiness for $\sigma^2 = 2.25$, 2.5 and 2.75 (co_1, co_2 and co_3 respectively) is given in the following figure.

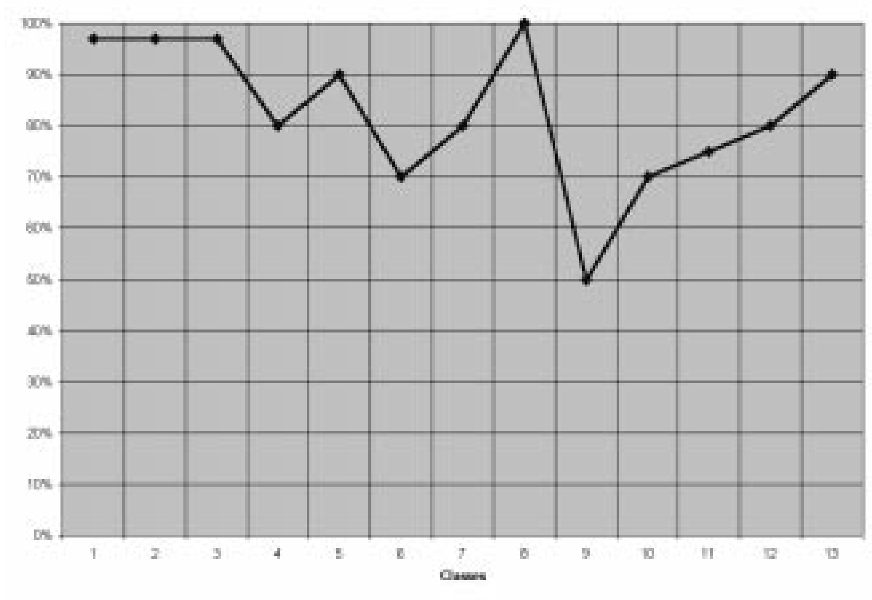


Remark 10 *We would like to point out that our equilibrium concept depends on a particular selection criterion that could be disputable. However, in the single crossing case all equilibrium selections lead to MP. In the non single crossing case, depending on the selection criterion chosen, one could have MP or not even though riskiness is always monotone. This is enough to guarantee that our test is not dependent on the selection criterion.*

5.1 Empirical Evidence

As Chiappori and Salanié (2000), Dionne *et al.* (2001) provide a test for MP using other data set. The basic idea was to take into account the nonlinearity of the risk classification variables to avoid spurious conclusions. The figure below is an adaptation of the one presented in Dionne *et al.* (2001) and it gives the relation between observed deductible choices according to classes of risk. More precisely, for each group of risk (1-13), it gives the percentage of people that prefers a $250 deductible (high coverage) instead of a $500 deductible (low coverage).

As we can see in the figure, this relation is non-monotone under zero correlation (or conditional independence) between coverage and riskiness. Therefore, using the model developed in this paper, this evidence of non-monotonicity is in fact related with asymmetric information with non single crossing instead of no residual adverse selection on risk types.



6 Concluding Remarks and Extensions

In this paper we have focused on insurance markets. However, the model developed here is naturally applicable to labor markets. Below we provide two applications of these ideas in labor market situation.

It is straightforward to adapt the above results to a labor market situation. The main difference is that the reservation utility of the employee can be considered constant since the random variable ω in this case is the return for the principal. More interestingly, if we interchange the roles of θ and σ^2, we are able to explain another stylized fact. More explicitly, suppose now that risk (σ^2) is the adverse selection parameter, θ is a known fixed parameter and the lenience (cost of effort) is negatively correlated with risk, i.e., industries with high risk are associated with diligent agents. Thus, for some intermediate levels of risk aversion, there exist equilibria in which low variance industries provide the same power as high variance ones. In particular, the correlation between incentive and risk may assume positive and negative values. For results in the same direction, see Prendergast (2000).

Heckman and Rubinstein (2001) gives an empirical evidence of the possibility of a non-monotonic relation between wage and cognitive ability in a data base of recipients of a specific educational exam. They argue that this exam is a mixed signal of cognitive ability and another (non-cognitive) ability in such way that if there is no control for the cognitive ability, the measure of workers' performance is biased. Araujo and Moreira (2000) provides an example in the monopolistic case that is compatible with their findings. In particular, for some schooling signals, cognitive and non-cognitive abilities are in conflict in the worker's utility function such that the SCP does not hold and, consequently, U-shaped wage contracts (with respect to the cognitive ability) may arise. In the sequel of this paper we will build on this framework to provide a multi-signalling model without the SCP, which will capture all important aspects of this evidence.

In sum, this paper has provided a theoretical insurance model where *ex-post* zero correlation between coverage and riskiness is consistent with asymmetric information. What drives the positive correlation is the SCP. When it is broken, there is a possibility of observing a non monotonic relationship between coverage and risk aversion and, consequently, between coverage and riskiness when riskiness is decreasing with risk aversion. We have presented a model with negative correlation between risk aversion and lenience which leads to non single crossing. Moreover, we have provided a testable implication of this model to disentangle single crossing and non single crossing under a zero correlation result: the monotonicity of coverage as a function of riskiness. And, as an important corollary, we showed the necessity of another instrument for controlling the effect of omitted variable when the SCP does not hold. We also have shown that this test can be used as a refinement to check the validity of asymmetric information and also of the SCP in insurance markets. Indeed, Dionne *et al.* (2001) could be used as potential evidence of the importance of this type of refinement test.

7 Appendix

7.1 Appendix A

Using the Envelope Theorem, the derivatives of v are (in general and under A2):

$$\begin{aligned}
\partial_\alpha v(\alpha,\theta) &= e - \theta\sigma^2\alpha \\
\partial_\theta v(\alpha,\theta) &= -\partial_\theta C(e,\theta) - \frac{\sigma^2}{2}\alpha^2 = \alpha\epsilon - \frac{\sigma^2}{2}\alpha^2 \\
\partial_{\alpha\theta} v(\alpha,\theta) &= \partial_\theta e(\alpha,\theta) - \sigma^2\alpha = \frac{c'(\epsilon)}{c''(\epsilon)} + \epsilon - \sigma^2\alpha
\end{aligned}$$

where $\epsilon = e/\theta$, $e = e(\alpha,\theta)$,

$$\begin{aligned}
\alpha &= \partial_e C(e,\theta) = \frac{c'(\epsilon)}{\theta}, \\
\partial_\theta C(e,\theta) &= -\frac{c'(\epsilon)}{\theta}\epsilon = -\partial_e C(e,\theta)\epsilon = -\alpha\epsilon
\end{aligned}$$

and, using the Implicit Function Theorem,

$$\begin{aligned}
\partial_\theta e(\alpha,\theta) &= -\frac{\partial_{\theta e} C(e,\theta)}{\partial_{ee} C(e,\theta)} = \frac{c'(\epsilon)}{c''(\epsilon)} + \epsilon, \\
\partial_\alpha e(\alpha,\theta) &= \frac{1}{\partial_{ee} C(e,\theta)} = \frac{\theta^2}{c''(\epsilon)}
\end{aligned}$$

since

$$\begin{aligned} \partial_{\theta e} C(e,\theta) &= -\frac{1}{\theta^2}\left(c'(\epsilon) + c''(\epsilon)\epsilon\right), \\ \partial_{ee} C(e,\theta) &= \frac{c''(\epsilon)}{\theta^2}. \end{aligned}$$

7.1.1 Proof of Proposition 2:

The cross derivative of v is calculated and it is:

$$\partial_{\alpha\theta} v(\alpha,\theta) = -\frac{\partial_{\theta e} C(e,\theta)}{\partial_{ee} C(e,\theta)} - \sigma^2 \alpha.$$

Then assumption A1 implies that this derivative is always negative, i.e., the SCP holds. Using standard arguments (see Guesnerie and Laffont (1984)), the proposition is equivalent to the first and second order conditions of the IC constraints being necessary and sufficient conditions for implementability. In this case, the first order condition is given by (2) and the second by the monotonicity of $\alpha(\cdot)$.

7.1.2 Proof of Proposition 3

The first order condition of the pure moral hazard problem can be written in the following way:

$$c'(\epsilon) = \frac{\theta}{1 + \frac{\sigma^2}{\theta} c''(\epsilon)}$$

where $\epsilon = e/\theta$ is the prevention effort in units of risk aversion. Observe that c' is increasing and the right hand side of the last equation is a non-increasing function of ϵ and an increasing function of θ. This implies that the implicit solution of this equation, $\epsilon(\theta)$, is an increasing function of θ and, therefore, $e(\theta) = \theta\epsilon(\theta)$ and $\alpha(\theta) = c'(\epsilon(\theta))$ are also increasing in θ.

7.2 Appendix B

The (IR) constraint is equivalent to $r(\theta) = \mathcal{V}^{(\alpha,\beta)}(\theta) - w_0(\theta) \geq 0$, for all $\theta \in [1,2]$. Taking the derivative with respect to θ and using (2) we have:

$$\begin{aligned} \dot{r}(\theta) &= \frac{\alpha(\theta) e(\alpha(\theta),\theta)}{\theta} - \frac{\sigma^2}{2}\alpha(\theta)^2 - \frac{e(1,\theta)}{\theta} + \frac{\sigma^2}{2} \\ &= (\alpha(\theta)\psi(\alpha(\theta)\theta) - \psi(\theta)) + (1-\alpha(\theta)^2)\frac{\sigma^2}{2} \end{aligned}$$

where ψ is the inverse of c' and $e(\alpha,\theta) = \theta\psi(\theta\alpha)$. If ψ is concave (or c' is convex) and $\psi(0) = 0$, then $\dot{r}(\theta) \geq \left(\frac{\sigma^2}{2} - \psi(\theta)\right)(1 - \alpha(\theta)^2)$.

In the case of A2 with quadratic cost we have that $\psi(x) = x$ and

$$\dot{r}(\theta) = \left(\frac{\sigma^2}{2} - \theta\right)(1 - \alpha(\theta)^2).$$

Therefore, if there is no overinsurance ($\alpha \leq 1$), $\dot{r}$ changes its sign at most one time, and then it is only necessary to check the IR constraint at one of the boundary values: 1 or 2.

If $\sigma^2 < 2$, one can also use the technique of Jullien (2000) here to conclude that the power is capped by 1. Otherwise, there exists $\theta^* \in (1, 2)$ such that the IR constraint is binding $r(\theta^*) = 0$, i.e., θ^* is a minimum for r which implies $\dot{r}(\theta^*) = 0$ or $\alpha(\theta^*) = 1$. Conditioning on $[1, \theta^*]$ and on $[\theta^*, 2]$, the first order condition of the respective relaxed programs gives a non-decreasing $\alpha(\cdot)$. However, if α is greater than 1, the expression of r would be negative, what is a contradiction. In this case, $\beta(\theta) = 0$, for all $\theta \in [\theta^*, 2]$ because $0 = r(\theta) = v(\alpha, \theta)|_{\alpha=1} + \beta(\theta) - v(1, \theta)$.

For $\sigma^2 \in [2, 4]$, from Araujo and Moreira (2000), α is implementable iff α is non-increasing on $[1, \sigma^2/2]$, non-decreasing on $[\sigma^2/2, 2]$, and if θ and $\widehat{\theta}$ are pooling (and α is not flat at these points), then θ and $\widehat{\theta}$ are symmetric with respect to $\sigma^2/2$. Therefore, the rent function $r(\cdot)$ is bell shaped and $r(1) = 0$ if and only if $\alpha(1) \geq \alpha(2)$.

Equation (2) and standard arguments in the literature of mechanism design imply that the monopolist's virtual profit function[17] is only dependent on α and θ:

$$\Pi^i(\alpha, \theta) = e(\alpha, \theta) - C(e(\alpha, \theta), \theta) - \frac{\sigma^2}{2}\theta\alpha^2 + R^i(\theta)\left(\partial_\theta C(e(\alpha, \theta), \theta) + \frac{\sigma^2}{2}\alpha^2\right) - \mathcal{V}^{(\alpha,\beta)}(i)$$

where $R^i(\theta) = \begin{cases} \frac{1-F(\theta)}{f(\theta)} & \text{if } i = 1 \\ -\frac{F(\theta)}{f(\theta)} & \text{if } i = 2 \end{cases}$ is the hazard rate. Depending on the case, $\mathcal{V}^{(\alpha,\beta)}(1) = 0$ or $\mathcal{V}^{(\alpha,\beta)}(2) = 0$.[18]

First, let us characterize the relaxed solution, i.e., when we only take into consideration the first and second order conditions of the IC constraint: $\partial_\alpha \Pi^i(\alpha, \theta) = 0$. Thus,

$$\begin{aligned} \alpha(\theta) &= \partial_e C(e(\theta), \theta) = \frac{1 + R^i(\theta)\partial_{\theta e} C(e(\theta), \theta)}{1 + \sigma^2(\theta - R^i(\theta))\,\partial_{ee} C(e(\theta), \theta)} \\ \beta(\theta) &= \beta(1) + \int_1^\theta \partial_\theta v(\alpha(\widetilde{\theta}), \widetilde{\theta})d\widetilde{\theta} - v(\alpha(\theta), \theta) \end{aligned}$$

where $e(\theta) = e(\alpha(\theta), \theta)$ and the last equation is obtained by solving (2).

[17] As usual in the literature of contract theory, the word 'virtual' means that the profit is discounted by the informational rent.

[18] For the general case, the technique developed in Jullien (2000) for dealing with countervailing incentives should be applied.

Under the SCP, only the monotonicity of (α, β) (the local second order condition of the IC constraint) is a necessary and sufficient condition for implementability. If the SCP does not hold, Araujo and Moreira (2000)[19] show other necessary conditions for implementability of (α, β): if θ and $\widehat{\theta}$, choose the same coverage $1-\alpha$, then their marginal utility with respect to α should coincide, i.e., $\partial_\alpha v(\alpha, \theta) = \partial_\alpha v(\alpha, \widehat{\theta})$ or $e(\alpha, \theta) - \theta\sigma^2\alpha = e(\alpha, \widehat{\theta}) - \widehat{\theta}\sigma^2\alpha$.

Moreover, this new implementability condition gives a new necessary condition for optimality: the ratio between the marginal (w.r.t. α) profit and the marginal (w.r.t. α and θ) informational rent weighted by the density are equalized across pooling types:

$$\frac{\partial_\alpha \Pi(\alpha, \theta)}{\partial_{\alpha\theta} v(\alpha, \theta)} f = \frac{\partial_\alpha \Pi(\alpha, \widehat{\theta})}{\partial_{\alpha\theta} v(\alpha, \widehat{\theta})} \widehat{f} \quad (3)$$

where hat means that the function is calculated at $\widehat{\theta}$.

In some cases these necessary conditions for implementability are also sufficient and we can characterize the optimal contract. This is precisely what happens in the example of the paper.

Let us characterize the optimal contract for A2 with quadratic cost and uniform distribution of risk aversion. Following the analysis of Araujo and Moreira (2000), we have the following optimal contract.

- $\sigma^2 \leq 2$: the second best power is

  $$\alpha^2(\theta) = \min\left\{ \frac{1}{-1 + 2\frac{(1+\sigma^2)}{\theta} - \frac{\sigma^2}{\theta^2}}, 1 \right\}$$

  i.e., it is the relaxed associated to the hazard function R^2 (the label '2' means that the IR is binding at the highest type). The power is increasing in σ^2 and capped by 1.

- $\sigma^2 \geq 4$: the second best power is

  $$\alpha^1(\theta) = \begin{cases} \frac{1}{-1+2\frac{(2+\sigma^2)}{\theta} - 2\frac{\sigma^2}{\theta^2}}, & \text{if } 1 \leq \theta \leq \theta_0 \\ \bar{\alpha}, & \text{if } \theta_0 < \theta \leq 2 \end{cases}$$

  where $\int_{\theta_0}^{2} \left[\begin{array}{c} (1 - \partial_e C(e(\bar{\alpha}, \theta), \theta))\partial_\alpha e(\bar{\alpha}, \theta) - \theta\sigma^2\bar{\alpha} \\ +R^1(\theta)\left(\partial_{\theta e} C(e(\bar{\alpha}, \theta), \theta)\partial_\alpha e(\bar{\alpha}, \theta) + \sigma^2\bar{\alpha}\right) \end{array} \right] f(\theta)d\theta = 0$, $\bar{\alpha} = \alpha^1(\theta_0)$ and α^1 is the relaxed associated to the hazard function R^1. This last equation is the "ironing principle" (see Guesnerie and Laffont (1984)). Also, the power is increasing in σ^2.

[19]In particular, the space of implementable contracts is the space of contracts that can be approximated by a sequence of continuous ones. For more details, see Araujo and Moreira (2000).

- $\sigma^2 \in [2,3)$

$$\alpha^{SB}(\theta) = \begin{cases} \alpha^u(\theta), \text{ if } 1 \le \theta \le \theta_0 \\ \alpha^2(\theta), \text{ if } \theta_0 \le \theta \le 2 \end{cases}$$

where θ_0 is such that $\alpha^u(1) = \alpha^u(\theta_0)$, from (3), α^u is characterized by

$$\left(\frac{1-(1+\theta\sigma^2\partial_{ee}C(e,\theta))\alpha}{\partial_{\theta e}C(e,\theta)+\sigma^2\alpha\partial_{ee}C(e,\theta)} + R^2\right) f = \left(\frac{1-(1+\widehat{\theta}\sigma^2\partial_{ee}C(\widehat{e},\widehat{\theta}))\alpha}{\partial_{\theta e}C(\widehat{e},\widehat{\theta})+\sigma^2\alpha\partial_{ee}C(\widehat{e},\widehat{\theta})} + \widehat{R}^2\right)\widehat{f}$$

and $\widehat{\theta}$ satisfies $\partial_\alpha v(\alpha,\theta) = \partial_\alpha v(\alpha,\widehat{\theta})$. For the case of assumption A2 with quadratic cost and uniform distribution we have

$$\alpha^u(\theta) = \frac{1+\left(\frac{\sigma^2}{\theta}-1\right)^2}{1+\frac{\sigma^2}{\theta}+\left(\frac{\sigma^2}{\theta}-1\right)^2\left(1+\frac{\sigma^2}{\sigma^2-\theta}\right)-\frac{4}{\theta^2}\left(\theta-\frac{\sigma^2}{2}\right)}$$

- $\sigma^2 \in (3,4]$:

$$\alpha^{SB}(\theta) = \alpha^1(\theta)$$

7.3 Appendix C

The following analysis detaches the possible features of the equilibria in the competitive case:

- separating equilibrium: $\mu(\cdot|c(\theta))$ is a singleton measure concentrated at θ. Then, the zero profit condition for the insurance company (condition 1. in the definition of the PBE) under full revelation of θ for the contract $(\alpha(\theta),\beta(\theta))$ is

$$\beta(\theta) = (1-\alpha(\theta))e(\alpha(\theta),\theta).$$

The first order condition of 2. in the definition of the PBE is:

$$0 = \dot{\beta} + \partial_\alpha v(\alpha,\theta)\dot{\alpha} = (1-\alpha)(\partial_\alpha e(\alpha,\theta)\dot{\alpha} + \partial_\theta e(\alpha,\theta)) - \sigma^2\theta\alpha\dot{\alpha}.$$

Using the expression of β we have the following ODE:

$$\dot{\alpha} = \frac{(1-\alpha)\partial_{\theta e}C(e(\alpha,\theta),\theta)}{1-\alpha(1+\sigma^2\theta\partial_{ee}C(e(\alpha,\theta),\theta))}$$

- continuous pooling equilibrium: suppose that there exists a non degenerated interval $[\theta_1,\theta_2]$ where the equilibrium contract is constant, i.e., all insuree with risk aversion coefficient in this interval choose the same contract $(\overline{\alpha},\overline{\beta})$. Thus, condition 1. becomes

$$\overline{\beta}(F(\theta_2)-F(\theta_1)) = \int_{\theta_1}^{\theta_2} e(\overline{\alpha},\widetilde{\theta})f(\widetilde{\theta})d\widetilde{\theta}$$

and 2. and 3. are easily satisfied in the given interval.

- discrete pooling equilibrium: suppose that there are two intervals $[\theta_1, \theta_2]$ and $[\widehat{\theta}_1, \widehat{\theta}_2]$ such that for each $\widetilde{\theta} \in [\theta, \theta + d\theta]$, there is exactly one $\widehat{\widetilde{\theta}} \in [\widehat{\theta} - d\widehat{\theta}, \widehat{\theta}]$ that chooses the same contract α. We have argued in the text that a necessary condition of the IC constraint is $\partial_\alpha v(\alpha, \widetilde{\theta}) = \partial_\alpha v(\alpha, \widehat{\widetilde{\theta}})$. So, let $\widehat{\theta} = \gamma(\alpha, \theta)$ be the implicit of the last equation. In other words, given α, there are exactly two types that choose this contract: $\theta(\leq \sigma^2/2)$ and $\widehat{\theta}$. Therefore, the updating belief of θ given α is[20]

$$\lambda = \lambda(\alpha, \theta) = \Pr(\theta|\alpha) = \lim_{d\theta \to 0} \frac{\Pr(\theta \leq \widetilde{\theta} \leq \theta + d\theta)}{\Pr(\theta \leq \widetilde{\theta} \leq \theta + d\theta) + \Pr(\widehat{\theta} - d\widehat{\theta} \leq \widetilde{\theta} \leq \widehat{\theta})}$$

where $\widehat{\theta} = \gamma(\alpha(\theta), \theta))$ and $\widehat{\theta} - d\widehat{\theta} = \gamma(\alpha(\theta + d\theta), \theta + d\theta)$ which implies[21]

$$\lambda = Pr(\theta|\alpha) = \frac{f(\theta)}{f(\theta) + |\partial_\theta \gamma(\alpha, \theta)| f(\gamma(\alpha, \theta))}$$

Then, the equilibrium is characterized by $\beta = (1 - \alpha)[\lambda e(\alpha, \theta) + (1 - \lambda) e(\alpha, \widehat{\theta})]$ and so α satisfies the following ODE:

$$\dot{\beta} + (e(\alpha, \theta) - \sigma^2 \theta \alpha)\dot{\alpha} = 0$$

In the text we presented the explicit ODE for the case of assumption A2 with quadratic cost and uniform distribution of risk aversion.

Now consider the following family of power distributions: $f_n(\theta) \cong (\overline{\theta} - \theta)^n$. In this case the ODE that characterizes the discrete pooling part is given by:

$$\frac{\dot{\alpha}}{\alpha} = \frac{(1 - \alpha)[(\widehat{\theta}^2 - \theta^2)\partial_\theta \lambda + 2((1 - \lambda)\widehat{\theta} - \lambda\theta)]}{(1 - 2\alpha)(\lambda\theta^2 + (1 - \lambda)\widehat{\theta}^2) + \widehat{\theta}\theta\alpha}$$

where $\widehat{\theta} = \sigma^2 - \theta$ and

$$\begin{aligned} \lambda(\theta) &= \frac{f_n(\theta)}{f_n(\theta) + f_n(\widehat{\theta})} \\ \partial_\theta \lambda(\theta) &= -n\lambda(\theta)(1 - \lambda(\theta)) \left(\frac{1}{\overline{\theta} - \theta} + \frac{1}{\overline{\theta} - \widehat{\theta}} \right). \end{aligned}$$

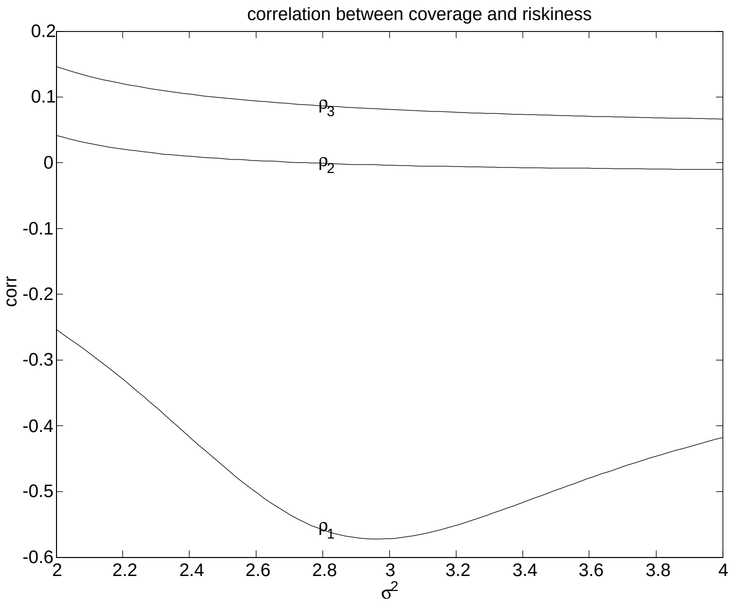
From this family of distributions we can build another example where zero correlation between coverage and risk is attained and the quasi separable equilibrium is discrete pooling. The basic intuition is that when n is larger there is

[20] Observe that for the given α there exist exactly two types choosing it in equilibrium. Thus, we make the limit of the updating of a non degenerated interval around these types to obtain the weights in each type.

[21] Since $\gamma(\alpha, \gamma(\alpha, \theta)) = \theta$, it is easy to see that $\Pr(\theta|\alpha)+\Pr(\widehat{\theta}|\alpha) = 1$.

more weight for low risk averse agents where riskiness is decreasing and coverage is increasing with respect to risk aversion. In order to do that, we will take the following parametrized family of examples in σ^2 and n: $\underline{\theta} = \frac{\sigma^2}{2} - \varepsilon$ and $\overline{\theta} = \frac{\sigma^2}{2} + \varepsilon$, where 2ε is the width of the interval centered at $\sigma^2/2$.

Several numerical examples were tried varying σ^2 and n. Two main features have appeared: (i) when n is large (typically, $n \geq .5$) there are U-shaped equilibria with total coverage for the middle risk averse type ($\sigma^2/2$) (which was not true for the uniform distribution case) and (ii) riskiness is bell-shaped, i.e., for very low risk averse type, riskiness increases with risk aversion. The figure below gives ρ_c for the following family of parameters: $\sigma^2/2 \in [1, 2]$, $\underline{\theta} = \frac{\sigma^2}{2} - \varepsilon$, $\overline{\theta} = \frac{\sigma^2}{2} + \varepsilon$ with $\varepsilon = .5$ and $n = .5$, $.95$ and 1.15 (ρ_1, ρ_2 and ρ_3, respectively). Observe that for $n = .5$ the correlation is negative, for $n = 1.15$ it is positive and for $n = .95$ it changes its sign.



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