Identifying the Source of Misbehavior in First-Price Auctions with Bounds on Overbidding

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Abstract

Experimental evidence shows that in first-price auctions bidders tend to "misbehave" (Harrison, 1989) by bidding more aggressively than predicted by basic theory. This notes considers the possibility that this is due to bidders making one of three possible types of *mistakes*: mistakes in their valuation assessment, mistakes in optimizing their best response, or mistakes in predicting the behavior of their opponents. Bounds on the distribution of bid-valuation pairs are obtained, and hypothesis testing is performed using the experimental data from Dyer, Kagel, and Levin (1989). The tests indicate that the most likely type of mistake is on predicting the behavior of the other bidders.

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1 Introduction

Experimental evidence has been widely used to test if subjects do follow the behavior prescribed by theory. When they do not, the standard next

*I thank Pat Bajari, Lanier Benkard, In-Koo Cho, Ali Hortaçsu and Roger Koenker for helpful comments. Besides the bidders, all mistakes are mine.

step is to redo the theory, by changing the assumptions made about the environment, including the hypothesis of full rationality. The pitfall of this endeavour is the risk of ending up with an ad hoc set of theories, that explain one empirical finding but not others.

This paper proposes an empirical methodology that uses experimental data not only to test the established theory, but also to identify in what step of the process of decision making the assumption of full rationality fails (if any). As such it can be used to inform the development of broader theories of boundedly rational behavior that are better grounded in empirical evidence.

This paper will develop and apply the methodology in the context of experiments of independent private values first-price auctions; in particular, the experimental data collected by Dyer, Kagel, and Levin (1989). It is however, broadly applicable to any social situation that can be conceived as a mechanism or as an imperfect information game.

A well-known empirical regularity in experiments involving independent private value auctions is that bidders do not play according to the theoretical Nash equilibrium, but rather tend to bid too aggressively. The most popular, albeit controversial, explanation is the one provided by Cox, Smith, and Walker (1988): risk aversion.[1] Studying the same data as in here, Bajari and Hortaçsu (2004) have concluded that a model with risk aversion fits the data better than the other alternatives considered there. In section 8 we shall compare our results with those obtained under risk aversion. Here we will seek an alternative explanation; we shall assume throughout that bidders are indeed risk neutral, but make *mistakes* of one several different types.

We shall consider three possible mistakes:

Mistakes in valuations: Bidders misunderstand their valuations, and act rationally given this misunderstanding;

Mistakes in maximization: Bidders do not maximize perfectly: they bid in a way that is nearly, but not fully, optimal.

Mistakes in beliefs: Bidders fail to predict exactly the behavior of their rivals, and therefore respond rationally to mistaken beliefs.

This objective of this paper is to study if those three different types of mistakes lead to different observed bidding patterns, and if these differences

[1]Kagel (1995, section I.G) provides a survey of the debate.

allows us to identify which of these alternatives is more empirically relevant.[2]

The rest of the paper is organized as follows: The next section reviews the standard First-price auction model under independent private values. Section 3 describes formally the three different types of mistakes considered in this paper and derives formulas for the bounds consistent with these mistakes. Section 7 is the heart of the paper: it tests the three bounds against the experimental data collected by Dyer, Kagel, and Levin (1989). Section 8 discusses the leading alternative explanation that bidders may be risk averse. Section 9 concludes.

2 The First-Price Auction

In an independent private values first price auction, a bidder's expect profit is

$$\pi = (v - b)Q(b)$$

where v is the value it assigns to the good, b is its bid, and Q the probability of winning the auction as a function of b (of course, it depends also on the other bidders strategies). We shall assume that valuations across different bidders are independent and identically distributed, with distribution F and density f.

In a symmetric Nash Equilibrium of this game, all players follow the same strategy $b = \beta(v)$. Once β has been shown to be an increasing and differentiable function, one can write $Q(b) = \Pr(\beta(v) < b)^{N-1} = F(\beta^{-1}(b))^{N-1}$, where N is the number of bidders.

Optimal bidding is therefore uniquely characterized by a differential equation,

$$-F(v)^{N-1} + (v - \beta(v))\frac{(N-1)f(v)F(v)^{N-2}}{\beta'(v)} = 0$$

as well as a boundary condition, $\beta(v_0) = v_0$, where v_0 is the lowest possible value for v.

[2]It is important to make clear that the term "mistake" is not meant in a derogatory sense. Here, bidders are assumed to be procedurally rational, and fail to follow the prescriptions of fully rational behavior because they face communicational or computational costs that lead to apparent mistakes. So mistakes here are meant to represent in a reduced form fashion the effect of such cost, just as stochastic errors are often used to capture unobserved covariates.

This is a relatively complex mathematical problem, and a skeptical reader may have the impression that experimental subjects may fail to solve this problem exactly. This possibility will be discussed in section 5 below. First, we shall discuss a simpler type of mistake, that arises if agents solve the right problem, but for the wrong input, namely, v.

3 Three mistakes

4 Mistakes in valuation assessment

In this section we assume bidders act thinking that their valuation is $\tilde{v} = v + \nu$, with $\nu \in [-A/2, A/2]$, that is, they play the bid $\tilde{b}$ that maximizes

$$\pi = (\tilde{v} - b)Q(b).$$

Suppose an econometrician has data such type of bidding and estimates the valuation distribution compatible with the (fully rational) Nash equilibrium. We are interested in studying how ν will bias this estimation. More precisely, we want to characterize the bounds for $\hat{V} = \beta^{-1}(\tilde{b})$, as a function of A.

We shall proceed by making a revealed preference argument. If b is played by a bidder with (true) valuation v, then

$$(v + \nu - b)Q(b) \geq (v + \nu - b')Q(b')$$

for all alternative bids b'. For $b' < b$, we can write

$$v - \nu \geq b + \frac{(b' - b)Q(b')}{Q(b') - Q(b)}$$

and likewise for $b' > b$, $v + \nu \leq b + \frac{(b'-b)Q(b')}{Q(b')-Q(b)}$. Therefore, $v \in [\underline{v}_V, \overline{v}_V]$, where

$$\underline{v}_V = \max_{b':b'<b} b + \frac{(b' - b)Q(b')}{Q(b') - Q(b)} - A/2$$

and

$$\overline{v}_V = \min_{b':b'>b} b + \frac{(b' - b)Q(b')}{Q(b') - Q(b)} + A/2$$

The tightest bounds are obtained when $b' \simeq b$:

Proposition 1 *For any point where Q is differentiable, $v \in [\underline{v}_V, \overline{v}_V]$, where*

$$\begin{aligned}\underline{v}_V &= b + \frac{Q(b)}{q(b)} - A/2,\\ \overline{v}_V &= b + \frac{Q(b)}{q(b)} + A/2,\end{aligned}$$

and $q(b) = Q'(b)$.

Proof: To find the b' that provides the tightest bounds, we solve the first order conditions for $\min_{b':b'>b} b + \frac{(b'-b)Q(b')}{Q(b')-Q(b)}$. We obtain $Q(b')(Q(b')-Q(b))+(b'-b)[q(b')(Q(b')-Q(b))-q(b')Q(b')] = Q(b')(Q(b')-Q(b))-(b'-b)q(b')Q(b) = 0$. Note that, since Q is continuous, $b' \to b$ satisfies this condition. Taking this limit yields the result. $\square$

Note that $\hat{V} = b + \frac{Q(b)}{q(b)}$ (Guerre, Perrigne, and Vuong, 2000). This means that, as expected, the econometrician will always find estimated values that are $A/2$ away from the truth.

5 Mistakes in maximization

A more complex calculation is required to investigate mistakes in maximizing π. Let us model mistakes in maximization as if instead of maximizing π, the bidder maximizes $\tilde{\pi}(b) = \pi(b)+\epsilon(b)$, where ϵ is a random process that assigns to each bid a disturbance with support on $[-B/2, B/2]$. This is in the spirit of discrete choice models that are popular in the literature of heterogenous good demand estimation. It can also be justified as way to incorporate a satisficing type of behavior: suppose a bidder is happy to play a given bid $\tilde{b}$ as long as there is no other bid b^* that improves its profits by more than B. Such bidder will act exactly as predicted by this model, with $\epsilon(\tilde{b}) = B/2$ and $\epsilon(b) = -B/2$ for all $b \neq \tilde{b}$.

Mistakes in maximization are also reminiscent of Harrison (1989). In his "flat maximum critique" Harrison argues that the relevant metric for evaluating deviations from rational behavior is the one of payoffs, rather than the one of messages (bids). The mistakes in maximization approach is in accordance with this view, in that it admits only behavior that is slightly suboptimal in the payoff metric.

Under mistakes in maximization by revealed preference we know that, from an observed bid b,

$$(v-b)Q(b)+\epsilon(b) \geq (v-b')Q(b')+\epsilon(b'),$$

for all other bids b', so that

$$v \leq b + \frac{(b'-b)Q(b')}{Q(b')-Q(b)} + \frac{B}{Q(b')-Q(b)},$$

for all b' such that $Q(b') > Q(b)$, and

$$v \geq b + \frac{(b-b')Q(b')}{Q(b)-Q(b')} - \frac{B}{Q(b)-Q(b')},$$

for all b' such that $Q(b') < Q(b)$. These provide upper and lower bounds to values of v compatible with the observed behavior and a limited scope for maximization error.

Proposition 2 *If Q is differentiable, $v \geq \underline{v}_M$, where*

$$\underline{v}_M = \max_{b':b'>b} b + \frac{(b'-b)Q(b')}{Q(b')-Q(b)} - \frac{B}{Q(b')-Q(b)}$$

and b' solves

$$\left(b'-b+\frac{Q(b')}{q(b')}\right)Q(b) - \frac{Q(b')^2}{q(b')} + B = 0.$$

Proof: The tightest bound $\underline{v}_M$ can be characterized using the first-order condition of $\max_{b':b'<b} V(b') = b + \frac{(b'-b)Q(b')}{Q(b')-Q(b)} - \frac{B}{Q(b')-Q(b)}$. Since the argument of the maximization program asymptotes to infinity as $b' \to b$ or ∞, the problem is well-defined and has an interior solution.

It is convenient to start with the condition that determines $V(b')$, namely

$$(V(b')-b)Q(b) - (V(b')-b')Q(b') + B = 0$$

Using the implicit function theorem, we have that the first-order condition is

$$\frac{\partial}{\partial b'}V(b') = -\frac{Q(b')-(V(b')-b')q(b')}{Q(b)-Q(b')} = 0$$

so b' solves

$$V(b') = b' + \frac{Q(b')}{q(b')}.$$

Substituting in the equation above, we obtain

$$\left(b' - b + \frac{Q(b')}{q(b')}\right) Q(b) - \frac{Q(b')^2}{q(b')} + B = 0,$$

as claimed. □

For example, if bids follow a uniform distribution between 0 and X and there are three bidders, $Q = (b/X)^2$ and the condition that determines the b' that achieves the highest lower bound for v is

$$\left(b' - b + \frac{b'}{2}\right) b - b'^3 + X^2 B = 0,$$

A cubic polynomial equation that must be solved for b'.

6 Mistakes in Beliefs

A third possibility is that bidders act on mistaken beliefs about the rivals' behavior. In a first price auction, such beliefs affect a bidder in two related ways: it determines $Q(b)$, the probability of winning given her current bid b, and how this probability changes if she bid b' instead, $Q(b') - Q(b) = q(b'')(b' - b)$, for some b'' between b and b'. In principle, a bidder may be mistaken in either one or both ways.

Here we shall assume that a bidder knows the correct Q, but is mistaken in q: she acts with $\tilde{Q}(b') = Q(b) + \lambda q(b'')(b' - b)$ instead of $Q(b)$, where $\log(\lambda) \in [-C/2, C/2]$. This assumption is made for several reasons; as will be seen below, it provides an analytically convenient bound; but also, if agents form their beliefs from past experience, then they would be able to obtain better estimates of Q than q, just like an econometrician would more easily estimate a distribution than a density. The multiplicative form is also convenient because it allows us to impose some conditions that would naturally be valid even for the wrong belief; for example, it does not allow $\tilde{Q}$ to be decreasing.

If a bidder know the right Q, but is mistaken about q, we have

$$(v - b)Q(b) \geq (v - b')\tilde{Q}(b')$$

so, for $b' < b$, $\tilde{Q}(b') < Q(b)$ we obtain a lower bound:

$$v \geq \frac{bQ(b) - b'\tilde{Q}(b')}{Q(b) - \tilde{Q}(b')} = b + \frac{(b-b')\tilde{Q}(b')}{Q(b) - \tilde{Q}(b')} = b + \frac{\tilde{Q}(b')}{\lambda q(b'')} = b' + \frac{Q(b)}{\lambda q(b'')}.$$

The worst case scenario is therefore when $\log \lambda = C/2$, and the bidder overestimates q. Notice that the derivative of the bound with respect to b' is $1 - Q(b)/(\lambda q'(b'')\frac{\partial}{\partial b'}b''$. If Q is regular[3], then the bound is strictly increasing in b' in a neighborhood of b, and as $b' \to b$ we obtain a local, and perhaps the global, maximum bound:

$$v \geq \underline{v}_B = b + \frac{Q(b)}{q(b)}e^{-C/2}.$$

7 Bringing the Bounds to Data

The three last sections provide three alternative theories of why and how bidders may deviate from predicted behavior in a first price auction. This section brings this analysis to data. We will use part of the experimental data from Dyer, Kagel, and Levin (1989) to investigate if the pattern of deviations conforms with which of the three alternative theories.

In this experiment, subjects were given a valuation drawn from the $U[0, 30]$ distribution, and ask to provide two provisional bids, one conditional on participating in an auction with 3 bidder, and the other with 6 bidders. Actual payoffs were based on the same rules, so for fully rational players this unusual design should not affect the analysis.[4] We shall focus mainly on the subsample of the bids for auctions with 3 bidders; results for the 6 bidder subsample are qualitatively identical, and are discussed in section 7.1.

Figure 1 shows the data. The solid line represents the Nash Equilibrium bid function (more precisely, β^{-1}, since bids are represented in the horizontal axis). Actual bids are the data points. Since the vast majority of points are to the right of the theoretical bid function, there seems to be systematic overbidding.

[3] Q is regular if $x + Q(x)/q(x)$ is increasing in x. It is a necessary condition for the identifiability of the valuation distribution (Guerre, Perrigne, and Vuong, 2000).

[4] There might be however a framing effect: since bidders are asked to solve two problems at the same time, and the other problem involves bidding more aggressively, that might lead to over-aggressive bidding in the 3-bidder auction.

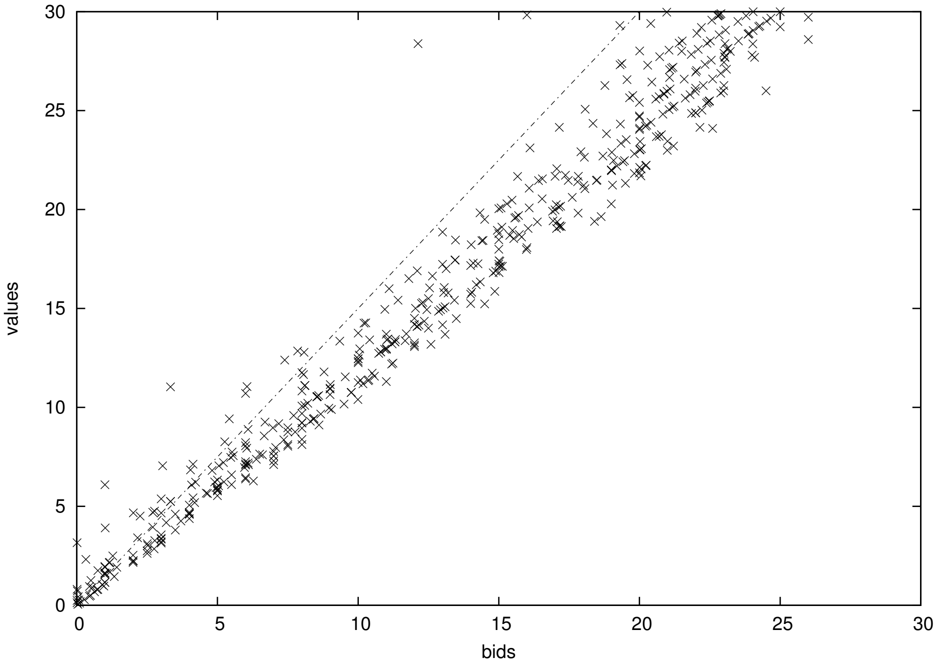

Figure 1: Dyer, Kagel, and Levin (1989) data, 3 bidder subsample

Before proceeding with the estimation of the bounds we must make a choice on which $Q(b)$ to use. The natural alternatives are either using the formula corresponding to the theoretical bid distribution coming out of the Nash Equilibrium without mistakes, or to use an estimate based on the empirical distribution of bids. The first alternative would lead to $Q(b) = (b/20)^2$, since Nash Equilibrium implies $b \sim U[0, 20]$ in this environment.

Since there is significant overbidding, the empirical distribution of bids departs from this prediction, but is still approximately uniform. Figures 2 and 3 show kernel estimates for the density of bids for a variety of bandwidths against the density of the $U[0, 26]$ distribution. 26 is the highest observed bid and is the maximum likelihood estimate of the upper bound of the bid distribution.[5]

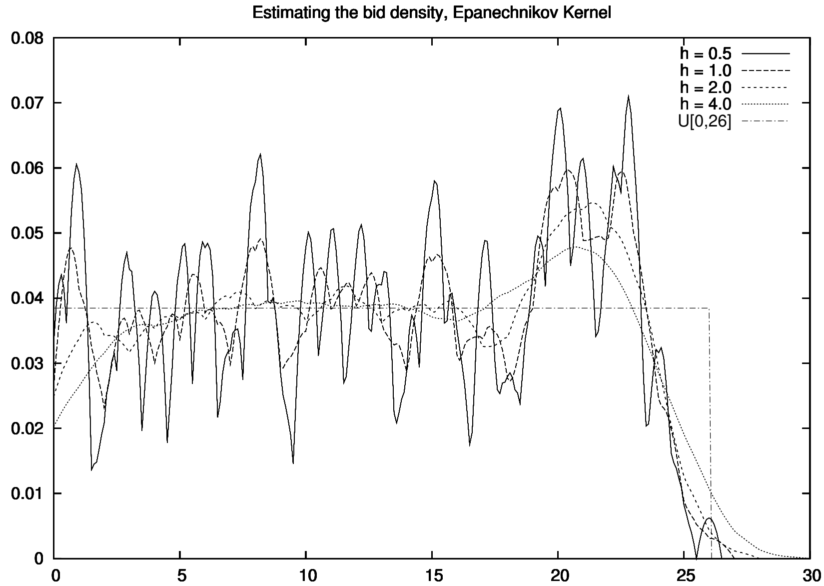

Figure 2: Estimates for the bid density in the Dyer, Kagel, and Levin (1989) data, Epanechnikov kernel.

Except for some concentration of bids between 20 and 23, and perhaps for the density to be decreasing at the right tail, the estimated density is remarkably similar to $U[0, 26]$. Besides being of independent interest, this is convenient since is allows us to nest the two alternatives into a class of distributions $U[0, X]$, where $X = 20$ or 26.

In this case $Q(b) = (b/X)^2$ and the lower bounds are

[5]The estimates in figure 2 use the Epanechnikov kernel. Because kernel estimators are based on the assumption that the density is continuous, they are biased around discontinuity points of the density, such as 0 and the upper bound of support in the case of the uniform distribution. Figure 3 is based on a Gaussian kernel augmented by a procedure that corrects for this effect, by reflecting data points around the boundaries.

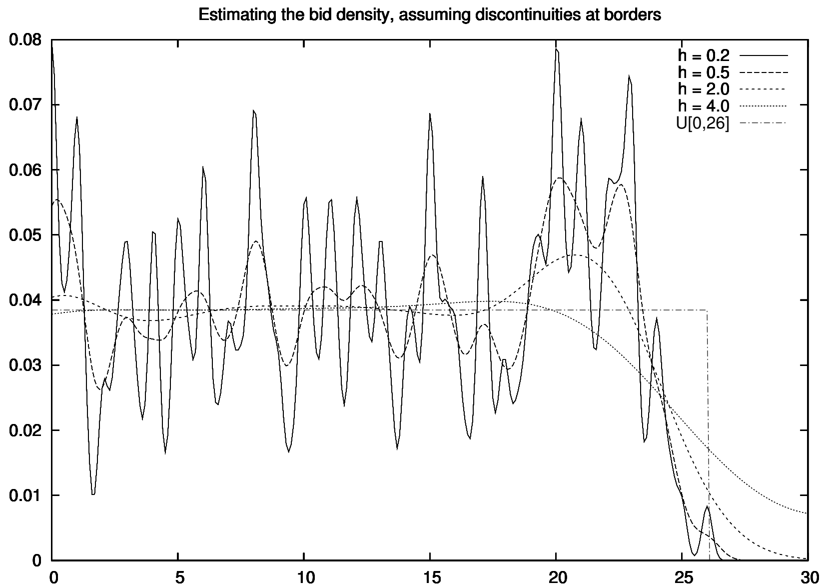

Figure 3: Estimates for the bid density in the Dyer, Kagel, and Levin (1989) data, modified Gaussian kernel.

$$
\begin{aligned}
\underline{v}_V &= b + \frac{b}{2} + \frac{A}{2} \\
\underline{v}_M &= \max_{b':b'<b} b + \frac{b - b'}{b^2 - b'^2} - \frac{BX^2}{b^2 - b'^2} \\
\underline{v}_V &= b + \frac{b}{2} e^{-C/2}
\end{aligned}
$$

Interestingly, since $Q(b)/q(b)$ does not depend on X, neither do $\underline{v}_V$ and $\underline{v}_V$, whereas the effect of X on v_M amounts to a change of the scale of B. So the following analysis does not depend in any significant way on whether X is chosen to be 20 or 26, and we proceed assuming that $X = 26$.

In order to estimate A, B and C we use the fact that the lower bounds $\underline{v}_j(b)$, $j = V, M, B$ are extreme quantile regressions, with quantile $\tau = 0$.

While in principle one would like to estimate the lower extreme quantile $\tau = 0$, it is convenient for statistical reasons to estimate instead a near extreme quantile, such as $\tau = 0.1$. Extreme quantile estimators that are consistent do exist (Chernozhukov, 2005), but are not asymptotically normal, and therefore they are ill-suited to standard hypothesis testing. Since the main focus of the inference is the shape of the conditional quantiles, rather than their location, such simplification is not likely to significantly distort the findings.

Also, because the three alternatives are not nested, we consider additionally a non-parametric quantile regression in order to nest the alternatives and facilitate testing. We fitted a second degree polynomial on bids. This nests the quantile regressions for the case of error in values and beliefs, and hopefully is flexible enough to nest the errors in maximization case as well.

Estimates of the bounds were obtained by minimizing the objective

$$S_j = \sum_t \rho_\tau(v_t - \underline{v}_j(b_t))$$

where $j = V, M$ or B, $\tau = .1$ and ρ_τ is the "tilt" function $\rho_\tau(x) = \tau x^+ + (1-\tau)x^-$. Likewise, the coefficients of the polynomial approximation $\hat{P}(b)$ to the nonparametric quantile regression minimize $S_0 = \sum_t \rho_\tau(v_t - \hat{P}(b_t))$.

The test performed is the first likelihood ratio test proposed by Koenker and Machado (1999),

$$L_j = \frac{2(S_0 - S_j)}{\tau(1-\tau)\hat{s}(\tau)}$$

which is asymptotically χ^2_2. $\hat{s}(\tau)$ is an estimator for the scarcity function $s(\tau)$. Here we shall follow Koenker and Machado (1999) and use $\hat{s}(\tau) = (\hat{P}(\tau+h) - \hat{P}(\tau-h))/(2h)$, where h is a bandwidth parameter, and $\hat{P}$ is evaluated at the mean bid. Using $h = 0.05$, $\hat{s}(.1) = 4.4645$.

Test results are as follows:[6]

Statistical testing strongly supports the hypothesis mistakes in beliefs; this is the only hypothesis that is not rejected at conventional significance levels.

Figure 4 redraws figure 1 with the estimated lower bounds implied by the three models. It is clear from the picture that the theory of mistakes in beliefs provides a much better fit to the data than the alternatives considered, as the formal hypothesis testing indicates.

[6] The estimated polynomial approximation to the unrestricted quantile regression is $\hat{P}(b) = 10.8756 + 1.1177(b-10) + 0.0031(b-10)^2$.

Model	Coefficient(s)	S_j	test statistic	p-value
Values	$A = 11.3900$	144.56	418.97	0.0000
Maximization	$B = 0.54236$	170.59	548.53	0.0000
Belief	$C = 3.3316$	61.189	3.9998	0.1353
Unrestricted		60.385	—	—

Table 1: Hypothesis testing, 3 bidders sample.

7.1 Evidence from the 6 bidder sample

Dyer, Kagel, and Levin (1989) have also collected bids from the same subjects under the assumption that they are participating in an auction with 6 bidders. The testing procedure can be applied to this sample as well, once the bound formulas are modified accordingly.

Test results for the 6 bidder sample are as follows:[7]

Model	Coefficient(s)	S_j	test statistic	p-value
Values	$A = 3.944$	63.509	210.09	0.0000
Maximization	$B = 1.19 \times 10^{-12}$	46.147	66.521	0.0000
Belief	$C = 4.8159$	38.332	1.8998	0.3868
Unrestricted		38.102	—	—

Table 2: Hypothesis testing, 6 bidders sample.

Again, statistical hypothesis testing strongly favour the mistakes in beliefs interpretation. Figure 5 graphically illustrates the test for the 6 bidder sample.

Bidding in the 6 bidder sample seems to be closer to the teoretical Nash equilibrium, and indeed estimated bounds are tighter. This might be due to the fact that there is less "room" for overbidding, as the distance between the theoretical predictions and the 45 degree line shrinks as the number of bidders grows large. It may also be due to a framing effect, as bidder neglect to react strongly enough to the increase in competition.[8]

[7]For 6 bidders the estimated polynomial approximation to the unrestricted quantile regression is $\hat{P}(b) = 10.4135 + 1.0557(b - 10) + 0.0015(b - 10)^2$.

[8]In 111/480 = 23% of the observations subjects have place the exact same bid for 3

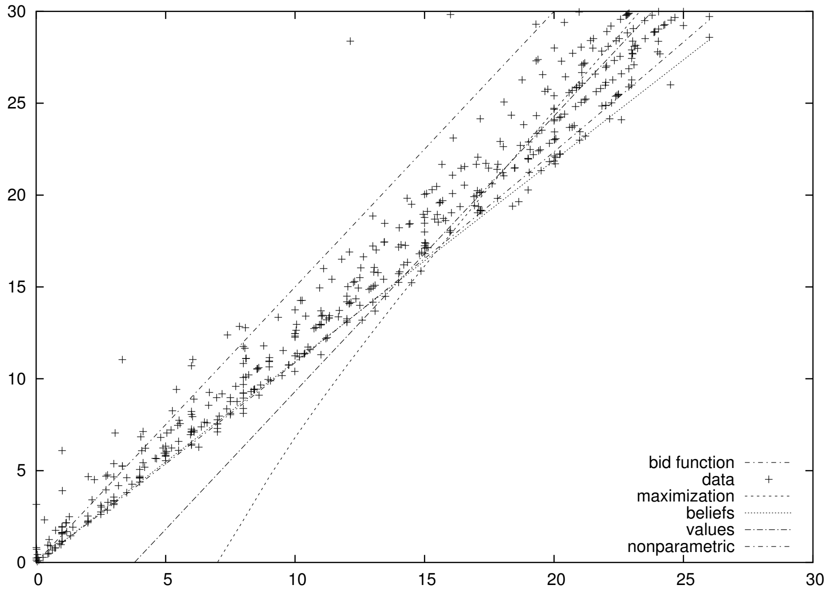

Figure 4: Bounds for Dyer, Kagel, and Levin (1989) data, 3 bidders sample

In any case, this comparison is not central to the main point of this paper. Here the main insight is that different types of mistakes would lead to different predictions on how overbidding varies with values. The mistakes in beliefs story is the one that predicts the pattern observed in both samples: the more serious bidders are the ones that do most of the overbidding.

8 Comparison with risk aversion

If bidders are risk averse, they seek to maximize

$$u(v-b)Q(b),$$

where u is a concave Bernoulli utility function. Assuming both u and Q are differentiable, the FOC is

$$\frac{u(v-b)}{u'(v-b)} = \frac{Q(b)}{q(b)}.$$

and 6 bidders; therefore about in about quarter of the sample such neglect is evident.

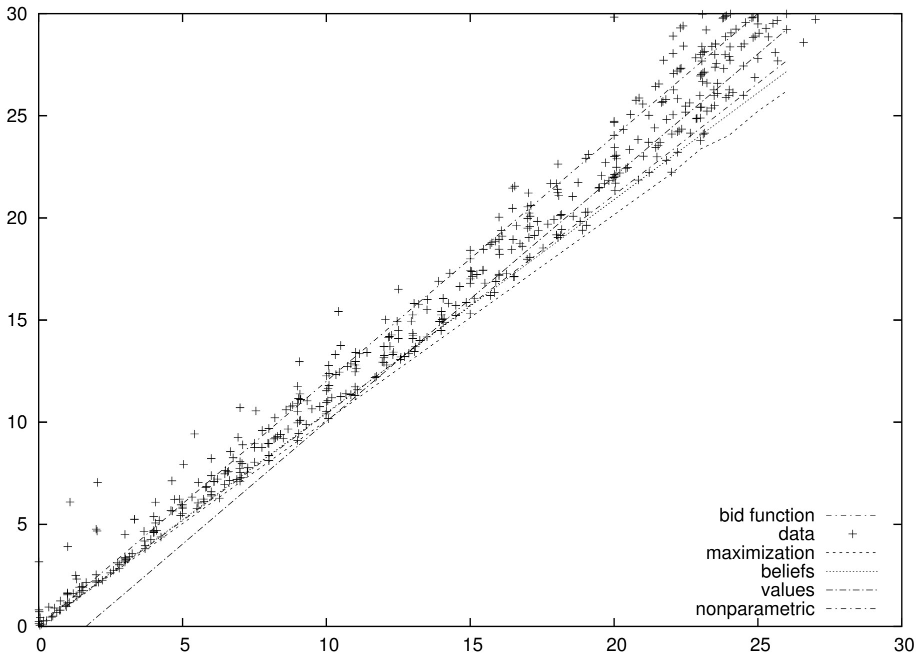

Figure 5: Bounds for Dyer, Kagel, and Levin (1989) data, 6 bidders sample

Cox, Smith, and Walker (1983, 1985, 1988); Bajari and Hortaçsu (2004) focus on constant relative risk aversion (CRRA) preferences, that have $u(x) = x^\eta$, for $\eta \in (0, 1]$. In that case, $u(v - b)/u'(v - b) = \eta(v - b)$, and one obtains

$$v = b + \frac{1}{\eta}\frac{Q(b)}{q(b)}.$$

Which is very similar to the expression for $\underline{v}_B$. It is no wonder that both models fit the experimental data well, since they predict similar patterns.

It must be pointed out that however the assumption of CRRA, while standard in financial economics, is by no means the most reasonable one in the context of auction experiments. The dollar amounts involved in experiments is a negligible fraction of subjects' wealth, and therefore it is not reasonable to expect that income effects exist in a range of 10–20 dollars (even if it did, the effect should not be proportional to receipt in the experiment).

One important difference between our and the standard risk-averse model is that the latter predicts bias, but not dispersion: v should be exactly $b +$

$(1/\eta)Q(b)/q(b)$, or at least $E[v]$ should be this, with nothing said about the variance of v given b. In paticular, the basic risk averse story does not explain the observed pattern that $v|b$ is more dispersed for higher b. In contrast, risk-neutral bidders with mistaken assessments of q would bid in a way that generates this heteroskedastic pattern.

One plausible extension of the risk-aversion hypothesis that would fit this feature of the data as well is heterogeneity on η, as advocated by Cox, Smith, and Walker (1985, 1988). If each bidder has a different η, then each observation (v, b) will fall into a different ray going through the origin, and that model would fit the data well.

A potential critique of introducing heterogeneity is that it makes the model "too rich": in principle one can rationalize any pattern of bids with it, by simply picking $\eta_i = (v_i - b_i)q(b_i)/Q(b_i)$. Without restrictions of what η_i can be, the theory does not provide any testable predictions.[9]

It is hard to distinguish these two competing theories based on the data used here only; one possibility is to augment the experiment with additional exercises devoted to measuring η independently. Harrison (1990) has done that, and have found that subjects in his experiments exhibit risk attitudes that are not consistent with the amount of risk aversion necessary to rationalize bidding behavior.

Another possibility is to redesign the experiment in a way that controls for risk attitudes. A clever way to do so is to pay bidders in lottery tickets instead of dollars (Roth and Malouf, 1979; Berg, Daley, Dickhaut, and O'Brien, 1986; Cox, Smith, and Walker, 1985). If the winner of the auction receives x_1 with probability (proportional to) $v_i - b_i$ and x_2 otherwise, his or her payoff is $u(x_2) + [u(x_1) - u(x_2)](v_i - b_i)Q(b_i) = A + B(v_i - b_i)Q(b_i)$, and he or she should behave as a risk neutral bidder, independently of the shape of u. Cox, Smith, and Walker (1985) have applied this idea to the context of independent private values first-price auction, and have found that the tendency to overbid still exists in auctions for lottery tickets: that evidence points against risk aversion being the only source of overbidding (Kagel and Roth, 1992).

[9] A natural restriction, that bidders are not risk lovers ($\eta_i \leq 1$), is violated by part of the data, since one does observe underbidding as well, as seen in Figure 1.

9 Concluding remarks

In summing up the evidence regarding overbidding in experiments involving independent private values first price auctions, Kagel (1995) has stated that "it is probably safe to say that risk aversion is one element, but far from the only element, generating bidding above the (risk-neutral) Nash equilibrium" (p. 525). I see this paper as contributing to this literature by providing one such other element, namely, the inability to form accurate beliefs about other players' behavior. Such element is not only reasonable a priori, but it has also found strong empirical support from the data investigated.

More generally, and beyond the debate about the sources of overbidding in first price auctions, this paper proposes a methodology to empirically identify what is the potential source of the breakdown when theory fails to predict economic behavior. The decomposition among errors in valuations/types, maximization and beliefs is conceivable in any social situation that can be framed as an incomplete information game or a mechanism; as such, the methodology proposed here can be applied to a variety of contexts.

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