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A Regression Tree Analysis of Real Interest Rate Regime Changes

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November 1996

This paper uses regression tree analysis to locate changes in the real interest rate process from the early 1950s to the early 1990s. We find important changes in the mean and variance of the process in 1972:Q4, 1980:Q1, and 1986:Q2. Removing the changing mean from the *ex post* real interest rate leaves a time series that is largely unpredictable — consistent with the view that it is a rational forecast error as predicted by the Fisher effect. This implies that the *ex ante* real interest rate is approximately a constant subject to infrequent but important changes.

1. Introduction

A number of studies have documented changes in the stochastic process followed by the real interest rate between the early 1960s and late 1980s. A variety of statistical techniques have been used, producing different estimates of the dates of the regime shifts. As a consequence, several different, although not necessarily competing, explanations of the regime shifts have been offered.

Huizinga and Mishkin [1986] use the maximum likelihood approach suggested by Quandt [1958, 1960] to identify shifts in the real interest rate process in October 1979 and October 1982. These dates coincide with changes in the Federal Reserve's operating procedures and Huizinga and Mishkin argue that these changes are responsible for the regime shifts. Walsh [1988] has criticized this dating as possibly reflecting shifts in the inflation process. Modifying the equation estimated by Huizinga and Mishkin accordingly, he finds shifts in October 1979 and April 1983 and concludes that changes in the conduct of monetary policy may not be the only force at work. An alternative explanation is the rise in level of Federal deficits in the early 1980s.

Garcia and Perron [1991] use the Markov switching method of Hamilton [1989] to locate shifts in the real interest rate in late 1972 and mid 1981. Based on the dating of the latter they argue that it is consistent with the deficit explanation. They also find that, once account is taken of the shifts, the *ex post* real interest rate exhibits little or no persistence. Note that the approaches of both Huizinga and Mishkin and of Garcia and Perron require *a priori* specification of the number of different regimes while the regression tree method that we use allows the data to determine the number of regimes.

In this paper we use regression tree analysis to locate the changes in the real interest rate process between the early 1950s and the early 1990s. We find a regime changes in 1972:Q4, 1980:Q1, and 1986:Q2. We argue that, apart from changes in an otherwise constant real interest rate at these times, the Fisher effect explains much of the behavior of nominal interest rates. Thus, we conclude that these changes represent shifts in the *ex ante* real interest rate. The paper is organized as follows: the next section discusses the regression tree method and presents the results of our application of this technique. Section 3 discusses the implications of the regime changes that we find for the Fisher hypothesis. Section 4 presents a summary and conclusions.

2. Regression Tree Estimates of Regime Changes

Let i_t be the one–period–ahead nominal interest rate, and π_{t+1} be the one–period–ahead inflation rate. We use the US 90–day Treasury Bill rate at the end of quarter t to measure i_t and the percentage change in the US CPI (not seasonally adjusted) from the last month of quarter t to that of quarter $t+1$ measure π_{t+1}. Until 1986:Q4 we use the Huizinga and Mishkin [19☠] CPI data to compute the inflation rate as it treats housing costs on a rental–equivalence basis.[1] The Treasury Bill rate and the CPI after 1986:Q4 are from CitiBase. The sample period is 1951:Q4 to 1991:Q4. The one–period–ahead *ex post* real interest rate is then defined as $r_{t+1} = i_t - \pi_{t+1}$. Our analysis models r_{t+1} as following an $AR(2)$ process as in Garcia and Perron but the regime changes we find are robust to both shorter and longer lag lengths.

Regression tree analysis is discussed in detail in Breiman, *et al.* [1984]—here we present a brief description of the method.[2] There are two stages to the regression tree procedure—growing and pruning. In the growing stage the $AR(2)$ model for r_t is estimated for each possible contiguous binary split of the sample. That is, letting T be the sample size, for each choice of τ with $4 \leq \tau \leq T-3$,[3] we fit separate models to the observations with $1 \leq t \leq \tau$ and to those with $\tau < t \leq T$. That split (choice of τ) minimizing the overall sum of squared residuals is chosen, dividing the sample into two subsamples. This procedure is repeated separately on each of these subsamples further splitting the sample into four subsamples. Growing continues in this way until all degrees of freedom are exhausted. The result is a representation of sample as a tree with each node containing a subsample. In the pruning stage, a cost of making a split is imposed. As this cost is increased from 0 to ∞ the tree from the growing stage is pruned until only the original sample remains. The optimal tree is one of the resultant sequence of pruned subtrees of the tree from the growing stage.

To choose the optimal tree “cross–validated" estimates of the error variance of each model represented by a tree in the sequence are required. These are found by calculating the residual for each observation using estimates of the model parameters computed from the sample net of that observation. These residuals are used to estimate the error variance using

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[1]One of the regime changes that we find is at 1986:Q2 but this apparently has little if anything to do with the change in our measure of the inflation rate at 1986:Q4. Changing from the Huizinga and Mishkin CPI data to the CitiBase data in any quarter from 1983:Q1 to 1986:Q4 still produces the split at 1986:Q4.

[2]A summary more detailed than that here may be found in the appendix to Durlauf and Johnson [1995] who use regression trees to test the convergence hypothesis of neoclassical growth theory.

[3]This restriction on τ ensures that the subsamples always have more observations than the number of explanatory variables.

the usual formula. There are two rules for selecting the optimal tree from this sequence. One is to choose that tree having the minimum cross–validated error variance. The other, which, of course, chooses a more parsimonious model, is to choose that tree having the largest cross–validated error variance that is less than the minimum plus its estimated standard error.[4] The procedure is consistent in the sense that, if there are finitely many splits, as the sample size goes to infinity, both rules will select the correct model. The main advantage of using a regression tree to look for regime shifts in the data is that the number of regimes need not be specified in advance. The procedure used by Garcia and Perron, for example, requires that the number of regimes be specified, although a test of the hypothesis that there are $n - 1$ regimes against the alternative that there are n regimes is possible. With a regression tree, the number of regimes is determined endogenously by the number of splits in the optimal tree.

We apply the regression tree procedure to the *ex post* real interest rate data described above and found regime changes in 1972:Q4, 1974:Q4, 1980:Q1, and 1986:Q2 using the minimum cross–validated error variance rule. That is, this rule breaks the 1951:Q4 to 1991:Q4 sample period into five regimes. The other rule breaks the sample into three regimes with splits in 1972:Q4 and 1980:Q1. To the extent that the samples overlap, these are similar to the regimes found by Garcia and Perron who find splits at the end of 1972 and in mid 1981.[5] While the regime change in 1980:Q1 conforms closely to that found by Huizinga and Mishkin in October 1979, we find no evidence of the subsequent split that they find in October 1982. For the period prior to 1972:Q4, our results are very different from those of Garbade and Wachtel [1978] who find that the *ex ante* real interest rate "varied substantially" over this period. The difference is most likely due to their modeling of the rate as a random walk — an assumption that has not proven robust [Litterman and Weiss, 198☠].

Figure 1 shows the two optimal tree representations of the sample. Inside each nonterminal node (those with descendants) is shown the split date at that node. Observations in the left descendant are those prior to the split date while observations in the right descendant are those at or after the split date. The terminal nodes (those without desendants) contain the number of observations, the subsample mean of r_t, and the subsample variance of r_t. Underneath each terminal node is indicated the subsample represented by that node. The two dashed terminal nodes give the data for the tree chosen by the minimum cross–validated error

[4]This is computed as $\frac{1}{N}[\sum_i (e_i^2 - \widehat{\sigma}_{CV}^2)^2]^{1/2}$ where the e_i are the cross–validated errors and $\widehat{\sigma}_{CV}^2$ is the cross–validated estimate of the error variance.

[5]The quarterly "Mishkin data set" used by Garcia and Peron covers the period 1961:Q1 to 1986:Q4 and applying the regression tree technique to this sample suggests shifts in 1972:Q4 and 1980:Q4. We present results based on the sample from CitiBase because it is longer.

variance plus one standard error rule—that is, if the nodes containing the splits in 1974:Q4 and 1986:Q2 are made terminal.

Table 1 presents the estimated models for each regime as well as for the entire sample. Observe that, while the AR terms are significant in the model estimated for the entire sample, only one of the ten AR terms are significant in the five subsamples defined by the larger tree and only two of the six are significant in the three subsamples defined by the smaller tree. At the same time, the constant terms are significant in the subsamples but vary substantially across the subsamples. This suggests that the differences between the regimes are largely differences in the mean *ex post* real rate across the regimes and that the changing mean accounts for most of the persistence in the rate over the sample period. To confirm this idea we computed a mean adjusted real rate, $\widehat{r}_t^5$, by subtracting from r_t the respective subsample means.[6] Regressing r_t on a constant and eight lagged values for the entire sample produces a statistically significant R^2 of 0.33. Doing the same for $\widehat{r}_t^5$ gives an insignificant R^2 of 0.025.[7] This has implications for the Fisher hypothesis which we pursue below. The changing mean also accounts for much of the variation in the *ex post* real rate—the regression of r_t on a constant and dummy variables for each of the second, third, fourth and fifth subsamples has an R^2 of 0.49.

The regime change in 1974:Q4 appears to account for little in these results. As Figure 1 demonstrates, it was the last split found in the larger of the optimal trees, indicating that it is the least important of the four in improving the fit of the model. We omitted the 1974:Q4 split by combining the second and third regimes into one covering the period 1972:Q4 to 1979:Q4 and computed a mean adjusted real rate, $\widehat{r}_t^4$, using the subsample means from the remaining four regimes. Regressing $\widehat{r}_t^4$ on a constant and eight lagged values for the entire sample produces a statistically insignificant R^2 of 0.022.[8] Further, as Table 1 shows, fitting the $AR(2)$ model to the combined subsamples produces no more significant coefficients than fitting it to the second and third subsamples separately. Furthermore, the four remaining regimes account for almost as much of the variation in r_t as do the five discussed above—the regression of r_t on a constant and dummy variables for three of the four has an R^2 of 0.48.

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[6]Suppose that the sample is partitioned into n subsamples. Then $\widehat{r}_t^n = r_t - \sum_{i=1}^{n} D_{i,t}\widehat{\mu}_i$ where $D_{i,t} = 1$ if observation t is in regime i and zero otherwise and $\widehat{\mu}_i$ is the estimated mean of r_t in subsample i.

[7]The test statistic used is $T \times R^2$, where T is the sample size, which, under the null hypothesis, is asymptotically distributed as χ_8^2. For r_t the value of the test statistic is 83.7, while for $\widehat{r}_t^5$ it is 3.91. The 5% critical value is 15.51.

[8]The test statistic, computed as described in the previous footnote, is 3.43.

quarter of this period, the low mean *ex post* real interest rate from then until 1974:Q3 is consistent with the idea that inflation was (*ex post*) systematically underforecast. Of course, this does not require that expectations be formed irrationally, but rather only that the subjective probability of a shift to a high inflation regime was sufficiently low during the 1972:Q4 to 1974:Q3 period. Consequently, our belief is that, of the four shifts that we find in the larger tree, the shift in 1974:Q4, is the least likely to represent a shift in the *ex ante* real interest rate. With this in mind, for the remainder of the paper we ignore the 1974:Q4 split.

The same conclusion is a bit more difficult to reach for the split found in 1986:Q2. We omitted this split by combining the last two regimes into one covering the period 1980:Q1 to 1991:Q4 and computed a mean adjusted real rate, $\widehat{r}_t^3$, using the subsample means from the remaining three regimes. Regressing $\widehat{r}_t^3$ on a constant and eight lagged values for the entire sample produces a statistically insignificant R^2 of 0.073.[9] Further, as Table 1 shows, fitting the $AR(2)$ model to the combined subsamples results in a significant coefficient on r_{t-2} which is not the case when the model is fit to the two subsamples separately. Furthermore, the three remaining regimes account for somewhat less of the variation in r_t than do the four discussed above—the regression of r_t on a constant and dummy variables for two of the three has an R^2 of 0.40. This and the evidence presented below on the implications of these regime shifts for tests of the Fisher hypothesis make us unwilling to ignore the 1986:Q2 shift.

Figure 2 plots the *ex post* real interest rate and shows the four remaining subsamples and the average rate for each—the means and variances were given in Figure 1. The four separate $AR(2)$ models representing these subsamples are identified by *s in Table 1. The average rate was 1.36% during the 1951:Q4 to 1972:Q3 period and then fell to -1.11%. In 1980:Q1 it rose to 5.09% before falling to 2.27% in 1986:Q2. The variance of the rate during the 1972:Q4 to 1979:Q4 period was about 60% higher that in the first subsample and higher again by about the same amount during the 1980:Q1 to 1986:Q1 period. During the last subsample the variance fell to about that in 1972:Q4 to 1979:Q4 period. So, in addition to the shifts in the mean *ex post* real rate the sample period also saw large changes in its variance. We argue below that this represents a rise in the variance of the unpredictable part of inflation and not in that of the *ex ante* real interest rate.

[9]The test statistic, computed as described in footnote 7, is 11.1.

3. Implications for Tests of the Fisher Hypothesis

Durlauf and Hall [1988, 1989a, 1989b] present a signal extraction framework for testing rational expectations models. Garcia [1993] uses this framework to test the Fisher hypothesis for Brazil and Johnson [1994] uses it to test the Fisher hypothesis for the United States. The idea behind the Durlauf–Hall framework is that the *ex post* real interest rate can be decomposed as $r_{t+1} = \rho_t + N_t - \nu_{t+1}$, where ρ_t is the *ex ante* real rate, ν_{t+1} is the one–period–ahead inflation forecast error and N_t is the "model noise". The model noise is the difference between the actual nominal interest rate and that predicted by the Fisher hypothesis.[10] It provides a measure of the extent to which the Fisher equation is misspecified. If the Fisher hypothesis is true then $N_t = 0$.

Since N_t is unobservable, an estimate is required to test the Fisher hypothesis. One way to construct an estimate is to assume that ρ_t is constant and let X_t be any set of variables known to the econometrician and the market at time t, including a constant. The projection of r_{t+1} onto X_t is then equal to the projection of N_t onto X_t, denoted $\widehat{N}_t$. Further, since $N_t - \widehat{N}_t$ is orthogonal to $\widehat{N}_t$, $Var(\widehat{N}_t) \leq Var(N_t)$, so that the variance of $\widehat{N}_t$ provides a lower bound for the variance of the model noise. It can be shown that, given X_t, this is the tightest available lower bound. The variance of $\widehat{N}_t$ thus provides a metric for evaluating the veracity of the Fisher equation. If it is small the Fisher equation may be a useful description of interest rate behavior and if it is large the Fisher equation reveals little about interest rate behavior. To make "small" and "large" operational some normalization of $Var(\widehat{N}_t)$ is required. One is to divide by $Var(r_{t+1})$ giving the R^2 from the regression of r_{t+1} on X_t.[11]

Table 2 gives the results of noise estimation using four different sets of regressors—r_{t-j}, i_{t-j}, π_{t-j}, and, i_{t-j} and π_{t-j}, for $j = 0 \ldots 7$.[12] The first column uses r_t as a regressand, the second $\widehat{r}_t^5$, the third $\widehat{r}_t^4$, and the fourth $\widehat{r}_t^3$. Reported are the R^2 values for each regression. A † indicates a rejection of the hypothesis that the regressors have zero coefficients at the 5% level of significance and a ‡ indicates a rejection at the 10% level. For r_t the most noise is found using i_t and π_t and their lags as regressors. For this X_t the model noise contributes at

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[10] More explicitly, Fisher's theory predicts that the nominal interest rate will be given by $i_t^* = \rho_t + E_t\pi_{t+1} = \rho_t + \pi_{t+1} - \nu_{t+1}$. The model noise is given by $N_t = i_t - i_t^* = i_t - \rho_t - \pi_{t+1} + \nu_{t+1}$. Rearranging yields $r_{t+1} = i_t - \pi_{t+1} = \rho_t + N_t - \nu_{t+1}$.

[11] This normalization is problematic in the event that N_t obeys an integrated process. In this case, if X_t is integrated and N_t and X_t are cointegrated then R^2 converges to unity. If they are not cointegrated, R^2 converges to a non–degenerate random variable. If X_t is not integrated R^2 converges to zero. In any case, R^2 is a uninformative metric for model evaluation if N_t obeys an integrated process.

[12] In each case we use an even number of regressors in case i_t or π_t or both obey integrated processes as argued by Mishkin [1991].

least 44% of the variation in the *ex post* real interest rate. Consistent with the large body of literature on this issue, this implies a rejection of the Fisher hypothesis for the 1951:Q4 to 1991:Q4 period.

The second, third and fourth columns indicate that when the mean adjusted rates are used as regressands much less model noise is found. For example, the regression of $\widehat{r}_t^5$ on i_t and π_t and their lags has a statistically insignificant R^2 of 0.14. Only in the case of $\widehat{r}_t^3$ does this set of regressors produce a significant R^2, and then only at the 10% level. Clearly, the mean adjusted rates contain much less model noise than r_t. That is, the shifts in the mean of r_t account for most of the model noise found above. In fact, in the case of the larger tree the variation in the mean of r_t explains 66% of the variation in the model noise found in r_t when i_t and π_t and their lags are used as regressors. For the smaller tree the figure is 56%.

This suggests that the unfavorable inference about the Fisher hypothesis drawn from the large amount of model noise found in r_t is erroneous in the sense that much of that noise can be attributed to variation in the *ex ante* real interest rate. Once that variation is removed from the data the Fisher hypothesis represents an accurate summary of interest rate behavior. Put another way, while we agree entirely with Garcia and Perron's conclusion that the *ex post* real interest rate is "… a random process around a mean that exhibits infrequent but important changes" (p8), we can say something stronger. Apart from the changes in the mean of the *ex post* real interest rate, very little of the variation in the random process is predictable and so much of it may be considered to be a rational forecast error. This means that nominal interest rates can be well described as the sum of a rational forecast of inflation and an infrequently changing *ex ante* real interest rate.[13]

[13]Of course, this conclusion rests on the assumption that the observed shifts in the mean *ex post* real interest rate represent shifts in the mean *ex ante* real interest rate. An alternative assumption is that the shifts represent systematic inflation forecast errors. Under this assumption the fall in the mean *ex post* rate in 1972:Q4 would be due to a persistent underprediction of inflation and the rise in 1980:Q1 would be due to a persistent overprediction. It is difficult to believe, however, that, even in an environment where agents had to learn of a change in the inflation process, such forecast errors could persist for as long as they would have to in order to justify this explanation.

4. Conclusions

In this paper we have used regression tree analysis to locate shifts in the stochastic process followed by the *ex post* real interest rate from the early 1950s to the early 1990s. This method has the advantage of not requiring us to specify the number of different regimes *a priori* as do those used by Huizinga and Mishkin and by Garcia and Perron. That is, regression tree analysis allows the data to determine the number of regime changes as well as their dates. Nevertheless, our findings are similar to those of Garcia and Perron in that we find two shifts at dates close to the dates they find when they assume that there are only two shifts.

We find important changes in the mean and variance of the real interest rate process in 1972:Q4, 1980:Q1, and 1986:Q2. Apart from the changing mean, the *ex post* real rate seems to be an uncaused process — a finding consistent with the Fisher hypothesis. That is, the remaining variation in the *ex post* real interest rate is largely unpredictable as it would be if it were a rational forecast error as predicted by the Fisher effect. This implies that the changes in the mean of the *ex post* real interest rate are due to changes in the *ex ante* real interest rate which is well modeled as a constant subject to infrequent but important changes.

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Table 1: Estimates of $r_{t+1} = \beta_0 + \beta_1 r_t + \beta_2 r_{t-1} + \epsilon_{t+1}$

	Number of Observations	β_0	β_1	β_2	σ_ϵ^2	R^2
			Entire Sample			
1951:Q4 to 1991:Q4	161	$0.66^\dagger$ (0.22)	$0.24^\dagger$ (0.07)	$0.37^\dagger$ (0.07)	2.25	0.27
			Minimum Cross–Validated Error Variance Tree			
1951:Q4 to 1972:Q3*	84	$1.40^\dagger$ (0.26)	-0.14 (0.10)	0.11 (0.10)	1.55	0.04
1972:Q4 to 1974:Q3	8	$-3.14^\dagger$ (1.08)	-0.46 (0.33)	-0.13 (0.33)	1.92	0.28
1974:Q4 to 1979:Q4	21	$-1.41^\dagger$ (0.35)	-0.01 (0.16)	$-0.76^\dagger$ (0.16)	1.39	0.55
1980:Q1 to 1986:Q1*	25	$3.88^\dagger$ (1.19)	-0.05 (0.17)	0.31 (0.17)	2.52	0.13
1986:Q2 to 1991:Q4*	23	$2.64^\dagger$ (0.86)	-0.19 (0.22)	0.04 (0.18)	2.05	0.04
			Minimum Cross–Validated Error Variance plus One Standard Error Tree			
1951:Q4 to 1972:Q3	84	$1.40^\dagger$ (0.26)	-0.14 (0.10)	0.11 (0.10)	1.55	0.04
1972:Q4 to 1979:Q4*	29	$-1.67^\dagger$ (0.42)	-0.07 (0.17)	$-0.48^\dagger$ (0.17)	1.84	0.23
1980:Q1 to 1991:Q4	48	$2.15^\dagger$ (0.73)	0.10 (0.13)	$0.34^\dagger$ (0.13)	2.55	0.15

This table shows estimates of $AR(2)$ models for the *ex post* real interest rate for the entire sample and the regimes identified by the regression tree procedure described in the text. Estimated standard errors are given in parentheses below parameter estimates. A † indicates significance at the 5% level. A * identifies those equations relevant to the 4-regime split of the sample discussed in the text. Data sources are described in the text.

Table 2

Noise Estimates

Regressors ($j = 0, \ldots, 7$)	r_{t+1}	$\widehat{r}^5_{t+1}$	$\widehat{r}^4_{t+1}$	$\widehat{r}^3_{t+1}$
r_{t-j}	$0.33^{\dagger}$	0.05	0.04	0.04
i_{t-j}	$0.21^{\dagger}$	$0.09^{\ddagger}$	$0.09^{\ddagger}$	$0.11^{\dagger}$
π_{t-j}	0.06	0.07	0.06	0.07
i_{t-j}, π_{t-j}	$0.44^{\dagger}$	0.14	0.13	$0.15^{\ddagger}$

This table shows estimates of $Var(\widehat{N}_t)/Var(x_{t+1})$, the R^2's from the regression of x_{t+1} on the indicated regressors, for $x_{t+1} = r_{t+1}$, $\widehat{r}^5_{t+1}$, $\widehat{r}^4_{t+1}$ and $\widehat{r}^3_{t+1}$. A † indicates a rejection of the hypothesis that the regressors have zero coefficients at the 5% level of significance and a ‡ indicates a rejection at the 10% level. Data sources and definitions are given in the text.

Figure 1: Regression Tree for $r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \varepsilon_t$

1951:Q4 to 1991:Q4