Pontifícia Universidade Católica
do Rio de Janeiro

Tiago Cytryn Collett Solberg

A stochastic simulation/calibration of the cash flows between FAT and BNDES

Better understanding the cash flow projections for the fund

Dissertação de Mestrado

Masters dissertation presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Márcio Gomes Pinto Garcia

Rio de Janeiro
May 2025

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To my parents, my brother, and my wife
for their support
and encouragement.

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Abstract

Solberg, Tiago Cytryn Collett; Garcia, Márcio Gomes Pinto (Advisor). **A stochastic simulation/calibration of the cash flows between FAT and BNDES**. Rio de Janeiro, 2025. 57p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

The Workers' Support Fund (FAT) is a central source of funding for Brazil's National Development Bank (BNDES), holding 69% of the bank's liabilities by the end of 2023. It plays a critical role in financing long-term investments and development initiatives, while charging BNDES below-market interest rates. However, FAT's sustainability has come under increasing scrutiny due to fiscal pressures, rigid repayment rules, and exposure to macroeconomic volatility. This study develops a Monte Carlo simulation model to evaluate FAT's financial resilience under uncertainty. It incorporates stochastic dynamics in FAT's tax revenue, expenditure commitments, interest earnings, and cash flows involving BNDES and the National Treasury. By simulating alternative political behaviors and interest rate scenarios, the analysis quantifies the risk of underfunding and the frequency of Treasury bailouts. Financial risk measures, such as Value at Risk (VaR) and Conditional Value at Risk (CVaR), are used to assess the fiscal burden under extreme outcomes. Results show that higher interest rates substantially improve FAT's financial health, but institutional constraints limit its autonomy to address deficits. The study contributes to the policy debate by underscoring the need for better-aligned lending rates and clearer fiscal responsibilities to ensure FAT's long-term viability.

Keywords

BNDES; FAT; National Treasury; Monte Carlo simulation; Fiscal risk; Subsidies; Value at Risk.

Resumo

Solberg, Tiago Cytryn Collett; Garcia, Márcio Gomes Pinto. **Uma simulação estocástica/calibração dos fluxos de caixa entre o FAT e o BNDES**. Rio de Janeiro, 2025. 57p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

O Fundo de Amparo ao Trabalhador (FAT) é uma fonte central de financiamento para o Banco Nacional de Desenvolvimento Econômico e Social (BNDES), respondendo por 69% do passivo do banco ao final de 2023. Ele desempenha um papel fundamental no financiamento de investimentos de longo prazo e iniciativas de desenvolvimento, ao mesmo tempo em que cobra do BNDES taxas de juros abaixo do mercado. No entanto, a sustentabilidade do FAT vem sendo alvo de crescente escrutínio devido a pressões fiscais, regras rígidas de reembolso e exposição à volatilidade macroeconômica. Este estudo desenvolve um modelo de simulação de Monte Carlo para avaliar a resiliência financeira do FAT sob condições de incerteza. O modelo incorpora dinâmicas estocásticas nas receitas tributárias do FAT, compromissos de despesa, rendimentos com juros e fluxos de caixa envolvendo o BNDES e o Tesouro Nacional. Ao simular diferentes comportamentos políticos e cenários de taxa de juros, a análise quantifica o risco de insuficiência de recursos e a frequência de socorros do Tesouro. Medidas de risco financeiro, como o Value at Risk (VaR) e o Conditional Value at Risk (CVaR), são utilizadas para avaliar o ônus fiscal em cenários extremos. Os resultados mostram que taxas de juros mais altas melhoram substancialmente a saúde financeira do FAT, mas restrições institucionais limitam sua autonomia para enfrentar déficits. O estudo contribui para o debate de políticas públicas ao destacar a necessidade de taxas de empréstimo mais alinhadas com o mercado e de uma definição mais clara das responsabilidades fiscais para garantir a viabilidade de longo prazo do FAT.

Palavras-chave

BNDES; FAT; Tesouro Nacional; Simulacão de Monte Carlo; Risco fiscal; Subsídios; Value at Risk.

Table of contents

1 Introduction **13**

2 BNDES **16**

3 FAT **18**
3.1 Sources of funding 18
3.2 FAT Constitutional and FAT Special Deposits 19
3.3 Liabilities and Obligations 23
3.4 Net assets 27

4 FAT, BNDES, and the Treasury **29**

5 Model and Calibration **33**
5.1 Assumptions 33
5.2 Tax revenue and costs 33
5.3 Interest rates, inflation, and financial revenue 36
5.4 Correlation Structure of Shocks 38

6 Simulation procedure **39**

7 Results **41**
7.1 Revenue dynamics 41
7.2 Cost pressure 42
7.3 Interest rate regimes & bailouts 43
7.4 VaR and CVaR 48

8 Who bears the cost? **52**

9 Conclusion **54**

10 Bibliography **56**

List of figures

Figure 3.1	Nominal interest rates	21
Figure 3.2	BNDES & FAT Constitutional (R$ thousand, 2023 values)	22
Figure 3.3	BNDES & FAT Special Deposits (R$ thousand, 2023 values)	22
Figure 3.4	FAT's revenue (R$ million, 2023 values)	23
Figure 3.5	Unemployment insurance beneficiaries (millions)	24
Figure 3.6	Real minimum wage (R$, 2023 values)	24
Figure 3.7	Salary bonus beneficiaries (millions)	26
Figure 3.8	FAT's costs excluding FC & FSD (R$ million, 2023 values)	27
Figure 3.9	FAT's assets (R$ million, 2023 values)	28
Figure 3.10	FAT's outstanding loans to BNDES (R$ thousand, 2023 values)	28
Figure 4.1	FAT Costs & Revenues (R$ million, 2023 values)	29
Figure 4.2	Subsidies conferred by the National Treasury (R$)	31
Figure 7.1	Baseline revenue (R$ billion, 2023 values)	42
Figure 7.2	Baseline costs (R$ billion, 2023 values)	43
Figure 7.3	Financial revenue evolution (R$ billion, 2023 values)	44
Figure 7.4	Outstanding subordinate loans (R$ billion, 2023 values)	45
Figure 7.5	Outstanding special loans (R$ billion, 2023 values)	45
Figure 7.6	Probabilities of negative net revenue	47
Figure 7.7	Expected net revenue (R$ billion, 2023 values)	47
Figure 7.8	Unconditional VaR (R$ billion, 2023 values)	49
Figure 7.9	Unconditional CVaR (R$ billion, 2023 values)	49
Figure 7.10	Conditional VaR (R$ billion, 2023 values)	50
Figure 7.11	Conditional CVaR (R$ billion, 2023 values)	50
Figure 7.12	Expected funding needed (R$ billion, 2023 values)	51
Figure 8.1	Tax revenue and workers beneifts (R$ billion, 2023 values)	52

List of tables

Table 3.1	TLP decomposition	20
Table 7.1	Parameter configurations for each simulation	41
Table 7.2	Probability of insufficient repayment	46
Table 7.3	Expected insufficient repayment amount (R$ billions)	46

List of Abbreviations

BNDES – Banco Nacional de Desenvolvimento Econômico e Social

C – Costs

CODEFAT – Conselho Deliberativo do FAT

CVaR – Conditional value at risk

FAT – Fundo de Amparo ao Trabalhador

FC – FAT Constitutional

FSD – FAT Special Deposits

GBM – Geometric Brownian Motion

LIBOR – London Interbank Offered Rate

MTE – Ministério do Trabalho e Emprego

NTN-B – Nota do Tesouro Nacional Série B

OMF – Off-Market Fund

OR – Other revenue

PASEP – Programa de Formação do Patrimônio do Servidor Público

PIS – Programa de Integração Social

PPP – Public-Private Partnership

SENAC – Serviço Nacional de Aprendizagem Comercial

SENAI – Serviço Nacional de Aprendizagem Industrial

SD – Special-purpose loans

SINE – Sistema Nacional de Emprego

SL – Subordinate loans

SLS – Subordinate loans - Selic rate

SLTJLP – Subordinate loans - TJLP rate

SLTLP – Subordinate loans - TLP rate

TB – Treasury bonds

TJLP – Taxa de Juros de Longo Prazo

TLP – Taxa de Longo Prazo

TR – Tax revenue

VaR – Value at risk

1
Introduction

The Banco Nacional de Desenvolvimento Econômico e Social (BNDES) is Brazil's main development bank, created to finance long-term investments in infrastructure, industry, and other strategic sectors. It has historically played a central role in Brazil's industrial and economic policy by offering subsidized loans and channeling resources to areas considered essential for national development by the government. Its funding model, however, has relied heavily on public resources, drawing from both the National Treasury and earmarked contributions from the Fundo de Amparo ao Trabalhador (FAT).

FAT was conceived in the late 1980s, but its roots trace back to policies implemented during Brazil's military dictatorship (1964–1985). In 1970, the regime introduced two mandatory contribution programs—Programa de Integração Social (PIS) for private-sector employees and Programa de Formação do Patrimônio do Servidor Público (PASEP) for public employees. Although initially intended as mechanisms for social integration and profit-sharing with workers, by 1974 these contributions were redirected to the Banco Nacional de Desenvolvimento Econômico e Social (BNDES) through Lei Complementar nº 19. This provided BNDES with a consistent source of funding for industrial and infrastructure projects, aligning with the regime's broader developmental strategies during Brazil's "economic miracle" era.

With the transition to democracy in the mid-1980s, there was growing pressure to expand social protections for workers, especially amid rising economic instability. In 1986, during the administration of President José Sarney, a provisional unemployment insurance program was introduced. This marked a turning point in the use of worker contributions, signaling a shift toward direct social welfare. The 1988 Constitution formalized this change, mandating that revenues from PIS-PASEP be used primarily to finance a new system of worker protection, including unemployment insurance and an annual income supplement for low-wage workers. However, the Constitution also preserved the previous arrangement by requiring that at least 40% of these funds continue to support economic development through BNDES (BNDES, 2018).

FAT was formally established in January 1990 under Federal Law Nº 7.998, consolidating the constitutional provisions into a structured fund under the Ministry of Labor. It institutionalized a dual mandate: to finance key labor programs and to provide long-term resources to BNDES.

Over the past two decades, the relationship between FAT and BNDES has been shaped by shifting political priorities, evolving institutional mandates, and persistent fiscal constraints. The resulting arrangement has created structural tensions within FAT, as its redistributive obligations often conflict with the demands of sustaining subsidized lending to BNDES. The 2000s and early 2010s were marked by a strong expansion in BNDES's operations, supported by subsidized interest rates and substantial Treasury loans. However, the rise in public debt and persistent primary deficits have raised concerns about the fiscal cost of these interventions. In response, the 2018 transition from the Taxa de Juros de Longo Prazo (TJLP) to the more market-aligned Taxa de Longo Prazo (TLP) sought to reduce implicit subsidies and improve the transparency and sustainability of BNDES financing.

During this same period, FAT began to face increasing financial pressure. While expenditures on social programs continued to grow, revenue, primarily tied to payroll taxes, remained stagnant. Combined with limited returns on its lending to BNDES and a rigid cost structure, FAT has recorded recurring deficits. These are frequently covered by transfers from the National Treasury, creating implicit fiscal risks and adding to the burden on public accounts.

The motivation of this research is to understand how FAT's financial health is shaped by interest rate dynamics, BNDES's repayment behavior, and political commitment to fiscal discipline, and how these factors translate into fiscal risk for the National Treasury. Although FAT is legally entitled to demand repayments from BNDES during periods of financial stress, these repayments are often deferred in practice, prompting the Treasury to intervene to preserve BNDES's liquidity. This pattern of implicit guarantees creates a contingent liability for the federal government, complicating debt management and increasing fiscal vulnerability in a context of limited fiscal space.

The study investigates the financial sustainability of FAT and its evolving relationship with BNDES and the National Treasury by using a dynamic Monte Carlo simulation to model FAT's revenues, expenditures, and financial flows under diverse macroeconomic and political scenarios. The model incorporates stochastic variation in interest rates, fiscal rules, and political behaviors to estimate how these factors affect FAT's net revenue, liquidity, and dependence on Treasury transfers over a ten-year horizon. Risk metrics such as Value at Risk (VaR) and Conditional Value at Risk (CVaR) are used to assess the likelihood and severity of FAT underfunding and Treasury bailouts, providing a forward-looking evaluation of tail risks and fiscal exposure.

The findings underscore the critical role of interest rate dynamics, the limited effectiveness of current repayment rules for subordinate debt, and the

persistent fiscal risks of recurring Treasury interventions. In doing so, the study contributes to the broader policy debate on how to reform development finance in Brazil in a more transparent and fiscally sustainable manner.

This thesis is structured as follows. Chapter 2 provides an overview of BNDES, detailing its historical evolution, funding structure, and recent strategic shifts. Chapter 3 focuses on FAT, examining its legal foundations, funding sources, expenditure obligations, and financial interactions with BNDES. Chapter 4 explores the relationship between FAT, BNDES, and the Treasury, focusing on fiscal accounting classifications and implicit subsidies embedded in current arrangements. Chapter 5 explains the model and calibration methodology used to estimate revenue, cost, and interest rate dynamics parameters. Chapter 6 introduces the Monte Carlo simulation model, outlining its structure. Chapter 7 presents simulation results across various scenarios, including analysis of VaR and CVaR metrics. Chapter 8 discusses an important question of who bears the cost of subsidized loans to BNDES. Finally, Chapter 9 offers concluding remarks and policy recommendations, reflecting on the implications of the findings for fiscal sustainability and the future of development finance in Brazil.

2
BNDES

BNDES has several funding sources, the two main ones being the FAT and the National Treasury. These sources represent approximately 75% of the bank's total liabilities on average, from 2002 to 2024, reaching the peak of 85% in the second quarter of 2014. FAT was historically the main funding source until the National Treasury began lending significant resources in 2008. With most of BNDES's debt to the Treasury repaid, FAT has reclaimed its role as the primary funding provider.

Furthermore, BNDES receives funding from various governmental sources, including the Fundo de Garantia do Tempo de Serviço (FGTS), Fundo de Investimento do Fundo de Garantia do Tempo de Serviço (FI-FGTS), Fundo da Marinha Mercante (FMM), and Fundo Amazônia (FA). The bank's other obligations include accounts for tax collection and receipt, trading and brokerage of securities, funds dedicated to specific purposes, and miscellaneous liabilities[1].

Beyond its primary funding sources, BNDES also raises funds internationally through bond issuances and loans from multilateral institutions such as the Inter-American Development Bank and development banks from other countries. Another funding mechanism involves repurchase agreements (repos), in which BNDES temporarily sells securities, which are typically government bonds, to financial institutions with a commitment to repurchase them at a later date for a predetermined price. This effectively functions as a short-term collateralized loan, where the securities serve as collateral. The proceeds from repos provide BNDES with short-term liquidity. Additionally, BNDES has issued debentures through its equity arm, BNDES Participações S.A. (BNDESPAR)[2], with the last issuance occurring in 2012 and maturing in 2019. Lastly, the PIS-PASEP Fund, established in the 1970s, remains a residual liability on BNDES's balance sheet. These are social contribution programs funded by employer contributions to provide benefits such as salary bonuses and unemployment insurance for private (PIS) and public sector (PASEP) workers.

Over the years, BNDES's role has changed a lot. Between 2002 and 2015, under the Partido dos Trabalhadores (PT) administration, the bank

[1]Equity comprises share capital, reserves, retained earnings, and other comprehensive income

[2]BNDESPAR supports the capitalization of Brazilian companies by acquiring shares and other securities.

significantly increased its disbursements and expanded its lending operations. During this period, a substantial share of financing was directed toward long-term investment projects, with extended grace periods and repayment terms, and also support for micro, small, and medium-sized enterprises (MSMEs).

After the impeachment of President Rousseff (PT) in May 2016, the bank's role changed. BNDES began assembling a portfolio focused on privatization projects, public-private partnerships (PPPs), and concessions, collaborating closely with clients including the Federal Government, states, and municipalities. The objective is to attract long-term investments and modernize the Brazilian state. In 2019, the first year of President Bolsonaro's administration, BNDES had 22 projects under development. By 2022, the number had increased to over 160 projects, mobilizing an estimated R$ 440 billion in capital. At the same time, BNDES's disbursements decreased from 1.4% of GDP in 2016 to 0.7% in 2021 (GIAMBIAGI; FAVERET, 2022).

As a facilitator of PPPs, concessions, and privatizations, BNDES plays a central role in designing, planning, and structuring large-scale infrastructure projects for private sector participation or complete privatization. Its responsibilities include identifying infrastructure needs, outlining technical and financial project details, performing risk assessments, allocating responsibilities between public and private parties, preparing legal, regulatory, and contractual frameworks, arranging financing, and managing competitive auctions or bids to select private operators.

Privatization involves permanently transferring ownership of state-owned enterprises or assets to the private sector. Concessions grant private entities the temporary responsibility for providing public services. This group encompasses traditional concessions, where user fees and revenues fully compensate the private operator, and PPPs, which address projects lacking sufficient revenue. PPPs can take the form of administrative concessions, where the government pays the service provider fully or partially without user fees, or sponsored concessions, where user fees exist but are complemented by government payments to guarantee financial viability.

3
FAT

The FAT is a compulsory public savings fund established in 1988 to finance workers' welfare programs, particularly unemployment insurance and salary bonuses for low-wage employees. Additionally, it provides BNDES with a stable funding source, enabling better long-term expenditure planning. FAT is a special accounting-financial fund linked to the Ministry of Labor and Employment (MTE).

3.1
Sources of funding

The primary revenue source of FAT comes from tax collections linked to two public funds: the Programa de Integração Social (PIS) and the Programa de Formação do Patrimônio do Servidor Público (Pasep). PIS is funded by private sector contributions, while Pasep is sustained by public sector employers. Both are financed through mandatory contributions levied on gross revenue, and the combined resources flow into the PIS-Pasep Fund, which finances FAT.

The applicable tax rate and calculation method depend on the taxpayer's chosen tax regime. Under the cumulative regime—generally used by firms that adopt the presumed profit method—the rate is 0.65% of gross revenue, with no deduction of taxes paid on inputs. Under the non-cumulative regime, typically used by companies under the actual profit method, the rate is higher at 1.65%, but companies are allowed to deduct taxes paid on input purchases, resulting in a lower effective tax burden. Additionally, nonprofit entities contribute 1% of payroll, rather than on gross revenue.

Historically, this tax revenue has been impacted by several factors, such as the Desvinculação de Receitas da União (DRU), a fiscal mechanism that allowed the federal government to reallocate part of earmarked revenues to general expenses. Initially 20% of PIS-Pasep's revenue, DRU reached 30% in 2016 but was discontinued for PIS and Pasep in 2019. Tax relief measures, often intensified during economic downturns, further reduce revenues. Additionally, since 2019, there can be discretionary allocation of a portion of the tax revenue to the social security system. This allocation has happened every year since then, with values close to BRL 17 billion, but slowly increasing.

Besides these contributions, FAT has a financial revenue. It generates income from financial returns on its net assets, which include FAT Constitu-

tional, FAT Special Deposits, and the off-market fund. The off-market fund comprises short-term liquidity reserves, primarily invested in treasury bonds, to meet immediate obligations. This fund is overseen primarily by Banco do Brasil. By regulation, this liquidity reserve must exceed the total unemployment insurance plus 50% of salary bonuses paid over the previous three months, both adjusted for inflation.

3.2 FAT Constitutional and FAT Special Deposits

FAT Constitutional receives a fixed share of net PIS-Pasep revenues. Until 2019, this share was 40%, but with the extinction of the DRU over PIS-Pasep revenue, the share went down to 28%. In contrast, FAT Special Deposits receives surplus revenues after FAT's obligations are fully met, and these surplus funds are lent not only to BNDES but also to other government financial institutions.

A crucial difference between FAT Constitutional and Special Deposits is their repayment structure. FAT Special Deposits require monthly principal amortization ranging between 1% and 2%. FAT Constitutional acts as subordinate debt without a defined repayment deadline, demanding only interest payments twice a year, in January and July. Principal repayment is required only if FAT lacks sufficient resources to fulfill its obligations, following guidelines set by CODEFAT [1]: up to 20% of the balance in the first two fiscal years, 10% from the third to fifth year, and 5% thereafter.

In 2018, FAT Constitutional and Special Deposits transitioned from being remunerated at the TJLP (Taxa de Juros de Longo Prazo) to the TLP (Taxa de Longo Prazo). The TJLP, a long-term interest rate set quarterly by Brazil's National Monetary Council, was primarily used by BNDES to offer subsidized financing for investment projects. As it was typically below market rates, it effectively embedded implicit subsidies for borrowers.

The TLP was introduced to reduce fiscal costs and bring credit operations more aligned with market conditions. It is calculated based on the 90-day average real yield of 5-year NTN-B bonds, adjusted monthly for inflation. To smooth the transition, a discount factor was applied to the NTN-B component of the TLP, also called the real component. This multiplier increased yearly, reaching one by 2023, as shown in Table 3.1. The government opted to adopt

[1] CODEFAT is a tripartite body composed of government, employer, and worker representatives that oversees the management of FAT. It sets policy guidelines, monitors program effectiveness, and ensures proper use of FAT resources for employment and income-support initiatives.

the TLP over the Selic because the latter is more volatile, and using the TLP reduces refinancing risk for long-term investment projects.

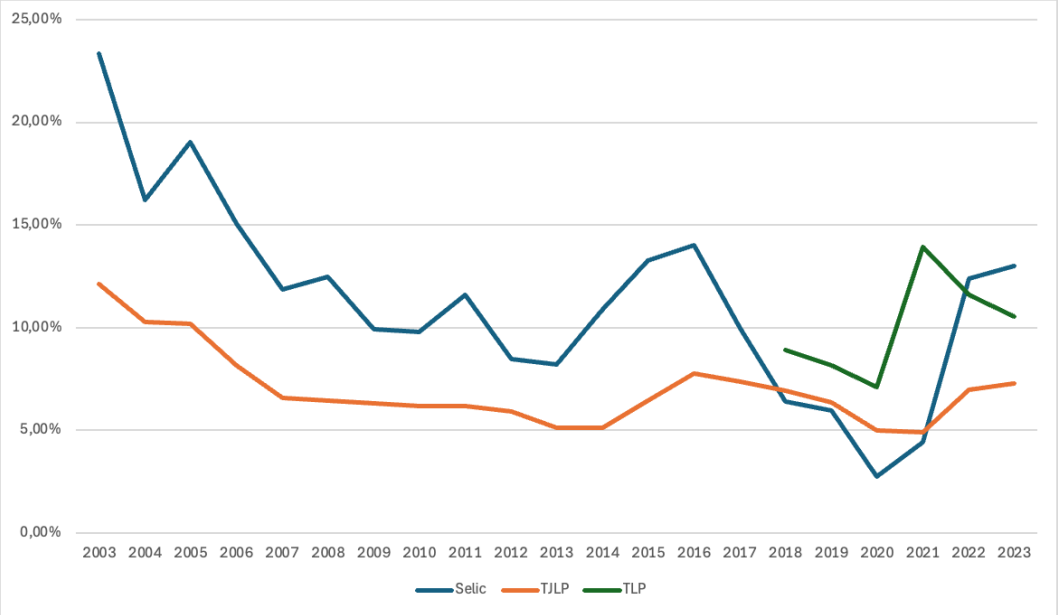
Table 3.1: TLP decomposition

Year	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**
Real component (%)	4.98	3.68	2.49	3.51	5.51	5.65
Multiplier	0.57	0.63	0.74	0.83	0.91	1.00
Effective real component (%)	2.84	2.32	1.84	2.91	5.01	5.65
IPCA (%)	3.75	4.31	4.52	10.06	5.79	4.62
TLP (%)	6.70	6.73	6.44	13.26	11.09	10.53

As can be seen in Figure 3.1, which disregards the multiplier since its implementation, the TLP was higher than Selic for the years 2018 to 2021. Selic is a short-term interest rate, set by the Central Bank to guide monetary policy. In contrast, the TLP reflects five-year market interest rates, which are determined by investor expectations regarding inflation, fiscal risk, and term premiums. Consequently, the TLP incorporates long-term political and fiscal uncertainty: the higher the perceived future fiscal risk, the higher the interest investors demand to finance the government.

During this period, the country was adopting an expansionary monetary policy while facing fiscal uncertainty due to elections, Covid-19, and important fiscal legislation under congressional debate. This led to a higher TLP than Selic.

Figure 3.1: Nominal interest rates



Source: FAT

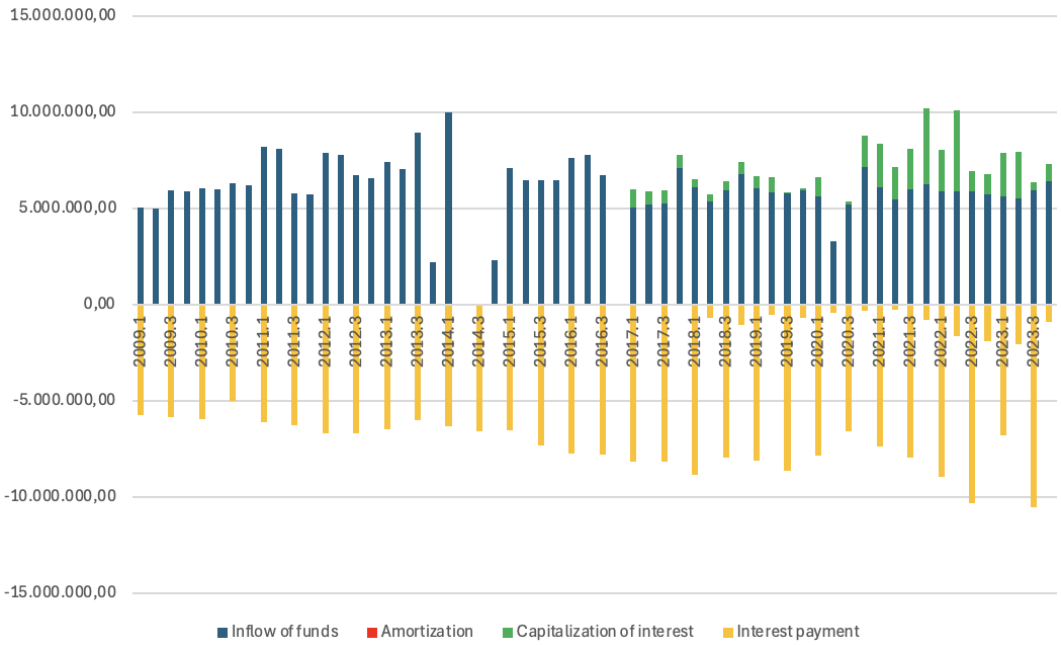
Loans repaid using TJLP or TLP are subject to an annual remuneration cap of 6%; any interest accrued above this threshold is capitalized into the outstanding loan balance. Since 2018, unused funds from these deposits held by BNDES have been remunerated at the Selic rate, whereas previously, FAT Constitutional unused funds earned returns based on the TJLP. A small share of FAT Constitutional funds, known as FAT Cambial, is denominated in foreign currency and remunerated according to international benchmarks, such as LIBOR. Due to low foreign interest rates, this share hasn't earned interest in more recent years.

Figure 3.2 presents the quarterly cash flows between BNDES and FAT related to the subordinate loans, adjusted to 2023 values. Each quarter, new loans are disbursed to the bank, while interest payments occur only in the first and fourth quarters of each year. In recent years, adopting the TLP has led to higher interest rates, which can be observed through the capitalization of interest beyond the 6% nominal cap. Notably, these loans have never been amortized; whenever amortization would have been required, the National Treasury intervened by providing the necessary funds to FAT.

Figure 3.3 presents the quarterly cash flows between BNDES and FAT for the Special Loans. In many periods, no new loans were issued. This happens when the fund faces negative net revenue. Meanwhile, amortizations and interest payments occur regularly. Despite the increase in interest rates, total interest payments have been declining over time, as fewer new loans are made

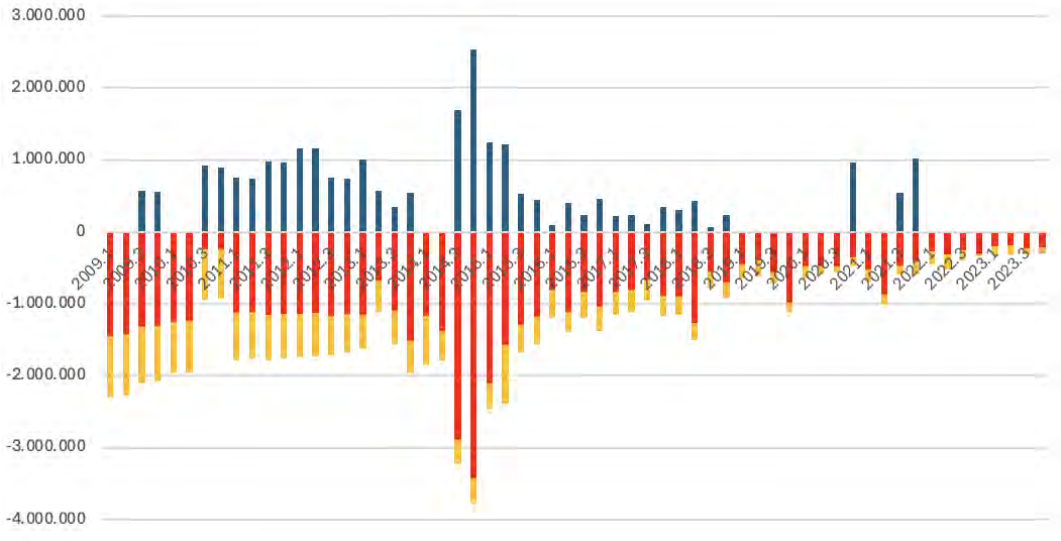
and principal repayments continue, gradually reducing the stock of outstanding loans.

Figure 3.2: BNDES & FAT Constitutional (R$ thousand, 2023 values)



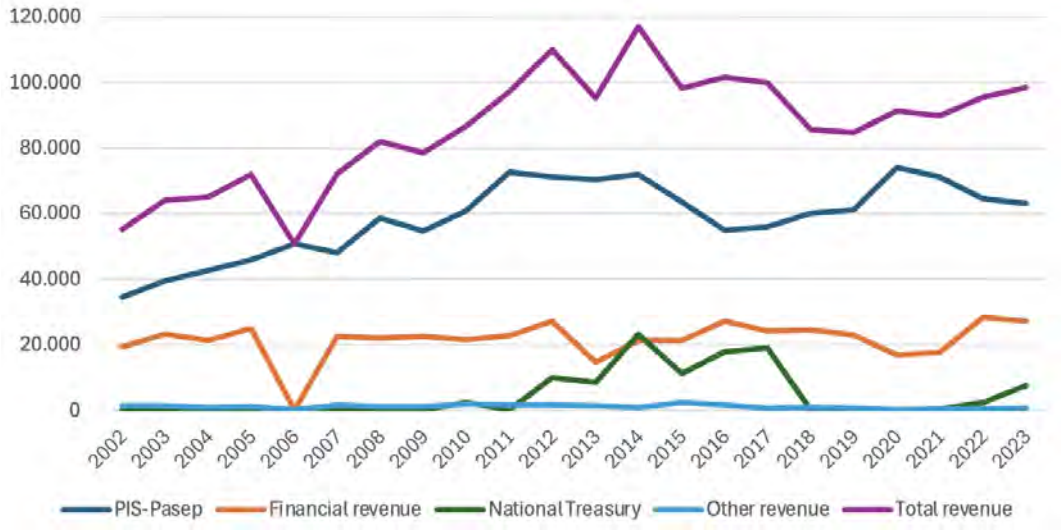
Source: FAT

Figure 3.3: BNDES & FAT Special Deposits (R$ thousand, 2023 values)



Source: FAT

Figure 3.4: FAT's revenue (R$ million, 2023 values)



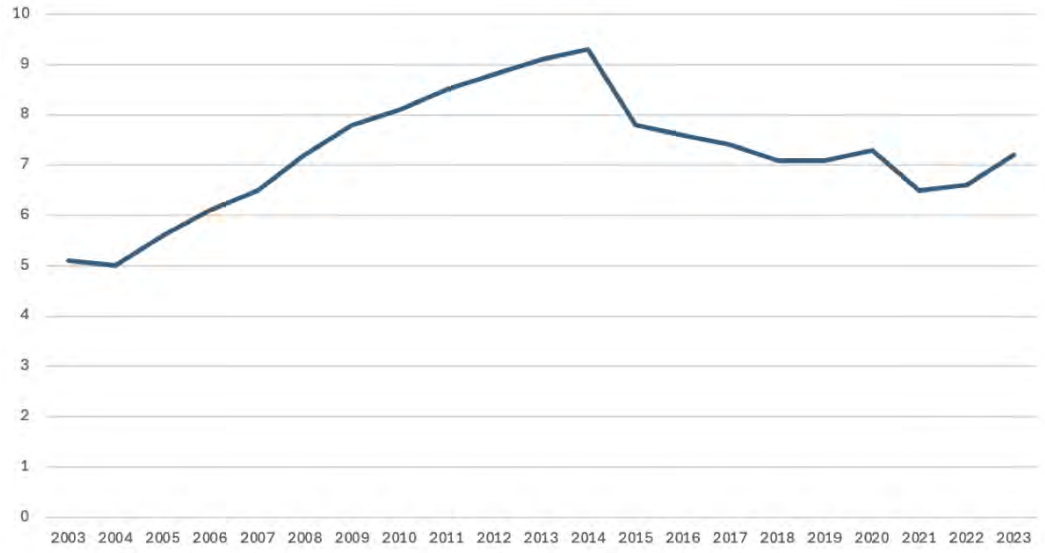
Source: FAT

3.3 Liabilities and Obligations

Beyond funding BNDES, FAT's primary obligations are unemployment insurance and salary bonuses. Unemployment insurance benefits workers dismissed without cause. It can last for up to five months, with a minimum value equal to the minimum wage. Legislation changes, such as Law No. 13.134/2015, increased qualification requirements, reducing FAT expenses. Similarly, Law No. 13.467/2017 introduced mutual termination agreements, which exclude eligibility for unemployment insurance.

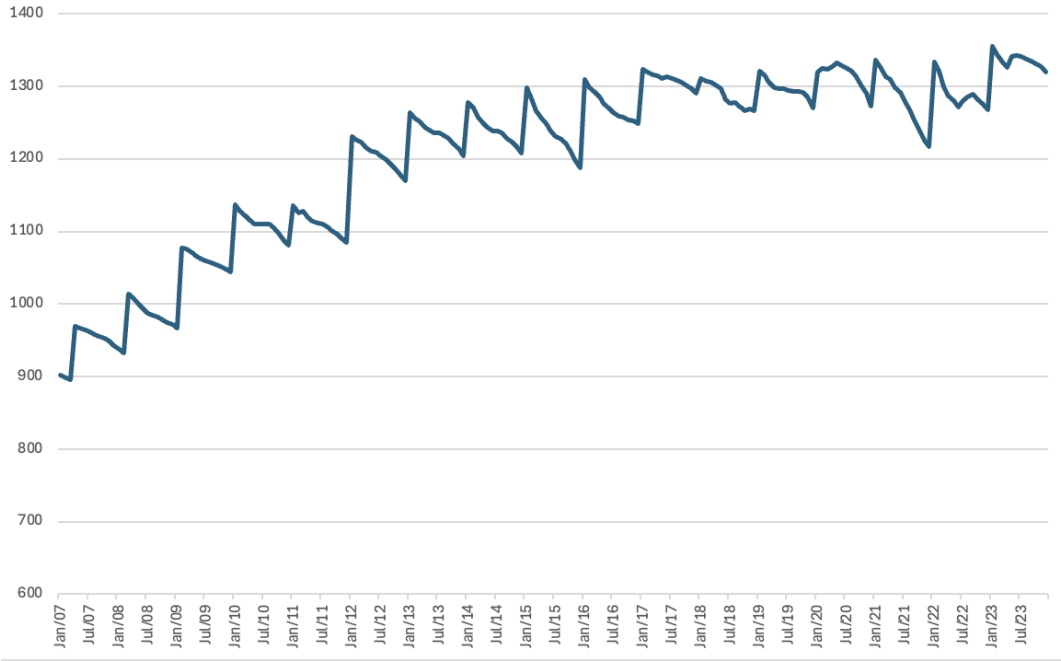
Interestingly, unemployment insurance expenditures rose even during periods of falling unemployment due to expanding formal employment and increasing minimum wages, offsetting potential savings from lower unemployment rates. As formal employment expanded, more workers became eligible for benefits, and rising minimum wages made the benefit itself more costly. Conversely, economic downturns often led to reduced unemployment insurance claims, as many displaced formal workers transitioned into informal employment, thereby losing eligibility, as shown in Figure 3.5. Additionally, real minimum wage growth typically stagnated during recessions, further limiting benefit expenditures (Figure 3.6).

Figure 3.5: Unemployment insurance beneficiaries (millions)



Source: FAT

Figure 3.6: Real minimum wage (R$, 2023 values)



Source: MTE

This seemingly paradoxical dynamic reveals a deeper fiscal structure: unemployment insurance costs in Brazil exhibit a pro-cyclical pattern, increasing during economic expansions rather than downturns. This arises primarily due to Brazil's high turnover rates among formal workers (GONZAGA; PINTO, 2014) and its substantial informal labor market. Although this arrangement

may appear counterintuitive or even undesirable from a macroeconomic stabilization perspective—contrary to conventional unemployment insurance that increases payouts precisely during economic downturns—it nonetheless fulfills a distinct fiscal smoothing function reminiscent of Arrow-Debreu's complete markets framework. According to this framework, resources are ideally allocated to states of the world when they are most abundant; thus, in prosperous times, higher formal employment and rising wages increase both government revenue and unemployment insurance obligations simultaneously. During downturns, fiscal resources shrink, but the program's expenditures also decline naturally due to decreased eligibility.

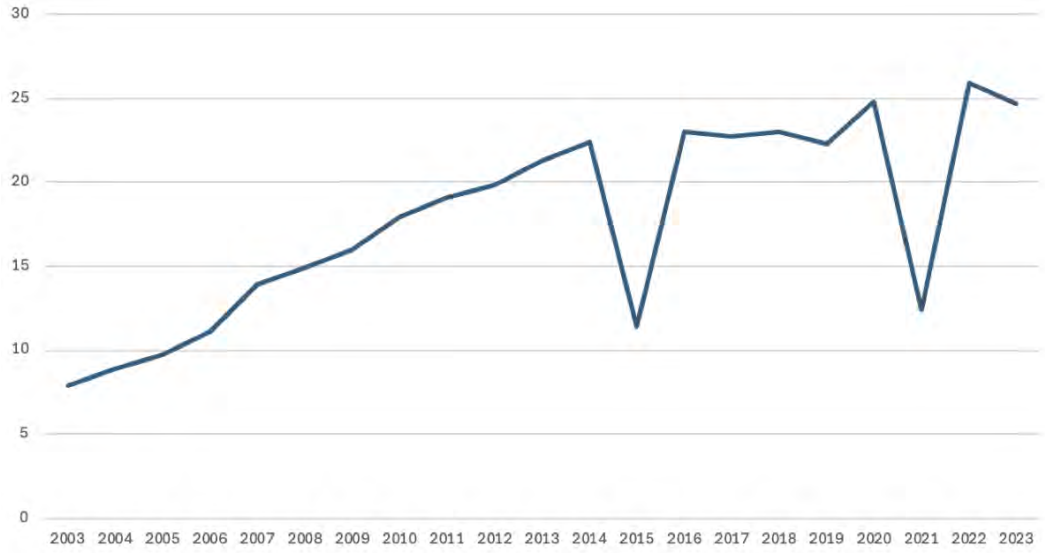
From the individual worker's standpoint, the Brazilian system still operates as genuine unemployment insurance: workers receive benefits precisely when they become unemployed. However, at the aggregate level, this system behaves more like a contingent income transfer linked to formal employment dynamics rather than a typical counter-cyclical stabilizer. It provides limited macroeconomic stabilization precisely because total expenditures rise when aggregate unemployment risks are lower and decline when these risks increase. Thus, while the system aligns with a form of contingent risk-sharing consistent with Arrow-Debreu principles, its pro-cyclical aggregate structure fundamentally contrasts with the counter-cyclical automatic stabilization expected from standard unemployment insurance schemes.

A simple stylized model helps illustrate this: if total cost is approximately $U = N \times \pi \times b$, where N is the number of formal workers, π the job separation rate, and b the average benefit, then even as unemployment falls (π decreases), growth in N and b during expansions can still drive costs upward. Importantly, however, the system is not without risks. Periods of high employment and generous eligibility can invite fraud or misuse, such as collusive layoffs or benefit cycling. These practices inflate claims and erode the program's insurance logic. In response, recent legal reforms have tightened access criteria and enhanced oversight.

Salary bonuses benefit workers earning less than two minimum wages, registered in the PIS-Pasep Participation Fund or National Worker Registry for at least five years. Law No. 13.134/2015 altered benefit calculation from a full minimum wage to a proportional monthly value of work in a given year. Eligibility now requires at least six months of uninterrupted employment in a year. Rising formal employment and increased minimum wages amplified the cost of salary bonuses to FAT. Furthermore, payments were delayed in 2015 to ease fiscal pressures and again in 2021 due to reallocation for COVID-19 relief programs, as seen by the downward spike in the number of beneficiaries

in those years in Figure 3.7.

Figure 3.7: Salary bonus beneficiaries (millions)

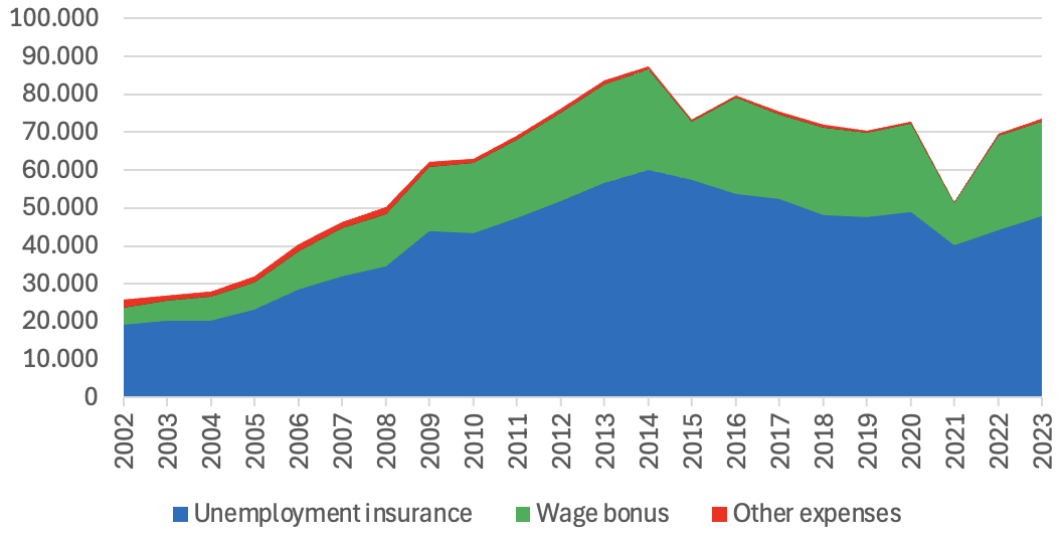


Source: FAT

FAT incurs smaller additional costs related to administrative operations, oversight activities by CODEFAT, and funding training and professional qualification programs, including the National Qualification Plan (PNQ) and job placement services through Sistema Nacional de Emprego (SINE), in collaboration with entities like the Serviço Nacional de Aprendizagem Industrial (SENAI) and Serviço Nacional de Aprendizagem Comercial (SENAC)[2]. Figure 3.8 shows FAT's obligations throughout the years, excluding the loans made out of the tax revenue to FAT Constitutional.

[2]SENAI provides technical and vocational training focused on Brazil's industrial sector, while SENAC offers professional education for commerce and service industries like tourism, IT, and administration. SINE is a federal government program that connects job seekers with employers and manages unemployment benefits and labor market services.

Figure 3.8: FAT's costs excluding FC & FSD (R$ million, 2023 values)



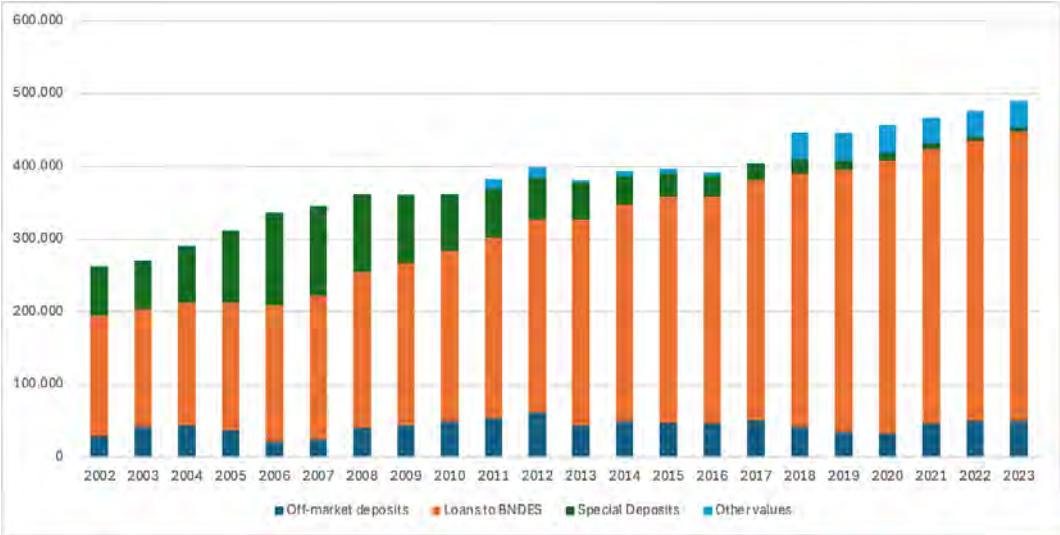
Source: FAT

3.4 Net assets

FAT's main net assets are the ones that affect the financial revenue: FAT Special Deposits, FAT Constitutional, and Fundo Extramercado. Additionally, FAT maintains an "other" category containing PIS-Pasep tax credits, bonds, and receivables—essentially rights to collect unpaid taxes formally registered as active debt. Since 2018, following recommendations by the TCU[3], these credits have been transparently registered in Federal Revenue accounts, with active debt managed by the National Treasury Attorney General's Office. This last category is negligible compared to the other three, which compose the financial revenue.

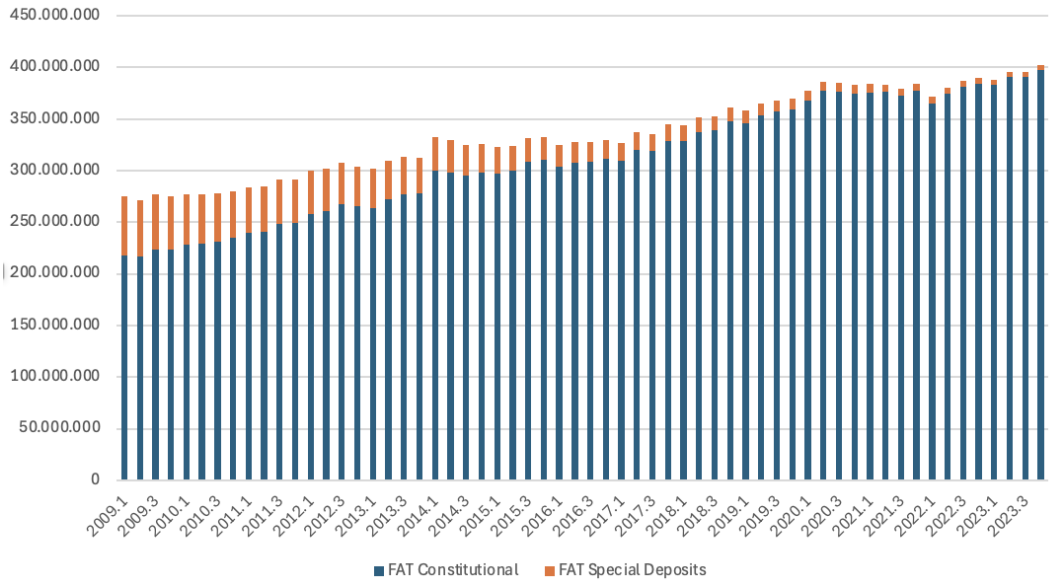
[3]TCU (Tribunal de Contas da União) is Brazil's Federal Court of Accounts, an independent government body responsible for overseeing the use of public funds. It audits and monitors federal government expenditures, evaluates the performance of public programs, and supports Congress in holding public officials accountable for financial management.

Figure 3.9: FAT's assets (R$ million, 2023 values)



Source: FAT

Figure 3.10: FAT's outstanding loans to BNDES (R$ thousand, 2023 values)



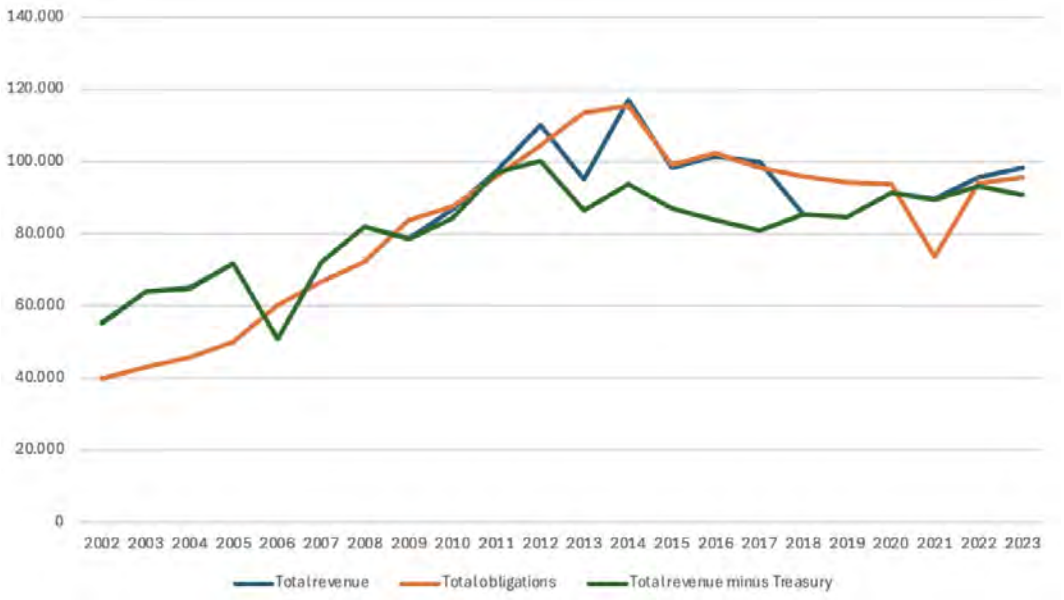
Source: FAT

4
FAT, BNDES, and the Treasury

In recent years, FAT has consistently experienced budget deficits due to expenditures outpacing revenue growth. This imbalance has been exacerbated by historically low interest rates, tax relief measures, and reallocations via the DRU and social security contributions, all negatively impacting FAT's revenues. Concurrently, rising minimum wages have expanded both the number and average benefits of beneficiaries, intensifying fiscal pressures. From 2002 to 2023, total revenue grew by 64%, while total obligations increased by 139% (in 2023 values).

Figure 4.1 illustrates FAT's persistent deficits since 2009, highlighting an unsustainable fiscal trajectory despite periodic Treasury interventions.

Figure 4.1: FAT Costs & Revenues (R$ million, 2023 values)



Source: FAT

In response, FAT ceased issuing new loans via FAT Special Deposits, further constraining its financial revenues. To address liquidity shortfalls, the National Treasury began transferring funds to FAT, initially justified by losses due to DRU reallocations—a mechanism now discontinued. This raises uncertainty about continued Treasury support, prompting debates on whether BNDES should start repaying principal on FAT Constitutional loans.

While intra-governmental transfers do not directly alter the government's consolidated net worth, they highlight underlying fiscal stress. Without Trea-

sury support, FAT would face tough choices: reducing benefits, increasing PIS-Pasep taxes, or demanding higher interest from BNDES, each with significant distributional implications. Consequently, Treasury support indirectly constitutes economic subsidies, carrying real consequences for taxpayers, workers, and investment decisions.

Accounting practices clarify these fiscal impacts and pressures to halt transfers to FAT and BNDES loans. Treasury loans to BNDES, such as those between 2008 and 2014, are recorded as financial assets (*below-the-line* operations), thus excluded from primary expenditures but increasing gross debt. Conversely, Treasury transfers to FAT directly increase both primary deficits and public debt. For instance, FAT's $R10.7 billion deficit in 2015 was entirely Treasury-funded, elevating gross debt and worsening the primary fiscal balance. Such intra-government reallocations often have tangible external fiscal consequences.$

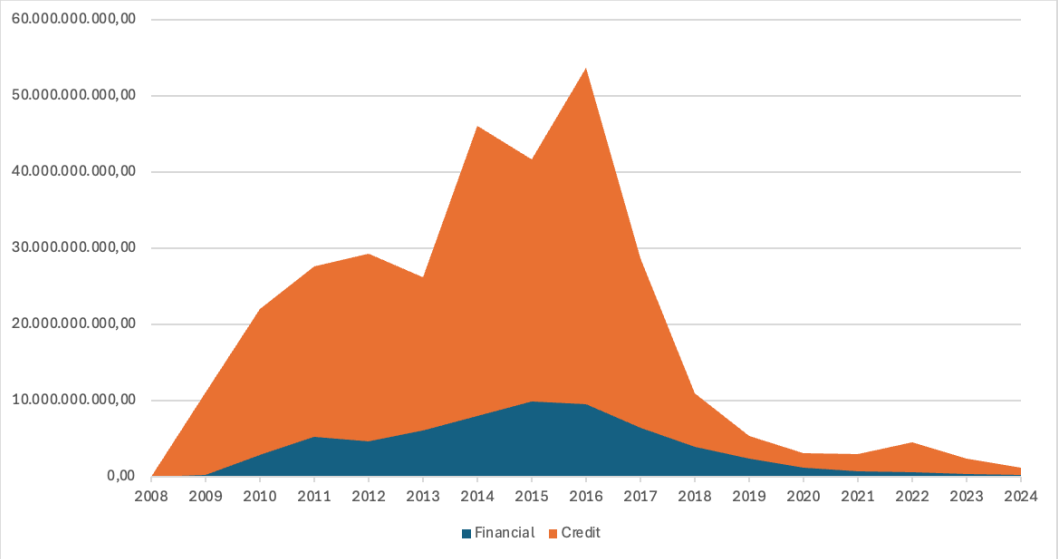
This illustrates a crucial distinction: intra-government flows among FAT, BNDES, and Treasury affect public sector accounting but do not inherently impact government net wealth. Real economic effects emerge when funds flow to the private sector, such as wage supplements or subsidized credits to firms.

Treasury dividends from BNDES count as primary revenue, reflecting operational profits, whereas interest and principal repayments from BNDES constitute financial revenue, thus excluded from primary balances. PIS-Pasep revenues allocated to FAT (72%) count as primary government revenue, while the 28% directed to BNDES remain off public accounts.

Recent fiscal reforms, notably the 2019 pension reform (PEC 6/2019), highlighted these dynamics by proposing FAT resource reallocations from BNDES financing to direct social security expenditures. According to the Independent Fiscal Institution (IFI), this would increase transparency of primary spending without affecting overall fiscal neutrality. Though broader pension reform passed, this specific FAT resource proposal was not approved.

Implicit subsidies from Treasury and FAT to BNDES contribute notably to fiscal pressures, carrying substantial opportunity costs even without immediate primary balance or gross debt implications. Resources allocated to BNDES could instead offset Treasury borrowing at higher market rates (closer to the Selic rate). The federal budget explicitly recognizes these implicit subsidies (Demonstrative Statement of Financial and Credit Benefits—DBFC). These arise from below-market-rate loans and from Treasury covering FAT liquidity shortfalls, converting intra-governmental imbalances into real fiscal transfers.

Figure 4.2: Subsidies conferred by the National Treasury (R$)



Source: Boletim de Subsídios do Tesouro Nacional no âmbito do PSI e nos empréstimos ao BNDES

Figure 4.2 shows Treasury subsidies to BNDES (2008–2024, adjusted to 2024 values). These include explicit financial subsidies (interest rate equalization) and implicit credit subsidies (below-market-rate loans). Financial subsidies arise from gaps between lender and borrower rates, requiring regular Treasury transfers to BNDES and FINEP. Credit subsidies reflect the difference between Treasury borrowing costs and BNDES loan rates, totaling BRL 360.4 billion as of December 2024.

Transitioning from TJLP to TLP may enhance FAT's financial sustainability by linking interest rates more closely to inflation and market-driven parameters, thereby increasing FAT's income. However, full financial benefits were delayed until 2023 due to applied discount factors. Some subordinate loans still use TJLP, but these are diminishing. This shift reduces government burdens and subsidies, realigning incentives and shifting costs partially to borrowing firms.

Ultimately, intra-governmental flows among FAT, BNDES, and Treasury represent substantial fiscal transfers to non-government entities. Whether through worker benefits, subsidized firm financing, or Treasury debt issuance, the FAT-BNDES financial structure significantly impacts income distribution, private investment, and fiscal sustainability.

In summary, FAT's financing role through PIS-Pasep involves significant implicit fiscal costs. Persistent deficits and rising debt amplify political and fiscal pressures, highlighting the need to reassess implicit subsidies and financial

arrangements. While recent reforms may improve sustainability, risks remain substantial and require continued Treasury attention.

5
Model and Calibration

The model employs a Monte Carlo simulation to estimate FAT's revenues, costs, and cash flows between FAT, BNDES, and the Treasury. FAT's lending to BNDES is driven entirely by its internal revenue and cost dynamics, independent of BNDES's specific financial parameters.

FAT's revenues come primarily from two sources: tax revenues (TR) and financial revenues (FR). Nearly all FAT costs arise from salary bonuses, unemployment insurance, and new subordinate loans to BNDES. Salary bonus and unemployment insurance costs are modeled separately and then combined into a single category—Costs (C). New subordinate loan costs are set at 28% of the tax revenue each period.

At each period-end, net revenue determines FAT's actions: positive net revenue results in new special, while negative net revenue leads either to the Treasury lending money to FAT (probability 'p') or FAT demanding repayment from BNDES on subordinate loans (probability '1-p'). The probability 'p' is determined by the government type. Every four years, there is a 50/50 chance that a *type a* candidate will win the election or lose it to a *type b* candidate.

5.1
Assumptions

The simulation starts with 2023 values for outstanding subordinate loans, special loans, and treasury bonds, as well as initial monthly tax revenue, unemployment insurance and salary bonus. All these values come from the observed values for December 2023.

No significant structural changes are anticipated in population growth, economic expansion, labor force participation, or the adjustment of the minimum wage. As a result, it is reasonable to rely on historical values for costs and revenue without explicitly accounting for potential shifts in these variables.

5.2
Tax revenue and costs

This section describes the econometric modeling and calibration of the three key financial variables for FAT: the tax revenue (TR), the unemployment insurance costs (UI), and the salary bonus costs (SB). All variables are modeled in real terms, indexed to 2023 values and seasonally adjusted. Due to excessive volatility in monthly data, the initially estimated regression

equations use annual data. The annual coefficients were then adjusted to a monthly frequency when necessary; specifically, drift and volatility parameters were scaled, while elasticity parameters remained unchanged. Below, the final calibrated equations used in the monthly Monte Carlo simulation model is presented.

5.2.1 Log-Difference Calibration for GDP and Minimum Wage

The evolution of real GDP and real minimum wage (MW) is modeled using log-differences calibrated from monthly, real, and seasonally adjusted data. The following stochastic processes are used:

$$\Delta \log(GDP_t) = 0.0028 - 0.7845\Delta \log(GDP_{t-1}) - 0.3527\Delta \log(GDP_{t-2}) + 0.5039\varepsilon_{gdp,t-1} + \varepsilon_{gdp,t} \tag{5-1}$$

This ARMA(2,1) specification captures cyclical GDP dynamics, reflecting mean reversion and inertia in GDP growth rates.

$$\Delta \log(MW_t) = 0.00241 + \varepsilon_{mw,t} \tag{5-2}$$

This random walk with drift implies that minimum wage changes are persistent with an average monthly growth of approximately 0.241%, capturing the policy-driven nature of minimum wage adjustments.

5.2.2 Tax Revenue (TR)

The monthly equation for the tax revenue growth is:

$$\Delta \log(TR_t) = -0.00155 + 1.528 \cdot \Delta \log(GDP_t) + \varepsilon_{tr,t}, \tag{5-3}$$

where

$$\varepsilon_{pp,t} \sim N(0, 0.0183^2) \tag{5-4}$$

This specification implies that a 1% increase in monthly real GDP growth translates, on average, into approximately a 1.53% increase in monthly tax revenue.

5.2.3
Unemployment Insurance Costs (UI)

The monthly unemployment insurance costs equation is given by:

$$\Delta \log(UI_t) = -0.00499 + 1.730 \cdot \Delta \log(MW_t) + 1.340 \cdot \Delta \log(GDP_{t-1}) + \varepsilon_{ui,t}, \tag{5-5}$$

where

$$\varepsilon_{ui,t} \sim N(0, 0.0117^2) \tag{5-6}$$

This indicates that unemployment insurance costs are highly responsive to real minimum wage growth and moderately responsive to lagged GDP growth.

5.2.4
Salary Bonus Costs (SB)

The salary bonus costs are modeled monthly as follows:

$$\Delta \log(SB_t) = -0.000681 + 1.703 \cdot \Delta \log(MW_t) + 1.031 \cdot \Delta \log(GDP_t) + \varepsilon_{sb,t}, \tag{5-7}$$

where

$$\varepsilon_{sb,t} \sim N(0, 0.0203^2) \tag{5-8}$$

The salary bonus thus significantly increases with both real GDP growth and real minimum wage growth, reflecting its dependence on macroeconomic and labor market conditions.

These calibrated equations are integrated into the monthly simulation framework, ensuring consistency with historical annual trends and accurately capturing the stochastic evolution of FAT's revenues and costs over the 10-year forecast horizon.

When estimating tax revenue parameters, the model incorporates amounts historically retained by the government through DRU, reflecting current policy where DRU no longer applies. Furthermore, specific corrections were implemented for 2015 and 2021 to account for one-time adjustments in the salary bonus payment schedule. These corrections rest on the simplifying assumption that similar extraordinary adjustments will not recur.

5.3
Interest rates, inflation, and financial revenue

One of the key challenges in this work was deciding whether to use nominal or real interest rates in the model. Using real interest rates initially seemed intuitive, as everything in the model is expressed in real terms. However, an important complication arises from a specific contractual detail: the existence of a nominal interest rate cap.

The FAT Constitutional payments are subject to a fixed 6% nominal interest rate cap. Any interest exceeding this cap in a given period is recapitalized into the outstanding debt. Crucially, this nominal cap does not adjust for inflation.

This creates an important methodological issue. If it was used a real, Fisher-adjusted interest rate, it would introduce an inflation adjustment to the cap itself, something not reflected in the actual contractual conditions. Therefore, employing real interest rates would distort the realistic constraints of the nominal 6% cap.

Given this contractual reality, the model utilizes nominal interest rates and the nominal cap. This choice maintains historical consistency, accurately reflecting both contractual obligations and actual observed repayments. Historically, when past nominal payments are adjusted for inflation to 2023 BRL terms, the ratio of real interest payments to real outstanding debt demonstrates that the nominal 6% cap remained binding in real terms. Thus, even after inflation adjustment, the cap remains a relevant and effective constraint historically.

The modeling strategy adopted here directly tracks real cash flows on a period-by-period basis by converting actual nominal payments into their real purchasing power in 2023. Importantly, this approach does not directly apply the Fisher equation to infer growth rates or perform discounting explicitly. Instead, it focuses on how inflation directly affects the real value of nominal cash flows. This approach matches precisely how the FAT fund receives cash flows: nominally, without any indexation to constant purchasing power.

Thus, the focus shifts from evaluating "What real rate of return does this lending generate?" to "How much real money will the fund earn and spend, and when might it face deficits?". This perspective aligns closely with the practical reality faced by financial institutions and firms that lend and receive nominal interest payments, subsequently experiencing the erosion of purchasing power due to inflation. The model accurately reflects this reality by applying nominal interest rates first and then deflating the resulting cash flows.

Since the debt stock is modeled in real terms, the following calculations are employed:

– **Nominal interest paid:**

$$P_t^{\text{nom}} = \min(r_t^{\text{nom}}, c) \times D_t^{\text{real}} \times (1 + \pi_t) \tag{5-9}$$

– **Recapitalized nominal interest:**

$$R_t^{\text{nom}} = \max(0, r_t^{\text{nom}} - c) \times D_t^{\text{real}} \times (1 + \pi_t) \tag{5-10}$$

Then, deflate both:

– **Real paid interest:**

$$P_t^{\text{real}} = \frac{P_t^{\text{nom}}}{1 + \pi_t} = \min(r_t^{\text{nom}}, c) \times D_t^{\text{real}} \tag{5-11}$$

– **Real recapitalized interest:**

$$R_t^{\text{real}} = \frac{R_t^{\text{nom}}}{1 + \pi_t} = \max(0, r_t^{\text{nom}} - c) \times D_t^{\text{real}} \tag{5-12}$$

Therefore, the model simplifies calculations by directly applying nominal interest rates and the nominal cap to real debt, avoiding explicit inflation correction.

The model features three sources of financial revenue: treasury bonds, special-purpose loans, and subordinate loans. Treasury bonds yield returns based on the Selic rate, which evolves according to a Cox-Ingersoll-Ross (CIR) process calibrated using nominal monthly Selic values from 2002 to 2023. Special-purpose loans generate revenue using the TLP rate, while subordinate loans include portions with different compensation methods: some carry no interest, others are compensated with Selic, TJLP, or TLP.

The TJLP also evolves according to a CIR process calibrated with nominal historical values from the same 2002-2023 period. The TLP rate has two components: a pre-fixed rate linked to Treasury Notes (NTN-B) and a post-fixed component representing inflation. The transition from TJLP to TLP in 2018 involved a smoothing multiplier that gradually adjusted rates over five years. The model computes the pre-fixed TLP component using a moving average approach to accurately reflect this. Initially, this average incorporates historical data from the available previous 72 months plus the current value generated using a Vasicek model calibrated on historical NTN-B data. Each month, as new values are generated, the average expands until reaching 120

months, representing an assumed average loan maturity of 10 years. After reaching 120 months, the oldest value is discarded monthly, and the most recent value is added, maintaining a rolling 120-month window. Inflation, required to complete the TLP calculation, is similarly modeled using a Vasicek process calibrated with historical inflation data.

Interest rate update equations:

$$r_{tlp} = r_{tlp} + a_{tlp}(b_{tlp} - r_{tlp}) + \sigma_{tlp} z_{tlp} \tag{5-13}$$

$$r_s = r_s + a_s(b_s - r_s) + \sigma_s \sqrt{\max(r_s, 0)} z_s, \quad r_s \geq 0 \tag{5-14}$$

$$r_{tjlp} = r_{tjlp} + a_{tjlp}(b_{tjlp} - r_{tjlp}) + \sigma_{tjlp} \sqrt{\max(r_{tjlp}, 0)} z_{tjlp}, \quad r_{tjlp} \geq 0 \tag{5-15}$$

5.4 Correlation Structure of Shocks

To capture the joint behavior of the innovations in interest rates and FAT cash flows, I correlate their residuals using a Cholesky decomposition.

A Cholesky decomposition is applied to this correlation matrix, producing a lower triangular matrix L. At each step of the simulation, a vector of independent standard normal shocks $\mathbf{u}_t \sim \mathcal{N}(0, I)$ is drawn, and the correlated shocks are generated as:

$$\mathbf{z}_t = L \cdot \mathbf{u}_t$$

This ensures that the simulated innovations for interest rates and FAT variables replicate the empirical correlation structure observed in the data.

6
Simulation procedure

The simulation operates monthly, updating each period:

1. **GDP and minimum wage:** Each period the log-difference of real GDP and real minimum wage are estimated.

2. **Compute revenues and costs:** Tax revenue (TR), salary bonus, and unemployment insurance are generated using their respective regression equations and the estimated values for the log-difference of GDP and MW. SB and UI are then combined into Costs (C).

3. **Calculate financial revenue (FR):** Financial revenue includes interest on special-purpose loans (SD), treasury bonds (TB), and subordinate loans (SL), along with monthly principal repayments (1.5%) for special-purpose loans and treasury bonds. Subordinate loans are only paid twice a year, in January and July, and are divided into four components: a non-interest-bearing portion, a Selic-linked portion (SLS), a TJLP-linked portion (SLTJLP), and a TLP-linked portion (SLTLP). Interest on the TJLP and TLP portions is capped at an annual nominal rate of 6%, with excess interest recapitalized into the respective outstanding balances.

   Financial revenue calculation is:

$$\begin{aligned} FR_{t+\Delta t} = {} & r_{tlp} SD_t + r_s TB_t + r_s SLS_t + \min(r_{tjlp}, 3\%) SLTJLP_t \\ & + \min(r_{tlp}, 3\%) SLTLP_t + 0.015(SD_t + TB_t) \end{aligned} \tag{6-1}$$

   with the SLTLP and SLTJLP terms only applicable on the first and seventh months of each year.

4. **Update financial asset balances:**

$$TB_{t+\Delta t} = 0.985 \times TB_t \tag{6-2}$$

$$SD_{t+\Delta t} = 0.985 \times SD_t \tag{6-3}$$

   For the TJLP and TLP-linked subordinate loans (when applicable), recapitalized interest updates the balances:

$$SLTJLP_{t+\Delta t} = [1 + \max(0, r_{tjlp} - 3\%)]\, SDTJLP_t \tag{6-4}$$

$$SLTLP_{t+\Delta t} = [1 + \max(0, r_{tlp} - 3\%)]\, SDTLP_t \tag{6-5}$$

5. **New subordinate loan issuance:** Any new subordinate loans made within the period (28% of the tax revenue) are allocated entirely to the TLP-linked subordinate loan component:

$$SLTLP_{t+\Delta t} += 0.28 \times TR_{t+\Delta t} \tag{6-6}$$

6. **Monthly net income calculation:**

$$NI_{t+\Delta t} = (TR_{t+\Delta t} + FR_{t+\Delta t}) - (C_{t+\Delta t} + 0.28 \times TR_{t+\Delta t}) \tag{6-7}$$

7. **Treasury bond holdings** must remain above minimum liquidity—approximately 84% of costs over the last three months (historically approximation of actual liquidity rules). If the current outstanding treasury bonds value falls below this threshold, the month's net income is allocated to bring it up to the required level. This step concludes the monthly evaluation.

8. **Annual adjustments and repayments:** At the end of each fiscal year:

   – A fixed amount of 8,000 is transferred annually from the TJLP-linked subordinate loan component to the TLP-linked subordinate loan component.
   – Cumulative annual net income, discounted by 17000 for the social security, determines the issuance of new special-purpose loans (if positive). If annual net income is negative, support is provided via Treasury loans (with probability 'p') or repayment from BNDES on the TLP-linked subordinate loan component (with probability '1-p'). Repayments from BNDES are capped annually based on outstanding subordinate loan balances (20% during the first two years, 10% from the third to fifth years, and 5% thereafter).
   – The non-interest-bearing and Selic-linked subordinate loan portions remain unchanged throughout the simulation. Any principal repayments and new subordinate loan issuance exclusively affect the TLP-linked component.

7
Results

Several simulations were done to analyze how changes in key parameters affect FAT's projected cash flows. In the Base model, interest rates and inflation evolve based on their historical behavior. Additional simulations explored the effects of higher (High model) and lower (Low model) interest rates and inflation paths.

Using this calibration, different scenarios were created by varying the probability that the government would avoid repaying the principal on subordinate loans.

Table 7.1 summarizes the parameter configurations for each simulation. In the table, θ denotes the long-term mean level of interest rates and inflation, while $P(A)$ and $P(B)$ represent the probabilities that governments of type A and B, respectively, will intervene to support the fund. The numbers on each model's name represent $P(A)$ and $P(B)$, respectively.

Table 7.1: Parameter configurations for each simulation

Model\Parameter	**P(A)**	**P(B)**	θ_{r_s}	$\theta_{r_{tjlp}}$	$\theta_{r_{tlp}}$	θ_{infl}
0.0/0.0 Base	0%	0%	0.84%	0.50%	0.48%	0.45%
0.5/0.5 Base	50%	50%	0.84%	0.50%	0.48%	0.45%
1.0/1.0 Base	100%	100%	0.84%	0.50%	0.48%	0.45%
0.0/0.0 High	0%	0%	1.04%	0.65%	0.63%	0.60%
0.5/0.5 High	50%	50%	1.04%	0.65%	0.63%	0.60%
1.0/1.0 High	100%	100%	1.04%	0.65%	0.63%	0.60%
0.0/0.0 Low	0%	0%	0.64%	0.35%	0.33%	0.30%
0.3/0.3 Low	30%	30%	0.64%	0.35%	0.33%	0.30%
0.5/0.5 Low	50%	50%	0.64%	0.35%	0.33%	0.30%

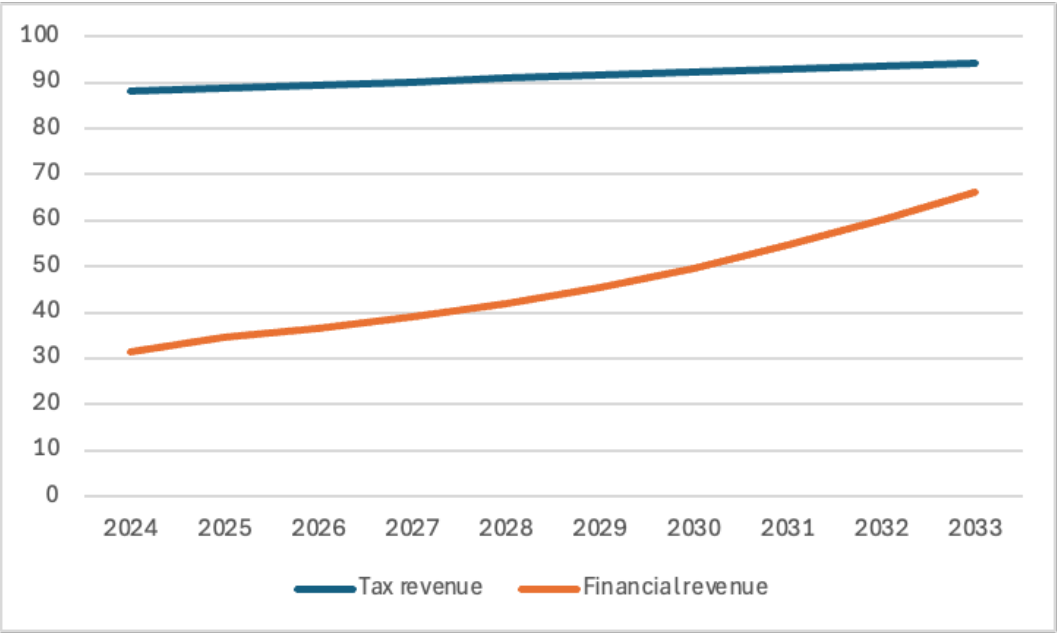
Note that costs, tax revenue, other revenue, and subordinate loans don't change throughout the models.

7.1
Revenue dynamics

Throughout the simulations, parameter changes do not affect all components of the model. They only impact financial revenue and, consequently, net revenue. Figure 7.1 illustrates the evolution of each of the revenue components in the 0.5/0.5 Base simulation. While tax revenue remains the largest source

of income throughout the simulation, financial revenue increasingly dominates over time, growing by 111% in real terms by year 10, compared to a 7% increase in tax revenues.

Figure 7.1: Baseline revenue (R$ billion, 2023 values)



Source: Author

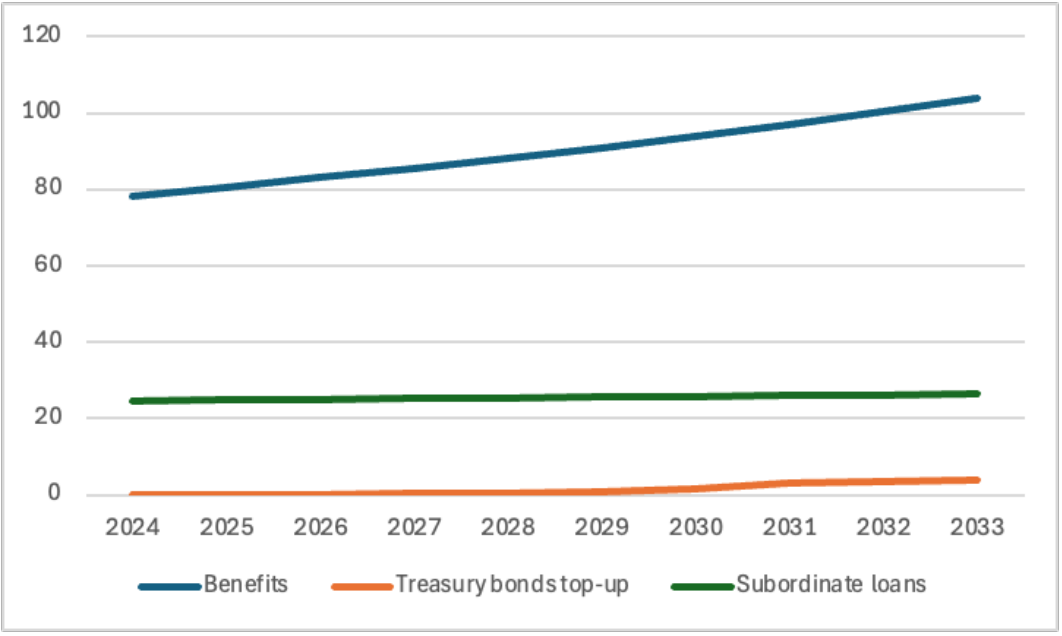
The key driver is the compounding effect of recapitalized interest on subordinate loans, which was greatly amplified with the TLP. This leads to larger balances and even greater interest accrual in future periods. Financial revenue doesn't just reflect higher rates; it amplifies them via dynamic feedback. While in the first year of the simulation tax revenue was responsible for approximately 74% and financial revenue for 26%, by the end of the simulation the shares are 60% and 40%, respectively.

As FAT earns more from TLP-linked loans, its dependence on tax revenue and Treasury support diminishes. However, this also exposes FAT to heightened interest rate volatility. Should rates fall, financial revenue would contract disproportionately, reversing the fiscal improvement.

7.2 Cost pressure

Figure 7.2 presents the corresponding trajectories for each cost category for the 0.5/0.5 Base model.

Figure 7.2: Baseline costs (R$ billion, 2023 values)



Source: Author

Over the simulation period, costs increase by 33%, outpacing tax revenue growth. This is consistent with the historical data and reflects the fund's risk of negative yearly results.

Costs associated with subordinate loans rise by the same 7% as tax revenue. This relationship creates a link between revenue and lending, which undermines FAT's ability to moderate outflows in response to fiscal constraints.

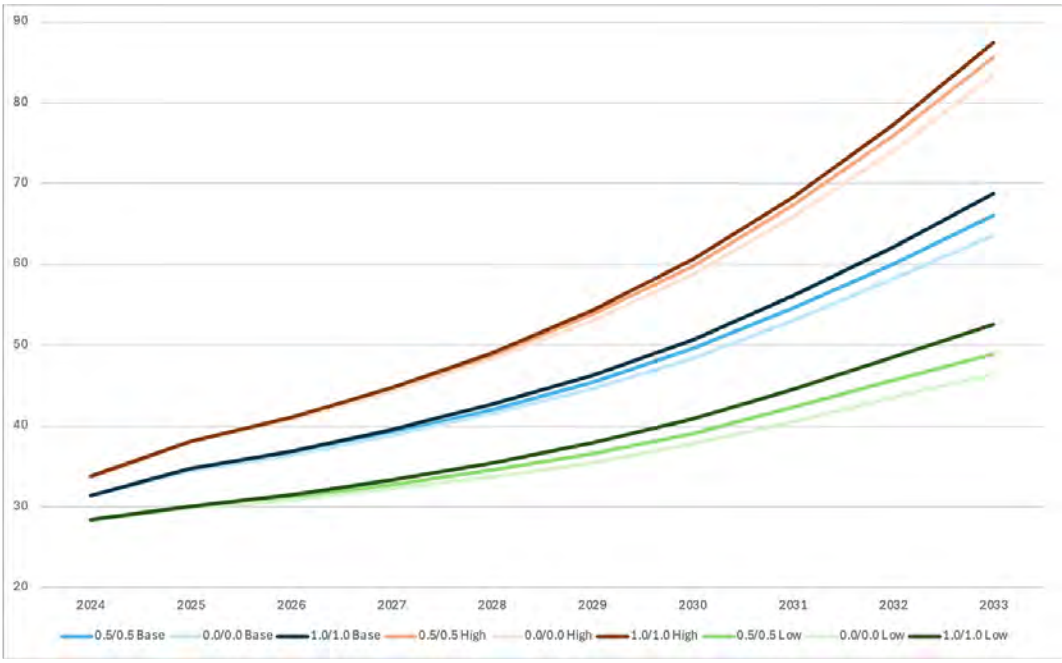
The rigid nature of FAT's obligations, especially regarding unemployment insurance and salary bonuses, means there is little endogenous cost adjustment in response to revenue fluctuations.

The result is a structural imbalance: FAT's fixed outflows grow faster than its discretionary income, creating a tension that leads to either a decrease in liquid assets or reliance on external transfers.

7.3 Interest rate regimes & bailouts

Figure 7.3 illustrates the evolution of financial revenue across all simulated scenarios. The results suggest that interest rate dynamics have a greater impact on financial revenue than the government's decision to bail out BNDES. Within each interest rate group, varying the probabilities $P(A)$ and $P(B)$ has a relatively minor effect, whereas differences across interest rate scenarios lead to significant changes in outcomes. This reinforces the empirical claim that the switch from TJLP to TLP fundamentally altered FAT's solvency issues.

Figure 7.3: Financial revenue evolution (R$ billion, 2023 values)

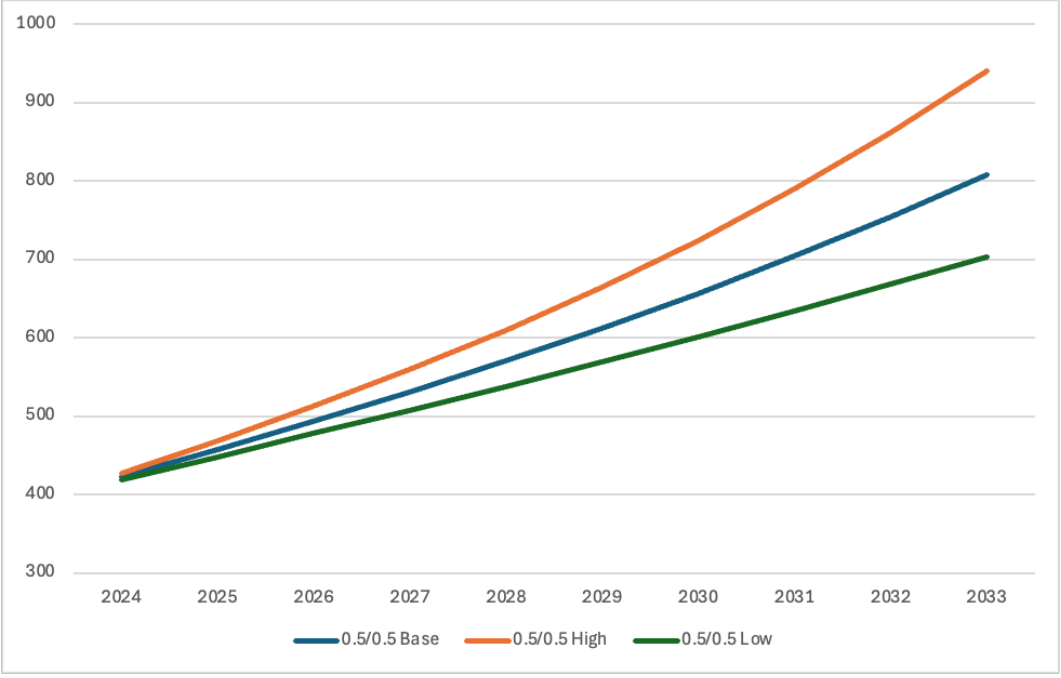


Source: Author

Unlike costs and most other revenue components, which evolve more linearly over time, financial revenue grows exponentially. This pattern arises from the mechanism of recapitalization. The increase in loan balance leads to higher interest payments and further recapitalization in subsequent periods, all else equal.

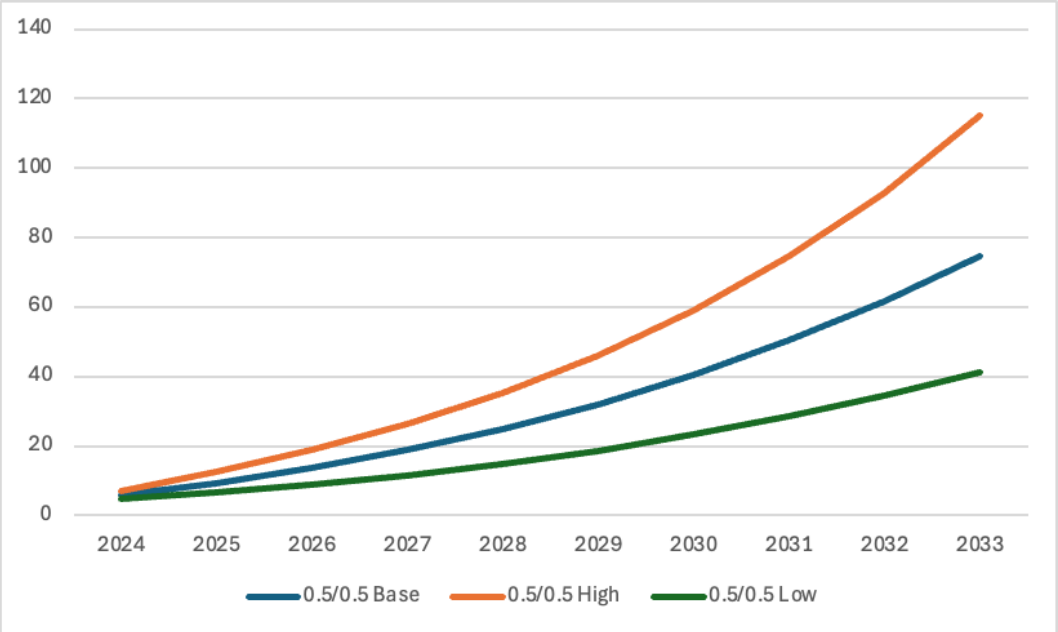
This compounding effect is illustrated in Figures 7.4 and 7.5, which plot the trajectories of subordinate and special loans under the baseline, high, and low interest rate scenarios, each assuming $P(A) = P(B) = 50\%$.

Figure 7.4: Outstanding subordinate loans (R$ billion, 2023 values)



Source: Author

Figure 7.5: Outstanding special loans (R$ billion, 2023 values)



Source: Author

As interest rates increase, financial revenue increases accordingly and exhibits a steeper growth trajectory. Higher probabilities of Treasury intervention also lead to greater financial revenue, particularly in the long run. However,

as previously discussed, CODEFAT imposes a cap on the amount BNDES is required to repay FAT[1]. When the Treasury ceases to bail out BNDES, periods of negative fund results without sufficient support emerge due to this cap. This is shown in Tables 7.2 and 7.3. It implies that the Treasury would likely need to intervene regardless, to prevent underfunding and a halt to the workers' benefits, granting new implicit subsidies. It can be observed that for a BNDES' "default", not only the interest rates but also the probability of bailout have a big impact.

Table 7.2: Probability of insufficient repayment

Year	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2033**
0.0/0.0 Base	0.00%	0.00%	0.00%	0.02%	0.01%	3.89%	6.53%	9.71%	11.47%	13.28%
0.5/0.5 Base	0.00%	0.00%	0.00%	0.00%	0.00%	0.15%	0.17%	0.24%	0.21%	0.32%
1.0/1.0 Base	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.0/0.0 High	0.00%	0.00%	0.00%	0.00%	0.00%	1.05%	2.28%	3.44%	3.92%	4.35%
0.5/0.5 High	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.03%	0.07%	0.06%	0.08%
1.0/1.0 High	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.0/0.0 Low	0.00%	0.00%	0.00%	0.03%	0.25%	12.07%	16.90%	22.48%	25.92%	28.63%
0.5/0.5 Low	0.00%	0.00%	0.00%	0.00%	0.01%	0.20%	0.38%	0.50%	0.69%	0.97%
1.0/1.0 Low	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

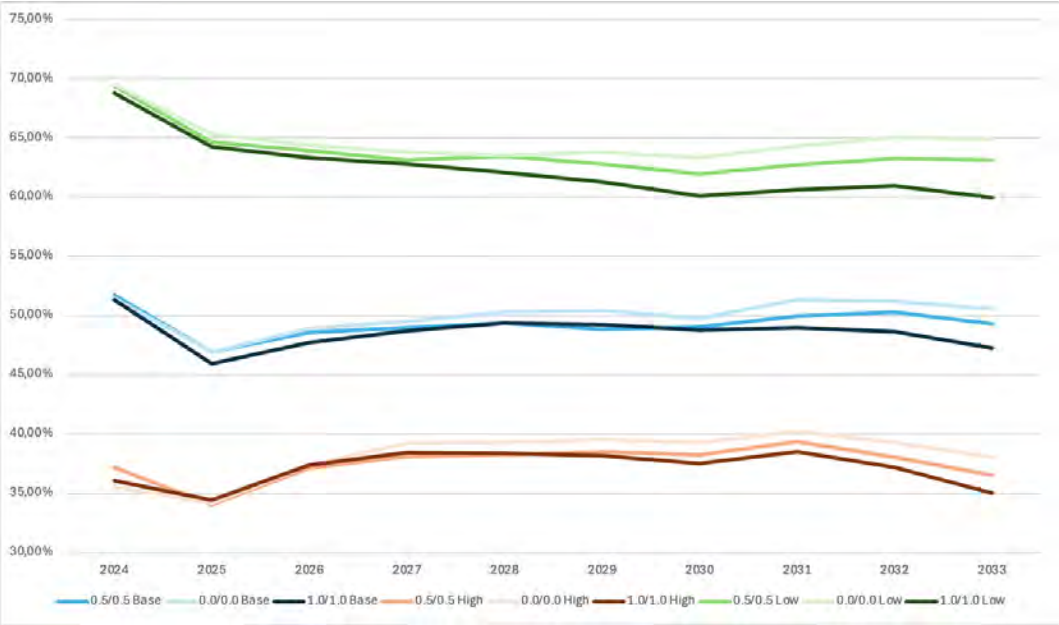
Table 7.3: Expected insufficient repayment amount (R$ billions)

Year	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2033**
0.0/0.0 Base	0.00	0.00	0.00	0.54	0.81	277.70	606.02	1,049.76	1,404.13	1,830.10
0.5/0.5 Base	0.00	0.00	0.00	0.00	0.00	10.81	17.40	26.98	21.08	49.35
1.0/1.0 Base	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0/0.0 High	0.00	0.00	0.00	0.00	0.00	68.90	187.24	306.26	393.74	497.79
0.5/0.5 High	0.00	0.00	0.00	0.00	0.00	1.27	3.06	7.66	7.49	9.70
1.0/1.0 High	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0/0.0 Low	0.00	0.00	0.00	0.74	10.17	1,103.54	1,905.50	2,961.41	3,852.96	4,751.08
0.5/0.5 Low	0.00	0.00	0.00	0.00	0.22	15.79	36.58	56.44	74.73	133.86
1.0/1.0 Low	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Since the only cash flow that changes from model to model is the financial revenue, and they mostly change based on interest rates, the probability of having a negative net revenue each year also depends mostly on the interest rates, as shown in Figure 7.6.

[1]Up to 20% of the outstanding balance in the first two fiscal years, 10% from the third to fifth year, and 5% thereafter.

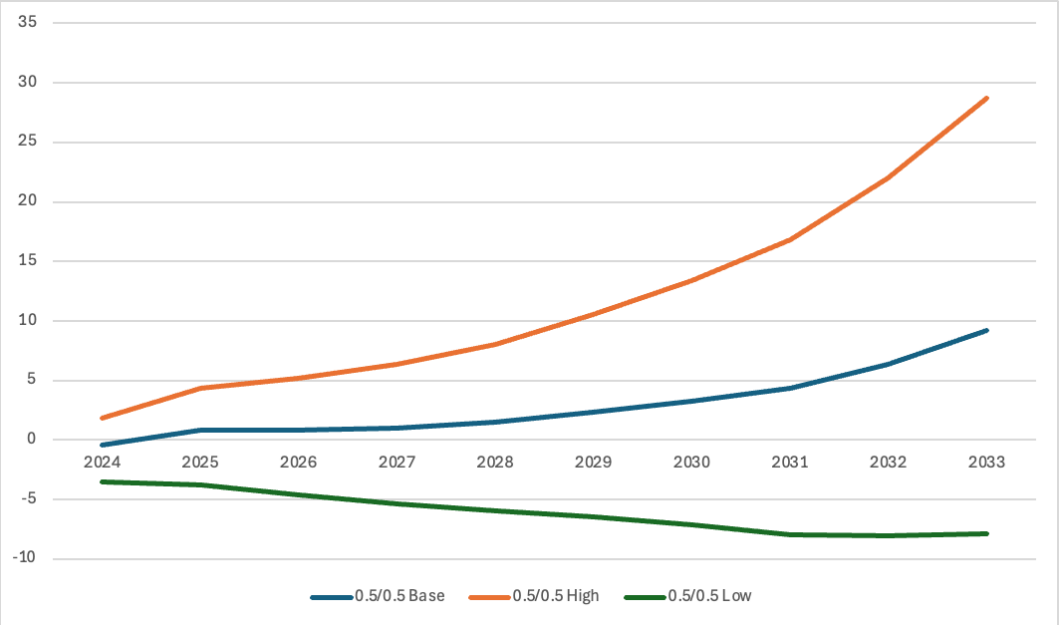
Figure 7.6: Probabilities of negative net revenue



Source: Author

Figure 7.7 shows the expected net revenue for each year for the baseline, high, and low interest rate scenarios, each with $P(A) = P(B) = 50\%$.

Figure 7.7: Expected net revenue (R$ billion, 2023 values)



Source: Author

Because of the compounding effect, the influence of interest rates grows significantly over time, making the fund's expected net revenue highly sensitive to interest rate changes.

7.4 VaR and CVaR

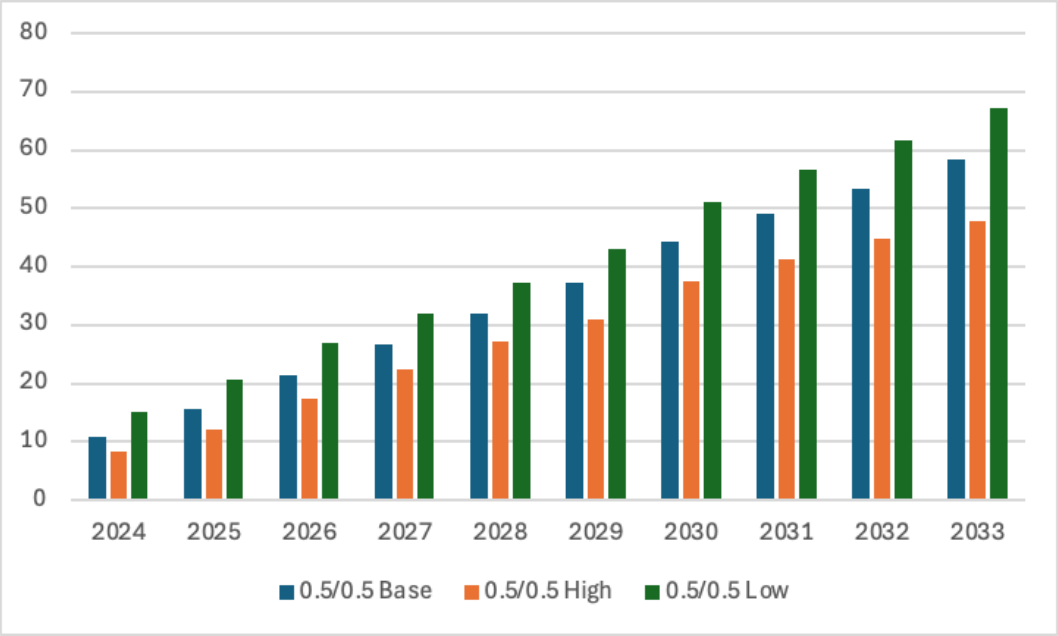
Lastly, we calculate the Value at Risk (VaR) and Conditional Value at Risk (CVaR) for three models in which both government types A and B have a 50% probability of bailing BNDES out: 0.5/0.5 High, 0.5/0.5 Base, and 0.5/0.5 Low. Both conditional and unconditional versions of VaR and CVaR are reported: the unconditional versions are computed across all simulations, while the unconditional ones consider only scenarios where there was a negative net revenue.

In this study, VaR represents the funding shortfall threshold that is not exceeded in 95% of all simulated outcomes. The unconditional VaR (Figure 7.8) is computed across all simulations, including both those in which no additional funding is needed and those where a bailout occurs. In contrast, the conditional VaR (Figure 7.10) considers only the adverse scenarios. This conditional measure thus provides a clearer picture of the "bad-case" outcomes: if intervention is necessary, the conditional VaR indicates the maximum expected shortfall in 95% of these problematic instances.

Similarly, CVaR captures the average funding shortfall in the worst 5% of outcomes. The unconditional CVaR (Figure 7.9) averages these tail losses across all simulations, while the conditional CVaR (Figure 7.11) focuses only on the scenarios where a bailout was needed. As a result, the conditional CVaR provides a sharper estimate of the expected severity in the most critical cases, as it excludes the many simulations where no shortfall occurs.

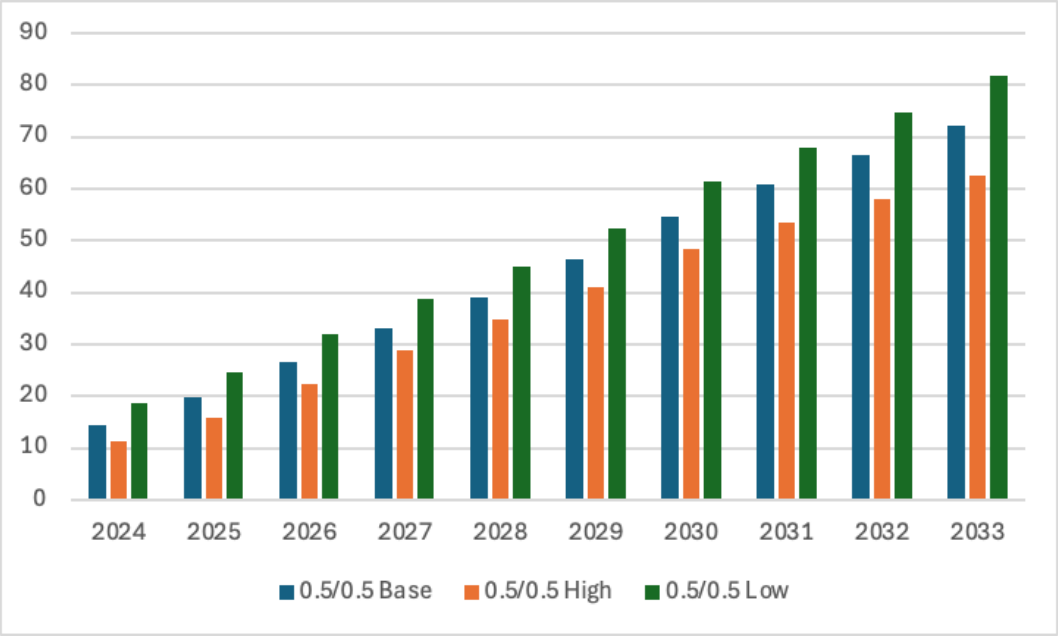
Together, these metrics offer a two-sided perspective: while the unconditional measures reflect the overall risk profile of the fund, the conditional figures focus on the burden of intervention when trouble arises. This distinction is crucial when comparing models, as it highlights not only the potential fiscal cost of a bailout, but also its frequency and severity.

Figure 7.8: Unconditional VaR (R$ billion, 2023 values)



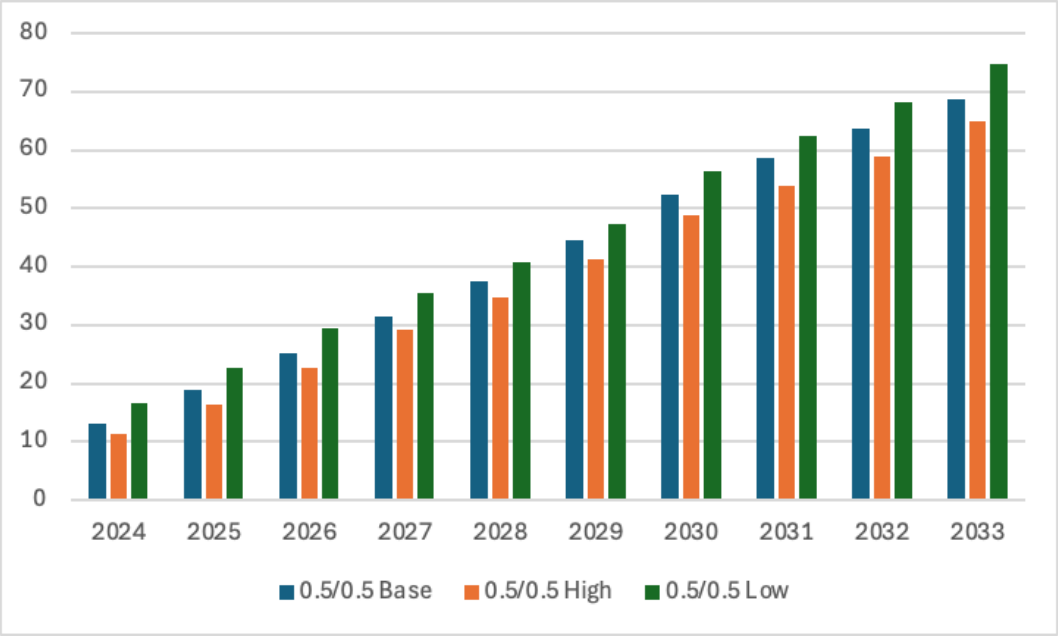
Source: Author

Figure 7.9: Unconditional CVaR (R$ billion, 2023 values)



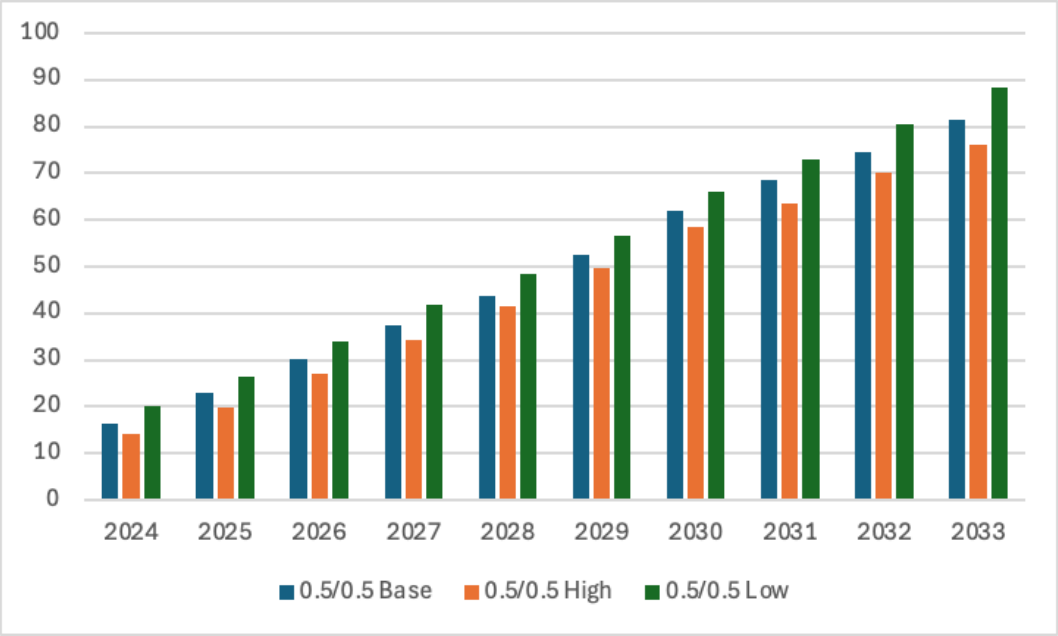
Source: Author

Figure 7.10: Conditional VaR (R$ billion, 2023 values)



Source: Author

Figure 7.11: Conditional CVaR (R$ billion, 2023 values)

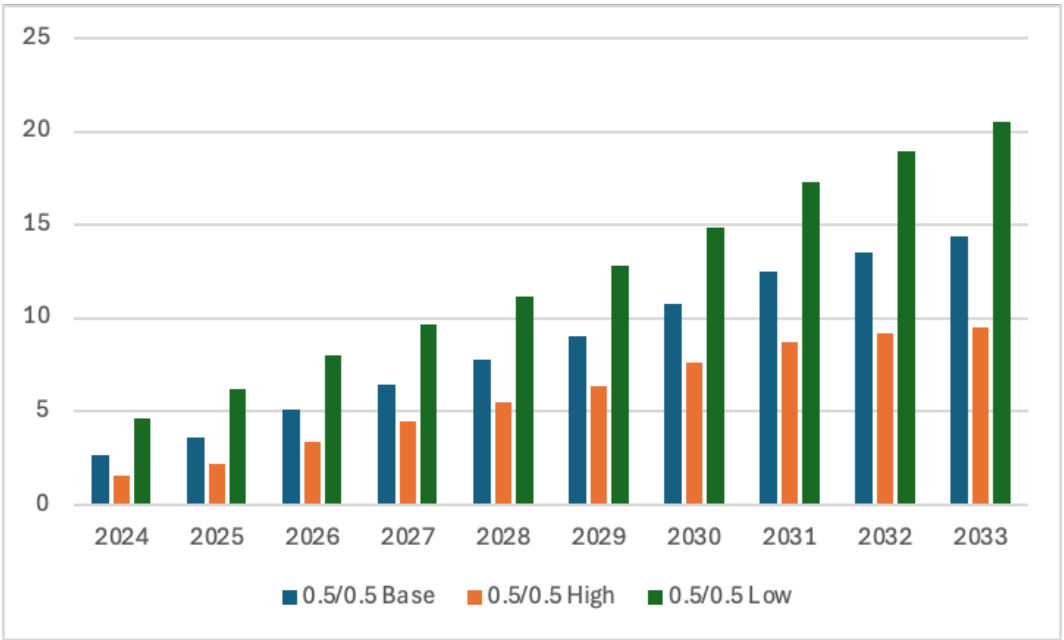


Source: Author

These values increase rapidly over the simulation horizon, indicating that risk escalates significantly over time. For the Unconditional VaR, the 0.5/0.5 Base model shortfall threshold grows by 433%, for example.

By the end of year 10, the magnitude of losses in the worst 5% of scenarios may exceed the capacity of either the government or the bank to intervene, potentially compromising the fund's solvency and threatening the workers' benefits.

Figure 7.12: Expected funding needed (R$ billion, 2023 values)



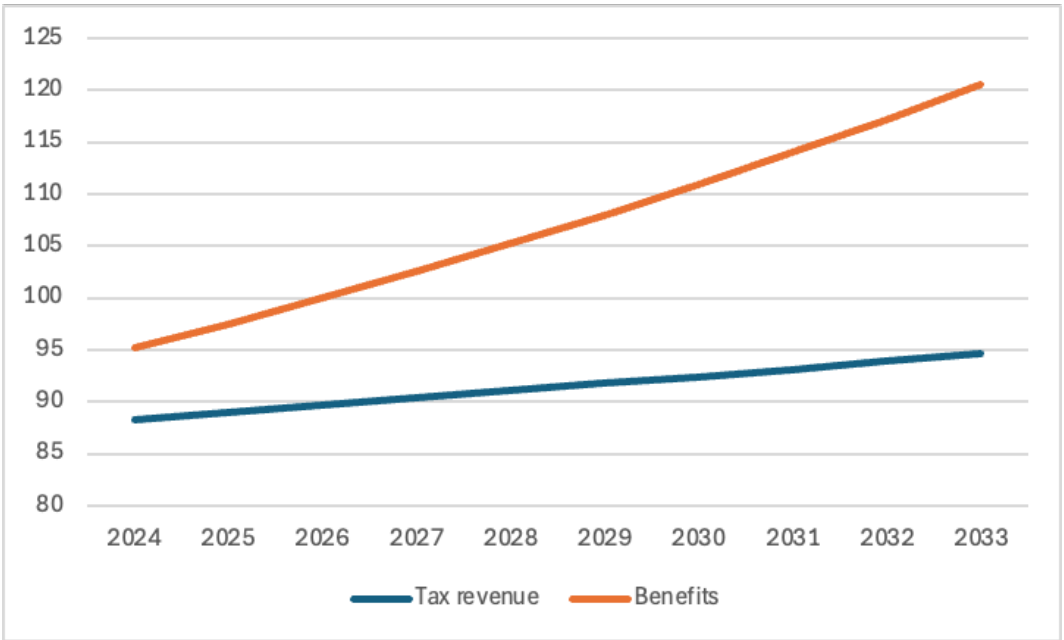
Source: Author

Figure 7.12 presents the expected funding required throughout the simulation period. Although these values are not as high as those observed in the worst 5% of outcomes, they exhibit a similarly rapid growth trajectory. For the 0.5/0.5 Base model, the expected funding required grows by 446% over the 10-year period. This reinforces the concern that, over the long term, the fund may become financially unsustainable.

8
Who bears the cost?

While this thesis focuses primarily on FAT's financial sustainability and the contingent fiscal risks it poses, the simulation results also bring an important discussion: who ultimately bears the cost of sustaining FAT's deficits and, by extension, the subsidies to BNDES? The analysis shows that the total value of unemployment insurance, salary bonus and social security consistently exceeds the tax revenue FAT collects, as shown in Figure 8.1. Despite this structural gap, these benefits continue to be paid, either by drawing on FAT's financial revenue, charging BNDES the principal owed, or through transfers from the National Treasury. From the worker's perspective, this implies a net positive return on their contributions: they receive more in benefits than the system collects through earmarked taxes. The fund itself does not have the autonomy to alter either its revenue sources or its legal obligations.

Figure 8.1: Tax revenue and workers beneifts (R$ billion, 2023 values)



Source: Author

This contradicts any interpretation that workers are implicitly "taxed" to support BNDES. They are not the ones bearing the fiscal cost of subsidized development finance. Instead, that burden falls primarily on general taxpayers, and increasingly on future generations, as Treasury interventions are financed through broader tax revenue or new debt. These bailouts, while intra-governmental on paper, represent real fiscal transfers, and show that the

cost of BNDES subsidies is not internalized by the bank itself, nor by the workers, but by the public budget.

Nonetheless, this does not settle the broader equity or efficiency implications of FAT's institutional design. Although workers are net recipients from the system, 28% of their earmarked tax contributions is not used to finance direct benefits, but instead channeled to BNDES at below-market rates. This has historically been justified by the dual mandate of FAT to support both workers and economic development. However, the simulation shows that FAT continues to rely on Treasury transfers even with this lending mechanism in place. This raises a key opportunity cost: could the entire flow of PIS-Pasep contributions generate higher returns if invested differently? If so, the current structure may underperform in both fiscal and social terms.

Moreover, while PIS-Pasep is legally paid by employers, its economic incidence may partially fall on workers, whether through lower wages or reduced formal employment opportunities. If workers ultimately bear part of the tax burden while a portion of those funds subsidize credit to firms, the system may generate regressive redistribution, transferring income from labor to capital. This would contradict FAT's original purpose and raises further distributional concerns. Analyzing the fair-value cost of BNDES loans, combined with a tax incidence model of PIS-Pasep, would allow a more explicit identification of who ultimately pays for development finance under this structure.

While these questions are beyond the scope of the current thesis, they mark an underexplored area for future research. Understanding the true economic incidence of FAT's financing model, both in terms of equity and efficiency, would deepen the policy debate surrounding earmarked taxes, intergovernmental transfers, and the appropriate role of public development banks in Brazil's fiscal architecture.

9
Conclusion

In conclusion, this thesis examined FAT's financial sustainability and related fiscal risks by analyzing its cash flow projections, sensitivity to interest rate fluctuations, and its financial relationship with BNDES and the National Treasury. The main question addressed was how interest rate dynamics, institutional constraints, and political behaviors affect FAT's resilience and the frequency of Treasury bailouts.

The findings showed that FAT's financial health depends significantly on interest rate movements. Higher interest rates, particularly after the shift from TJLP to TLP, substantially strengthen FAT's financial position by increasing revenues from its loans to BNDES. Nevertheless, this greater dependency on financial income also amplifies FAT's vulnerability to interest rate volatility, exposing the fund to new risks.

Additionally, institutional rigidities, especially inflexible repayment terms on BNDES loans and limited fiscal autonomy, frequently push FAT into situations where Treasury interventions become unavoidable. These interventions operate effectively as implicit subsidies to BNDES, generating considerable contingent liabilities and increasing fiscal pressures on the government. Although FAT is legally permitted to request loan repayments from BNDES during financial difficulties, in practice, these mechanisms are rarely fully applied, reinforcing the reliance on recurrent Treasury support.

Chapter 8 complemented these insights by analyzing who ultimately bears the costs of FAT's structural deficits and implicit subsidies to BNDES. The chapter demonstrated a crucial distributive issue: benefits paid by FAT to workers consistently surpass the earmarked PIS-Pasep tax revenues, resulting in a net gain for workers relative to their contributions. Consequently, the fiscal burden of these implicit subsidies does not fall on workers but rather on taxpayers broadly through repeated Treasury interventions. This highlights that FAT's pro-cyclical spending pattern acts more as a conditional income transfer than traditional counter-cyclical unemployment insurance, bringing significant implications for macroeconomic stabilization and income distribution.

Therefore, the analysis emphasizes the importance of policy adjustments, such as aligning lending rates more closely with market conditions, clarifying fiscal responsibilities, increasing institutional flexibility, and enhancing transparency. Addressing these structural issues is crucial to ensuring FAT's long-term sustainability, reducing fiscal vulnerabilities, and fostering balanced eco-

nomic growth and equitable fiscal management in Brazil.

10
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