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Price Uncertainty and the Exchange-Rate Risk Premium*

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1. Introduction

The role of asset markets in the determination of exchange rates has been an important research topic in recent years. In a world where all international assets are perfect substitutes, open interest parity holds, and there is no role for sterilized intervention or for current account imbalances (as wealth transfers) in the determination of exchange rates. When assets are not perfect substitutes, however, open parity will, in general, be violated. In this case, exchange rates will depend on the composition of outside assets outstanding, and on the distribution of world wealth[1].

The initial tests for open parity (see Levich, 1985, and references therein) tended not to reject it, concluding that the forward exchange rate provided an unbiased forecast of the future spot rate. More recently, however, work by Cumby and Obstfeld (1981), Hansen and Hodrick (1980), and Meese and Singleton (1982) using more powerful techniques has shown that in fact the assumption of open parity is rejected by the data.

The next question is then whether any systematic deviations can be related to macroeconomic policy variables. The standard approach here has been to follow finance theory in deriving a model of expected relative returns on assets denominated in different currencies. Departures from open parity are then said to be due to a "risk premium". Frankel (1982) tested for the presence of a risk premium applying this theory, but was unable to reject the hypothesis that it was not there.

In the theoretical front, many papers have been written. Among the more recent, Dornbusch (1980) discusses the real interest rate differential across countries, and Krugman (1981) discusses departures from open parity, both in a micro, portfolio context[2].

This note extends this literature by analysing the determination of the risk premium in a two-country world where inflation uncertainty is explicitly introduced into the model, along with exchange rate uncertainty. The risk premium will be shown to depend on the composition of outside assets, on the distribution of world wealth, and on the variance-covariance matrix of the two inflation rates and the exchange rate. With this structure, we can then analyse the consequences of purchasing power parity holding exactly and on an expected basis only. In the first case, the risk premium does not depend on the distribution of world wealth, whereas in the second it does.

The next section sets up the framework, and discusses each country's asset demands. Section 3 derives an expression for the risk premium, and discusses PPP. Some extensions are provided in the appendix.

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[1] See Branson and Henderson (1985), for a comprehensive discussion of all these issues.

[2] See also the earlier, and related work by Fama and Farber (1979), and the review by Adler and Dumas (1983).

2. Asset Demands[3]

Each of our two countries issues a security denominated in its currency. The nominal returns on the home security (B) and on the foreign security (F) follows:

(1) $$\frac{dB}{B} = i dt$$

(2) $$\frac{dF}{F} = i^* dt$$

The exchange rate (E) is defined as the home currency price of foreign currency, and is assumed to follow geometric Brownian motion:

(3) $$\frac{dE}{E} = e dt + S_e dz_e$$

where e and S_e are the instantaneous mean and variance of the process, and dz_e is a standard Wiener process[4].

Home and foreign net wealth are defined as:

(4) $$W = B + EF$$
$$W^* = \frac{B^*}{E} + F^*$$

where B and B^* are the respective home and foreign holdings of home bonds, and similarly for foreign bonds. The total stocks of each country's bonds are denoted by bars. Equations (4) can be combined to yield:

(5) $$W + EW^* = \bar{B} + E\bar{F}$$

On the consumption side, we assume there are two goods, each priced in the respective home currency. These prices are also assumed to follow geometric Brownian motion:

(6) $$\frac{dP}{P} = p dt + S_p dz_p$$

[3] This section follows Branson and Henderson (1985). A derivation of asset demand functions from first principles is contained in the appendix.

[4] See Merton (1971) for a discussion of these stochastic processes, and Adler and Dumas (1983, pp. 940-941) for a discussion of their use in international finance.

$$\frac{dp^*}{p^*} = p^* dt + S_{p^*} + dz_{p^*}$$

Consumers maximize expected utility over an infinite horizon, subject to the usual wealth constraints. The utility functions for both consumers (and countries, since I assume individuals are identical within each country) are time separable, and exhibit constant relative risk aversion (equal to R). Within each period, utility is Cobb-Douglas, with $\alpha(\alpha^*)$ being the home (foreign) consumer's share of the home good in consumption.

Under these conditions, it can be shown that consumption and investment decisions are independent; and that the optimal portfolio shares coincide with those obtained from the maximization of a mean-variance utility function on real wealth changes, where nominal wealth is deflated with the appropriate (dual) price index. For example, the home consumer's problem is to maximize

(7) $$V = \varepsilon\left(\frac{dW}{\widetilde{W}}\right) - \frac{1}{2} R \text{var}\left(\frac{d\widetilde{W}}{\widetilde{W}}\right)$$

where $\widetilde{W}$ is real wealth $\left[\frac{W}{P^\alpha}(EP^*)^{1-\alpha}\right]$, ε denotes the expectation operator, and the foreign investor's problem is defined analogously.

Let $\lambda(\lambda^*)$ be the share of the home asset in the home (foreign) investor's portfolio:

(8) $$\lambda \equiv \frac{B}{W}$$

$$\lambda^* \equiv \frac{B^*}{EW^*}$$

Under the above assumptions, Branson and Henderson (1985) show that:

(9) $$\lambda = \frac{1}{RS_e^2}\left[i - i^* - e - (R-1)\left(\alpha S_{ep} + (1-\alpha)\left(S_e^2 + S_{ep^*}\right)\right) + RS_e^2\right]$$

where $S_{ep}\left(S_{ep^*}\right)$ is the instantaneous covariance between the exchange rate and the home (foreign) good price.

Following the same procedure, it can be shown that:

(10) $$\lambda^* = \frac{1}{RS_e^2}\left[i - i^* - e - (R-1)\left(\alpha^* S_{ep} + (1-\alpha^*)\left(S_e^2 + S_{ep^*}\right)\right) + RS_e^2\right]$$

Thus, even though both investors follow the same portfolio rule, their portfolio demands may differ on account of their differing consumption preferences. In particular, we can state the condition under which "home asset preference" will hold:

(11) $$\lambda > \lambda^* <=> (R-1)(\alpha-\alpha^*)\left(S_e^2 + S_{ep^*} - S_{ep}\right) > 0$$

This condition ís frequently invoked in the macroeconomics literature (see Branson and Henderson, 1985).

Assuming that $R > 1$, and that "home good preference" ($\alpha > \alpha^*$) holds, home asset preference obtains when:

(12) $$S_e^2 + S_{ep} - S_{ep^*} > 0$$

This condition requires that the variance of the exchange rate be sufficiently higher than the variances of prices, and it is usually thought to hold[5].

When the expression in (12) is equal to zero, home and foreign investors will hold the same portfolio. In this case, the distribution of wealth across countries will not influence the equilibrium in international asset markets. The next section will discuss this result in more detail.

3. Market Equilibrium

The market clearing conditions for the home asset is:

(13) $$\lambda W + \lambda^* E W^* = \bar{B}$$

This equation can be rewritten as:

(13') $$\lambda w + \lambda^*(1-w) = b$$

where w is the share of home wealth in world wealth, and b is share of the home asset in all assets.

Substituting for λ and λ^*, and solving for the expected return differential (the 'risk premium'), we get:

[5] This is Macedo's (1982) condition for an increase in a to lead to an increase in α to lead to an increase in λ.

$$(14) \qquad i - i^* - e = c + RS_e^2 b - (R-1)(\alpha - \alpha^*)\big(S_e^2 + S_{ep^*} - S_{ep}\big)w$$

$$c \equiv (R-1)\big[\alpha^*\big(S_{ep} - S_e^2\big) + (1-\alpha^*)S_{ep^*}\big] - S_e^2$$

This is the result mentioned in the introduction. Notice that signs of the coefficients of b and w (when home asset preference holds) correspond to those usually assumed in macro models.

Let us now turn to a discussion of purchasing power parity. Given our Cobb-Douglas assumption, the real exchange (π) can be written as:

$$(15) \qquad \pi = \left(\frac{EP^*}{P}\right)\alpha - \alpha^*$$

Assuming that the law of one price holds, and that there are no transactions costs, PPP will hold when either there is no home good preference ($\alpha = \alpha^*$), or the terms of trade $\left(\frac{EP^*}{P}\right)$ are constant. If the latter is the case, the stochastic processes of E, P, and P^* will not be independent, and we can use Ito's lemma (see Merton 1971) to obtain:

$$(16) \qquad S_e^2 + S_{ep^*} - S_{ep} = 0$$

Going back to equation (11), we see that when PPP holds home asset preference does not obtain, and, as equation (14) indicates, the risk premium is independent of the distribution of wealth across countries. In this case, the expression for the risk premium reduces to:

$$(17) \qquad i - i^- - e = RS_e^2 b + c$$

When $\alpha = \alpha^*$, this equation can be rewritten as:

$$(17') \qquad i - i^* - e = S_e^2[(R-1)(b - \lambda_m) - (1-b)],$$

where

$$(18) \qquad \lambda_m \equiv -\frac{1}{S_e^2}\big[\alpha\big(S_{ep} - S_e^2\big) + (1-\alpha)S_{ep^*}\big]$$

is the share of the home asset in the minimum-variance portfolio (obtained from the minimization of var $\left(\frac{d\overline{W}}{\overline{W}}\right)$).

From equation (17’) we see that when PPP holds there is still a risk premium and it will be higher the higher the excess supply of home bonds over its share in the minimum-variance portfolio[6]. Note also that even when $b = \lambda_m$ there remains a deviation from open interest parity. This follows from the fact that investments in foreign currency are more attractive in the presence of uncertainty on account of Jensen’s inequality, as Krugman (1981) pointed out.

The assumption of exact PPP, however, is clearly too strong. Let us look instead at a case where PPP is only expected to hold[7].

Applying Ito’s lemma we can derive an expression for the stochastic process followed by the real exchange rate (let $a \equiv \alpha - \alpha^*$):

(19) $$\frac{d\pi}{\pi} = \Big[a(e + p^* - p) + a^2\big(S_{ep*} - S_{ep} - S_{pp*}\big) + \frac{1}{2}\big(a(a-1)\big(S_{p*}^2 + S_e^2\big) + a(1 + a)S_p^2\Big]dt + a\big(S_e dz_e + S_{p*}dz_{p*} - S_p dz_p\big)$$

If PPP is expected to hold, the term in square brackets will be equal to zero. To develop some intuition, let us assume that:

$e + p^* - p = 0$ and $S_{p^*}^2 = S_p^2$

Under these assumptions, (10) implies:

$$a\big(S_e^2 + S_{ep^*} - S_{ep}\big) + a\big(S_p^2 - S_{pp^*}\big) - \frac{1}{2}(1 + a)S_e^2 = 0$$

The first term in this expression has to be positive if home asset preference is to hold at all. A sufficient condition for this is given that $0 < a < 1$:

(20) $$S_e^2 > S_p^2 - S_{pp^*}$$

Thus, if PPP is only expected to hold, a restriction similar to the one imposed on equation (12) is sufficient to guarantee home asset preference.

[6] Dornbusch (1980) provides a similar interpretation in his discussion of the differential on real interest rates.
[7] This is consistent with the recent finding of Adler and Lehman (1984) that deviations from *PPP* follow a martingale.

APPENDIX

A.l. The Demand for Foreign Bonds

This section derives equation (9) in text from first principles, using standard techniques of portfolio selection in continuous-time models. (See Merton 1971, for details and references, and Ingersoll 1982 for a nice exposition.) The derivation is done for a general price index Q.

Consider a risk-averse consumer who maximizes the expected utility of a continuous flow of consumption $C(t)$ over an infinite horizon. Assume his utility function is concave, time-separable, and state-independent (but not necessarily time-independent). More formally, he will want to:

$$\max E_0 \int_0^\infty u(C(t),t)ft \tag{1}$$

subject to a wealth constraint. Let us further assume, for simplicity, that the consumer receives no income, and that he is born with an amount of wealth, a part of which he must invest each period to provide for future consumption. As the scale of his initial wealth is not relevant, let us focus on the flow budget constraint faced by the individual each period. This constraint can be approximated by:

$$W(t+\Delta t) = [W(t) - C(t)\Delta t]\left[\sum \lambda_i(1+r_i)\right] \tag{2}$$

where $W(t)$ is real wealth ($\widetilde{W}$ in the text).

This equation States that at time t an amount $C(t)\Delta t$ is set aside for consumption (at a constant rate) during the subsequent interval of length Δt. Simultaneously, the remainder W(t) - C(t)At is invested, leading to next period's wealth. The investment decision consists of deciding what share X. of wealth is to be allocated to each asset, given the variance-covariance properties of their returns, r_i.

In our case the choice has to make over two assets: a domestic bond, and a foreign currency bond. Their nominal returns in their currencies of denomination are reproduced here for convenience.

$$\frac{dB}{B} = idt \tag{3}$$

$$\frac{dF}{F} = i^* dt \tag{4}$$

Both assets have non-stochastic nominal returns in their currencies, but this does not mean they are riskless: for a home consumer, the returns on the home asset are subject to (home) inflation risk, and the returns on the foreign asset are subject to exchange rate risk. The relevant domestic price index and the exchange rate will be assumed to follow Geometric Brownian notions, represented by:

$$\frac{dQ}{Q} = qdt + S_q dz_q \tag{5}$$

$$\frac{dE}{E} = edt + S_e dz_e \tag{6}$$

In order to solve the optimization problem, we must convert the nominal returns into real returns. For example, the real value of the house asset at a point in time is given by:

$$\tilde{B}(t) = B(t)/Q(t) \tag{7}$$

To calculate its real return, we apply Itô's lemma (see Merton, 1971, p. 375), obtaining:

$$r_b \equiv \frac{d\tilde{B}}{\tilde{B}} = \left(i - q - S_q^2\right)dt - S_q dz_q \tag{8}$$

Following the same route, we can obtain the real return on the foreign asset:

$$r_f \frac{d(E\tilde{F})}{E\tilde{F}} = \left(i^* + e - q - S_{eq} + S_q^2\right)dt + S_e dz_e - S_q dz_q \tag{9}$$

where S_{eq} is the instantaneous covariance between E and Q.

The share of investment placed in the foreign asset is denoted by;

$$\lambda(t) = \frac{E(t)F(t)}{W(t) - C(t)\Delta t}$$

We can now rewrite the wealth constraint (2) as:

$$\Delta W \equiv W(T + \Delta t) - W(t) = -C(t)\Delta t + [W(t) - C(t)\Delta(t)]\left\{\lambda(t)r_f + [(1 - \lambda(t)]r_b\right\} \tag{2'}$$

Substituting for the real returns, and taking expectations, we obtain (dropping the time notation):

$$E_t(\Delta W) = \left[\lambda\left(i^* + e - i - S_{eq}\right) + i - q + S_q^2\right]W\Delta t - C\Delta t \tag{10a}$$

$$\mathrm{var}_t(\Delta W) = \left[\lambda^2 S_e^2 - 2\lambda S_{eq} + S_q^2\right]W^2\Delta t \tag{10b}$$

We can now turn to the optimization problem. Define a maximized utility of wealth function as follows:

$$J(W(t), t) \equiv \max E_t \int_0^\infty u(C, s)ds \tag{11}$$

Using this definition, and the properties of conditional expectations, we get:

$$J(W(t), t) = \max E_t \left[\int_t^{t+\Delta t} u(C, s)ds + \int_{t+\Delta t}^\infty u(C, s)ds\right] = \max E_t \left[\int_t^{t+\Delta t} u(C, s)ds + E_{t+\Delta t} \int_{t+\Delta t}^\infty u(C, s)ds\right] = \max E_t \left[\int_t^{t+\Delta t} u(C, s)ds + J(W(t + \Delta t), t + \Delta t)\right] \tag{12}$$

Expand $J(W(t + \Delta t), t + \Delta t)$ around $J(W(t), t)$:

$$J[W(t + \Delta t), t + \Delta t] = J(W, t) + J_t\Delta t + J_W\Delta W + \frac{1}{2}J_{WW}\Delta W^2 + \frac{1}{2}J_{tt}\Delta t^2 + J_{tW}\Delta t\Delta W + R \tag{13}$$

Taking expectations, and disregarding small terms, we get:

$$E_t[J(W(t + \Delta t), t + \Delta t)] = J(W, t) + J_t\Delta t + J_W E_t(\Delta W) + \frac{1}{2}J_{WW}\mathrm{var}_t)\Delta W)$$

Substituting back into (11), we obtain:

$$0 = \max\left\{u(C,t)\Delta t + J_t \Delta t + J_W E_t(\Delta W) + \frac{1}{2} J_{WW} \mathrm{var}_t(\Delta W)\right\} \tag{14}$$

where we have approximated the utility obtained in the time interval.

Substituting (10) into (14), and dividing by Δt, we get:

$$0 = \max_{c,\lambda}\{u(C,t) + J_t + J_W W\left[\left(i^* + e - i - S_{eq}\right) + i - q + S_q^2 - C/W\right]\}$$

$$+ \frac{1}{2} J_{WW} W^2 \left[\lambda^2 S_e^2 - 2\lambda S_{eq} + S_q^2\right] \tag{15}$$

The first-order conditions for this maximization are:

$$\frac{\partial J}{\partial C} = 0 \;\; \rightarrow \;\; u_c = J_W \tag{16a}$$

$$\frac{\partial J}{\partial \lambda} = 0 \;\; \rightarrow \;\; J_W\left[i^* + e - i - S_{eq}\right] + J_{WW} W\left[\lambda S_e^2 + S_{eq}\right] = 0 \tag{16b}$$

The first is known as the 'envelope condition', and it states that the marginal utility of consumption should be equated to an 'expected, discounted marginal utility of wealth' (re: the utility function may include a discount factor). The second condition implies an optimal share λ^*:

$$\lambda^*(t) = \frac{-J_W}{J_{ww} W S_e^2}\left[i^* + e - i - \left(\frac{J_{WW} W}{J_W} + 1\right) S_{eq}\right] \tag{17}$$

where λ^* is written as function of time to remind us that J will in general be time-dependent. This equation is already very similar to the foreign-bond demand in the paper, and will in fact reduce to that when we assume that utility has constant relative risk aversion. To see this, let us proceed with the analysis, assuming:

$$u(C,t) = e^{-pt\, C^\gamma}/\gamma \tag{18}$$

where p is a subjective discount factor, and $1 - \gamma$ is equal to the coefficient of relative risk aversion (and hence we assume $\gamma < 1$). In this case we have, from (16a),

$$C^*(t) = \left(C^{pt} J_W\right)^{1/(\gamma - 1)} \tag{16a'}$$

Using the Solutions for C and λ, we can rewrite (15) as (after some manipulations):

$$0 = e^{-pt}\left(\frac{1}{\gamma} - 1\right)\left(e^{pt} J_W\right)^{\frac{\gamma}{\gamma - 1}} + J_t + rWJ_W - \frac{1}{2}\frac{x^2}{S_e^2}\frac{J_W^2}{J_{WW}} + \frac{1}{2} J_{WW} S W^2 \tag{15'}$$

where:

$$x \equiv i^* + e - i - S_{eq}$$

$$r \equiv \frac{S_{eq}}{S_e^2} x + i - q + S_q^2$$

$$S \equiv \frac{\left(S_{eq}\right)^2}{S_e^2} + S_q^2$$

Equation (15') is quite similar to the partial differential equation that appears in the standard model of portfolio selection. There, r is the risk-free interest rate, instead of x^2/S_e^2 we have the square of the expedted excess return on the market over the risk-free rate divided by the variance of the market, and the last term is absent. To solve (15') we employ the same trick, defining a convenient function of W as follows:

$$I(W) \equiv e^{pt}J(W,t)$$

Substituting into (15') we get:

$$0 = \left(\frac{1}{\gamma} - 1\right)(I')^{\frac{\gamma}{\gamma-1}} - pI + \gamma WI' - \frac{1}{2}\frac{x^2}{S_e^2}\frac{(I')^2}{I''} + \frac{1}{2}SI''W^2 \tag{15''}$$

If we guess the form AW^γ/γ for $I(W)$, we obtain:

$$0 = AW^\gamma\left[\frac{1-\gamma}{\gamma}A^{\frac{1}{\gamma-1}} - \frac{p}{\gamma} + r - \frac{1}{2}\frac{x^2}{S_e^2}\frac{1}{\gamma-1} + \frac{1}{2}S(\gamma-1)\right] \tag{15'''}$$

The non-trivial solution is then:

$$A = \left\{\frac{\gamma}{1-\gamma}\left[\frac{p}{\gamma} - r + \frac{1}{2}\frac{x^2}{S_e^2}\frac{1}{\gamma-1} - \frac{1}{2}S(\gamma-1)\right]\right\}^{\gamma-1} \equiv a^{\gamma-1}$$

Now, remember that:

$$J(W) = e^{-pt}I(W) = e^{-pt}a^{\gamma-1}\frac{W^\gamma}{\gamma}$$

From (16a') and (17) we obtain the optimal choices:

$$C^*(t) = aW(t) \tag{19a}$$

$$\lambda^* = \frac{1}{(1-\gamma)S_e^2}\left[i^* + e - i + \gamma S_{eq}\right] \tag{19b}$$

where we have shown that λ is time-independent, and corresponds to equation (10) in the paper.

Finally, to rule out Solutions with negative consumption we impose $a > 0$; and to rule out the possibility of plans which generate infinite utility, we impose $p > 0$. These two conditions can be combined into a 'transversality condition',

$$p > \max\left\{0, \gamma\left[r - \frac{1}{2}\frac{x^2}{S_e^2}\frac{1}{\gamma-1} + \frac{1}{2}S(\gamma-1)\right]\right\}$$

which is not easily interpretable, and is assumed to hold.

References:

Ingersoll, J. (1982), 'Continuous-Time Portfolio Selection', unpublished lecture note, University of Chicago.

Merton, R. C. (1971), 'Optimum Consumption and Portfolio Rules in a Continuous-Time

Model', *Journal of Economic Theory*, 3: 373-413.

A.2. Extensions of the Basic Model

The unifying theme of this section is the notion that the basic model presented in the text provides us with some insight into the functioning of international asset markets, but is far too narrow to be considered realistic. As a consequence, the portfolio-balance/asset-markets approach to exchange rates will only be empirically successful when we employ specifications that are more general than the ones that have been used up to now, including the extension of Frankel's (1982) work derived in section A.2.1.

The first additional extension will change or expand the menu of assets available to our investors. The asset demand functions derived in the text are crucially dependent on the assumption that there are only two assets available: a domestic and a foreign asset. Section A.2.2 will relax this assumption in two different directions; it will first assume that the domestic asset is indexed to the domestic price index, and, alternatively, it will include a third asset that is available only to domestic residents (a 'non-traded asset'). Section A.2.3 will discuss in more detail the consequences of purchasing power parity, extending the discussion in the text to the case where PPP does not hold exactly, but is expected to hold. Section A.2.4 analyzes changes in the degree of relative risk aversion R, allowing for different $R's$ at home and abroad. Finally, section A.2.5 provides some further thoughts on asset demands.

A.2.1. Empirical Implications of the Model in the Text

In a recent paper, Frankel (1982) has attempted to test for the existence of a risk premium. He derives asset demand functions from mean-variance optimization, and estimates a model that includes six different currencies. The only random variables in his model are the exchange rates, and all investors are assumed to have the same consumption preferences.

In my simple two country example Frankel's equation can be rewritten as:

$$i_t^* - i_t + \bar{e}_{t+1} = RS_e^2(\lambda_t - \beta) + u_{t+1} \qquad (21)$$

where λ_t is the actual share of Fr bonds in the market, β is the share of Fr goods in consumption (which in the absence of price uncertainty coincides with the minimum variance portfolio), $\bar{e}_{t+1}$ is the actual depreciation, and u_{t+1} is an error term which reflects the rational expectations assumption, i.e.:

$$\bar{e}_{t+1} = e_{t+1} + u_{t+1} \qquad \varepsilon(u_{t+1}/I_t) = 0 \qquad (22)$$

Eguation (21) is used to estimate the coefficient of risk aversion (R), In order to maximize econometric efficiency, Frankel notes that the variance of the error term is exactly the variance that appears in the coefficients in the regression. He imposes this restriction, and concludes that the hypothesis of risk neutrality ($R = 0$) cannot be rejected. When he allows for different preferences across countries, and for the more precise continuous time derivation, positive values of R become slightly more plausible than zero or negative values. Frankel points out, however, that all these results must be interpreted with care because the tests have very low power.

This note suggests that these tests may be improved if price uncertainty is allowed for. The equation to be estimated is the one implied by (31), and it should be jointly estimated with inflation equations for each country. One possible system would be the following:

$$\begin{aligned} \bar{e}_{t+1} + i_t^* - i_t &= a_0 + a_1 b_t - a_2 w_t + u_{t+1} \\ P_{t+1} &= c_1(L)P_t + c_2(L)X_t + v_{t+1} \\ P_{t+1}^* &= d_1(L)P_t^* + d_2(L)X_t^* + \eta_{t+1} \end{aligned} \tag{23}$$

where $c_1(L)$, $c_2(L)$, $d_1(L)$, and $d_2(L)$ are polynomials in the lag operator, and X_t and X_t^* are any other variables that may enter each country's inflation equation.

To maximize efficiency, the variance-covariance restrictions between the coefficients in the risk premium equation and the error terms in all equations should be imposed. This natural extension of Frankel's work should increase the power of the test for the existence of a risk premium. As we mentioned above, however, this specification is still not broad enough. We now turn to a discussion of some further extensions.

A.2.2. Asset Demands with a Domestic Indexed Bond or with a 'Non-Traded' Bond

Let us assume first that the domestic asset is perfectly indexed to the domestic price level, yielding a non-stochastic return. Applying the methods used in the text and in Appendix I, we can calculate the real returns on our two assets, obtaining:

$$\frac{d(B/Q)}{B/Q} = rdt \tag{1}$$

(by assumption)

$$\frac{d(EF/Q)}{EF/Q} = \left(e + i^* - q - S_{eq} + S_q^2\right)dt + S_e dz_e - S_q dz_q \tag{2}$$

Real wealth ($\widetilde{W}$) dynamics are now given by:

$$\frac{d\widetilde{W}}{\widetilde{W}} = (1-\lambda)\frac{d(B/Q)}{B/Q} + \lambda\frac{(EF/Q)}{EF/Q} = \left[\lambda\left(i^* + e - r - q - S_{eq} + S_q^2\right)\right]dt + \lambda\left(S_e dz_i - S_q dz_q\right)$$

Assuming, as before, we can use mean-variance, we get:

(4)

$$\lambda = \frac{(i^* + e) - \left(r + q + S_{eq}\right) + S_q^2}{R\left(S_e^2 + S_q^2 - 2S_{eq}\right)}$$

It can be shown that the expected nominal return on the indexed bond (i) will equal the second term in parenthesis in (4), which can then be rewritten as:

$$\lambda = \frac{i^* + e - i + S_q^2}{R\left(S_e^2 + S_q^2 - 2S_{eq}\right)} \qquad (4')$$

For my purposes we can stop here, without doing the same calculation for the foreign investor. It should be clear that this demand differs from equation (10) in the text, and that therefore tests for a risk premium based on (10) would potentially run into misspecification problems.

Let us now examine asset demands when the home investor is given the additional choice of a non-traded asset. In principle one could examine the problem in its general form, including a non-tradable good in the domestic price index. To keep things manageable, however, I will assume that prices are non-stochastic, and that the returns on the new asset are given by:

$$\frac{dm}{m} = m dt + S_m dz_m \qquad (5)$$

Let its portfolio share be denoted by δ, and the share of the foreign bond by λ. The usual calculations imply:

$$\delta = \frac{1}{R\Delta}\left[(S_e^2 + S_{em})(i - m) + S_{em}(i^* + e - m)\right] \qquad (6)$$

$$\lambda = \frac{S_m^2}{R\Delta}\left[i - i^* - \frac{S_{em}}{S_m^2}(i - m)\right] \qquad (7)$$

$$\Delta \equiv -[S_m^2 S_e^2 + (S_{em})^2 + 2S_m^2 S_{em}] \qquad (8)$$

To rule out asset demands that depend negatively on the own rate of return, I will assume throughout that $\Delta < 0$. This amounts to assuming that S_{em} is not too large and negative. If, for example, the non-traded asset is land (foreigners are not allowed to own domestic land), and the returns on land are positively correlated with the exchange rate (say, through export earnings), then the condition will be satisfied.

In this case, the demand equations will be well behaved, and the assets will be gross substitutes. The foreign bond demand will now depend on m and S_{em}, implying that the use of (10) in the text will lead to the exclusion of a relevant variable in the test for a risk premium.

In practice, other problems are likely to arise. For example, it is very hard to obtain an appropriate measure of each country's wealth. Clearly, the definition used here is too narrow; we should try to include the stock market, human capital, etc. If we do so, and that is not easy, we then have to considerably broaden our vector of assets, leading to non-trivial data difficulties.

A.2.3. Implications of Purchasing Power Parity (PPP)

If the impact of current account disequilibria on exchange rates is modelled because of the wealth transfer aspect of the imbalance, then, as we saw in the text, a necessary condition is that PPP does not hold exactly. If it does hold exactly, then I have demonstrated that investors in both countries will hold the same portfolio, and therefore no change in asset prices (the exchange rate) will result from a wealth transfer. Moreover, even when PPP does not hold, we saw that the following condition had to hold to make sure that the wealth transfer would lead to right change in the exchange rate:

$$S_e^2 + S_{eq^*} - S_{eq} > 0 \tag{9}$$

Macedo (1982, p. 84) obtains essentially the same condition when discussing the relationship between goods and asset preferences (see Branson and Henderson 198u, for a discussion). He claims this condition is likely to hold "because the variance of exchange rate changes tends to be much larger than the variance of inflation rates". This is better seen when we rewrite (9) using correlation coefficients:

$$S_e + \rho_{eq^*} S_{q^*} \rho_{eq} S_q > 0 \tag{9'}$$

(This is Macedo's p. 84 equation.)

While the assumption that PPP holds exactly is certainly too strong, the alternative of PPP holding only in expected terms is more plausible. To examine this case let us look at the Cobb-Douglas example used in the text. Under this assumption, the real exchange rate π can be defined as:

$$\pi = \frac{EQ^*}{Q} = \left(\frac{EP^*}{P}\right) a$$

$$a \equiv 1 - \alpha - \beta > 0$$

(home goods preference)

Applying Ito's lemma we can derive an expression for the stochastic process followed by the real exchange rate:

$$\frac{d\pi}{\pi} = \left[a(e + p^* - p) + a^2\left(S_{ep^*} - S_{ep} - S_{pp^*}\right) + \frac{1}{2}\left(a(a-1)\left(S_{p^*}^2 + S_e^2\right) + a(1+a)S_p^2\right)\right] dt$$
$$+ a\left(S_e dz_e + S_{p^*} dz_{p^*} - S_p dz_p\right) \tag{10}$$

If PPP is expected to hold, the term in square brackets will be equal to zero. To develop some intuition, let us assume that:

$$e + p^* - p = 0$$

and

$$S_{p^*}^2 = S_p^2$$

Under these assumptions, (10) implies:

$$a\left(S_e^2 - S_{ep^*} - S_{ep}\right) + a\left(S_p^2 - S_{pp^*}\right) - \frac{1}{2}(1+a)S_e^2 = 0$$

The first term in this expression has to be positive if home asset preference (HAP) is to hold at all. A sufficient condition for this is given that $0 < a < 1$:

$$S_e^2 > (1 - \rho_{pp^*})S_p^2$$

where I made use of the assumptions above.

We can therefore conclude that a similar condition for HAP applies for the case of expected PPP as well – the variance of the exchange rate has to be sufficiently higher than the variance of inflation.

A.2.4. Changes in the Degree of Relative Risk Aversion (R, R^*)

The argument that the German stock market is relatively small because Germans are more risk-averse than Americans is sometimes brought up in (very) informal discussions. This section follows this 'conjecture' by checking whether $R^* > R$ has any implications for the forward premium on the dollar.

To analyse this problem we derive an expression for θ assuming that $R^* = \alpha R$, and compare across equilibria when a changes. Tedious calculations show that:

$$\frac{\partial\theta}{\partial\alpha} = \frac{-(1-w)\left(S_e^2 + S_{eq^*}\right) - (1-w)\theta}{w + \alpha(1-w)}$$

This could, in general, be positive or negative. If, however, we make the plausible assumptions that HAP holds, and that $S_{eq} > 0$ ("an appreciation of the dollar helps lower inflation"), we obtain for small θ that:

$$\frac{\partial\theta}{\partial\alpha} < 0$$

This means that higher R^* should correspond to higher θ, or that the fact that we observe a (risk) premium on the dollar is consistent with the fact that Germans are thought to be more risk-averse than Americans. The reader should naturally take this conclusion with a grain of salt.

A.2.5. A Final Note on Asset Demands

In survey of the literature on the specification and influence of asset markets [on exchange rates] Branson and Henderson (1984) show that the portfolio of domestic and foreign assets can be decomposed into a minimum-variance and a speculative 'fund'. For the world as a whole, however, this CAPM-type separation does not hold, because these 'funds' will not be the same for the domestic and foreign investors. This follows from the fact that in a world where foreign and domestic investors care about differing price indices there is no unique minimum-variance portfolio (each investor will have his own).

We can, however, obtain some further information about asset demands by substituting the equilibrium value of the risk premium into the asset demands derived in the text. When we do this, we obtain (for the shares of the home, i.e., Br asset):

$$1-\lambda = b + \frac{(R-1)S(1-w)}{RS_e^2} \tag{11}$$

$$\delta = b - \frac{(R-1)S_w}{RS_e^2} \tag{12}$$

$$S \equiv S_e^2 + S_{eq^*} - S_{eq}$$

When PPP holds ($S = 0$), each investor will, given equilibrium rates of return, hold each asset in the proportion that it is available in the market. When PPP does not hold, and home asset preference exists (S > 0), $1-\lambda > b$, i.e., the share of the investor's home assets in his portfolio will be higher than the assetts share in the market, the dífference being a function of each country's wealth.

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