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Abstract

In product markets, there exists substantial dispersion in prices for transactions of physically identical goods, and incumbent sellers sell at higher prices than entrants. This study develops a theory of dynamic pricing that explains these facts as results from the same fundamental friction: Buyers are imperfectly aware of which sellers are operating, and the degree of awareness about a seller is endogenous. The equilibrium is unique and efficient, and features randomized pricing strategies where incumbents post higher prices than entrants. If buyers' memory depreciation is low, then the equilibrium of the industry tends to approximate perfectly competitive conditions over time.

Keywords: Buyer Awareness, Price Dispersion, Customer Capital, Industry Life Cycle, Information Frictions.

JEL codes: C78, D11, D40, D83.

1 Introduction

In the textbook model of perfect competition, each decision maker in the economy has access to the same price vector that allows him or her to trade any quantity of

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goods desired at fixed prices, which are equal to the marginal costs of production. The current state of the empirical evidence paints a very distinct picture in product markets: There exists substantial dispersion in the prices for transactions of identical goods.[1] After entering a market, sellers slowly accumulate demand for their product, and prices for transactions with incumbent sellers tend to be higher than those with entrants.[2] Further, the markup over marginal cost tends to be substantial and varies across industries.[3] This study develops a model of dynamic price formation in an industry that explains these deviations from competitive conditions as the result of an imperfect awareness friction among buyers. This model of imperfect awareness also nests the classical model of competition as its frictionless limit.

This study features a novel approach to customer capital accumulation based on the diffusion of information that provides microfoundations for the literature that examines pricing as an investment (such as Foster, Haltiwanger, and Syverson (2016) [22] and Besanko et al. (2019) [6]). I consider an environment where at a given point in time buyers have imperfect awareness of the sellers; that is, each buyer is aware of only a subset of the sellers from which she can purchase the good in that period. The buyers retain some memory of their past awareness and can discover additional sellers over time through word-of-mouth, which means that a seller's previous sales activity is the driving force of customer capital accumulation.[4] As buyers have memory, it also means that the age of the industry as a whole and buyers' awareness dynamics are determinants of the degree of frictions of trading that are present at a point in time in that industry.[5]

The particular trading mechanism that is used in this study proceeds as follows: At each period, sellers choose which prices to post, and buyers choose whether to purchase the good among the set of sellers that buyers are aware of.[6] I examine only rational expectations equilibria where sellers know the distribution of the number of other sellers that their customers might know and choose prices to post to maximize expected

[1] Sorensen (2000) [51].

[2] Foster, Haltiwanger and Syverson (2008,2016) [21, 22].

[3] See, for example, De Loecker et al. (2018) [11] for evidence on the markup levels of the US and De Loecker and Eeckhout (2018) [12] for international evidence.

[4] In the marketing literature, it has been argued that word-of-mouth is a dominant form of customer capital accumulation, for example, Trusov et al. (2009) [54].

[5] In contrast to Dinlersoz and Yorukoglu (2012) [15] who find that increasing the memory of their agents has little effect on the model's equilibrium.

[6] This price posting mechanism is similar to the trading mechanism used in Satterthwaite and Shneyerov (2007, 2008) [49, 48] and Lauermann et al. (2018) [37], where agents trade through first price auctions: Buyers post offers, and the seller sells to the highest posted offer.

discounted lifetime profits. Buyers observe only the prices of the sellers that they are aware of, while sellers know the distribution of the sellers' consumer bases, as well as the distribution of posted prices across the whole industry. However, I find it is a reasonable assumption to suppose that sellers in an industry know more about their competitors than their customers do.[7]

The model has a unique symmetric equilibrium. The assumption that buyers have imperfect awareness of the sellers, and that the degree of the buyer's awareness is dynamic, has important consequences for the properties of the equilibrium. Demand for a seller's product can grow over time thanks to the diffusion of awareness through word-of-mouth imparted by the sellers' sales activity. Imperfect awareness also results in price dispersion caused by mixed pricing strategies, as a pure strategy is inconsistent with equilibrium: Sellers have an incentive to undercut other sellers at any pure strategy higher than marginal cost, and a price equal to the marginal cost is not an equilibrium, as imperfect awareness of competitors means that some buyers are captive and still continue to shop at the seller if prices are increased. However, different pricing strategies imply that there are dynamic gains in demand accumulation, and they are asymmetric across sellers of different ages (a seller's "age" is defined as time after entry). Entrants find it more profitable to sacrifice present profits to grow their consumer bases, while incumbents have less room to further increase their consumer bases, and instead, focus on maximizing present profits. Therefore, incumbent sellers post higher prices than entrants.

The basic version of the model assumes that the rate that sellers exit the industry is an arbitrary function of the seller age (where "seller age" means the time a seller has been operating in the market). If the seller exit rate function is constant, then as the industry matures over time, the quantity traded increases, and the average markup decreases. Those properties are caused by the diffusion of awareness regarding the sellers among the buyers over time, which makes the industry more competitive. That means that this type of model can replicate the stylized fact that markups and average profitability levels vary across industries without any variation in the underlying environment of different industries or in the entry value for firms across industries. That is, asymmetric markups and profitability levels can be consistent with a free-entry condition across industries.

[7] A possible avenue for further research in this framework is to relax this assumption by allowing sellers to not know the population and the distribution of the consumer bases of competitors operating in the industry. The solution of the model would include a description of the evolution of the sellers' beliefs in addition to the evolution of buyers' awareness and the resulting allocation and prices.

These properties imply a tendency for the equilibrium of an industry to converge to competitive conditions over time. This property of the model is related to the concept of competitive equilibrium being the limit of random matching and trading games when the frictions of trading become small (see Osborne and Rubinstein (1990) [44], Gale (2000) [26], Mortensen and Wright (2002) [43], Lauermann (2013) [36]). In the present study, the frictions of trading are buyers' imperfect awareness regarding sellers in the market, but this friction is dynamic in itself as buyers' awareness changes over time. This dynamic property means that this tendency is also related to Walras' original concept of tatonnement as a dynamic process of market adjustment toward a perfectly competitive equilibrium where output increases and profit margins fall (Walker (1987) [56]). That is, this model explains the deviations from competitive conditions suggested by the empirical evidence are features of the price formation process that, under certain conditions, converges to the competitive equilibrium.[8]

I also show that any symmetric equilibrium is constrained efficient in the sense that equilibrium strategies maximize aggregate surplus among feasible allocations at each period.[9] Additionally, by allowing the sellers to exit endogenously, this model can also explain the stylized fact that exit rates are declining in the seller's age: The longer a seller is in the industry, the greater is the degree of buyer awareness of the seller. Thus, the opportunity cost of leaving the industry increases, which lowers the exit rate.

In the following section, I discuss the literature related to this paper. The description of the model begins in Section 3 in which I describe the environment of the game being studied. In Section 4.4, I describes the game, and in Section 5 I present properties of the equilibrium. In Section 6 I discuss the incorporation of endogenous choice of seller exit in the model, and in Section 7 I present concluding remarks.

2 Related literature

The study of pricing and competition is one of the oldest fields in economics, so there is a very large amount of literature related to the present study. As a result a brief

[8]This model is related to Arrow (1977) [4] in that all endogenous variables including prices are deliberately chosen by rational agents, and is a distinct notion of tatonnement from its use in the stability literature (see Fisher (1983) [18] for a survey), where the mechanism of price formation is exogenously imposed on the economy.

[9]As in Gilbukh and Roldan (2018) [27], this model can be interpreted as a theory of efficient markups.

literature review required its own section in the paper. In this section, I relate this study to the existing literature that is perhaps the closest to this model.

Buyers' imperfect awareness of the sellers operating in a product category has been previously used in studies such as Butters (1977) [10], McAffe (1994) [40], and more recently, Perla (2019) [46]. In particular, the present study presents a model that is closely related to Butters (1977), but this study incorporates a dynamic process of awareness diffusion among buyers similar in concept to Perla's (2019) paper. However, in this study awareness diffusion occurs through a word-of-mouth matching process, which is closely related to the way the concept is employed in Fishman and Rob (2005) [20]. Another closely related paper, Guthmann (2020) [29], provides a deeper treatment of the concept of buyer awareness by relating it to the decision theoretical literature that distinguishes unawareness from uncertainty (see, e.g., Modica and Rustichini (1999) [41], Karni and Vierø (2013)[34], and Heifetz, Meyer, and Schipper (2013) [31]).

Existing models of equilibrium price dispersion (such as Butters (1977) [10], Varian (1980) [55], Burdett and Judd (1983)[8], and Stahl (1989) [52][10]) feature symmetric price posting strategies for all sellers and a distribution of prices posted with a strictly decreasing and convex density that peaks at the lower bound of the support of the distribution. These properties of the distribution of prices follow from the logic that to keep sellers indifferent between prices in the support, the absolute value of the elasticity of demand must be decreasing to compensate for the lower profit margins at the lower prices in the support. The predictions of existing price dispersion models can be made consistent with the observed distribution of prices only with explicit or implicit assumptions of product heterogeneity, such as postulating that buyers assign different reservation prices to the good if purchased from different sellers.[11] This paper shows that such assumptions do not need to be made to explain the morphology of price dispersion.

Regarding the literature on demand or customer-capital accumulation, Foster et al. (2016) [22] use the term "demand accumulation by doing" to describe the effect of past sales activity on the present demand for a seller's product, as opposed to "de-

[10]There are various recent applications of such models as Kaplan and Menzio (2016) [32], Moraga-Gonzáles et al. (2017) [42] (in the online appendix), Braido and Ledo (2018), and Burdett and Menzio (2018) [9] for recent examples in the literature.

[11]Some authors explain this assumption of heterogeneity by using the concept of "amenities" provided by individual sellers, which can be used to explain the apparent dispersion of average prices posted by different sellers (see, e.g., Sorensen (2000) [51] and Kaplan and Menzio (2016) [32]).

mand accumulation by being", which represents the accumulation of demand due to the continued presence of the seller in the industry. They show that the seller's "demand accumulation by doing" is the dominant factor in demand accumulation. This assertion is also corroborated by studies in the marketing literature, such as Trusov et al. (2009) [54]. The demand accumulation mechanism used in this study is similar to the mechanism used in other studies such as Luttmer (2006) [38] and Perla (2019) [46] but in this study, the seller's previous sales activity directly influences the diffusion of information regarding sellers in a product market among the buyers in the market to incorporate Foster et al.'s (2016) empirical findings.

Closely related concepts of customer capital accumulation are featured in studies such as Fishman and Rob (2003) [19], Dinlersoz and Yorukoglu (2012) [15], Gourio and Rudanko (2014) [28], and Paciello et al. (2019) [45].[12] However, this study models buyer capital in a distinct way from previous papers. Dinlersoz and Yorukoglu's (2012) work is similar to the present paper in that both papers offer dynamic versions of Butters' (1977) price competition model. In Dinlersoz and Yorukoglu (2012), the customer base consists of buyers who shopped at the seller in the last period, and the seller can attract new customers by sending ads to other buyers. Dinlersoz and Yorukoglu examine a stationary equilibrium and time enters their model through an assumption of the buyers' path dependency: Buyers can always shop at the last seller they shopped at which creates intertemporal incentives for sellers to retain customers. Gourio and Rudanko (2014) present an environment where the buyers have full awareness of the sellers operating and their pricing strategies, but each buyer is matched to only one supplier at a time and are constrained from changing suppliers due to coordination frictions.[13] The models in Fishman and Rob (2003) and Paciello et al. (2019) are similar to that in Dinlersoz and Yorukoglu (2012) in that each buyer is matched to one firm at a time but in the two former papers, buyers can shift from one supplier to another through a random search mechanism instead of receiving ads from the firms as in Dinlersoz and Yorukoglu (2012).

The present study presents a tractable model of the dynamics of price formation in the non-stationary equilibrium caused by the diffusion of awareness regarding sellers.

[12] Klemperer's (1987) [35] model of switching costs also has a form of customer capital accumulation due to previous sales activity.

[13] More specifically, in their model, buyers have a matching probability to the specific seller they choose to shop at. This probability is given by a function of the queue length, which is the ratio of buyers to sales representatives that the seller chooses to employ.

Buyers are said to be part of a seller's consumer base if they are aware of the seller; buyers have memory, so a buyer's consideration sets is durable. A buyer can be part of any seller's consumer base simultaneously as there is no limit on the number of sellers a buyer can be in contact with at a point in time. The relative sizes of sellers' consumer bases yield relative matching probabilities which is the empirical measure of the consumer base in Allen et al. (2019) [1], where, as in this paper, incumbency advantage is associated with a larger consumer base.

3 Environment

Consider an industry with a continuum of buyers and sellers, both of measure one. Time is discrete, denoted by $t = 0, 1, 2, 3, \ldots$. Buyers are assumed to be infinitely lived; sellers might enter and exit the market. There is a single perishable good. At each period, each buyer has unit demand for the good and a reservation price equal to 1, and each seller can produce a quantity $q \in \mathbb{R}_+$ of the good with a constant marginal cost, normalized to 0[14]. Sellers compete by posting prices, and buyers would like to shop at the lowest priced seller as long as the posted price is lower than their reservation price. Sellers discount future profits according to discount factor $\beta \in (0, 1)$.

3.1 Imperfect buyer awareness

Buyers' consideration sets are constrained by imperfect awareness, and buyers can become aware of (discover) additional sellers: At a given period, each buyer's choice set of sellers the buyer can purchase the good from is constrained to the set of sellers the buyer is aware of.

At first, consider an environment with $N > 1$ many sellers. Let $\alpha_t^j \in [0, 1]$ be the measure of buyers aware of seller $j \in \{1, \ldots, N\}$ in period t and let $A_t(i)$ be an assignment correspondence from the set of buyers to the set of sellers that represents the subset of sellers in each buyer i's consideration set. Thus α_t^j is the fraction of all buyers such

[14]That is, in each period each buyer can generate a unit surplus by matching with any seller. The products sold by each seller do not need to be interpreted as physically identical but only as potentially perfect substitutes (if the buyers are aware of both products offered by a pair of sellers). For example, two brands of smartphones might yield the exact same utility to tech-savvy buyers, but naïve buyers might prefer the phone brand they are used to. In this model, these types of buyers are distinguished by different awareness levels.

that $j \in A_t(i)$. See Figure 3.1 for an example where there are only two sellers in the industry.

Buyers can become aware of additional sellers through a word-of-mouth awareness diffusion process: At the end of each period, buyer $i \in [0,1]$ who is unaware of seller j can become aware of j by meeting a customer of j with probability $D(s) \in [0,1]$. Where $s \in [0,1]$ is the measure of buyers who shop at j, and $D : [0,1] \to [0,1]$ is a strictly increasing function such that $D(0) = 0$. That is, the probability that a buyer discovers a seller is strictly increasing in the number of buyers who are actively shopping at j. That is, an increase in the number of customers of a seller increase the probability that the buyer hears about the seller through word-of-mouth.

Buyers have imperfect memory and forget the sellers with probability $\delta \in [0,1)$, which means that the seller is not in the buyer's consideration set in the next period. Therefore, a seller's α_j^t evolves according to

$$\alpha_{t+1}^j = (1-\delta)\alpha_t^j + D(n)(1-\alpha_t^j). \tag{3.1}$$

In other words, the "consumer base" of a seller represented by α_t^j grows by the fraction of buyers who were previously unaware of the seller and discover seller $D(n)(1-\alpha_t^j)$ minus the measure of buyers who forget the seller $\delta\alpha_t^j$.

For buyer i, and sellers j and h with $h \neq j$, the probability of discovery is independent. That is, the probability P that i discovers j in period t conditional on being aware of h is the same as the unconditional probability: $\Pr(j \in A_{t+1}^i \setminus A_t^i \mid h \in A_t^i) = \Pr(j \in A_{t+1}^i \setminus A_t^i)$. The loss of memory is independent, which means that the probability an arbitrary buyer i forgets seller j is the same whether i was aware or not of seller h, $h \neq j$: $\Pr(j \in A_{t+1}^i \setminus A_t^i \mid h \in A_t^i) = \Pr(j \in A_{t+1}^i \setminus A_t^i)$. In period 0, awareness is assumed to be independent. Then, as argued in Lemma 1 below, awareness is independent for any period t. That is, the probability of buyer i being aware of seller j is α_j^t conditional on buyer i being aware or not of another seller h; thus $\Pr(j \in A_t^i \mid h \in A_t^i) = \Pr(j \in A_t^i) = \alpha_j^t$.

Lemma 1. *If discovery and memory loss are independent, and awareness is independent in period 0, then at any period $t > 0$, awareness is independent. That is, for any period t the probability that buyer i who is aware of some seller j is also aware of another seller h is α_t^h; that is, $Pr(h \in A_t^i \mid j \in A_t^i) = Pr(h \in A_t^i) = \alpha_t^h$.*

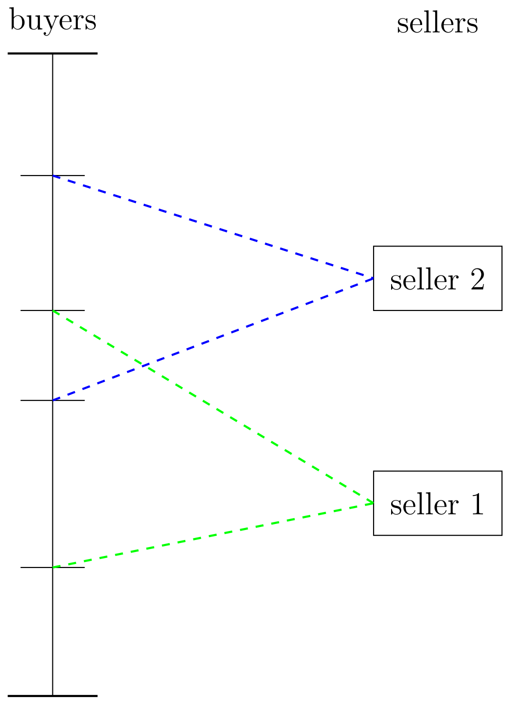


Figure 3.1: Consumer bases of sellers 1 and 2, when $\alpha_t^1 = .35$, and $\alpha_t^2 = .40$.

Proof. This and subsequent proofs are contained in the appendix. $\square$

3.2 Distribution of conditional probabilities of awareness when the sellers are small

This model represents a industry in which each individual seller is small and has no influence on industry aggregates by itself but instead takes them as given. Suppose that there are $K \geq 1$ types of sellers in the industry some period t, distinguished by their buyer awareness parameter $m_t^k \geq 0$, which I call **consumer base** for the remainder of the paper. Let $\boldsymbol{m}_t = (m_t^k)_{k=1}^K$ be the profile of consumer bases and let ζ_t^k be the fraction of sellers of type $k \in \{1, \ldots, K\}$, so $\sum \zeta_t^k = 1$.

Consider a sequence of assignment correspondences $\{A^z(i)\}_{z=1}^{\infty}$ such that the set of sellers of each type $k \in \{1, \ldots, K\}$ is partitioned into z partitions $\{J_n^k\}_{n=1}^z$, each of equal measure, ζ_t^k/z, and buyers are assigned to at most one seller of each partition (i.e. $|A^z(i) \cap J_n^k| \in \{0, 1\}$). The assignment correspondence $A^z(i)$ is such that a measure $\alpha_z^k = (m_t^k \zeta_t^k)/z$ of buyers are assigned to sellers in partition J_n^k, so $\Pr\{A^z(i) \cap J_n^k \neq$

$\emptyset\} = \alpha_z^k$.[15]

As buyers' awareness regarding sellers is independent, the probability that a buyer i is aware of $n \in \{0, 1, \dots, z\}$ sellers of type k is distributed according to a binomial distribution with parameters z for the number of trials and α_z^k for the probability of success for each trial. Suppose buyer i is aware of at least one seller of type k, then the probability that a buyer is aware of $n \in \{0, 1, \dots, l-1\}$ other sellers of type k is distributed according to a binomial distribution with parameters $z-1$ for the number of trials and α_z^k for the probability of success for each trial.

As $z\alpha_z^k = m_t^k \zeta_t^k$ and $\lim_{z\to\infty}(z-1)\alpha_z^k = m_t^k \zeta_t^k$, the Poisson limit theorem implies that as $z \to \infty$ both the unconditional and conditional Binomial distributions converge to the Poisson distribution with parameter $\overline{m}_t^k = m_t^k \zeta_t^k$. Take $A_t(i)$ to be an assignment correspondence $A^z(i)$ with z large so the distribution of buyer's awareness regarding sellers of each type is approximated by Poisson distributions of parameter $\overline{m}_t^k$. Let $\pi(n, \overline{m}_t^k)$ be the probability mass function at n of a Poisson distribution with parameter $\overline{m}_t^k$.

In addition, note that that the number of sellers of any type that a buyer is aware of in period t is distributed according to a Poisson with parameter $\hat{m}_t = \sum_k \overline{m}_t^k$, as it is the sum of Poisson distributed random variables for each type, thus $\hat{m}_t$ represents the average number of sellers in the consideration sets of buyers.

Diffusion of awareness when sellers are small

In this case the individual seller's consumer base is small relative to the market, and as the number of active customers of a seller is bounded by the buyers base, it implies that the diffusion of awareness does not immediately face the logistical constraint described in the law of motion 3.1. In this case, the law of motion for the consumer base can be written as:

$$m_{t+1}^k = (1-\delta)m_t^k + \Phi(q_t^j), \tag{3.2}$$

where $q_t^j \in [0, m_t^k]$ is the (normalized) size of the active clientele of a seller of type k, and the awareness diffusion function Φ is the normalized discovery function D.[16]

Assumption 1. (Decreasing Returns to Word-of-Mouth) The awareness diffusion

[15] For example, let $[0,1]$ be the set of sellers. Suppose there is one seller type and 10 partitions, so $J_n^1 = [(n-1)/10, n/10)$ for $n \in \{1, \dots, 10\}$. Then, $\Pr\{A^{10}(i) \cap J_n^1 \neq \emptyset\} = m_t^1/10$.

[16] For example, consider the function $\Phi(q) = \sqrt{q} + (1-\beta)q$ and suppose that D is described by $D(x) = \lim_{N\to\infty} \frac{1}{N}\Phi(Nq)$. Then the corresponding discovery function is $D(x) = (1-\beta)x$. In addition, note that $\Phi(q)/q = 1/\sqrt{q} + (1-\beta)$.

function Φ is strictly increasing and strictly concave, and satisfies the condition that $\lim_{x\to\infty} \frac{\Phi(x)}{x} \leq 1-\beta$.

The intuition behind this assumption is that as more buyers become aware of the seller, there exists congestion of the diffusion of awareness through word-of-mouth from customers of the seller to buyers who are unaware of the seller. The assumption that there are decreasing returns to word-of-mouth is consistent with the empirical evidence that sellers grow at slower rates as they become older and larger (Luttmer (2011) [39], Dinlersoz and Yorukoglu (2012) [15], Argente et al. (2019) [2]). The regularity condition that $\lim_{x\to\infty} \frac{\Phi(x)}{x} \leq 1-\beta$ prevents the consumer base from growing too quickly so the present value of a seller is well defined.

3.3 Sellers' entry and exit

At the end of each period, a fraction $\lambda \in (0,1)$ of all sellers exit and are replaced by new sellers. When a sellers enters the industry, the seller has a starting consumer base size $m_s > 0$ which can be though as due to an initial investment in advertising that reaches an initial mass of buyers. Then, further growth in their consumer base is achieved through word-of-mouth. A seller has an age a which measures the time since entry (thus new sellers have age $a = 0$).

The industry in period 0 is assumed to be inactive, so all sellers have a zero consumer base, except for the first cohort of sellers with positive consumer base $m_c > 0$, that just entered. Thus at period t there might be sellers of age $a \in \{0, 1, \ldots, t\}$ operating in the industry. Exit rates depend on a seller's age, given by a profile of exit rates by seller age $\{\lambda^a\}_{a=0}^{\infty}$, where for each $a \in \{0, 1, \ldots, t\}$ a fraction $\lambda^a \in (0,1)$ of the sellers of age a exit the industry at the end of the period. Therefore, the fraction of operating sellers who are of age $a \leq t$ is $\lambda \prod_{i=0}^{a-1}(1-\lambda^i)$. As $t \to \infty$, the distribution of active sellers by age converges to a stationary distribution.[17]

I could extend to model to also seller exit rates to change over the life cycle of the industry. However, if that assumption is made, then additional assumptions are required for the distribution of sellers by age to converge to a stationary distribution. Also, exit rates are exogenously given, but I show in an extension in Section 6, that if sellers

[17]To be consistent with the average replacement rate λ, the profile of the exit rates satisfies $\sum_{a=0}^{\infty} \lambda^a [\lambda \prod_{i=0}^{a-1}(1-\lambda^i)] = \lambda$ for active sellers, and inactive sellers exit at a rate $\lambda_t^I = \frac{\sum_{h>t} \lambda^h \prod_{i=0}^{h-1}(1-\lambda^i)}{\sum_{h>t} \prod_{i=0}^{h-1}(1-\lambda^i)}$ in period t.

are allowed to endogenously exit, the equilibrium implies that sellers with smaller consumer bases exit more frequently than sellers with larger consumer bases. As younger sellers, under certain assumptions regarding buyer memory depreciation, have smaller consumer bases than older sellers younger sellers exit more frequently. The reason is that the decision to exit is determined by the opportunity cost of staying in the industry, which is assumed to be a random variable that is realized every period and assumed to be identically distributed for all seller types. The present value of operating in the industry is increasing in the size of the consumer base of the seller, and older sellers have had more time to accumulate more customers. Therefore, older sellers have higher value in staying in the industry and thus, lower probability of exiting. This replicates the empirical observation that seller exit rates are higher for young sellers than for older sellers (Evans (1987) [17], Dunne, Roberts, and Samuelson (1989) [16]).

4 The price posting game

4.1 The determination of sellers' profits

At each period, each seller chooses price $p \in \mathbb{R}$ to post to all buyers in his or her consumer base.[18] Sellers can randomize: A seller chooses a randomized price posting strategy described by a cumulative distribution function F_t in period t. Buyers observe prices from the set of sellers they are aware of and can purchase zero or one unit of the good, which means that a consumer purchases a unit of the good at the lowest observed price conditional on that price not being higher than 1; otherwise, no purchase is made.[19] I am interested in symmetric equilibrium where sellers of the same type follow the same strategy, so the profile of sellers' strategies can be written as $\boldsymbol{F}_t = (F_t^k)_{k=1}^K$ for each type of seller $k \in \{1, \dots, K\}$.

Consider a consumer base profile $\boldsymbol{m}_t = (m_k^t)_{k=1}^K$ and a profile of strategies $\boldsymbol{F}_t$. Suppose a seller of type k posts a price p. The probability that a buyer in the seller's consumer base purchases is the probability that p is the lowest price observed by the buyer. Let $P(p, \boldsymbol{F}_t, \boldsymbol{m})$ be this probability, it is determined as follows:

[18] Implicitly, this means that buyers' awareness is private information, as the sellers cannot discriminate among different buyers in their consumer base.

[19] That is, a buyer's surplus is $u(x) = \begin{cases} 1 - p^j & \text{if } x = 1, p^j = \min\{p^h \mid h \in A_t^i\} \\ 0 & \text{if } x = 0 \end{cases}$.

Each seller competes against K types of other sellers. Suppose that $F_t^h, h \in \{1, \dots, K\}$ does not have an atom at p (that is F_t^h does not have strictly positive mass at p)[20], then, conditional on observing a price posted by another seller of type h, the probability that p is lower than that price is $1 - F_t^h(p)$. If the buyer observes n prices $\{p_1, \dots, p_n\}$ from n seller of type h the probability that p is lower than those prices is

$$\begin{aligned}\Pr(p < \min\{p_1, \dots, p_n\}) &= \Pr(p < \min p_1) \times \dots \times \Pr(p < \min p_n) \\ &= [1 - F_t^h(p)]^n.\end{aligned}$$

The probability a buyer observes n prices from competing sellers that are of type k is $\pi^n(\overline{m}_t^h)$. Thus, the probability $\Pr(p < \min\{p_j\} \mid j \text{ is type } h)$ that p is the lower than the prices the buyer observes from competing sellers of type h satisfies

$$\Pr(p < \min\{p_j\} \mid j \text{ is type } h) = \sum_{n=0}^{\infty} \pi(n, \overline{m}_t^h)[1 - F_t^h(p)]^n.$$

As $P(p, \boldsymbol{F}_t, \boldsymbol{m}_t)$ is the probability that the price the buyer observes is lower than prices posted by competing sellers of all types, it is the product of the vector of probabilities $(\Pr(p < \min\{p_j\} \mid j \text{ is type } h))_{h=1}^K$:

$$P(p, \boldsymbol{F}_t, \boldsymbol{m}_t) = \prod_{h=1}^{K} \Pr(p < \min\{p_j\} \mid j \text{ is type } h), \tag{4.1}$$

$$= \sum_{n=0}^{\infty} \pi(n, \overline{m}_t^1)[1 - F_t^1(p)]^n \times \sum_{n=0}^{\infty} \pi(n, \overline{m}_t^2)[1 - F_t^2(p)]^n \times \dots \times \sum_{n=0}^{\infty} \pi(n, \overline{m}_t^K)[1 - F_t^K(p)]^n. \tag{4.2}$$

[20] If p is an atom for some distribution of prices F_t^k, I assume a tie breaking rule where the buyer chooses to purchase from each seller with equal probability among sellers who post the same price. However such event never occurs in equilibrium in this model as the equilibrium distributions are atomless.

Because $\pi(.,.)$ is a Poisson probability mass function,

$$\begin{aligned}\sum_{n=0}^{\infty}\pi(n,\overline{m}_t^k)[1-F_t^k(p)]^n &= \sum_{n=0}^{\infty}\frac{\left(\overline{m}_t^k\right)^n\exp(-\overline{m}_t^k)}{n!}[1-F_t^k(p)]^n \\ &= \left[\sum_{n=0}^{\infty}\frac{\left(\overline{m}_t^k[1-F_t^k(p)]\right)^n}{n!}\right]\exp(-\overline{m}_t^k) \\ &= \exp(\overline{m}_t^k-\overline{m}_t^kF_t^k(p))\exp(-\overline{m}_t^k) \\ &= \exp(-\overline{m}_t^kF_t^k(p)).\end{aligned}$$

Therefore, the right hand side of equation 4.1 can be written as

$$\begin{aligned}P(p,\boldsymbol{F}_t,\boldsymbol{m}_t) &= \exp(-\overline{m}_t^1F_t^1(p))\times\exp(-\overline{m}_t^2F_t^2(p))\times\ldots\times\exp(-\overline{m}_t^KF_t^K(p)) \\ &= \exp\left(-\sum_k\overline{m}_t^kF_t^k(p)\right).\end{aligned} \tag{4.3}$$

Profits in the current period are given by the quantity sold, which is the probability of sale multiplied by the consumer base (which is a continuum of buyers) multiplied by the price:

$$\Pi(p,m,\boldsymbol{F}_t,\boldsymbol{m}_t)=pmP(p,\boldsymbol{F}_t,\boldsymbol{m}_t). \tag{4.4}$$

4.2 The determination of the profile of seller types

As sellers have identical technology seller types are differentiated by their consumer base.

Consider two sellers of type k. They have the same consumer base and that follow the same randomized price posting strategy described by a continuous cumulative distribution F_t^k. If each seller posted price is a realization from the distribution F_t^k then each seller would post a different price and the quantity sold by each seller would be different. This implies that consumer bases of sellers of the same type would diverge over the next period. Assumption 2 stated below prevent this divergence and implies that in symmetric equilibrium their consumer bases depend only on the age of the sellers.

Assumption 2. If the pricing strategy F_t^k of a seller of type k is non-degenerate, then each buyer in the seller's consumer base draws a different price from F_t^k.

Assumption 2 prevents sellers who play a randomized strategy from posting a single price to all potential customers. Instead, each buyer who has the seller in their consideration set observes a different realization from the distribution of prices. Thus a fraction $F_t^k(p)$ of buyers in each seller's consumer base would observe prices from that seller to be lower than p. As the distribution of prices observed by the buyers in the consumer bases of each seller is the same the probability a buyer purchases from each seller, conditional on being in the sellers' consumer base, is also the same. Therefore, two sellers with the same consumer base and who post prices according to the same distribution sell the same quantity. Thus, Assumption 2 implies that in symmetric equilibrium the evolution of the consumer base size depend only on a seller's type. As sellers enter the industry with the same consumer base size $m_s > 0$ the set of different seller types can be described by their age.[21]

In period 0, all sellers are inactive, with a zero consumer base, except the first cohort that enters, and there is only one type of seller who is said to be "active" in the industry. In period 1, by construction, all sellers who entered in period 0 posted prices according to the same strategy, and as they have the same consumer base size, sold the same quantity. Therefore, all sellers who entered in period 0 have the same consumer base in period 1. Thus, there are two types of sellers in period 1. By the same logic, if in some period r there are $r+1$ types, then in period $r+1$ there are $r+2$ types of sellers in the industry with strictly positive consumer bases. By induction, in period t there are $t+1$ types of sellers with non-zero consumer bases, and given the rates of entry and exit in the industry, a fraction $\lambda(1-\lambda)^{a-1}$ of sellers are age a. The set of active types are sellers of age $a \in \{0, 1, \ldots, t-1\}$.

Therefore, the set of seller types in some period t can be described as a sequence $\boldsymbol{m} = \{m_t^a\}_{a=0}^{\infty}$ where $m_t^a = 0$ for $a \geq t$. That is, the cohort of age a in period t has a consumer base m_t^a. If $a > t$ (the sellers in that cohort would be older than the industry),

[21] An alternative formulation of the model can allow for sellers to post a single realized price from their randomized strategy in each period and preserves all the positive features of the model presented here. It consists of countable set of possible seller types $\{m^1, m^2, m^3, \ldots\}$ with $m^k < m^{k+1}$ for all k, $\lim_{k\to\infty} m^k = \infty$, and the law of motion of awareness diffusion in this case defines a probability that the seller type "mutates" from k to $k+1$ and from k to $k-1$ (in the case of buyer's memory loss) as a function of sales quantity. For decreasing returns to word-of-mouth to hold in this environment is suffices to assume that $m^{k+1} - m^k$ is constant and that the probability that the type mutates from k to $k+1$ is increasing and concave on sales.

it means it this cohort is not operating yet, so $m_t^a = 0$. Note also that

$$\overline{m}_t^a = \zeta^a m_t^a, \tag{4.5}$$

where $\zeta^a = [\lambda \prod_{s=0}^a (1-\lambda^s)]$ is the fraction of all sellers who have been operating in the market for a periods, for $a \leq t$. In addition, the strategy profile in each period is then described by $\boldsymbol{F} = (F_t^a)_{a=0}^t$ for the set of seller types active in the industry described by the time they have been operating since entry (their "age").

4.3 The sellers' problem

In a given period t, the state of the industry is fully specified by consumer base profile $\boldsymbol{m}_t$. The consumer base profile in some period t is fully determined by the entry consumer base size m_s and the quantities sold implied by the pricing strategies of the sellers in previous periods $\{\boldsymbol{F}_y\}_{y<t}$.

Given a profile of strategies for each period $\boldsymbol{F} = \{\boldsymbol{F}_t\}_{t=0}^{\infty}$ and the sequence of consumer base profiles $\boldsymbol{m} = \{\boldsymbol{m}_t\}_t$ (determined by the strategy profile) the problem of a seller in period t is to choose a sequence of prices that maximize profits, taking into account the effects of its pricing strategy on the trajectory of the seller's consumer base:

$$\begin{aligned} &\sup_{\{p_k\}_{k\geq 0}\subset\mathbb{R}} \sum_{k\geq 0} \beta^k p_k m_{t+k}^k P(p, \boldsymbol{F}_{t+k}, \boldsymbol{m}_{t+k}) \\ &\text{s.t. } m_{t+k+1}^j = m_{t+k}^j + \Phi[m_{t+k}^k P(p, \boldsymbol{F}_{t+k}, \boldsymbol{m}_{t+k})], \forall k \geq 0. \end{aligned} \tag{4.6}$$

4.4 Equilibrium

The solution concept is symmetric Markov perfect equilibrium (SMPE). Symmetry in this setting means that sellers with the same consumer base play the same strategy. Equilibrium is Markov perfect as sellers do not condition their strategy on the previous history of play. Instead the sellers maximize only their present value, given by 4.6. Equilibrium is also anonymous: Sellers do not condition their strategies on other individual sellers.[22] Finally, as the buyers are assumed to not be strategic, this formulation

[22] As shown in Osbourne and Rubinstein (1990) [44] and Gale (2000) [26], anonymity is an important assumption for random matching games to yield an outcome equivalent to a competitive equilibrium.

avoids the issues with solution concepts for games without unawareness (see Heifetz, Meier, and Schipper (2013) [31], Schipper (2018) [50]).[23]

Definition 1. An SMPE is a profile of price posting strategies $F^* = \{(F_t^a)_{a=0}^t\}_{t\geq 0}$ such that for any period t and cohort a any price in the support of F_t^a is consistent with maximization of the present value of profits. That is, a price $p \in \text{supp}(F_t^a)$ is a solution to the seller's a problem, written in recursive fashion as

$$V(m_t^a, \boldsymbol{m}_t) = \max_{p\in\mathbb{R}} p m_t^a P(p, \boldsymbol{F}_t, \boldsymbol{m}_t) + \beta V(m^+, \boldsymbol{m}_{t+1}) \tag{4.7}$$
$$\text{s.t. } m^+ = (1-\delta)m_t^a + \Phi[m_t^a P(p, \boldsymbol{F}_t, \boldsymbol{m}_t)]$$

given that other sellers are playing $\boldsymbol{F}^*$ and with $\boldsymbol{m}_t$ as the profile of consumer bases induced by the sellers posting prices according to $\boldsymbol{F}^*$.

Theorem 1. *If Assumption 1 (decreasing returns to word-of-mouth) and Assumption 2 hold, then there exists a unique SMPE $\boldsymbol{F}^* = \{\boldsymbol{F}_t\}_t$. In this SMPE, for each period $t \geq 0$ and each cohort of sellers operating $a \in \{0, 1, \ldots, t\}$, F_t^a is continuous and has support $[\underline{p}_t^a, \overline{p}_t^a]$. The interiors of the supports are disjoint for two cohorts with different consumer bases: For two seller cohorts of ages a and b, if $m_t^a < m_t^b$, then $\overline{p}_t^a \leq \overline{p}_t^b$ and $\max\{\overline{p}_t^a\}_{a=0}^t = 1$. As $t \to \infty$, for each cohort a, F_t^a converges to a stationary equilibrium distribution F^a as the distribution of active sellers by age converges to a stationary distribution.*

If the memory depreciation parameter of buyers $\delta \in [0, 1)$ is small enough, then $b > a$ implies that $m^a < m^b$; that is, older sellers accumulate a larger consumer base and post higher prices than younger sellers. In addition, in this case the profile of the consumer bases for the operating sellers is stationary: There is a profile of consumer bases $\{m^a\}_a$ such that for each $t \geq 0$, $a \leq t$, $m_t^a = m^a$.

In equilibrium, there is a partitioning of the supports of the equilibrium strategies by type. This means that sellers randomize, but if the buyers' memory does not depreciate

[23] Although the equilibrium of this game where buyers are passive players can also be though as a memoryless sequence of self-confirming equilibria (in Schipper's (2018) [50] definition) of pricing games for each period. In a given period, sellers post prices first; buyers respond by choosing an offer given their imperfect awareness. Sellers' payoffs in the terminal node of the pricing game incorporate future payoffs. A buyer's rational response to the sellers' strategy is to shop at the lowest price he or she is aware of. As buyers do not, endogenously, discover other sellers by purchasing from the sellers they know, this is a self-confirming equilibrium according to Schipper's (2018) model.

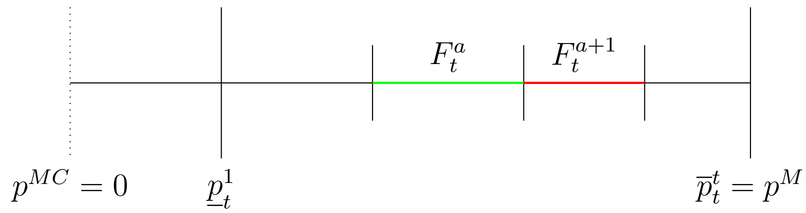


Figure 4.1: Partition of the supports of the strategy profile $\boldsymbol{F}_t$ in the equilibrium if memory depreciation is low.

too fast, the sellers of older cohorts, who have a consolidated position in the industry, sell at prices that are always higher than those of the sellers of younger cohorts who are trying to penetrate the industry.[24] The reasoning behind the result is simple: As there are decreasing returns to word-of-mouth when a seller is well-known by the buyers, then inexperienced sellers which are known by a smaller consumer base have greater potential for growing their consumer bases. Thus, these sellers' pricing is lower than incumbent sellers, who maximize the profits that they currently extract from their consumer base. This model then provides theoretical foundation for the empirical regularity that entrants sell at lower prices than incumbent firms because the entrants are "investing" in building up future demand (Foster, Haltiwanger, and Syverson 2008, 2016 [21, 22]).

Mixed pricing strategies follow from (1) The existence of captive buyers. That is, there are always some buyers in a seller's consumer base who have only the seller in their consideration sets, which means that marginal cost pricing is not consistent with equilibrium. (2) The incentive to undercut: In equilibrium, each cohort a of sellers has a strictly positive parameter $\overline{m}^a$. Therefore, sellers have incentives to deviate from any strategy profile where some cohort of sellers posts some price p with strictly positive probability. The uniqueness of the mixed-strategy equilibrium $\boldsymbol{F}^*$ follows from the fact that given a profile of consumer bases, and a value function that is strictly concave and increasing in the seller's consumer base, then there is a unique profile of pricing strategies consistent with maximization of the present value of profits, and the value function consistent with equilibrium is unique.

Theorem 1 states that the consumer bases of sellers of age a are constant over the industry life cycle. Thus, for each $a = 0, 1, 2, \ldots$ there is an $m^a > 0$ such that $m_t^a = m^a$,

[24] Armstrong and Vickers (2019)[3] have also shown partitioning in equilibrium pricing strategies in a static model. It has similarities to the result of Burdett and Coles (1997) [7] for the marriage market, which is partition matchings of couples into different classes.

$\forall t \geq a$. To see that note that the quantity sold by a seller of cohort a in equilibrium is the consumer base m_t^a multiplied by the probability the seller is not undercut by a competitor. The undercut probability in equilibrium is the probability that a customer is aware of younger cohorts that post lower prices than cohort a plus the probability a customer is aware of a competing seller of cohort a and this competitor post lower prices. Sellers of the same cohort post prices according to the same distribution. Assumption 2 implies for a seller j of cohort a that the probability that another seller of the same cohort undercuts j is one half. As pricing is increasing with seller age, only sellers from cohort 0 can undercut other sellers of cohort 0. Thus, $m_t^0 = m_s, \forall t$ imply that $m_t^1 = m^1$ for some $m^1 > 0$ for all t. Note that only cohorts 0 and 1 can undercut sellers from cohort 1 and that $\overline{m}_t^0, \overline{m}_t^1$ are constant in t. Therefore, $m_t^2 = m^2$. By induction, $m_t^a = m_{t+1}^a, \forall a \leq t, \forall t$.

5 Equilibrium properties

5.1 Pricing dynamics

Under the assumptions of constant entry and exit rates, in the symmetric non-stationary equilibrium that is being studied the average price for transactions is strictly decreasing over time. To show that, first I show that the average number of sellers that buyers know is strictly increasing over time. Second, I show that the distribution of prices for transactions is monotone decreasing over time if the average number of sellers that buyers know is strictly increasing over time.

Let S_t be the cumulative distribution function of sales prices in equilibrium in period t. It is a function of $\boldsymbol{F}_t$ and $\boldsymbol{m}_t$, as a buyer who knows k sellers buys at the lowest price among these k sellers who post prices according to $\boldsymbol{F}_t$. Let $P(p \geq \min p^j, \forall j \in A_t^i)$ be the probability for buyer i that p is higher than the minimum of the prices posted by

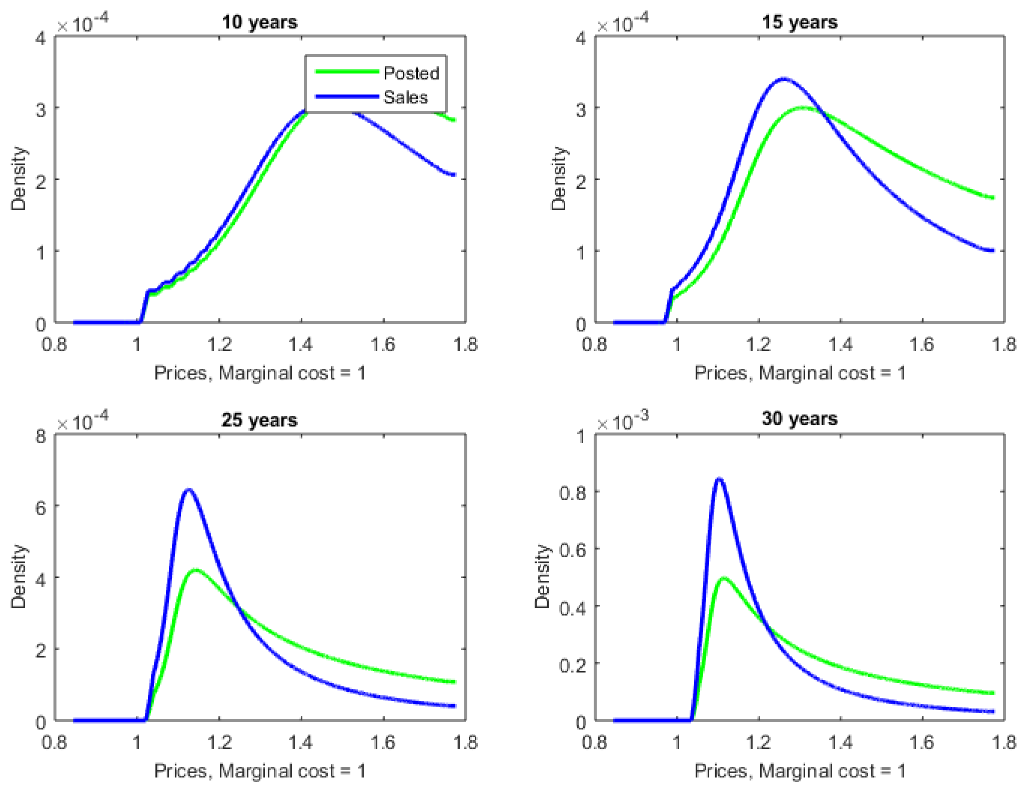


Figure 5.1: An example of evolution of the distribution of prices posted and for sales over time. A period is 1 year, with $\lambda^a = .1, \forall a, \delta = .005, \beta = .965$, and $\Phi(x) = .064\sqrt{x}$. The marginal cost is set at 1.28 times the buyer-seller surplus so that the standard deviation of prices is 10% at 20 year old industry.

sellers in A_t^i. Then

$$\begin{aligned}
S_t &= P(p \geq \min p^j, \forall j \in A_t^i \mid A_t^i \neq \emptyset) \\
&= 1 - P(p \leq \min p^j, \forall j \in A_t^i \mid A_t^i \neq \emptyset) \\
&= 1 - \sum_{n=1}^{\infty} P(|A_t^i| = n \mid A_t^i \neq \emptyset) P_t(p \leq p^j, \forall j \in A_t^i \mid |A_t^i| = n) \\
&= 1 - \sum_{n=1}^{\infty} P(|A_t^i| = n \mid A_t^i \neq \emptyset) \left(\sum_{a=0}^{t} P(j \text{ has age } a \mid j \in A_t^i) P(p \leq p^j \mid p^j \sim F_t^a) \right)^n \\
&= 1 - \sum_{n=0}^{\infty} P(|A_t^i| = n+1 \mid A_t^i \neq \emptyset) \left(\sum_{a=0}^{t} \left(\frac{\overline{m}_t^a}{\hat{m}_t} \right) [1 - F_t^a(p)] \right)^{n+1} \\
&= 1 - \sum_{n=0}^{\infty} P(|A_t^i| = n) [1 - \hat{F}_t(p)]^{n+1},
\end{aligned}$$

where $P(j \text{ has age } a \mid j \in A_t^i) = \overline{m}_t^a / \hat{m}_t$, as $\overline{m}_t^a / \hat{m}_t$ is the fraction of buyer consideration sets regarding operating sellers that consists of sellers of age a,[25] and $\hat{F}_t(p) = \sum_{a=0}^{t} \left(\frac{\overline{m}_t^a}{\hat{m}_t} \right) F_t^a(p)$ is the average of all prices posted by sellers weighted by the fraction of the buyer consideration sets of each seller age. Using Butters' (1977) terminology, $\hat{F}_t(p)$ is the distribution of prices advertised to buyers.

Theorem 2. *Given Assumptions 1 and 2, for δ small enough in equilibrium the average margin for transactions is strictly decreasing over time.*

Theorem 2 states that if memory depreciation is low enough the prices for transactions are decreasing over time. The reasoning for this result is as follows: Theorem 1 implies that consumer bases of different cohorts are stationary if buyer's memory depreciation is low enough. The stationarity of the consumer base profile result implies that $\hat{m}_t$ is

[25] Formally, for finitely many sellers

$$\begin{aligned}
P(j \text{ has age } a \mid j \in A_t^i) =& \frac{P(j \in A_t^i \mid j \text{ has age } a) P(j \text{ has age } a \text{ in period } t)}{P(j \in A_t^i)} \\
=& \frac{\alpha_t^a (n_t^a / N)}{\sum_{b=0}^{t} \alpha_t^b \left(\frac{n_t^b}{N} \right)} = \frac{\alpha_t^a n_t^a}{\sum_{b=0}^{t} \alpha_t^b n_t^b}.
\end{aligned}$$

Note $\overline{m}_t^a = n_{t,l}^a \alpha_{t,l}^a$, and therefore, for all l,

$$P(j \text{ has age } a \mid j \in A_t^i) = \frac{n_{t,l}^a \alpha_{t,l}^a}{\sum_{b=0}^{t} n_{t,l}^b \alpha_{t,l}^b} = \overline{m}_t^a / \sum_{b=0}^{t} \overline{m}_t^b = \overline{m}^a / \hat{m}_t.$$

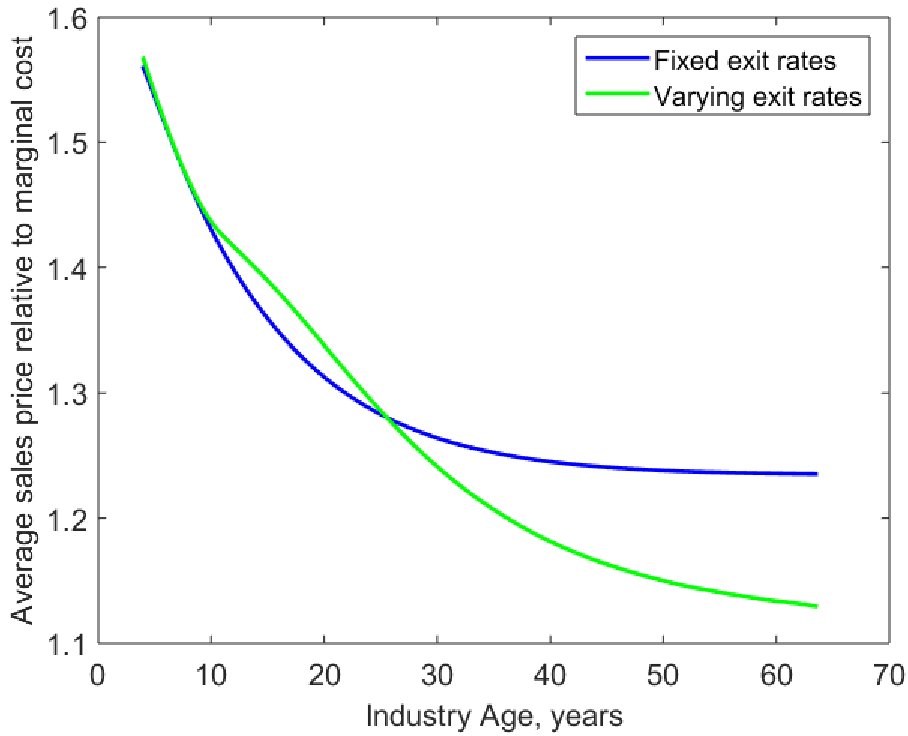


Figure 5.2: Comparative dynamics of average sales price over time: The blue line depicts the average sales price in the industry relative to marginal cost if the exit rate fixed at 10% per year. The green line depicts the average sales price as the exit rate varies from 16% per year for the first 10 years and decreases to 4% a year for the later years. The other parameter values are the same as in Figure 5.1.

strictly increasing in t. To see that note that in period t, there is a cohort of age t which was not present in period $t-1$, plus all cohorts of ages $a \in \{0, 1, \ldots, t-1\}$ which have the same measure and consumer base sizes as in period $t-1$. Therefore, $\hat{m}_t > \hat{m}_{t-1}$. This implies that the degree of competition between sellers is also increasing over time. I show that this increase in competition decreases the feasible profits rates sellers can obtain in the present and in the future. Otherwise, if the present value of future profits per buyer increased over time, then the degree of competition between sellers would decrease which is contradicted by the increase in average buyers' awareness of the sellers.

5.2 Convergence to perfect competition

The competitive equilibrium that corresponds to the physical environment described in this paper trivially satisfies the following properties: First, in every period, all sellers post a price equal to the marginal cost. Second, each buyer purchases a unit of the good; thus, the aggregate quantity sold in every period is 1, profits are zero, and buyers capture all the surplus. The set of core allocations in this environment consists of the competitive equilibrium allocation. To see that, note that the marginal product of a subset of buyers of measure μ is also μ, while the marginal product of additional sellers is 0 as sellers produce the good at constant marginal cost without capacity constraints.

Theorem 3 states conditions such that the equilibrium converges to the competitive equilibrium where all buyers purchase the good for the marginal cost. Intuitively, if sellers stay in the industry forever, and the speed of diffusion of awareness about them among the buyers is sufficiently fast, then the increase in the intensity of the sellers' competition is sufficiently fast to drive prices down to the marginal cost as $t \to \infty$.

Theorem 3. *If Assumptions 1 and 2 hold, buyers have perfect memory so $\delta = 0$, and if the frictions in awareness diffusion are sufficiently low (in particular, Φ satisfies the condition that for some $y > 0$ and $z > 0$, $\Phi(x) > yx$ for $x \in [0, z]$), then the equilibrium distribution of posted prices converges in probability to the marginal cost, and the equilibrium allocation corresponds to the competitive equilibrium as seller entry rate λ and exit rates λ^a converge to zero for all a.*

5.3 Efficiency

There is unrealized surplus in this industry due to the presence of imperfect awareness: The total surplus realized in the equilibrium of period t is $1 - \pi^0(\hat{m}_t)$, where $\pi^0(\hat{m}_t)$ is the population of buyers who cannot purchase the good.[26] As the evolution of awareness is endogenous to the assignment of buyers to sellers, there exists the possibility for this assignment to be inefficient in the sense that $1 - \pi^0(\hat{m}_t)$ in a given period is lower than the highest feasible value. However, as is shown in this section, the symmetric equilibrium yields an efficient assignment.

To formally define efficiency in this environment, I must first define the set of feasible allocations. Allocations are defined as an assignment rule of buyers to sellers, that is, an order of preference for the assignment of buyers to the sellers they might be aware

[26] As buyers' valuations are public information, there is no deadweight loss due to pricing above marginal cost: If they are aware of at least one seller, they will transact and realize the surplus. Clearly, if buyers' valuations are private information, then the equilibrium only approximates efficiency when the posted prices approximates the prices in competitive equilibrium.

of. I restrict attention to assignment rules that do not discriminate between buyers (who are identical) and sellers of the same type, where the types are sellers with the same consumer base (note that in the symmetric equilibrium, the age of a seller is his or her type).

Definition 2. An allocation in some period t is described by assignment rule $R : N \to \mathbb{R}_+$, where N is the set of seller types. If $R(k) > R(k')$, it implies that buyers are assigned to sellers of type k' over type k if they are aware of both types. If $R(k) > 1$, it implies that buyers are not assigned to the seller of type k even if k is the only seller the buyer knows (that is, buyers do not trade in that case). Let $k(j)$ be the type of seller $j \in J$. If the buyer knows multiple sellers of the preferred assignment type (i.e., $|\arg\min_{j \in A^i}\{R[k(j)] : R[k(j)] < 1\}| > 1$), then the assignment rule follows a tie-breaking rule where the buyer has equal probability of being assigned to each of the preferred sellers.

Example 1. Consider an industry with two types of sellers $a \in \{1, 2\}$, each of measure 1/2,

$$m_t^k = \begin{cases} 1 & \text{if } a = 1 \\ 2 & \text{if } a = 2; \end{cases}$$

thus, $\hat{m}_t = 3/2$. Consider an assignment rule R such that $R(1) < R(2) < 1$. In this case, the quantities sold by sellers of types 1 and 2, respectively, are

$$q_t^a = \begin{cases} m_t^a \exp(-1/4) & a = 1 \\ m_t^a \exp(-1) & a = 2. \end{cases}$$

As all sellers enter with the same consumer base, then any assignment rule implies in a profile of the types of active sellers that can be described by the seller's age. An efficient allocation in this environment in some period t is defined as a sequence of assignment rules for each period that maximizes the present value of the sequence of aggregate industry surpluses, as stated below:

Definition 3. A sequence of assignment rules $\{R_t\}_t$ is efficient if it yields a sequence of average awareness levels $\{\hat{m}_t\}$ and corresponding industry surpluses $\{1 - \pi^0(\hat{m}_t)\}_{t=0}^{\infty}$ such that there is no feasible sequence of assignment rules that implies in a higher present value (i.e., $\sum_t \beta^t[1 - \pi^0(\hat{m}_t)]$) of industry surpluses.

Theorem 4. *Given Assumption 1, the equilibrium is efficient.*

Efficiency is obtained by an assignment of buyers among sellers' ages that maximizes the average consumer base in each period, $\hat{m}_t$. The reason is that there is no time inconsistency in the maximization of the present value of industry surpluses: A marginal increase in the consumer base of sellers of age a in period t increases the consumer base of the same sellers in period $t + r$ by the marginal rate of $(1 - \delta)^r$. That means that the marginal rates of substitution of the consumer bases of sellers of different ages in any period $t + r \geq t$ to the changes in buyer assignments in period t are constant for any $r \geq 0$.

The equilibrium is efficient because each seller can extract the same surplus from their customers $(1 - \pi^0(\hat{m}_t))$ in each period as other sellers. Therefore, sellers fully internalize their relative potential contribution to surplus growth when they attract more customers. In equilibrium, buyers purchase from the sellers with the smallest consumer base among the sellers they know, which are the sellers with the greatest marginal gain in the consumer base to an increase in sales. The symmetry of the equilibrium is essential for the efficiency result because efficiency requires that sellers with the same consumer base should sell the same quantity. Otherwise, the strict concavity of the buyer accumulation function implies that it is possible for an assignment rule to improve efficiency through a sharing rule that equalizes sales between sellers with the same consumer base.

5.4 Density of prices at the lower bound of the support

In the canonical models of equilibrium price dispersion for identical goods, the density of prices for sales in the industry peaks at the lower bound of the support of the distribution of prices and gradually decreases as prices increase. For example, if the profits margin is 1 dollar per unit at the upper bound of the support, and the elasticity of demand is -1, then 1-cent decrease in the price should increase the quantity sold by 1% for the equal profit condition to hold. If a price 50 cents lower is in the support, then elasticity should be -2; if a price 75 cents lower is also in the support, then the profit margin at that price is 25 cents which implies that to maintain the equal profit condition the elasticity of demand should be -4. Therefore, the density of the prices for transactions in equilibrium must be strictly decreasing and convex, but the empirical evidence suggests that although skewed, the distribution of prices is roughly symmetric around a mode that is substantially higher than the lower bound of the support of the

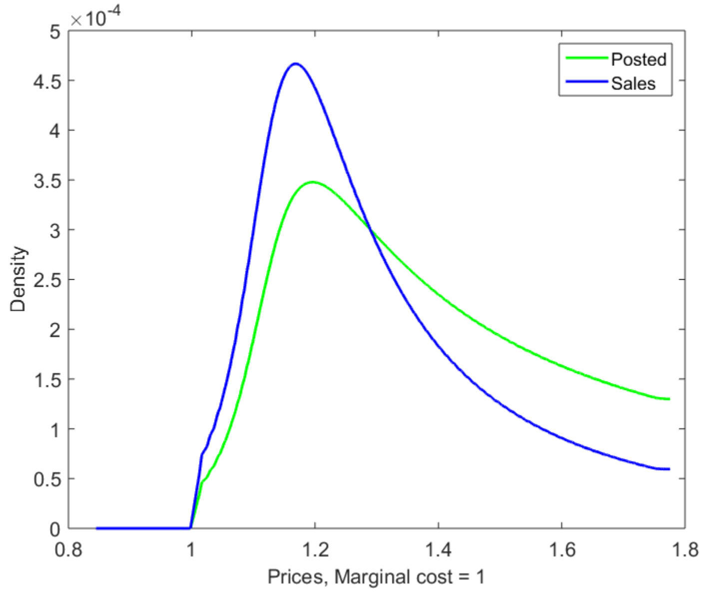


Figure 5.3: An example of the distribution of posted prices and for sales prices at $t = 20$. The parameter values are the same as in Figure 5.1.

distribution of prices (see Kaplan and Menzio (2015) [33]).

One desirable characteristic of this model is that it can generate this kind of distribution through the heterogeneity in the buyers' bases. The reasoning behind this result is that older sellers who have larger consumer bases post higher prices. However, buyers naturally are more likely to transact with sellers with larger consumer bases. By varying the entry/exit rates, the consumer base size upon entry, and the growth dynamics given by the awareness diffusion function, it is possible to change the fraction of transactions with each cohort of sellers so that the mode of transactions is at any price in the interior of the support of $\hat{F}_t$, and that the density of transactions at the lower bound of the support of $\hat{F}_t$ is arbitrarily smaller than at the mode. I show that in the context of an example with three periods in Subsection 8.6 in the Appendix.

6 Extension: Endogenous exit

As noted, empirical evidence suggests exit rates fall as sellers become older. This model can replicate this fact by allowing sellers to exit endogenously. Suppose any seller j operating in the industry can exit, and let E_t^j be the value of exiting which is a random variable distributed according to a continuous cumulative distribution function G on an interval $[\underline{E}, \overline{E}]$. The randomness represents the changing opportunities to do business outside this industry. A seller exits at the end of the period if the opportunity cost of staying in the industry is higher than the return.

The state of the industry must incorporate information regarding the population of sellers who decided to stay, given fixed entry rate α. Let $\boldsymbol{s} = (\boldsymbol{m}, \boldsymbol{\mu})$ be the state of the industry where $\boldsymbol{m} = \{m^a\}_{a=0}^{\infty}$ is the consumer base of each cohort, and $\boldsymbol{\lambda} = \{\lambda^a\}_{a=0}^{\infty}$ is the profile of exit rates by cohort. The average number of sellers that buyers are aware of is $\hat{m}(\boldsymbol{s}) = \sum_a \zeta^a m^a$. The value of a seller consumer base m is

$$v(m, \boldsymbol{s}) = \pi(m, \boldsymbol{s}) + \beta v(m^+, \boldsymbol{s}^+),$$

where $\pi(m, \boldsymbol{s})$ is the profits at the symmetric equilibrium if his or her consumer base is m, and $(m^+, \boldsymbol{s}^+)$ is the consumer base of the seller and the state of the industry in equilibrium in the next period given $(m, \boldsymbol{s})$ are the consumer base and the state of the industry in the present, respectively.

Sellers choose to exit if $v(m^+, \boldsymbol{s}^+) < E_t^j$ (the opportunity cost of staying in the industry in the next period is higher than the return). Therefore, the fraction of sellers of age a who exits the industry is given by

$$\begin{aligned}
\text{Prob}[v(m^{a+1}, \boldsymbol{m}_{t+1}) < E_t^j] &= 1 - \text{Prob}[v(m^{a+1}, \boldsymbol{s}_{t+1}) > E_t^j] \\
&= 1 - G[v(m^{a+1}, \boldsymbol{s}_{t+1})].
\end{aligned}$$

Thus, the exit rate of a seller of age a in period t is $\alpha_t^a = 1 - G[v(m^{a+1}, \boldsymbol{s}_{t+1})]$. Note that v is strictly increasing in consumer base m which implies in the equilibrium that $\alpha_t^a < \alpha_t^{a-1}$ for $a > 1$ as the consumer base is increasing with the seller's age. As the profile of exit rates $\{\lambda^a\}$ is set arbitrarily in Subsection 3.3 all the properties of the equilibrium of the model described in Section 5 hold.

7 Concluding remarks

This paper presented a theory of dynamic price formation in an industry that considers the effects of imperfect awareness and seller discovery through word-of-mouth among buyers in the market. The model has a unique symmetric equilibrium which converges over time to a stationary equilibrium. The properties of the equilibrium include the following: (1) Price dispersion occurs, as sellers randomize their price posting strategies. (2) Entrant sellers post lower prices than incumbents. (3) If seller exit rates do not change over the industry life cycle, the average markup over the marginal cost falls over time as the industry converges to a stationary equilibrium. In addition, an extension shows that the model can account for endogenous seller exit and replicate the empirical finding that the exit rate decreases with the time the seller has been operating in the industry.

Under certain assumptions, the frictions of trading vanish over time, and the equilibrium converges to the allocation that corresponds to perfect competition. This is not a trivial result as demonstrated in numerous studies.[27] These studies have shown that the choice of trading mechanisms and particular details of the environment can imply that markets might not converge to perfect competition as frictions vanish.[28] A candidate avenue for further research would be to extend the model to other trading mechanisms beyond price posting. However, classic results obtained in cooperative game theory (e.g., Debreu and Scarf (1963) [13] and Aumann (1964) [5]) and the mechanism design literature (e.g., Hammond (1979) [30]), suggest that in large economies (i.e., economies with a continuum of agents) without frictions of trading and any restrictions on specific trading mechanisms should arrive at allocations that correspond to competitive equilibria. These results appear to suggest that satisfactory models that describe frictions of trading should utilize a trading mechanism that implies in an allocation that approximates the competitive equilibrium allocation as frictions vanish, as is the case in the model presented here.

Many price indexes are constructed based on average posted prices and not on average prices for transactions. A possible application of this model is that it can estimate the average transaction price from the average posted price given sufficient data on the market's characteristics, to allow for higher precision of the estimates of price changes over time. Another candidate for further research is the extension of the awareness model developed in this paper to a more general environment that incorporates awareness

[27] Such as Diamond (1971) [14], Rubinstein and Wolinsky (1985) [47], Gale (1986a, 1986b ,1987 ,2000) [26, 25, 23, 24], Mortensen and Wright (2002) [43], Satterthwaite and Shneyerov (2007, 2008) [49, 48], Lauermann (2013) [36], and Lauermann et al. (2018) [37].

[28] In particular, Gale (2000) [26] shows that the assumption of anonymity, that is, that traders do not condition their strategies on a particular individual agent in the market, is critical for the convergence to perfect competition to occur when market frictions converge to zero. Lauermann (2013) [36] provides a general approach to the set of conditions required for random matching and bargaining games to converge to the competitive outcome as frictions vanish.

frictions on the supply side as well as on the demand side (as it is implicitly assumed that sellers have full awareness of the competitors operating to estimate their demand curves), and in a general equilibrium environment (so there is an endogenous decision for sellers to enter individual industries).

Finally, it seems important to emphasize that this study does not present a model for firm growth (unlike Luttmer (2006, 2011) [38, 39] or Dinlersoz and Yorukoglu (2012)[15]). For example, the sellers in the model can be interpreted to be individual product lines in an industry as I have considered only a market for a homogeneous good (in the sense that any buyer-seller pair can generate a unit surplus, thus, from the perspective of a buyer other sellers are perfect substitutes). Empirical studies indicate that firm usually sell many products, and that firm growth tends to involve expansion of the firm's activities into additional products beyond the expansion of their sales in markets in which they currently operate (Argente, Lee, and Moreira (2019) [2]). However, this study provides a model of industry consolidation that can be used to construct a model of firm growth that incorporates varied product portfolios.

8 Appendix

8.1 Proof of Lemma 1

Proof. To see that the assumption of independent discovery implies that the evolution of $\{\alpha^j\}_t$ does not affect the independence of awareness consider the following case: Without loss of generality, there are two sellers $1, 2$ and suppose that independence of awareness holds in some period t: Buyers who were aware of seller 1 are aware of seller 2 with probability $\alpha_t^2 \in (0, 1)$, so $\Pr(2 \in A_t^i \mid 1 \in A_t^i) = \Pr(2 \in A^i \mid 1 \notin A_t^i)$, and both are equal to $\Pr(2 \in A_t^i)$, which is equal to α_t^2. Also, suppose that $\delta > 0$ and $D(n) \in (0, 1)$ for $n \in [0, 1)$. Which means that, from period t to period $t + 1$, a positive measure of buyers who were aware of seller 2 forget seller 2 and also that a positive measure discovers seller 2.

Consider the partition the buyers into the following subsets in period $t + 1$:

$$
\begin{aligned}
R_{t+1}^1 &= \{i : 1 \in A_t^i \cap A_{t+1}^i\} \\
F_{t+1}^1 &= \{i : 1 \in A_t^i \cap (A_{t+1}^i)^c\} \\
U_{t+1}^1 &= \{i : 1 \in (A_t^i)^c \cap (A_{t+1}^i)^c\} \\
D_{t+1}^1 &= \{i : 1 \in (A_t^i)^c \cap A_{t+1}^i\}.
\end{aligned}
$$

In words, R_{t+1}^1 is the subset of buyers who were aware of seller 1 in period t and still remember 1 in period $t + 1$, F_{t+1}^1 is the subset of buyers who were aware of seller 1 in period t and forgot about 1 in period $t + 1$, U_{t+1}^1 is the subset of buyers who were unaware of seller 1 in period t and are still unaware in period $t + 1$, and D_{t+1}^1 is the subset of buyers who were unaware of seller 1 in period t and discover seller 1 by period $t + 1$.

As $\Pr(2 \in A^i \mid 1 \in A_t^i) = \Pr(2 \in A^i \mid 1 \notin A_t^i)$, and by the assumption that discovery and memory loss are independent, the fractions of buyers in each of these four subsets who discover 2 or forget 2 are the same. Thus,

$$\begin{aligned}
\Pr(2 \in A^i_{t+1} \mid i \in R^1_{t+1}) &= \Pr(2 \in A^i_{t+1} \mid i \in F^1_{t+1}) \\
&= \Pr(2 \in A^i_{t+1} \mid i \in U^1_{t+1}) \\
&= \Pr(2 \in A^i_{t+1} \mid i \in D^1_{t+1}) \\
&= \Pr(2 \in A^i_{t+1}).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Pr(2 \in A^i_{t+1} \mid 1 \in A^i_{t+1}) &= \Pr(2 \in A^i \mid 1 \notin A^i_{t+1}) \\
&= \Pr(2 \in A^i_{t+1}) \\
&= \alpha^2_{t+1}.
\end{aligned}$$

Therefore, independence holds for the period $t+1$. Independence is assumed to hold in period 0 so, by induction, it holds for all periods. $\square$

8.2 Proof of Theorem 1

I divide the proof of Theorem 1 into steps parts.

Step 1. Setup

Let K be the set of all sequences $\boldsymbol{m} = \{m^a\}_{a=0}^{\infty}$ in $\mathcal{R}_+$ such that there exists a unique $t(\boldsymbol{m})$ where $m^k > 0, \forall k \leq t(\boldsymbol{m})$ and $m^k = 0, \forall k > t(\boldsymbol{m})$. Let $v(m, \boldsymbol{m})$ be a value function that maps the seller's consumer base $m \in \mathbb{R}_+$ and the space of consumer base profiles $K^{\mathbb{N}}$ into a real number. I want to show that in the symmetric markov perfect equilibrium the value function v is unique.

Let $\mathcal{F}$ be the space of functions that map $\mathcal{R}_+ \times K$ into $\mathcal{R}_+$ that are continuous, non-decreasing and concave on $\mathcal{R}_+$. Consider the operator T that maps $\mathcal{F}$ into a functional space. For $V \in \mathcal{F}$, the operator $T(V)$ satisfies

$$T(V)(m, \boldsymbol{m}) = \sup_{p \in (-\infty, 1]} pmP(p, \boldsymbol{F}, \boldsymbol{m}) + \beta V[m^+(p, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})], \tag{8.1}$$

where the terms of the right hand side of 8.1 are defined as follows: Prices are restricted to values equal or smaller than 1 as revenues are zero with prices above one, and $\boldsymbol{F}$ is a symmetric profit maximizing strategy profile for the cohorts which are operating in the market $\boldsymbol{F} = (F^a)_{a=0}^{t(\boldsymbol{m})}$. The profile $\boldsymbol{F}$ is such that for any cohort $a \leq t(\boldsymbol{m})$, a price $p \in \text{supp}(F^a)$ satisfies

$$pm^a P(p, \boldsymbol{F}, \boldsymbol{m}) + \beta V[m^+(p, m^a, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] = T(V)(m^a, \boldsymbol{m}), \tag{8.2}$$

note that V being continuous, non-decreasing and concave implies that such profile of strategies $\boldsymbol{F}$ exists if q is continuous on p, which is true if $\boldsymbol{F}$ is a profile of atomless strategies. Finally,

$$m^+(p, m, \boldsymbol{F}, \boldsymbol{m}) = (1 - \delta)m + \Phi[mP(p, \boldsymbol{F}, \boldsymbol{m})]$$

and$\boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m}) = \{m^a\}_{a \geq 0}$ is given by

$$\{m^a\}_{a \geq 0} = \begin{cases} (1 - \delta)m^a + \Phi[m^a \int P(p, \boldsymbol{F}, \boldsymbol{m}) dF^a] & \text{if } a \leq t(\boldsymbol{m}) \\ m_s & \text{if } a = t(\boldsymbol{m}) + 1 \\ 0 & \text{if } a > t(\boldsymbol{m}). \end{cases}$$

A fixed point of T is a value function V that is consistent with equilibrium for any profile of consumer bases. We need to show that that any value function $V \in \mathcal{F}$ implies in a unique profile $\boldsymbol{F}$ of profit maximizing symmetric strategies for the sellers and show that T is a contraction.

Step 2. A function $V \in \mathcal{F}$ implies in a unique symmetric strategy profile $\boldsymbol{F}$ which satisfies 8.2.

The proof of Step 4.2 is organized in (additional) steps. Let $\boldsymbol{F}$ be a symmetric profit maximizing strategy profile generated by $V(., \boldsymbol{m})$.

Step 2.1. For each cohort $a \in \{0, \ldots, t\}$, F^a is atomless. Hence, q is continuous on p so $\boldsymbol{F}$ exists given V continuous, non-decreasing and concave on $\mathcal{R}_+$.

Proof. To note that F^a is atomless suppose that it has an atom at some $p > 0$, then as V is non-decreasing, it is easy to see that sellers have incentive to undercut by posting a price $p - \epsilon$ for some $\epsilon > 0$. A contradiction. □

Step 2.2. For cohorts $a, b \in \{1, \ldots, t\}$, if $m^a < m^b$ then the supports of F^a and F^b have disjoint interiors and $\max[\text{supp}(F^a)] \leq \min[\text{supp}(F^b)]$.

Proof. Let $F(x)$ be the equilibrium price posting strategy of a seller with consumer base x. Consider two sellers with consumer bases m, m' such that $m' \neq m$. Suppose $\text{Int}\{\text{supp}[F(m)]\} \cap \text{Int}\{\text{supp}[F(m')]\} \neq \emptyset$, then the profile of price posting distributions $\boldsymbol{F}$ must make seller with consumer base m indifferent between posting prices in $\text{Int}\{\text{supp}[F(m)]\} \cap \text{Int}\{\text{supp}[F(m')]\}$, however, due to the concavity of V and strict concavity of Φ in respect to sales quantity, a seller with consumer base m' will not be indifferent in posting prices in the same support as a seller with consumer base m as the equal profit conditions cannot not hold for both since

$$\frac{\Phi[m'P(p, \boldsymbol{F}, \boldsymbol{m})]}{m'} \neq \frac{\Phi[mP(p, \boldsymbol{F}, \boldsymbol{m})]}{m},$$

while relative profit margins in the present are the same. Which is contradiction with $\boldsymbol{F}$ being consistent with 8.2.

To see that the seller with the larger consumer base posts higher prices suppose that $m' > m$ and that there is some price p in $\text{supp}[F(m')]$ which is strictly smaller than any price in $\text{supp}[F(m)]$. $\boldsymbol{F}$ is such that the seller with consumer base m' strictly prefer posting prices on $\text{supp}[F(m')]$ over $\text{supp}[F(m)]$, combined with the concavity of V and strict concavity of Φ it implies that the seller with consumer base m obtain higher payoffs by posting p rather than prices in $\text{supp}[F(m)]$. A contradiction with $\boldsymbol{F}$ being consistent with 8.2. □

Step 2.3. For each cohort a, the support of F^a is a closed interval.

Proof. Suppose that the support of F^a is not convex therefore there is an open interval $(\underline{p}, \overline{p}) \nsubseteq \text{supp}(F^a)$ such that $\exists p, p' \in \text{supp}(F^a)$ and $p < \underline{p}$ while $\overline{p} < p'$. Then, it is simple to check that the equal profit condition to support a mixed strategy implies that posting a price $z \in (\underline{p}, \overline{p})$ will yield strictly higher profits than posting p. A contradiction. □

Step 2.4. The maximum upper bound of the supports of the strategy profile $\boldsymbol{F}$ is 1, that is $\overline{p} = \max\{\text{supp}(F^a)_{a=0}^{t(\boldsymbol{m})}\}$ is equal to 1.

Proof. Suppose that $\overline{p} \neq 1$. Clearly, $\overline{p} \leq 1$ since no seller has incentive to post prices higher than buyers are willing to pay, therefore $\overline{p} \neq 1$ implies that $\overline{p} < 1$. Then, probability of sale $P(p, \boldsymbol{F}, \boldsymbol{m})$ is the same if $p = 1$ or $p = \overline{p}$, therefore all sellers find that posting 1 strictly preferable than to post $\overline{p}$. A contradiction with $\boldsymbol{F}$ being profit maximizing. □

Step 2.5. Then the profile of equilibrium strategies $\{F^a\}_a$ is unique.

Proof. Let's order the $t(\boldsymbol{m})$ active seller cohorts by consumer base size with age indexed by $a_s \in \{a_{s_0}, a_{s_1}, \ldots, a_{s_{t(\boldsymbol{m})}}\}$ where $m^{a_{s+1}} > m^{a_s}$ for all s. If two cohorts have the same consumer base size then I group then into the same index a_s.

The upper bound 1, and the continuity, monotonicity of q and V on m implies that for each cohort a, for a given upper bound $\overline{p}^a$, the equal profit condition8.2 for prices in the support of the equilibrium strategy plus the fact the supports are connected, disjoint and monotone increasing on the seller's consumer base implies in a unique support $[\underline{p}^a, \overline{p}^a]$ and in a unique c.d.f. of prices σ^a that maintains the equal profit condition at $[\underline{p}^a, \overline{p}^a]$. If the support $[\underline{p}^{a_s}, \overline{p}^{a_s}]$ is determined, then $[\underline{p}^{a_{s-1}}, \overline{p}^{a_{s-1}}]$ is also determined. Note that Step 4.2.2 implies that $\overline{p}^{a_{s_{t(\boldsymbol{m})}}} = 1$, and following this reasoning the profile of supports $\{[\underline{p}^a, \overline{p}^a]\}_a$ is uniquely determined and hence the profile of profit maximizing strategies $\boldsymbol{\sigma}$ given V. $\square$

Step 3. To show that the equilibrium is unique it suffices to show that T maps $\mathcal{F}$ into itself, that T is a contraction, and that its fixed point v implies in a unique symmetric strategy profile.

Proof. First, I want to show that T maps $\mathcal{F}$ into itself, to show that I apply the maximum theorem: By previous steps there is a unique strategy profile $\boldsymbol{F}$ determined by V and $\boldsymbol{m}$. Note that the present level of profits $\Pi(p, m, \boldsymbol{F}, \boldsymbol{m}) = pmP(p, \boldsymbol{F}, \boldsymbol{m})$ is continuous in m and p. To apply the maximum theorem it remains to show that there exists a continuous and compact valued correspondence $C(m)$ such that the argument p for the right hand side of 8.1 is in $C(m) = [\underline{p}(m), 1]$, that is, there is a continuous function $\underline{p}(m)$ such that given a value function V prices that solve the right hand side of 8.1 are bounded below by $\underline{p}(m)$.

Consider a function $p(m)$ such that $p(m)m = -V[m^+(p(m), m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})]$. Then $C(m) = [\underline{p}(m), 1]$ is compact valued and as V is continuous on m then $C(m)$ continuous. Any price $p < \underline{p}(m)$ implies

$$\begin{aligned}&\Pi(p, m, \boldsymbol{F}, \boldsymbol{m}) + \beta V[m^+(p(m), m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] \\ &= p(m)m + \beta V[m^+(p(m), m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] < 0,\end{aligned}$$

but for $p' \geq 0$,

$$\Pi(p', m, \boldsymbol{F}, \boldsymbol{m}) + V[m^+(p', m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] \geq 0.$$

Therefore, any price that solves the right hand side 8.1 is in $C(m)$ and hence we can write the operator 8.1 as

$$T(V)(m) = \max_{p \in C(m)} \Pi(p, m, \boldsymbol{F}, \boldsymbol{m}) + \beta V[m^+(p, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})], \tag{8.3}$$

and, as Π is continuous in m and p and C is compact valued and continuous, by the maximum theorem, $T(V)$ is continuous. Note that $mP(p, \boldsymbol{F}, \boldsymbol{m})$ is concave and strictly increasing in m, V is concave and non-decreasing in m and $m^+(p, m)$. Hence, $T(V)$ is strictly increasing and strictly concave on m, and since $\Pi(p, m, \boldsymbol{F}, \boldsymbol{m}) \leq N$ therefore $T(V) \in \mathcal{F}$ and thus T maps $\mathcal{F}$ into itself.

To show that T is a contraction it suffices to show that it satisfies Blackwell's sufficiency conditions for a contraction: monotonicity and discounting.

Monotonicity: Let $f, g \in \mathcal{F}$ such that $f(m) \geq g(m)$ for all $m \in \mathbb{R}_+$. I want to show that $T(f) \geq T(g)$, let $m \in \mathbb{R}_+$, $\{\underline{p}^{a,f}, \overline{p}^{a,f}\}$ and $\{\underline{p}^{a,g}, \overline{p}^{a,g}\}$, then there is some cohort a such that either $m \in [m^{a-1}, m^a)$ or $m < m^0$, then

$$\begin{aligned}
T(f)(m) &= \Pi(\underline{p}^{a,f}, m, \boldsymbol{F}, \boldsymbol{m}) + \beta f[m^+(\underline{p}^{a,f}, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] && (8.4)\\
&\geq \Pi(\underline{p}^{a,g}, m, \boldsymbol{F}, \boldsymbol{m}) + \beta f[m^+(\underline{p}^{a,g}, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] && (8.5)\\
&\geq \Pi(\underline{p}^{a,g}, m, \boldsymbol{F}, \boldsymbol{m}) + \beta g[m^+(\underline{p}^{a,g}, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] && (8.6)\\
&= T(g)(m). && (8.7)
\end{aligned}$$

where the inequality 8.5 follows from 8.3 and Φ being concave.

Discounting: Let $f \in \mathcal{F}(\mathbb{R}_+ \times K^{\mathbb{N}})$ and $b > 0$, for $m \in \mathbb{R}_+$,

$$\begin{aligned}
T(f+b)(m) &= \Pi(\underline{p}^{a,f}, m, \boldsymbol{F}, \boldsymbol{m}) + \beta\{f[m^+(\underline{p}^{a,f}, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] + b\} && (8.8)\\
&= \Pi(\underline{p}^{a,f}, m, \boldsymbol{F}, \boldsymbol{m}) + \beta f[m^+(\underline{p}^{a,f}, m, \boldsymbol{F}, \boldsymbol{m}), \boldsymbol{m}^+(\boldsymbol{F}, \boldsymbol{m})] + \beta b && (8.9)\\
&= T(f)(m) + \beta b && (8.10)\\
&< T(f)(m) + b. && (8.11)
\end{aligned}$$

Therefore, T is a contraction and hence has a unique fixed point. It is easy to check that the fixed point of T is a value function v that is consistent with the profile of supports$\{[\underline{p}^{a,v}, \overline{p}^{a,v}]\}$ for the mixed pricing strategies in the stationary equilibrium.

Finally, to show that the fixed point of T implies in a unique strategy profile note that for a v fixed point of T then, as $T(V)$ for $V \in \mathcal{F}$ is is continuous, strictly increasing, and strictly concave then the fixed point v is continuous, strictly increasing, and strictly concave in m. By Step 4.2, v is continuous, strictly increasing, and strictly concave in m implies there is a unique strategy profile for each consumer base profile. □

Step 4. The fixed point v is consistent with the sellers' sequential problem: The recursive problem 4.7 and the sequential problem 4.6 of the seller are equivalent.

Let $(\boldsymbol{F}, \boldsymbol{m})$ be the pair of sequences of strategy profiles and consumer base profiles in symmetric equilibrium. Note that $P(p, \boldsymbol{F}, \boldsymbol{m})$ is bounded above by 1 and the properties of Φ from Assumption 1 imply that for some $\underline{m} > 0$ large enough, $m_{t+1}/m_t \in [1, 2-\beta]$ if $m_t \geq \underline{m}$. Therefore, the present value of profits for a seller of age a are bounded above by

$$\sum_{t \geq a} \beta^{t-a}(2-\beta)^t \underline{m} = \left[\frac{(2-\beta)^a \underline{m}}{1-\beta(2-\beta)}\right] = \overline{\Pi}^a < \infty$$

for any feasible sequence of consumer bases and any profile of pricing strategies played by the other sellers.

Hence, for a seller of age a, a solution $\{p_k^*\}_{k=0}^{\infty}$ to the sellers' sequential problem 4.6 and associated sequence of consumer bases $\{m_k^*\}$, the present value of the seller's stream of profits satisfies

$$\sum_{k=0}^{\infty} \beta^k \Pi(p_k^*, m_k^*, \boldsymbol{F}_{t_k}, \boldsymbol{m}_{t_k}) \in [0, \overline{\Pi}^a].$$

Therefore, the conditions for applying Theorems 4.2-4.4 of Stokey and Lucas (1989) [53] are satisfied which imply that the sequential problem 4.6 in period t and the functional equation 4.7 are equivalent if posting prices according to $\boldsymbol{F} = \{\boldsymbol{F}_t\}_t$ is profit maximizing for the sellers.

Step 5. The equilibrium has the following features:

(i) The profile of consumer bases is deterministic and stationary: there is a profile of consumer bases $\{m^a\}_a$ for each seller age a, $m_t^a = m^a, \forall t$.

(ii) For each period t, for each seller age $a \leq t$ the pricing strategyF_t^a converges uniformly to a cumulative distribution F^a as $t \to \infty$.

(iii) There is a $\overline{\delta} \in [0, 1)$ such that for any memory depreciation parameter $\delta \leq \overline{\delta}$ then $a > b$ implies $m^a > m^b$.

Proof. Consider a profile of consumer bases for each cohort $\{m^a\}_{a \geq 0}$, given the seller replacement rate λ and exit rates $\{\lambda^a\}_a$, then by equation 4.5 implies that $\overline{m}^a = [\lambda \prod_{i \leq a-1}(1 - \lambda^i)]m^a$ for each $a \in \{0, 1, 2, 3, \ldots\}$.

Let q^a be the quantity sold by a seller of cohort a, it is given by $q^a = m^a P(F^a_t, \boldsymbol{F}, \boldsymbol{m})$. The quantity sold by a seller of cohort a is the consumer base minus buyers aware of sellers with smaller consumer bases (which is the set of seller cohorts $\{b \leq t : m^b < m^a\}$) as these sellers post lower prices than cohort a. Cohorts with larger consumer bases post higher prices so they do not enter into $P(F^a_t, \boldsymbol{F}, \boldsymbol{m})$. Against other sellers of cohort a, since all sellers post prices according to the same continuous distribution by Assumption 2 half of the time it undercuts a competing seller of the same cohort and half of the time it does not. Therefore, the probability of sale $P(F^a_t, \boldsymbol{F}, \boldsymbol{m})$ is given by

$$\begin{aligned} P(F^a_t, \boldsymbol{F}, \boldsymbol{m}) &= \prod_{\{b \leq t : m^b < m^a\}} \pi^0(\overline{m}^b) \times \left\{ \int_{\underline{p}^a_t}^{\overline{p}^a_t} \sum_{n=0}^{\infty} \pi^n(\overline{m}^a)[1 - F^a_t(p)]^n dF^a_t(p) \right\} \\ &= \exp\left(- \sum_{\{b \leq t : m^b < m^a\}} \overline{m}^b \right) \int_{\underline{p}^a_t}^{\overline{p}^a_t} \exp(-\overline{m}^a F^a_t(p)) dF^a_t(p) \\ &= \exp\left(- \sum_{\{b \leq t : m^b < m^a\}} \overline{m}^b \right) \int_0^1 \exp(-\overline{m}^a x) dx. \end{aligned}$$

Thus, q^a satisfies

$$q^a = m^a \exp\left(- \sum_{\{b \leq t : m^b < m^a\}} \overline{m}^b \right) \int_0^1 \exp(-\overline{m}^a x) dx. \tag{8.12}$$

Therefore, the quantities sold do not depend on the specific forms of the pricing strategies $\{F^a_t\}_a$ but only on the profile of consumer bases and profile of seller entry and the exit rates. Since sellers enter with the consumer base $m_s > 0$, the consumer base of sellers of age 1 is

$$m^1 = (1-\delta)m_s + \Phi(q^0). \tag{8.13}$$

$$= (1-\delta)m_s + \Phi[m_s \int_0^1 \exp(-\lambda m_s x)dx]. \tag{8.14}$$

For sellers of age 2, their consumer base m^2 then is given by m^1. By induction, for each $a \in \{0, 1, 2, 3, \ldots\}$, m^{a+1} is given by m^a.

As the profile of active sellers operating converges to a stationary distribution, it is easy to see that the profile of consumer bases also converges to some $\boldsymbol{m}$. Therefore, the value function $V(m, \boldsymbol{m})$ converges to a function $v(m)$, a function of only m. That implies that the strategies of individual sellers only depend on their consumer bases, which are $\{m^a\}_a$. Finally, note that $m^{a+1} = (1-\delta)m^a + \Phi(q^a)$, then for $\delta \geq 0$ low enough, $m^{a+1} > m^a$ for all $a \geq 0$. Which concludes the proof of Theorem 1. □

8.3 Proof of Theorem 2

Want to show that S_t first order stochastically dominates S_{t+1}. The proof is divided into steps.

Step 1. The expected price paid by buyers is distributed according to S_t which can be written as

$$\begin{aligned}
S_t(p) &= 1 - \sum_{n=0}^{\infty} P(|A_t^i| = n)[1 - \hat{F}_t(p)]^{n+1} \\
&= 1 - \sum_{n=0}^{\infty} \frac{(\hat{m}_t)^n \exp(-\hat{m}_t)}{n!}[1 - \hat{F}_t(p)]^{n+1} \\
&= 1 - [1 - \hat{F}_t(p)] \exp(-\hat{m}_t) \sum_{n=0}^{\infty} \frac{\left[\hat{m}_t[1 - \hat{F}_t(p)]\right]^n}{n!} \\
&= 1 - [1 - \hat{F}_t(p)] \exp(-\hat{m}_t \hat{F}_t(p)).
\end{aligned} \tag{8.15}$$

Step 2. Given Assumptions 1 and 2, in symmetric equilibrium $\hat{m}_t$ is strictly increasing in the "age" t of the industry, that is $\hat{m}_{t+1} > \hat{m}_t$.

Proof. The average consumer base parameter is given by

$$\hat{m}_t = \sum_{a=0}^{t} \overline{m}_t^a,$$

note that for each $a \in \{0, 1, \ldots, t\}$,

$$\begin{aligned}\overline{m}_t^a &= \left[\lambda \prod_{i=0}^{a}(1-\lambda^a)\right] m_t^a \\ &= \left[\lambda \prod_{i=0}^{a}(1-\lambda^a)\right] m^a.\end{aligned}$$

Therefore there exists a stationary profile of consumer base parameters $\{\overline{m}^a\}$ such that for each t and $a \leq t$, $\overline{m}_t^a = \overline{m}^a$. Note that $m_c > 0$, $\delta \in (0,1)$, and $\Phi(q) \geq 0$ imply that $m^a > 0$ for all a. Therefore $\hat{m}_t$ is strictly increasing in t. □

Step 3. $\partial v(m, \boldsymbol{m})/\partial m$ is non-increasing in each element of $\boldsymbol{m}$ that is for each $a \in \{0, 1, 2, \ldots\}$ an increase in m^a decreases $\partial V(m, \boldsymbol{m})/\partial m$ if $\hat{m}\hat{F}(p)$ is non-decreasing in each element of $\boldsymbol{m}$**.**

Proof. Let K be the set of all sequences $\boldsymbol{m} = \{m^a\}_{a=0}^{\infty}$ in $\mathcal{R}_+$ such that there exists a unique $t(\boldsymbol{m})$ where $m^k > 0, \forall k \leq t(\boldsymbol{m})$ and $m^k = 0, \forall k > t(\boldsymbol{m})$. Let $\mathcal{F}$ be the set of functions $V : \mathcal{R}_+ \times K \to \mathcal{R}_+$ that are differentiable, strictly increasing, strictly concave on $\mathcal{R}_+$, and such that $\partial V(m, \boldsymbol{m})/\partial m$ is non-increasing on any element of $\boldsymbol{m}$.

Consider the operator T that maps $\mathcal{F}$ into a functional space. For $V \in \mathcal{F}$, the operator $T(V)$ satisfies

$$T(V)(m, \boldsymbol{m}) = \sup_{p \in (-\infty, 1]} pmP(p, \tilde{\boldsymbol{F}}, \boldsymbol{m}) + \beta V[m^+(p, m, \tilde{\boldsymbol{F}}, \boldsymbol{m}), \boldsymbol{m}^+(\tilde{\boldsymbol{F}}, \boldsymbol{m})], \tag{8.16}$$

where

$$m^+(p, m, \tilde{\boldsymbol{F}}, \boldsymbol{m}) = (1-\delta)m + \Phi[mP(p, \tilde{\boldsymbol{F}}, \boldsymbol{m})]$$

and $\boldsymbol{m}^+(\tilde{\boldsymbol{F}}, \boldsymbol{m}) = \{m^a\}_{a\geq 0}$ is given by

$$\{m^a\}_{a\geq 0} = \begin{cases} (1-\delta)m^a + \Phi[m^a \int P(p, \tilde{\boldsymbol{F}}, \boldsymbol{m}) dF^a] & \text{if } a \leq t(\boldsymbol{m}) \\ m_s & \text{if } a = t(\boldsymbol{m}) + 1 \\ 0 & \text{if } a > t(\boldsymbol{m}), \end{cases}$$

and $\hat{m}\hat{F}(p) = \sum_{a=0}^{t(\boldsymbol{m})} \overline{m}^a \tilde{F}^a(p)$ is assumed to be consistent with profit maximization (i.e. for $a \leq t(\boldsymbol{m})$ any price on the support of $\tilde{F}^a$ solves 8.16).

T maps $\mathcal{F}$ into the set of functions are differentiable (as $\partial pmP(p, \tilde{\boldsymbol{F}}, \boldsymbol{m})/\partial m = pP(p, \tilde{\boldsymbol{F}}, \boldsymbol{m})$), and as in the proof of Theorem 1, strictly increasing, strictly concave on $\mathcal{R}_+$, and it has a fixed point v. It remains to show that $\hat{m}\hat{F}(p)$ is non-decreasing in any element of $\boldsymbol{m}$ so that T maps $\mathcal{F}$ into itself. $\square$

Step 4. The object $\hat{m}\hat{F}(p)$ is non-decreasing in any element of $\boldsymbol{m}$.

Proof. Suppose not then there is some $p \in [\underline{p}_t, \overline{p}_t)$ such that $\hat{m}_t\hat{F}_t(p) > \hat{m}_{t+1}\hat{F}_{t+1}(p)$. First note that$\hat{m}_t\hat{F}_t(p) > \hat{m}_{t+1}\hat{F}_{t+1}(p)$ can be written as

$$\sum_{a=0}^{t} \overline{m}^a F_t^a(p) > \sum_{a=0}^{t+1} \overline{m}^a F_{t+1}^a(p). \tag{8.17}$$

In addition, $\hat{m}_{t+1} > \hat{m}_t$, $\overline{p}_t^t = \overline{p}_{t+1}^{t+1} = 1$, and $\hat{F}_t(1) = \hat{F}_{t+1}(1) = 1$ imply that $\hat{m}_t\hat{F}_t(\overline{p}_t^t) < \hat{m}_{t+1}\hat{F}_{t+1}(\overline{p}_t^t)$. Since $\boldsymbol{F}_t$ is a profile of continuous functions thus by the intermediate value theorem there exists a price $p' \in (p, \overline{p}_t^t)$ such that

$$\hat{m}_t\hat{F}_t(p') = \hat{m}_{t+1}\hat{F}_{t+1}(p').$$

The intermediate value theorem implies that exists a cohort a such that $\hat{m}_t\hat{F}_t(p) > \hat{m}_{t+1}\hat{F}_{t+1}(p)$ for some $p \in [\underline{p}_t^a, \overline{p}_t^a)$ and $\hat{m}_t\hat{F}_t(p') = \hat{m}_{t+1}\hat{F}_{t+1}(p')$ for some $p' \in (p, \overline{p}_t^a]$. By Theorem 1, δ small enough implies that older cohorts post higher prices than younger cohorts. Therefore,

$$\hat{m}_t\hat{F}_t(p') = \sum_{b=0}^{a-1} \overline{m}^b + \overline{m}^a F_t^a(p) = \sum_{b=0}^{a-1} \overline{m}^b + \overline{m}^a F_{t+1}^a(p) = \hat{m}_{t+1}\hat{F}_{t+1}(p')$$

which implies $p' \in [\underline{p}_{t+1}^a, \overline{p}_{t+1}^a]$.

Therefore, equal profit condition on the interior of the support of cohort a in periods t and $t+1$ imply

$$
\begin{aligned}
& T(V)(m^a,\boldsymbol{m}_t) = \Pi(p,m^a,\boldsymbol{F}_t,\boldsymbol{m}_t) + \beta V[m^+(p,m^a,\boldsymbol{F}_t,\boldsymbol{m}_t),\boldsymbol{m}_{t+1}] \qquad (8.18)\\
& = \Pi(p',m^a,\boldsymbol{F}_t,\boldsymbol{m}_t) + \beta V[m^+(p',m^a,\boldsymbol{F}_t,\boldsymbol{m}_t),\boldsymbol{m}_{t+1}]\\
& \Pi(p,m^a,\boldsymbol{F}_{t+1},\boldsymbol{m}_{t+1}) + \beta V[m^+(p,m^a,\boldsymbol{F}_{t+1},\boldsymbol{m}_{t+1}),\boldsymbol{m}_{t+2}] \qquad (8.19)\\
& \leq \Pi(p',m^a,\boldsymbol{F}_{t+1},\boldsymbol{m}_{t+1}) + \beta V[m^+(p',m^a,\boldsymbol{F}_{t+1},\boldsymbol{m}_{t+1}),\boldsymbol{m}_{t+2}] = T(V)(m^a,\boldsymbol{m}_{t+1}).
\end{aligned}
$$

Note that

$$\Pi(p,m^a,\boldsymbol{F}_t,\boldsymbol{m}_t) = p[m\exp(-\hat{m}_t\hat{F}_t(p))]$$

thus the gain in sales (the variation of $m\exp(-\hat{m}_t\hat{F}_t(p)))$ is higher in period t over period $t+1$ hence these two conditions imply that $\partial V/\partial m$ is strictly increasing in the consumer base of a cohort $t+1$. A contradiction with $\partial V(m,\boldsymbol{m})/\partial m$ being non-increasing in each element of $\boldsymbol{m}$. □

Step 5. S_t first order stochastically dominates S_{t+1}.

Proof. Therefore, at the fixed point v, $\hat{m}\hat{F}(p)$ is non-decreasing in each element of $\boldsymbol{m}$. Which implies that $\hat{m}_t\hat{F}_t(p)$ first order stochastically dominates $\hat{m}_{t+1}\hat{F}_{t+1}(p)$. The fact that $\hat{m}_t\hat{F}_t(p)$ first order stochastically dominates $\hat{m}_{t+1}\hat{F}_{t+1}(p)$ combined with equation 8.15 implies that the distribution of prices for sales S_t first order stochastically dominates S_{t+1}, so prices for sales must be decreasing over time. Which concludes the proof. □

8.4 Proof of Theorem 3

Proof. A constant exit rate of λ implies that the average age of the sellers operating in the industry converges to $(1-\lambda)/\lambda$ as $t\to\infty$. Let $\lambda^a\to 0,\forall a$ and by similar argument the average age of operating sellers converges to infinity as $t\to\infty$.

Let $q^a(\boldsymbol{\lambda}), m^a(\boldsymbol{\lambda})$ be, respectively, the quantity sold and the consumer base of a seller of age a in the equilibrium corresponding to a profile of seller exit rates $\boldsymbol{\lambda}=\{\lambda^a\}_{a=0}^{\infty}$. Consider an awareness diffusion function that satisfies Assumption 2 and is such that $\Phi(x) > yx$ for $x\in[0,z]$ for some $y>0$ and $x>0$. Then the equilibrium consumer base of cohort a satisfies

$$\lim_a m^a(\boldsymbol{\lambda}) = \infty \tag{8.20}$$

for all $\boldsymbol{\lambda}$ in $(0,1)$. To see that suppose, taking a subsequence if necessary, that $\lim_a m^a(\boldsymbol{\lambda}) = c$ for some $c > 0$. Since $m^a(\boldsymbol{\lambda}) \leq c$ for all cohorts, then quantity sold by a seller is at least

$$m^a(\boldsymbol{\lambda}) \exp(-c) \geq m_s \exp(-c) > 0,$$

as the fraction of captive buyers is at least as large as $\exp(-c)$ and prices posted in equilibrium are never higher than 1. That is, $q^a(\alpha)$ is bounded below by $m_s \exp(-c)$ which implies that

$$m^{a+1}(\boldsymbol{\lambda}) - m^a(\boldsymbol{\lambda}) \geq y \min\{z, m_s \exp(-c)\} > 0$$

which is a contradiction with $m^a(\boldsymbol{\lambda})$ converging to $c > 0$. Therefore, 8.20 is true.

Since $m^a(\boldsymbol{\lambda})$ diverges to infinity for every $\boldsymbol{\lambda} = \{\lambda^k\}_{k=0}^{\infty}, \lambda^k \in (0,1)$ then let $\boldsymbol{\lambda} \downarrow 0$ mean that $\lambda^k \downarrow 0$ for every k, then

$$\lim_{\boldsymbol{\lambda} \downarrow 0} \lim_{a \to \infty} m^a(\boldsymbol{\lambda}) = \infty.$$

Let $\hat{m}_t(\boldsymbol{\lambda})$ be the average equilibrium consumer base in period t, that is $\hat{m}_t(\boldsymbol{\lambda}) = \sum_{a=0}^{t} \overline{m}^a(\boldsymbol{\lambda})$. Since Φ is strictly concave, Jensen's inequality implies that $\hat{m}_t(\boldsymbol{\lambda})$ satisfies

$$\lim_{t \to \infty} \hat{m}_t(\boldsymbol{\lambda}) > m^{\hat{a}(\boldsymbol{\lambda})}(\boldsymbol{\lambda}), \tag{8.21}$$

where $\hat{a}(\boldsymbol{\lambda}) = \arg\min\{|x - \sum_{a=0}^{\infty} a\zeta^a(\boldsymbol{\lambda})| : x \in \mathbb{N} \cup \{0\}, x \leq \sum_{a=0}^{\infty} a\zeta^a(\boldsymbol{\lambda})\}$. That is, the average consumer base is larger than the consumer base of the seller of closest age that is not higher to the average age operating in the industry in the stationary equilibrium (when the age of the whole industry is infinity). As that $\hat{a}(\boldsymbol{\lambda}) \to \infty$ as $\boldsymbol{\lambda} \downarrow 0$ then equation 8.20 implies that $m^{\hat{a}(\boldsymbol{\lambda})}(\boldsymbol{\lambda})$ diverges to infinity when $\boldsymbol{\lambda} \downarrow 0$ and therefore the average consumer base diverges to infinity as $t \to \infty$ and as seller exit rates converge to 0, that is

$$\lim_{\boldsymbol{\lambda} \downarrow 0} \lim_{t \to \infty} \hat{m}_t(\boldsymbol{\lambda}) = \infty.$$

Let $\hat{F}_t(\boldsymbol{\lambda})$ be the average c.d.f. of prices posted in the equilibrium of period t if seller exit rate is $\boldsymbol{\lambda}$. If the average buyer awareness diverges to infinity then

$$\lim_{\boldsymbol{\lambda} \downarrow 0} \lim_{t \to \infty} \hat{F}_t(\boldsymbol{\lambda}, p) = 1, \forall p > 0,$$

as the fraction of captive buyers is given by $\exp[-\hat{m}_t(\boldsymbol{\lambda})]$ the incentives to undercut competing sellers imply that, as $\hat{m}_t(\boldsymbol{\lambda})$ increases to infinity, the probability of undercutting any strictly positive price increases to 1. Which concludes the proof. $\square$

8.5 Proof of Theorem 4

Proof. First note that any assignment rule R can be implemented by a pricing strategy $p(a) = R(a)$ for each seller age a. Second, note any efficient assignment rule never implies that buyers will not shop at any sellers as future consumer bases are monotone increasing in sales quantity and present surplus is reduced if some buyers do not shop. Hence, in optimal assignment, if the buyer knows at least one seller the buyer is going to purchase the good. Thus, the realized surplus in the industry in a given period is just 1 minus the population of buyers who do not know a single seller, $\pi^0(\hat{m})$. As consumer bases are durable, a marginal increase in consumer base of cohort a in period t does imply in the marginal increase of $(1-\delta)$ for cohort $a+1$ in period $t+1$. Which implies that the marginal rates of substitution of the consumer bases of sellers of different ages to the changes in buyer assignment in period t are constant for any period $t+r$ for $r \geq 0$. As there is no time inconsistency in the maximization of the present value of market surpluses, therefore, for any discount factor $\beta \in (0,1)$, an efficient profile of sales among sellers is such that it minimizes $\pi^0(\hat{m})$, which implies that it maximizes the average size of the seller's consumer bases in each period.

Therefore, given a profile of consumer bases $\{m_t^a\}_a$ in period t, efficiency in that period is obtained when the profile of sales quantities $\{q_t^a\}_a$ implied by R maximizes $\sum_{a=0}^{t+1} \zeta^a m_{t+1}^a$: sellers with smaller consumer bases have greater potential for growth than sellers with larger consumer bases, therefore, the efficient assignment of buyers among sellers is such that buyers purchase from a type a seller over a' as long as, by inverting the assignment rule, the marginal increase in that type's consumer base is higher than the marginal increase in the another type a' consumer base.

In equilibrium between any two sellers, the seller with smaller consumer base will post lower prices than the seller with the larger consumer base with probability 1. This is equivalent to an assignment rule R_t such that $R_t(a) < R_t(a')$ for two cohorts a, a' such that $a' > a$ and $R(a) \leq 1$. Additionally, in equilibrium, smaller sellers are always smaller than larger sellers (as $m_t^a < m_t^{a+1}, \forall a \in \{1, \ldots, t-1\}, \forall t \geq 2$). Thus, the marginal increase in their next period consumer bases to additionally assigned buyers is always strictly higher than for larger sellers, which implies that the assignment rule R_t is efficient. $\square$

8.6 A Three-Period Example

To illustrate the property 5.4 of the equilibrium of the model consider a simplified environment with three periods: $0, 1$ and 2. Period 2 is the last trading "day" in the industry, so there are only three types $a \in \{0, 1, 2\}$. Suppose there is an entry rate $\lambda = 1/3$ and all sellers exit in the last period. In period 2 the equilibrium is static (as in Butters (1977) [10]): All cohorts post prices according to a distribution F_2 which satisfies the equal profit condition

$$p \exp[-\hat{m}_2 F_2(p)] = \exp(-\hat{m}_2),$$

which implies that the value $v_2(m, \boldsymbol{m})$ of a consumer base m in period 2 given a profile of consumer bases $\boldsymbol{m} = (m_2^a)_{a=0}^2$ is

$$v_2(m, \boldsymbol{m}) = m \exp(-\hat{m}_2).$$

Therefore, in period 1 a seller of age $a \in \{0, 1\}$ has to solve the following profit maximization problem

$$\max_{p \leq 1} p m_1^a P(p, \boldsymbol{F}_1, \boldsymbol{m}_1) + \beta v(m_2^a, \boldsymbol{m}_2) \tag{8.22}$$

$$\text{s.t. } m_2^a = m_1^a + \Phi[m_1^a P(p, \boldsymbol{F}_1, \boldsymbol{m}_1)], \tag{8.23}$$

which can be written as

$$\max_{p \leq 1} p m_1^a P(p, \boldsymbol{F}_1, \boldsymbol{m}_1) + \beta \exp(-\hat{m}_2) \left\{ m_1^a + \Phi[m_1^a P(p, \boldsymbol{F}_1, \boldsymbol{m}_1)] \right\}. \tag{8.24}$$

As there is only one cohort and all sellers post prices according to the same distribution, the quantity sold in period 0 is given by

$$q_0^0 = m_s \int_0^1 [1 - \exp(-\lambda m_s x)] dx.$$

The consumer bases in period 1 are

$$(m_1^0, m_1^1) = [m_s, m_s + \Phi(q_0^0)].$$

In period 1 there are two cohorts, and cohort of sellers born in date 0 has consumer base m_1^1 strictly larger than cohort born in date 1. Therefore, the concavity of Φ implies that the equilibrium distributions of prices have supports $[\underline{p}_1^0, \overline{p}_1^0]$ and $[\underline{p}_1^1, \overline{p}_1^1]$ with $\overline{p}_1^1 = 1$ and $\underline{p}_1^1 = \overline{p}_1^0$.

Proposition 1. *In period* 1 *equilibrium consists of a unique pair of distributions of prices* (F_1^0, F_1^1) *that satisfies, respectively, the following equal profit conditions*

$$\begin{aligned} &p \exp\left[-\lambda m_s F_1^0(p)\right] + \frac{\beta}{m_s} \exp(-\hat{m}_2) \left((1-\delta)m_s + \Phi\left\{m_s \exp\left[-\lambda m_s F_1^0(p)\right]\right\}\right) \\ &= \underline{p}_1^0 + \frac{\beta}{m_s} \exp(-\hat{m}_2) \left\{(1-\delta)m_s + \Phi[m_s]\right\}. \end{aligned}$$

and

$$\begin{aligned} &p \exp\left[-\lambda m_s - \overline{m}_1^1 F_1^1(p)\right] + \frac{\beta}{m_s} \exp(-\hat{m}_2) \left((1-\delta)m_1^1 + \Phi\left\{m_1^1 \exp\left[-\lambda m_s - \overline{m}_1^1 F_1^1(p)\right]\right\}\right) \\ &= \underline{p}_1^1 \exp(-\lambda m_s) + \frac{\beta}{m_1^1} \exp(-\hat{m}_2) \left\{(1-\delta)m_1^1 + \Phi[m_1^1 \exp(-\lambda m_s)]\right\}. \end{aligned}$$

Proof. Equation 8.24 implies that equal profit conditions for prices in the supports of best response strategies for sellers of ages 0 and 1 are, for $p \in [\underline{p}_1^0, \overline{p}_1^0]$ and $p \in [\underline{p}_1^1, \overline{p}_1^1]$ respectively,

$$\begin{aligned} &p \exp\left[-\lambda m_s F_1^0(p)\right] + \frac{\beta}{m_s} \exp(-\hat{m}_2) \left[(1-\delta)m_s + \Phi\left\{m_s \exp\left[-\lambda m_s F_1^0(p)\right]\right\}\right] \\ &= \overline{p}_1^0 \exp(-\lambda m_s) + \frac{\beta}{m_s} \exp(-\hat{m}_2) \left\{(1-\delta)m_s + \Phi[m_s \exp(-\lambda m_s)]\right\}, \qquad (8.25) \\ &p \exp\left[-\lambda m_s - \overline{m}_1^1 F_1^1(p)\right] + \frac{\beta}{m_1^1} \exp(-\hat{m}_2) \left[(1-\delta)m_1^1 + \Phi\left\{m_1^1 \exp\left[-\lambda m_s - \overline{m}_1^1 F_1^1(p)\right]\right\}\right] \\ &= \exp(-\hat{m}_1) + \frac{\beta}{m_1^1} \exp(-\hat{m}_2) \left\{(1-\delta)m_1^1 + \Phi[m_1^1 \exp(-\hat{m}_1)]\right\}. \qquad (8.26) \end{aligned}$$

This pair of equal profit conditions imply that $\underline{p}_1^0$ and $\underline{p}_1^1$ (which is equal to $\overline{p}_1^0$) satisfy, respectively,

$$\underline{p}_1^0 = \overline{p}_1^0 \exp(-\lambda m_s) - \frac{\beta}{m_s} \exp(-\hat{m}_2) \left\{ \Phi(m_s) - \Phi\left[m_s \exp(-\lambda m_s)\right] \right\} \tag{8.27}$$

$$\underline{p}_1^1 = \frac{1}{\exp(-\lambda m_s)} \left[\exp(-\hat{m}_1) - \frac{\beta}{m_1^1} \exp(-\hat{m}_2) \left\{ \Phi\left[m_1^1 \exp(-\lambda m_s)\right] - \Phi\left[m_1^1 \exp(-\hat{m}_1)\right] \right\} \right]. \tag{8.28}$$

To check that the pair of distributions of posted prices (F_1^0, F_1^1) is an equilibrium note that for any $p \in [\underline{p}_1^0, \overline{p}_1^0)$, the equal profit condition that cohort of age 1 is indifferent between p and $\overline{p}_1^0$ implies that cohort of age 0 strictly prefers $\overline{p}_1^0$. The reason for that is as follows: The equilibrium distribution of prices for cohort of age 0, F_1^0, makes sellers of age 0 indifferent between p and $\overline{p}_1^0$. Given that Φ is strictly concave and both cohorts have the same discount rates, then F_1^0 makes the present value gain in future profits from consumer base accumulation for sellers of age 1 smaller than the loss in present profits from posting a lower price p vis $\overline{p}_1^0 = \underline{p}_1^1$.

Uniqueness of the equilibrium follows from the equilibrium being trivially unique in periods 0 and 2, and that any symmetric equilibrium in period 1 must be in mixed strategies with supports that are convex and without "gaps" between supports of different cohorts. The concavity of Φ then implies that cohort 0 will post prices that are always equal or greater than prices posted by cohort 1, which then implies that the pair of distributions (F_1^0, F_1^1) constitute the unique subgame perfect equilibrium for the pricing game in period 1. □

The proposition below states that the environment I study in this example can

Proposition 2. *Consider the three-period example from Subsection 8.6. In the equilibrium of that example:*

(1) The maximum density of prices that are observed by the buyers ("posted prices") can be at any point $p(z) = z + \underline{p}_1(1 - z)$ for $z \in [0, 1)$ in $[\underline{p}_1, 1)$.

(2) The density of posted prices at the lower bound of the distribution $\underline{p}$ can be arbitrarily lower than at the maximum density of prices $p(z)$.

Proof. (1) Note that if $\overline{m}_1^1/\hat{m}_1$ is close to 1 the majority of prices observed by buyers are prices posted by cohort of age 1, whose distribution has support $[\underline{p}_1^1, 1]$ with peak density at $\underline{p}_1^1$. Hence, for $\overline{m}_1^1/\hat{m}_1$ close to 1, $\underline{p}_1^1$ is the peak density for the aggregate distribution of prices $\hat{F}_1$.

Let the consumer base accumulation function Φ satisfy

$$\Phi(x) = \begin{cases} \overline{n}_0 m_s & x = m_s \\ \underline{n}_0 m_s & x = m_s \exp(-\lambda m_s) \\ \overline{n}_1 m_1^1 & x = m_1^1 \exp(-\lambda m_s) \\ \underline{n}_1 m_1^1 & x = m_1^1 \exp(-\hat{m}_1). \end{cases}$$

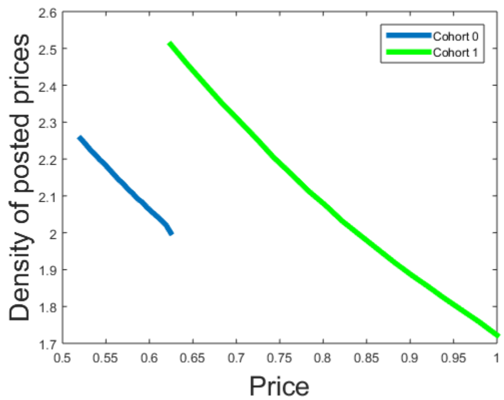
We can choose m_s and a strictly concave Φ such that m_1^1/m_s is large, and $\overline{n}_0/\underline{n}_0$ is very large but such that $\overline{n}_1/\underline{n}_1$ is not much larger than 1. That is, a very concave Φ, where gains from undercutting are overwhelmingly higher at the lower quantities sold. This implies that $1-\underline{p}_1^1$ is small relative to $\underline{p}_1^1-\underline{p}_1^0$ even though ω_1^1 is large. That implies that $z \in (0,1)$ is relatively closer to 1. Inversely, choose a m_s and an approximately linear Φ such that m_1^1/m_s is large, and $\overline{n}_1/\underline{n}_1$ is also large, then $1-\underline{p}_1^1$ is very large relative to $\underline{p}_1^1-\underline{p}_1^0$, that is, $z \in (0,1)$ is closer to 0.

Equations 8.28 and 8.27 imply in the continuity of the point of maximum density of prices $p(z)$ relative to different $\overline{n}_0, \underline{n}_0, \overline{n}_1$ and $\underline{n}_1$ which in turn yield the desired result for $z \in (0,1)$. For $z = 0$ note that it follows from $\underline{p}$ being the peak density, which is obtained if the growth in consumer base moving across the two cohorts m_1^1/m_s is not very large and $1-\underline{p}_1^1$ is large relative to $\underline{p}_1^1-\underline{p}_1^0$.

(2) This implies that $z > 0$, the argument above implies that we can choose m_s and a Φ such that m_1^1/m_s is large and $1-\underline{p}_1^1$ is small relative to $\underline{p}_1^1-\underline{p}_1^0$. Then, equations 8.25 and 8.26 imply that the density of prices observed by the buyers at $\underline{p} = \underline{p}_1^0$ is very low compared to $\underline{p}_1^1$. $\square$

Numerical example: Suppose that $\alpha = 0.1, m_s = 1, \Phi(q) = 3\sqrt{q}, \beta = .9, \delta = 0$. Then the equilibrium distributions features supports $\text{supp}(F_1^0) = [.521, .624], \text{supp}(F_1^1) = [.624, 1.000]$ for each cohort of sellers, and the density of prices posted (weighted by cohort's share of the consumer base) is described in Figure 8.1. Note that the peak density is not at the lower bound of the support $\underline{p}_1^0 = .521$ but at $\overline{p}_1^1 = .624$.

Figure 8.1: Densities of F_1^0 and F_1^1.



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