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# UNIT ROOTS IN THE PRESENCE OF ABRUPT GOVERNMENTAL INTERVENTIONS WITH AN APPLICATION TO BRAZILIAN DATA

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## SUMMARY

This paper considers econometric issues related to time-series data that have been subject to abrupt governmental interventions. The motivating example for this study is the Brazilian monthly inflation rate (1974:1–1993:6) which we use throughout for illustration. This series has been heavily influenced by the effect of so-called shock plans implemented by various governments starting in the mid-1980s. The plans act as 'inliers' in the sense that the series is temporarily brought down to low levels before returning to its previous trend path. We analyse the effects on standard unit root tests and measures of persistence caused by the presence of these 'inliers'. We show a substantial bias in favour of concluding that the series is stationary and that shocks have temporary effects. We then construct appropriately corrected statistics which take into account the presence of the plans. These show, unlike the standard tests, that the stochastic behaviour of the inflation rate was indeed unstable over this period. Simulation results are presented to support the adequacy of our corrected statistics. Copyright © 1999 John Wiley & Sons, Ltd.

## 1. INTRODUCTION

Non-stationarity in economic data can take various forms; for example, an autoregressive unit root or the presence of structural changes in a functional relation among a set of variables. In this paper, we discuss an alternative form of non-stationarity related to the effects of abrupt governmental interventions also referred to as 'shock plans'.

Our analysis is directly motivated by the time-series properties of the Brazilian inflation rate. This series is characterized by important increases starting in the early 1980s, turning into hyperinflation by the end of the 1980s. Yet this period of very high inflation has been marked by a few (five that are important until the early 1990s) 'shock plans' which have brought inflation to a low level for a short period of time. Intuition suggests that, in this highly volatile period with an ever-increasing trend path for inflation, standard statistical measures related to the issue of non-stationarity and the persistence of shocks would show the series to be highly persistent and non-stationary. Yet exactly the opposite occurs. Standard unit root tests suggest that inflation was stationary in that period and that shocks affected its level in a temporary manner. Indeed, standard measures suggest that inflation was 'more stationary' and less persistent in this hyperinflation period than in the 1970s when inflation was moderate.

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An issue we want to analyse is whether these results are the artifact of the presence of the temporary changes created by the shock plans. To get some intuition on this issue, we can view these shock plans as creating 'inliers' whose magnitude is related to the current level of the series. Hence, if the series truly has a stochastic trend (i.e. a unit root) or even an explosive path, the magnitude of these 'inliers' are, themselves, non-stationary random variables which have a tendency to increase as inflation increases. Since these shock plans have failed, the series exhibits a tendency to return to its old (non-stationary) trend path after each episode. This is basically what contaminates the standard statistical measures, since the failures of the shock plans create a kind of spurious mean-reverting aspect to the series.

This argument is fundamentally the flip-side of the argument exposed in Perron (1989, 1990) where permanent changes in the trend function of a series with a stationary noise bias standard unit root tests and persistence measures towards accepting the unit root hypothesis and concluding that shocks have persistent effects. Here, temporary, but large, changes bias these measures in the opposite direction.

The problem is somewhat related to the analysis of Franses and Haldrup (1994) who showed how unit root tests have liberal size distortions when a series with a unit root is contaminated by additive outliers (see also Vogelsang, 'Two simple procedures for testing for a unit root when there are additive outliers', forthcoming in *Journal of Time Series Analysis*). The issue is, however, qualitatively different in two aspects. First, the occasional events occur for more than a single period, lasting usually several months. Second, and more importantly, the magnitude of the 'inliers' or shock plans is directly related to the actual level of the series and is, hence, a non-stationary random variable.

The aim of the paper is first to provide a detailed analysis of the statistical effects of such 'inliers' on standard statistical tools such as unit root tests and measures of persistence. The second goal is to provide modifications to these standard tests that directly take into account the presence of the shock plans. As we shall see, the answers obtained are dramatically different.

The structure of the paper is as follows. Section 2 describes in detail the data used and Section 3 briefly discusses the historical settings surrounding the shock plans implemented by the various Brazilian governments. The results obtained from the application of standard unit root tests and measures of persistence are presented in Section 4. The bias of the unit root tests in the presence of occasional shock plans is analysed in Section 5. Our results show that standard unit root tests are severely biased by the shock plans towards a rejection of the unit root hypothesis in favour of stationary fluctuations around a stable linear trend function. Accordingly, Section 6 considers modified versions of the tests that explicitly take into account the presence of the shock plans. Some simulations show that these modifications yield tests with correct sizes and the empirical results show a very different picture. Indeed, we no longer reject the unit root hypothesis in favour of stationary alternatives as appears intuitively plausible in a period of hyperinflation.

## 2. DESCRIPTION OF THE DATA

The series used in this paper is the monthly Brazilian inflation rate for the period 1974:1 to 1993:6. Note that the choice of 1993:6 as the end of the sample is to avoid incorporating the Real Plan which is still in effect.[1] We use what is called the 'official inflation index'. This is actually a

[1] We also applied the same analysis to the Brazilian nominal interest rate, defined as the monthly compound overnight rate (basically the equivalent of the FED funds rate in the United States). The two series having similar general properties, the results obtained are qualitatively similar. Hence, we do not report them.

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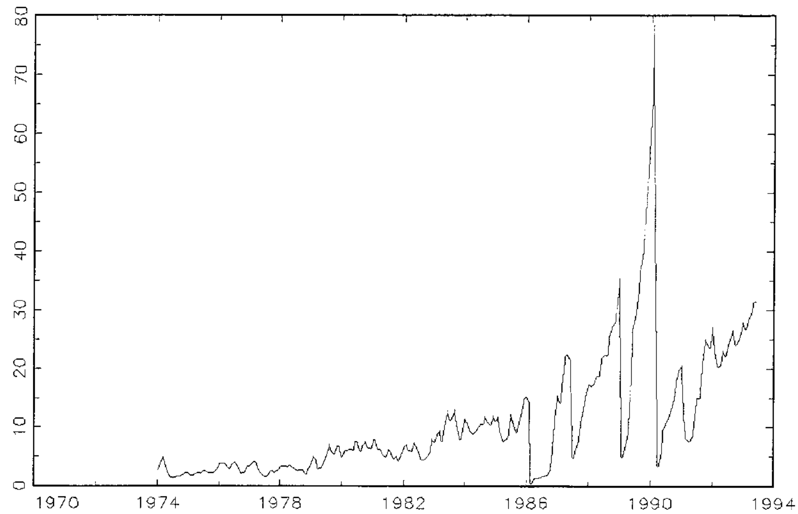


Figure 1. Brazilian inflation rate, 1974:1 to 1993:6

splice of several indices that was used by the government as the official index to all mandatory indexation schemes (for taxes, wages, etc.). This index was also widely employed by the financial markets and the central bank used it to calibrate the real interest rate. We applied two modifications to this 'official index'. First, since the price index is computed from an average of the daily prices from the beginning to the end of the month, the measured monthly inflation reflects price changes from the middle of the previous month to the middle of the current month. To obtain a better approximation of price changes from beginning to end of month, we used a geometric mean with equal weights of inflation over periods $t$ and $t+1$. Second, given the sudden and important changes in inflation caused by the shock plans, the usual continuity assumption that justifies the use of monthly averages breaks down. In order to mitigate the problems caused by averaging in this context, we used, for the months immediately following the plans, special price vectors computed by the government at the moment of each plan.

A graph of the inflation rate series is presented in Figure 1. It is clear that this series is characterized in the 1980s by several sudden drops that are important in magnitude. These are the outcome of the various shock plans instituted by the government in an attempt to stop the process of high and increasing inflation.

Table I presents a summary of the various plans along with the dates we retained to define them and the magnitude of the decrease in inflation. The starting date of a plan was decided as

Table I. List of shock plans in the Brazilian economy

| Name | Period | Length (months) | Decrease in inflation (%) |
|---|---|---|---|
| Cruzado | 86:3–86:10 | 8 | 14·03 (97·7) |
| Bresser | 87:7–87:9 | 3 | 16·75 (78·1) |
| Summer | 89:2–89:4 | 3 | 30·64 (86·4) |
| Collor I | 90:3–90:5 | 3 | 75·17 (95·8)[a] |
| Collor II | 91:2–91:6 | 5 | 9·53 (46·2) |

[a] For this plan, the decrease is computed over the two months from 90:02 to 90:04.

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the first month when over this month (and possibly the next one due to overlap) the decrease in inflation was at least 40% compared to its level in the preceding month. Choosing the ending date of a plan is somewhat more difficult. Our choice was guided both by historical records and by the use of dummy variables to create a real interest rate series with as few outliers as possible. More precisely, we estimated an error-correction model for the bivariate system that includes the inflation and nominal interest rates. The deterministic component included, besides the constant, various dummies for the plans. The ending dates of the plans were chosen so that the residuals from that error-correction model contained as few outliers as possible. It is important to note, however, that the results presented in this paper are not sensitive to minor variations in the choice of the ending dates for the plans.

Given the importance of the shock plans for the time-series behaviour of the inflation rate series, we start with a brief historical overview.

## 3. A BRIEF HISTORY OF THE SHOCK PLANS

After a short period of economic reforms in the mid-1960s the Brazilian economy grew sharply for almost a decade throughout the 1970s with a growth rate of GDP at 7·5%, on average. The yearly inflation rate was stable around 20% until the oil shock in 1973. The strategy adopted for economic development was a success because of a profitable combination of external financing and strong government support for private and public investment. This situation changed with the oil shocks in 1973 and in 1979 which were followed by an increase in the cost of external financing after the abrupt rise in interest rates at the beginning of the 1980s.

In contrast, high inflation rates and diminishing GDP growth rates were the norm starting in the 1980s (including negative rates in 1981 and, especially, in 1983). The first attempt to stabilize the economy was carried by a so-called orthodox economic team in 1982. The internal interest rate was raised above the international level; a plan for deficit reduction was proposed and a wage desindexation policy was adopted in order to restrict the internal aggregate demand. The external restriction imposed by the interest payments constraint and the lack of an international financial market that would provide financial aid inverted Brazil's former position in international trade. From 1984 on, the Brazilian trade balance was positive enough to meet international commitments. Despite the soundness of those economic decisions, the inflation rates did not fall below two digits a month. It was kept stable at around 150% per year. On the other hand, Brazilian GDP decreased by 2·0%, on average, in two years and it barely grew by 1·0%, on average, until the middle of the decade. Hence, this economic period was labelled as one of stagflation.

With the end of the dictatorship and the nomination of a new president at the beginning of 1985, the expectation was that a democracy would succeed in setting up a new economic order. However, high inflation rates and a lack of economic stability still persisted. After a long and deep recession, with high costs to the former government, the New Republic rulers decided to manage the situation without imposing more social costs. Hence, this time brought renewed discussions about the inflation rate and alternative proposals to manage it. The inertial inflation approach appeared as an alternative answer to the problem. Though the proponents of the inertial inflation approach agreed on a more orthodox diagnostic, no agreement was reached for the fight against inflation. For the inercialists, a traditional orthodox plan to stabilize the economy would imply high social costs for implementation. This alternative would require, probably, many years to bring the inflation rate down to a single-digit figure and there was reluctance to wait and accept more losses.

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A monetary reform based on a general desindexation and a change of currency was the core of the Cruzado Plan in 1986, Brazil's first heterodox attempt to stabilize the economy. A price freeze was also deemed necessary to avoid extra income gain or losses during the stabilization plan. Hence, the Cruzado Plan was also followed by a general price freeze. With hindsight, it is possible to criticize the Cruzado Plan in the way they established the initial level of some key economic variables. The interest rate was set below the international level and most of the time was negative in real terms. Then a consumption bubble and price pressures from the demand side imposed pressures to put aside the Plan against inflation. In July 1986, many commodities in the supermarket disappeared and goods such as gas, gasoline, etc. were subject to rationing. However, election and political pressures delayed changes until November. At that time, the government and its economic team could not keep the stabilization process under control, so the inflation rate again reached a two-digit figure per month (at about 14·5% in January 1987). The year following the Cruzado Plan showed high inflation rates, uncertainty and disagreement about the right economic policy to follow. At the same time, industrial production and investments started declining again.

By June, the inflation rates were out of control and the relative prices were disorganized. After a substitution of the Minister of Finance in April, a new stabilization plan was tried: the macroeconomic Consistency Plan, also called the Bresser Plan. Even though this was an attempt to correct the wrong path taken earlier it insisted on freezing prices again. As with the Cruzado Plan, the Bresser Plan was unable to solve the problem of the public deficit. A lack of control of this deficit and additional political pressures hampered any attempt to cut spending and the Finance Minister could do no more than watch further increases in inflation.

After another Finance Minister substitution in January 1988, Brazil witnessed the highest inflation rates in its history and another stabilization, called the Summer Plan, was tried a year later. It was an attempt analogous to that of Argentina. This Plan was based upon a tight monetary policy; interest rates were raised far above their historical levels and the government took the opportunity to change the feature of its internal debt (the government changed LFT (Treasury Financial Notes) by BBC (Central Bank Notes) and other longer-maturity debt instruments). Once more, the fiscal situation was not solved and the inflation rate rose again. By the end of Sarney's term, it had reached as high as 85% a month. The country was close to hyperinflation and economic chaos.

This path persisted until March 1990, when Fernando Collor de Melo became the new president. His first economic decision was a monetary reform that sequestered about 75% of all financial assets. They were converted into long-run deposits under the responsibility of the Brazilian Central Bank. The money supply fell sharply and, accordingly, so did the inflation rate, and economic activity registered a strong contraction. The plan was so efficient in the short run that, after two months, the inflation rate appeared stable at a low level. Internal debt and the interest payments decreased giving a short breathing space to the Treasury's financial operation. However, as the total debt problem was not solved and government spending was not managed, inflation went back to 20% a month in December 1990.

At the beginning of 1991, the economic team tried again to stabilize the economy through a mixture of price freezes and public spending cuts. That was the Collor II Plan which lasted less than four months. Due to a severe recession, the inflation rate stabilized around 20% a month for the rest of President Collar's term. Political problems, corruption and the increase in uncertainty are ingredients for a more general crisis that ended with the impeachment of President Collor in December 1992. He was replaced by the vice-president, Itamar Franco, who inherited an inflation

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rate close to 30% a month. This inflationary feature continued until July 1994 (at about 50% a month), when a new and, up to now, successful plan was introduced, the Real Plan.

## 4. EMPIRICAL RESULTS WITH STANDARD UNIT ROOT TESTS

In this section we discuss empirical results obtained with the application of some standard unit root tests. By that we mean tests that do not take into account the presence of the shock plans. We start with a description of the statistics as well as a measure of persistence. The empirical results are then discussed highlighting the potential problems of this standard approach.

### 4.1. The Test Statistics

For our analysis we use three tests for the presence of an autoregressive unit root. The first two are by now standard tools in the analysis of univariate data, namely the Augmented Dickey–Fuller (1979) test (labelled ADF) and the Phillips–Perron (1988) test based on the normalized bias in a first-order autoregression (labelled $Z_\alpha$). We also consider a new test suggested by Stock (1990) and further analysed by Perron and Ng (1996) which is a modification of the Phillips–Perron test that is less subject to size distortions in the presence of serial correlation in the first-differences of the data (this test is labelled $MZ_\alpha$). Finally, we consider a measure of the persistence of shocks based on an autoregressive spectral density estimator at frequency zero.

The class of processes considered, $y_t$, can be described as follows:

$$\begin{aligned} y_t &= \mu + \beta t + z_t \\ A(L)z_t &= B(L)e_t \end{aligned} \tag{1}$$

where $A(L) = 1 - a_1 L - a_2 L^2 - \cdots - a_p L^p$ is a $p$th-order autoregressive polynomial in the lag operator $L$ (defined such that $Lx_t = x_{t-1}$). Similarly $B(L)$ is a $q$th-order moving-average polynomial defined by $B(L) = 1 + b_1 L + b_2 L^2 + \cdots + b_q L^q$. The errors $\{e_t\}$ are assumed to be martingale differences (e.g. uncorrelated but not necessarily homoscedastic). System (1) simply describes a process that is the sum of a deterministic time trend (a first-order polynomial in time) and a noise function modeled as an ARMA process. Of course, more general processes are possible, but for simplicity of exposition we consider this leading case of interest.

The null hypothesis is that one autoregressive root is unity, i.e. we have the factorization $A(L) = (1 - L)A^*(L)$ with all the roots of $A^*(L)$ outside the unit circle. This implies that the sum of the autoregressive coefficients is unity. The usual alternative hypothesis is that this sum is less than one (in which case $z_t$ is stationary) but given the nature of the series analysed here we also consider the alternative hypothesis that this sum is greater than one, i.e. $z_t$ is an explosive process.

The ADF tests of Dickey and Fuller (1979) (also extended by Said and Dickey, 1984 to the case of data having an ARMA structure) is based on the idea that a stationary and invertible ARMA process can be approximated by an autoregression. Hence, the relevant regression estimated by OLS is:

$$y_t = \eta + \gamma t + \alpha y_{t-1} + \sum_{j=1}^{k} c_j \Delta y_{t-j} + v_t \tag{2}$$

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Here, $\alpha$ is the sum of the autoregressive coefficients and the null hypothesis can be tested using the $t$-statistic for $\alpha = 1$. To select $k$, we use Bayesian Information Criterion (BIC). This is adequate provided the noise component $z_t$ does not contain roots that are close to the unit circle, in particular strong negative MA components. This is indeed the case with the inflation series under investigation.[2]

The unit root test of Phillips and Perron (1988) is based on a non-parametric correction of the autoregressive estimate, $\hat{\alpha}$, in the following first-order autoregression:

$$y_t = \hat{\eta} + \hat{\gamma} t + \hat{\alpha} y_{t-1} + \hat{u}_t \tag{3}$$

Let

$$s_u^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2$$

and $\tilde{y}_{t-1}$ be the residuals from a regression of $y_{t-1}$ on a constant and a time trend. The test is defined as:

$$Z_\alpha = T(\hat{\alpha} - 1) - (s^2 - s_u^2) \Bigg/ \left( 2T^{-2} \sum_{t=1}^{T} \tilde{y}_{t-1}^2 \right) \tag{4}$$

where $s^2$ is a consistent estimate of the spectral density function at frequency zero of $\Delta z_t$ under the null hypothesis of a unit root, denoted $h_{\Delta z}(0)$. Conventional estimators are based on a kernel method that constructs a weighted sum of the empirical autocovariances of the estimated residuals $\hat{u}_t$ (see e.g. Andrews, 1991). However, Perron and Ng ('An autoregressive spectral density estimator at frequency zero for nonstationarity tests', forthcoming in *Econometric Theory*) found this estimator to be inferior to an autoregressive spectral density estimator based on the first-differences of the data, defined by:

$$s^2 = s_{ek}^2 / (1 - \hat{b}(1))^2 \tag{5}$$

with

$$s_{ek}^2 = T^{-1} \sum_{t=1}^{T} \hat{e}_{tk}^2, \quad \hat{b}(1) = \sum_{j=1}^{k} \hat{b}_j$$

where $\hat{b}_j$ and $\{\hat{e}_{tk}\}$ are obtained from a $k$th-order augmented autoregression in $\Delta y_t$:

$$\Delta y_t = c + b_0 y_{t-1} + \sum_{j=1}^{k} b_j \Delta y_{t-j} + e_{tk} \tag{6}$$

The consistency of $s^2$ for $h_{\Delta z}(0)$ under the null hypothesis of a unit root follows from the results of Said and Dickey (1984) and Berk (1974). Again, $k$ is selected as that value which minimizes the BIC criterion. An important point to note is that $s^2$ is bounded above by zero even under the alternative of a stationary noise function $z_t$. This ensures the consistency of the modified statistic which we now describe.

[2] An alternative method, following Campbell and Perron (1991) and Ng and Perron (1995), is to use a data-dependent method based on a general-to-specific recursive procedure. We also verified that the basic qualitative results are not sensitive to using this alternative strategy.

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Stock (1990) proposed a class of statistics which exploits the feature that a series converges with different rates of normalization under the null and alternative hypotheses. We consider one such test, referred to as $MZ_\alpha$, defined by

$$MZ_\alpha = (T^{-1}\tilde{y}_T^2 - s^2)\bigg/\left(2T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^2\right) \tag{7}$$

where again $\tilde{y}_t$ are the residuals from a regression of $y_t$ on a constant and a trend and $s^2$ is the autoregressive spectral density estimator defined by equations (5) and (6). We can view $MZ_\alpha$ as a modified Phillips–Perron test. These issues are examined in detail in Perron and Ng (1996) where, in particular, it is demonstrated that this test has superior size and power properties for a wide range of data-generating processes.

A topic that has received substantial attention recently is the measure of the persistence of shocks on the level of a given series. Here the concept of persistence relates to the long term effect of a shock $e_t$ in equation (1) on the level of $y_t$ (see e.g. Cochrane, 1988; Campbell and Mankiw, 1987). All the measures of persistence proposed are directly related to the normalized spectral density function at frequency zero of the first-differences of a series, $f_{\Delta y}(0) = h_{\Delta y}(0)/\sigma^2_{\Delta y}$ where $\sigma^2_{\Delta y}$ is the variance of the first differences of the series $y_t$. For example, if the series is trend-stationary, $f_{\Delta y}(0)$ is 0 and the series exhibits no persistence. For a random walk, it is one and shocks have a one-for-one effect on the long-term level of the series. When $0 < f_{\Delta y}(0) < 1$, shocks have a permanent effect but their influence is attenuated over time. If $f_{\Delta y}(0) > 1$, their effect is exacerbated over time. Hence, an estimate of $f_{\Delta y}(0)$ provides valuable information on the characteristics of a series. Here, we consider the estimate $\hat{f}_{\Delta y}(0) = \hat{h}_{\Delta y}(0)/\hat{\sigma}^2_{\Delta y}$ where

$$\hat{\sigma}^2_{\Delta y} = T^{-1}\sum_{t=1}^{T}(\Delta y_t - \overline{\Delta y})^2$$

the sample variance of $\Delta y_t$ and where $\hat{h}_{\Delta y}(0)$ is an autoregressive spectral density estimate at frequency 0 defined by

$$\hat{h}_{\Delta y}(0) = s^2_{ek}/(1 - \hat{d}(1))^2 \tag{8}$$

with

$$s^2_{ek} = T^{-1}\sum_{t=1}^{T}\hat{e}^2_{tk}, \quad \hat{d}(1) = \sum_{j=1}^{k}\hat{d}_j$$

where $\hat{d}_j$ and $\{\hat{e}_{tk}\}$ are obtained from the following $k$th-order augmented autoregression in $\Delta y_t$:

$$\Delta y_t = c + \sum_{j=1}^{k} d_j \Delta y_{t-j} + e_{tk} \tag{9}$$

Note that equation (9) differs from (6) in that the lagged level $y_{t-1}$ is not included. This ensures consistency under both the null and stationary alternative hypotheses and a more efficient estimator under the null hypothesis of a unit root. The truncation lag is again selected using the BIC criterion.

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### 4.2. Empirical Results

We applied the tests discussed above to the Brazilian monthly inflation rate series for the period 1974:1–1993:6. The results are presented in Tables II and III. The strategy adopted was to conduct the tests for the full sample and various subsamples with and without shock plans. Consider first the results for the full sample. All three unit root tests concur for an overwhelming rejection of the null hypothesis of a unit root in favour of stationary fluctuations. All statistics are significant at the 1% level (the critical values, from Fuller, 1976, are −29·5 for $Z_\alpha$ and $MZ_\alpha$ and −3·96 for ADF). We note, for further comparisons, that the measure of persistence given by the estimate of the spectral density function at the origin of the first-differences of the data is 0·97 with the order $k$ chosen by BIC, a value substantially above 0 which contrasts with the unit root tests. Note, however, that this measure of persistence is sensitive to the order $k$ selected. Indeed, we see a substantial decrease in $\hat{f}_{\Delta y}(0)$ as $k$ increases.

Consider now the results for various subsamples, starting with those that do not contain shock plans. For 1974:1 to 1979:12, all tests agree on a non-rejection of the unit root at any conventional significance level. For 1974:1 to 1984:12, the results are mixed; the ADF test does not allow for a rejection while the $Z_\alpha$ and $MZ_\alpha$ tests do so at the 5% and 10% levels, respectively. Overall, the results suggest that the period prior to the shock plans and the high inflation is characterized by stochastic non-stationarity and persistence of shocks. This is confirmed by the estimate $\hat{f}_{\Delta y}(0)$ which is 0·70 for the period 1974:1 to 1979:12, again well above 0 (for the period 1974:1 to 1984:12 it is 0·33).

Table II. Empirical results for unit root tests

| Sample | $Z_\alpha$ | $k$ | $MZ_\alpha$ | $k$ | ADF $t_\alpha$ | ADF $\hat{\alpha}$ | ADF $k$ |
|---|---|---|---|---|---|---|---|
| 74:1–93:6 | −41·55[a] | 2 | −37·57[a] | 2 | −6·61[a] | 0·75 | 1 |
| 74:1–79:12 | −9·12 | 2 | −7·55 | 2 | −2·43 | 0·79 | 2 |
| 74:1–84:12 | −22·92[c] | 4 | −19·17[d] | 4 | −2·63 | 0·84 | 4 |
| 74:1–89:8 | −45·20[a] | 1 | −38·25[a] | 1 | −5·09[a] | 0·72 | 1 |
| 80:1–89:8 | −31·22[a] | 1 | −26·67[b] | 1 | −3·75[b] | 0·75 | 0 |
| 80:1–93:6 | −30·65[a] | 1 | −27·87[b] | 1 | −5·65[a] | 0·73 | 1 |
| 85:1–93:6 | −19·04[d] | 1 | −17·38 | 1 | −4·48[a] | 0·73 | 1 |

*Note*: The superscripts a, b, c and d denote significance in favour of stationary alternatives at the 1%, 2·5%, 5% and 10% levels, respectively.

Table III. Empirical results for $\hat{f}_{\Delta y}(0)$

| Sample\\$k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | *kbic* ($k$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 74:1–93:6 | 1·58 | 0·97 | 0·86 | 0·65 | 0·59 | 0·51 | 0·41 | 0·35 | 0·25 | 0·20 | 0·97 (2) |
| 74:1–79:12 | 1·74 | 0·70 | 0·59 | 0·37 | 0·48 | 0·37 | 0·22 | 0·29 | 0·29 | 0·27 | 0·70 (2) |
| 74:1–84:12 | 1·08 | 0·43 | 0·57 | 0·33 | 0·31 | 0·29 | 0·34 | 0·29 | 0·25 | 0·21 | 0·33 (4) |
| 74:1–89:8 | 1·09 | 0·93 | 0·67 | 0·42 | 0·34 | | | | | | 1·09 (1) |
| 80:1–89:8 | 1·09 | 0·94 | 0·68 | 0·43 | 0·35 | | | | | | 1·09 (1) |
| 80:1–93:6 | 1·58 | 0·97 | 0·87 | 0·66 | 0·60 | 0·52 | 0·42 | 0·36 | 0·25 | 0·21 | 0·97 (2) |
| 85:1–93:6 | 1·61 | 1·00 | 0·88 | 0·68 | 0·62 | 0·53 | 0·43 | 0·37 | 0·26 | 0·22 | 1·00 (2) |

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We now turn to the results concerning the subsamples that contain shock plans. The results for the subperiods 1980:1 to 1993:6 and 1985:1 to 1993:6 are very similar to those for the full sample. Most tests agree for a rejection of the unit root, the rejections being stronger using the sample 1980:1 to 1993:6. It is of interest to note that the estimates of the measure of persistence $\hat{f}_{\Delta y}(0)$ are 0·97 and 1·00 (almost the same as that for the full sample). These values suggest substantial persistence of shocks contrary to the unit root tests. In particular, it is important to note that $\hat{f}_{\Delta y}(0)$ is higher when a strong rejection of the unit root occurs (i.e. when including the plans) than when a rejection is not possible (i.e. not including subperiods with plans). These results offer a conflicting picture of the properties of the data.

Finally, we also consider the subsamples 1974:1 to 1989:8 and 1980:1 to 1989:8 to assess whether the results are due to the presence of the very large increase in the inflation rate during the period surrounding the Collor I plan. Hence, for these two subsamples, only the first three plans are present. The results show the same pattern as with the full sample, namely a strong rejection of the unit root and a value of $\hat{f}_{\Delta y}(0)$ at 1·09 which suggests, on the contrary, high persistence. As with the full sample, this estimate decreases substantially as the order $k$ increases. Hence, the results are robust to excluding the most dramatic period associated with the Collor I plan.

The results of this section suggest the following perplexing conclusion. The inflation rate is characterized by stochastic non-stationarity and persistence of shocks prior to the emergence of very high levels of inflation and the institution of the various shocks plans. The opposite holds for the period of high inflation with occasional shocks plans. For that period, fluctuations in inflation appear as stationary deviations around a stable linear trend function and shocks accordingly have effects that dissipate quickly (given the low value of the sum of the autoregressive coefficients).

These results are perplexing because they are contrary to what intuition would suggest. Indeed, one would expect non-stationary (or erratic) behaviour to occur especially in a period of uncontrolled growth in inflation and a failed attempt at stabilizing its level. Yet standard tests suggest the opposite.

Our argument is that the results are simply artifacts created by the occasional presence of short but important shock plans. Indeed, the plans act in such a way that the level of the series is brought temporarily to a low level. Since the plans in the period considered have all failed quickly, inflation has returned to its old trend path. This is a manifestation of a mean-reverting behaviour that also characterizes a stationary series. Since the decreases and subsequent increases are so important they are likely to contaminate the statistical tests used.

The question of interest is then whether inflation, in periods when shock plans are not into effect, is characterized by a trend path that is unstable (stochastically non-stationary with a unit root) or is even of an explosive nature. To answer this question, the tests used so far must be modified to isolate the effect of the shock plans. These modifications are the object of the following sections. Before presenting them, we first turn to the issue of the possible bias on the unit root tests and the measure of persistence caused by the shock plans.

## 5. BIASES CAUSED BY THE PRESENCE OF 'INLIERS'

In this section we present simple simulation experiments that aim at quantifying the bias on the size of the unit root tests and the mean of the persistence measure created by the presence of shock plans (or 'inliers') that are short-lived but important in magnitude. The results will show how shock plans can create spurious mean-reverting behaviour that would lead an investigator to conclude that the time series is stationary over the whole sample when using unit root tests.

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### 5.1. Description of the Experiments

The data are first generated according to the following simple random walk with drift interrupted by occasional 'inliers' or shock plans:

$$\begin{aligned} y_t &= y_0 + \mu t + S_t \quad t = 1, \ldots, T \text{ and } t \notin \{t_{i,j}\} \\ y_t &= a \text{ for } t \in \{t_{i,j}\} \ (j = 1, \ldots, p; i = 1, \ldots, n_j) \end{aligned} \tag{10}$$

where

$$S_t = \sum_{j=1}^{t} e_j$$

Here $t_{i,j}$ refers to the time index of the $i$th observation of plan $j$. There are $p$ shock plans and each contains $n_j$ observations. The series is a random walk with drift $\mu$ except when a plan is in effect, in which case the level of the series drops to a value $a$. To complete the specifications, the errors $\{e_t\}$ are independent N(0, 1) random variables, the initial condition is $y_0 = a$ (so that the plans, in effect, bring the level of inflation to its initial value).[3]

We used the following specific values for the parameters. First $a = 4$ which can be viewed as an initial level of 4% for the inflation rate. There are $p = 3$ plans irrespective of the sample size and each plan contains $n_j = 6$ ($j = 1, 2, 3$) observations corresponding to plans lasting 6 months. A key parameter is the drift $\mu$ which specifies how fast the deterministic trend component increases. We consider four values ranging from mild to rapid growth: $\mu = 0{\cdot}1$, 0·2, 0·4 and 0·8. The specification of this trend component is important because it basically dictates the magnitude of the decrease occurring with a shock plan. The faster the rate of growth, the larger the decrease and the likely importance of the spurious effect on the unit root tests. We consider three different sample sizes, $T = 150$, 250 and 500. Associated with each of these are the starting dates of the plans. These are {40, 70, 120} for $T = 150$, {150, 170, 220} for $T = 250$, and {250, 350, 450} for $T = 500$. It is important to note that as the sample size increases the number of plans remains the same but the decreases caused by the plans are more important since they occur when the level of the series is higher.

Given the possibility that the noise component for Brazilian inflation is explosive, we also considered experiments with such an explosive process interrupted by shock plans. The set-up is exactly the same as described above except that the process describing the behaviour of the series when shock plans are not in effect is given by:

$$\begin{aligned} y_t &= y_0 + \mu t + Z_t \\ Z_t &= \alpha Z_{t-1} + e_t \end{aligned} \tag{11}$$

with $Z_0 = 1$. In our experiments, we considered $\alpha = 1{\cdot}01$ and $\alpha = 1{\cdot}02$. All the other parameter configurations are exactly as for the unit root case.

While these data-generating processes are simple they are rich enough to obtain an overview of the bias on unit root tests caused by temporary shock plans or 'inliers'.

[3] We also considered a slight modification of this data-generating process when the plans bring the level of inflation to half its value the month before the plan. The results, reported in a previous version of the paper, are qualitatively similar and, hence, for brevity we omit them.

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### 5.2. Description of the Results

We used 1000 replications for each specification to compute the exact size of the unit root tests. The nominal size of the test is 5% and the critical values are taken from Fuller (1976) (−21·8 for $Z_\alpha$ and $MZ_\alpha$ and −3·41 for ADF). We also report the mean and standard deviation of the statistics. The program was coded in Gauss using the RNDNS routine to generate the random numbers. To allow proper comparison of differences across cases, we used the same starting seed for each entry, arbitrarily set at 12345. The results for the unit root case are presented in Table IV.

We first note that in all cases the tests are severely oversized, so much as to be useless to provide a characterization of the non-stationary nature of the series. Consider the case of $Z_\alpha$ presented in panel (a) of Table IV. With a sample size $T = 150$ and a small drift $\mu = 0{\cdot}1$, the test would incorrectly reject the unit root in favour of stationary deviations around a linear trend in 59% of the cases. This false rate of rejection increases as the drift $\mu$ and the sample size $T$ increases, and quickly reaches 100% (for example, when $\mu = 0{\cdot}4$ and $T = 250$, which roughly characterizes the Brazilian inflation series).

The same qualitative results hold for the tests $MZ_\alpha$ (panel (b)) and ADF (panel (c)). The rates of rejections are only marginally lower compared to those with $Z_\alpha$. It is interesting to note that the mean of the statistics seems to approach some limiting value as $\mu$ increases, keeping a fixed sample size. This limiting value is well below the respective 5% critical value. The concentration also increases given that the standard error decreases. This implies a limiting rate of rejections of 100% as $\mu$ increases keeping $T$ fixed but without the tests diverging to minus infinity. On the other hand, when $\mu$ is kept fixed and $T$ increases the means of the statistics grow more negative (perhaps diverging to minus infinity) but the standard errors also increase.

The results presented here clearly show that short but abrupt shock plans can bias the tests statistics against the unit root hypothesis in favour of stationary fluctuations around a stable linear trend function. This is an undesirable feature since the time span covered by

Table IV. Exact size of unit root tests in the presence of shock plans (5% nominal size)

| | $T = 150$ | | | $T = 250$ | | | $T = 500$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | Size | Mean | s.e. | Size | Mean | s.e. | Size | Mean | s.e. |
| (a) $Z_\alpha$ | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·59 | −23·21 | 8·03 | 0·78 | −34·66 | 13·82 | 0·92 | −58·93 | 22·70 |
| $\mu = 0{\cdot}2$ | 0·80 | −26·75 | 6·68 | 0·97 | −45·17 | 8·93 | 1·00 | −82·40 | 11·96 |
| $\mu = 0{\cdot}4$ | 0·98 | −30·39 | 3·06 | 1·00 | −51·46 | 3·08 | 1·00 | −93·63 | 3·90 |
| $\mu = 0{\cdot}8$ | 1·00 | −31·62 | 1·27 | 1·00 | −53·12 | 1·34 | 1·00 | −96·81 | 1·46 |
| (b) $MZ_\alpha$ | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·51 | −21·35 | 7·34 | 0·76 | −32·10 | 12·80 | 0·92 | −55·25 | 21·01 |
| $\mu = 0{\cdot}2$ | 0·75 | −24·74 | 6·15 | 0·97 | −41·85 | 8·08 | 1·00 | −76·70 | 10·75 |
| $\mu = 0{\cdot}4$ | 0·97 | −28·19 | 2·75 | 1·00 | −47·54 | 2·70 | 1·00 | −86·68 | 3·41 |
| $\mu = 0{\cdot}8$ | 1·00 | −29·34 | 1·13 | 1·00 | −49·03 | 1·16 | 1·00 | −89·47 | 1·27 |
| (c) ADF | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·48 | −3·25 | 0·67 | 0·76 | −4·03 | 1·00 | 0·91 | −5·38 | 1·44 |
| $\mu = 0{\cdot}2$ | 0·69 | −3·50 | 0·53 | 0·96 | −4·67 | 0·73 | 1·00 | −7·04 | 1·46 |
| $\mu = 0{\cdot}4$ | 0·94 | −3·76 | 0·21 | 1·00 | −5·00 | 0·48 | 1·00 | −9·01 | 0·94 |
| $\mu = 0{\cdot}8$ | 1·00 | −3·84 | 0·08 | 1·00 | −5·03 | 0·28 | 1·00 | −9·55 | 0·13 |

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the plans are very short compared to the whole sample (18 'months' in samples of 150 to 500 'months').

Consider now the behaviour of the persistence measure $\hat{f}_{\Delta y}(0)$. To better assess potential biases, we slightly generalized the data-generating process described by equation (10) to include serial correlation in the errors to permit some variations in the true values of the persistence of shocks. Hence, we now have

$$S_t = \sum_{j=1}^{t} v_t$$

with $v_t$ generated by the first-order autoregression $v_t = \rho v_{t-1} + e_t$. With $e_t \sim$ i.i.d. N(0, 1), $f_{\Delta y}(0) = (1-\rho^2)/(1-\rho)^2$. We used three values of $\rho$, namely 0·0, 0·5 and −0·5 with corresponding values of $f_{\Delta y}(0)$ given by 1·0, 3·0 and 1/3. The results are presented in Table V.[4] These are quite striking. For large $T$ and/or large $\mu$, the mean of $\hat{f}_{\Delta y}(0)$ is close to 1·0 when $k$ is less than 6 (the length of the plans used in the data-generating process) and close to 0·34 when $k$ is greater than or equal to 6, irrespective of the true value of $f_{\Delta y}(0)$. For small values of $T$ and/or $\mu$ the means are somewhat higher when $\rho = 0{\cdot}5$ and somewhat lower when $\rho = -0{\cdot}5$. The mean of the estimates when $k$ is selected by BIC correspond to that obtained with large $k$'s when $T$ and/or $\mu$ is large and to some value in between 1·0 and 0·34 when $T$ and $\mu$ are small. Some theoretical explanations for this peculiar bias are provided in the next subsection.

These simulation results go a long way to explain the empirical results discussed earlier for the inflation rate. We saw, indeed, that with any subsample that includes shock plans, the estimate $\hat{f}_{\Delta y}(0)$ was close to 1·0 when $k$ is small but that it decreased substantially with an increase in $k$.

Table VI presents the results when the data are generated by an explosive process. Here, we present the probability of rejecting the null hypothesis of a unit root in favour of stationary fluctuations (along with the mean and standard error of the statistics).[5] The results show again that, even with an explosive noise component, the presence of shock plans induces a strong bias in spuriously concluding that the process is trend-stationary. This bias increases as $\mu$ increases (in which case the shock plans are more important) but, unlike in the unit root case, decreases as $T$ increases. This false rejection in favour of stationary fluctuations also decreases as $\alpha$ increases.

### 5.3. Some Theoretical Explanations

To provide explanations for the simulation results, we consider data generated by

$$y_t = y_0 + \mu t + S_t \quad t \neq T_{Bj} + 1, \ldots, T_{Bj} + n$$
$$y_t = y_0 \quad t = T_{Bj} + 1, \ldots, T_{Bj} + n$$

[4] To conserve space, only the means and not the standard deviations are presented. The latter carry little useful information. They are generally quite small. They decrease with an increase in $T$ or $\mu$ and increase with an increase in the order $k$.

[5] The rejection rates against explosive alternatives are as follows. For $\alpha = 1{\cdot}01$, they are less than 1% for $T = 150$ and 250. At $T = 500$, they range from 0·03 for $\mu = 0{\cdot}1$ to 0·19 for $\mu = 0{\cdot}8$. When $\alpha = 1{\cdot}02$, they range from 0·03 to 0·09 for $T = 150$, and from 0·02 to 0·20 for $T = 250$. With $T = 500$, they reach 100% for any value of $\mu$.

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Table V. Mean of $\hat{f}_{\Delta y}(0)$ with shock plans

| | $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | *kbic* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) $\rho = 0{\cdot}0$ | | | | | | | | | | | | |
| $T = 150$ | $\mu = 0{\cdot}1$ | 0·99 | 0·98 | 0·98 | 0·97 | 0·96 | 0·46 | 0·45 | 0·44 | 0·44 | 0·43 | 0·73 |
| | $\mu = 0{\cdot}2$ | 1·00 | 1·00 | 1·00 | 1·01 | 1·01 | 0·40 | 0·39 | 0·39 | 0·39 | 0·39 | 0·53 |
| | $\mu = 0{\cdot}4$ | 1·01 | 1·01 | 1·02 | 1·02 | 1·03 | 0·36 | 0·36 | 0·36 | 0·36 | 0·36 | 0·36 |
| | $\mu = 0{\cdot}8$ | 1·01 | 1·01 | 1·02 | 1·03 | 1·03 | 0·35 | 0·35 | 0·35 | 0·36 | 0·36 | 0·35 |
| $T = 250$ | $\mu = 0{\cdot}1$ | 1·00 | 0·99 | 0·99 | 0·99 | 0·99 | 0·43 | 0·43 | 0·43 | 0·43 | 0·43 | 0·51 |
| | $\mu = 0{\cdot}2$ | 1·00 | 1·00 | 1·01 | 1·01 | 1·01 | 0·36 | 0·36 | 0·37 | 0·37 | 0·37 | 0·37 |
| | $\mu = 0{\cdot}4$ | 1·00 | 1·01 | 1·01 | 1·02 | 1·02 | 0·34 | 0·35 | 0·35 | 0·35 | 0·35 | 0·34 |
| | $\mu = 0{\cdot}8$ | 1·00 | 1·01 | 1·01 | 1·02 | 1·02 | 0·34 | 0·34 | 0·35 | 0·35 | 0·35 | 0·34 |
| $T = 500$ | $\mu = 0{\cdot}1$ | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 0·39 | 0·39 | 0·39 | 0·39 | 0·39 | 0·40 |
| | $\mu = 0{\cdot}2$ | 1·00 | 1·00 | 1·01 | 1·01 | 1·01 | 0·35 | 0·35 | 0·35 | 0·35 | 0·35 | 0·35 |
| | $\mu = 0{\cdot}4$ | 1·00 | 1·00 | 1·01 | 1·01 | 1·01 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 |
| | $\mu = 0{\cdot}8$ | 1·00 | 1·00 | 1·01 | 1·01 | 1·01 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 |
| (b) $\rho = 0{\cdot}5$ | | | | | | | | | | | | |
| $T = 150$ | $\mu = 0{\cdot}1$ | 1·19 | 1·25 | 1·26 | 1·24 | 1·23 | 0·56 | 0·63 | 0·64 | 0·63 | 0·63 | 0·82 |
| | $\mu = 0{\cdot}2$ | 1·15 | 1·20 | 1·21 | 1·20 | 1·19 | 0·52 | 0·56 | 0·57 | 0·57 | 0·57 | 0·70 |
| | $\mu = 0{\cdot}4$ | 1·06 | 1·08 | 1·09 | 1·10 | 1·10 | 0·41 | 0·43 | 0·43 | 0·43 | 0·44 | 0·45 |
| | $\mu = 0{\cdot}8$ | 1·01 | 1·02 | 1·03 | 1·04 | 1·04 | 0·35 | 0·36 | 0·36 | 0·37 | 0·37 | 0·35 |
| $T = 250$ | $\mu = 0{\cdot}1$ | 1·18 | 1·24 | 1·25 | 1·26 | 1·25 | 0·56 | 0·63 | 0·65 | 0·65 | 0·65 | 0·64 |
| | $\mu = 0{\cdot}2$ | 1·12 | 1·15 | 1·17 | 1·17 | 1·17 | 0·48 | 0·52 | 0·53 | 0·54 | 0·54 | 0·53 |
| | $\mu = 0{\cdot}4$ | 1·02 | 1·04 | 1·04 | 1·05 | 1·05 | 0·37 | 0·38 | 0·38 | 0·38 | 0·38 | 0·37 |
| | $\mu = 0{\cdot}8$ | 1·01 | 1·01 | 1·02 | 1·02 | 1·02 | 0·34 | 0·35 | 0·35 | 0·35 | 0·35 | 0·34 |
| $T = 500$ | $\mu = 0{\cdot}1$ | 1·16 | 1·22 | 1·24 | 1·25 | 1·25 | 0·54 | 0·61 | 0·63 | 0·63 | 0·64 | 0·57 |
| | $\mu = 0{\cdot}2$ | 1·06 | 1·08 | 1·09 | 1·09 | 1·10 | 0·40 | 0·43 | 0·44 | 0·44 | 0·44 | 0·41 |
| | $\mu = 0{\cdot}4$ | 1·01 | 1·01 | 1·02 | 1·02 | 1·02 | 0·35 | 0·35 | 0·35 | 0·35 | 0·35 | 0·35 |
| | $\mu = 0{\cdot}8$ | 1·00 | 1·01 | 1·01 | 1·01 | 1·01 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 |
| (c) $\rho = -0{\cdot}5$ | | | | | | | | | | | | |
| $T = 150$ | $\mu = 0{\cdot}1$ | 0·73 | 0·79 | 0·77 | 0·77 | 0·77 | 0·37 | 0·31 | 0·31 | 0·31 | 0·31 | 0·57 |
| | $\mu = 0{\cdot}2$ | 0·90 | 0·95 | 0·93 | 0·95 | 0·94 | 0·36 | 0·33 | 0·34 | 0·34 | 0·34 | 0·47 |
| | $\mu = 0{\cdot}4$ | 0·98 | 1·00 | 1·00 | 1·01 | 1·01 | 0·35 | 0·34 | 0·35 | 0·35 | 0·35 | 0·35 |
| | $\mu = 0{\cdot}8$ | 1·00 | 1·01 | 1·01 | 1·02 | 1·03 | 0·35 | 0·35 | 0·35 | 0·35 | 0·36 | 0·35 |
| $T = 250$ | $\mu = 0{\cdot}1$ | 0·83 | 0·88 | 0·87 | 0·88 | 0·87 | 0·36 | 0·32 | 0·33 | 0·32 | 0·33 | 0·39 |
| | $\mu = 0{\cdot}2$ | 0·96 | 0·98 | 0·98 | 0·99 | 0·99 | 0·35 | 0·33 | 0·34 | 0·34 | 0·34 | 0·35 |
| | $\mu = 0{\cdot}4$ | 0·99 | 1·00 | 1·00 | 1·01 | 1·01 | 0·34 | 0·34 | 0·34 | 0·34 | 0·35 | 0·34 |
| | $\mu = 0{\cdot}8$ | 1·00 | 1·01 | 1·01 | 1·01 | 1·02 | 0·34 | 0·34 | 0·34 | 0·35 | 0·35 | 0·34 |
| $T = 500$ | $\mu = 0{\cdot}1$ | 0·90 | 0·94 | 0·93 | 0·94 | 0·94 | 0·36 | 0·32 | 0·33 | 0·33 | 0·33 | 0·35 |
| | $\mu = 0{\cdot}2$ | 0·98 | 0·99 | 0·99 | 0·99 | 1·00 | 0·34 | 0·33 | 0·34 | 0·34 | 0·34 | 0·34 |
| | $\mu = 0{\cdot}4$ | 1·00 | 1·00 | 1·00 | 1·00 | 1·01 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 |
| | $\mu = 0{\cdot}8$ | 1·00 | 1·00 | 1·01 | 1·01 | 1·01 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 | 0·34 |

for $j = 1, \ldots, p$ and where

$$S_t = \sum_{j=1}^{t} u_t$$

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Table VI. Probability of rejecting in favour of stationarity when the noise function is explosive

| | $T=150$ | | | $T=250$ | | | $T=500$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | Size | Mean | s.e. | Size | Mean | s.e. | Size | Mean | s.e. |
| (a) $Z_\alpha$ | | | | | | | | | |
| (i) $\alpha=1{\cdot}01$ | | | | | | | | | |
| $\mu=0{\cdot}1$ | 0·68 | −24·12 | 7·68 | 0·82 | −35·27 | 12·35 | 0·39 | −21·74 | 12·46 |
| $\mu=0{\cdot}2$ | 0·78 | −26·12 | 6·92 | 0·83 | −38·69 | 13·94 | 0·58 | −24·93 | 18·45 |
| $\mu=0{\cdot}4$ | 0·96 | −29·72 | 4·30 | 0·91 | −45·24 | 12·85 | 0·64 | −31·06 | 24·87 |
| $\mu=0{\cdot}8$ | 1·00 | −31·59 | 1·32 | 1·00 | −51·74 | 3·39 | 0·71 | −41·86 | 30·14 |
| (ii) $\alpha=1{\cdot}02$ | | | | | | | | | |
| $\mu=0{\cdot}1$ | 0·25 | −17·66 | 6·80 | 0·09 | −14·99 | 6·74 | 0·00 | 2·67 | 0·42 |
| $\mu=0{\cdot}2$ | 0·41 | −18·96 | 8·34 | 0·18 | −16·32 | 10·17 | 0·00 | 2·66 | 0·82 |
| $\mu=0{\cdot}4$ | 0·71 | −22·18 | 9·60 | 0·33 | −18·92 | 13·79 | 0·00 | 2·62 | 1·52 |
| $\mu=0{\cdot}8$ | 0·90 | −27·12 | 7·12 | 0·60 | −24·08 | 16·60 | 0·00 | 2·49 | 2·66 |
| (b) $MZ_\alpha$ | | | | | | | | | |
| (i) $\alpha=1{\cdot}01$ | | | | | | | | | |
| $\mu=0{\cdot}1$ | 0·60 | −22·22 | 7·07 | 0·80 | −32·58 | 11·55 | 0·30 | −20·28 | 11·79 |
| $\mu=0{\cdot}2$ | 0·73 | −24·11 | 6·42 | 0·81 | −35·67 | 13·12 | 0·49 | −23·23 | 17·36 |
| $\mu=0{\cdot}4$ | 0·94 | −27·54 | 3·98 | 0·90 | −41·67 | 12·16 | 0·64 | −28·91 | 23·36 |
| $\mu=0{\cdot}8$ | 1·00 | −29·30 | 1·17 | 1·00 | −47·74 | 3·12 | 0·71 | −38·93 | 28·23 |
| (ii) $\alpha=1{\cdot}02$ | | | | | | | | | |
| $\mu=0{\cdot}1$ | 0·17 | −16·15 | 6·37 | 0·07 | −13·73 | 6·34 | 0·00 | 2·89 | 0·40 |
| $\mu=0{\cdot}2$ | 0·30 | −17·37 | 7·91 | 0·17 | −14·94 | 9·59 | 0·00 | 2·88 | 0·79 |
| $\mu=0{\cdot}4$ | 0·60 | −20·38 | 9·18 | 0·27 | −17·32 | 12·97 | 0·00 | 2·84 | 1·45 |
| $\mu=0{\cdot}8$ | 0·87 | −25·08 | 6·78 | 0·54 | −22·10 | 15·59 | 0·00 | 2·71 | 2·51 |
| (c) ADF | | | | | | | | | |
| (i) $\alpha=1{\cdot}01$ | | | | | | | | | |
| $\mu=0{\cdot}1$ | 0·53 | −3·30 | 0·66 | 0·78 | −3·96 | 0·88 | 0·16 | −2·83 | 0·95 |
| $\mu=0{\cdot}2$ | 0·69 | −3·45 | 0·57 | 0·80 | −4·14 | 1·03 | 0·32 | −2·98 | 1·48 |
| $\mu=0{\cdot}4$ | 0·91 | −3·71 | 0·34 | 0·90 | −4·55 | 1·01 | 0·53 | −3·35 | 2·10 |
| $\mu=0{\cdot}8$ | 1·00 | −3·83 | 0·09 | 1·00 | −4·96 | 0·30 | 0·70 | −4·04 | 2·57 |
| (ii) $\alpha=1{\cdot}02$ | | | | | | | | | |
| $\mu=0{\cdot}1$ | 0·11 | −2·68 | 0·74 | 0·05 | −2·27 | 0·65 | 0·00 | 0·70 | 0·10 |
| $\mu=0{\cdot}2$ | 0·21 | −2·75 | 0·90 | 0·12 | −2·31 | 1·02 | 0·00 | 0·70 | 0·18 |
| $\mu=0{\cdot}4$ | 0·44 | −2·97 | 1·10 | 0·21 | −2·44 | 1·36 | 0·00 | 0·70 | 0·24 |
| $\mu=0{\cdot}8$ | 0·79 | −3·44 | 0·77 | 0·39 | −2·80 | 1·62 | 0·00 | 0·68 | 0·39 |

with $u_t$ some 'mixing' process with $\sigma^2 = \lim_{T\to\infty} T^{-1}E(S_T^2)$. Hence, the series is trending with an integrated noise function. There are $p$ shock plans occurring at times $T_{Bj}$ that last $n$ periods. We posit that $T_{Bj}$ is a fixed proportion of the total sample such that $T_{Bj}/T = \lambda_j$. We simplify by considering the detrended version defined by:

$$\begin{aligned} y_t^{\mathrm{d}} &= S_t \qquad\quad t \neq T_{Bj}+1,\ldots,T_{Bj}+n \\ y_t^{\mathrm{d}} &= -\mu T_{Bj} \quad t = T_{Bj}+1,\ldots,T_{Bj}+n \end{aligned} \tag{12}$$

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Representation (12) helps to make clear the relationship between additive outliers and shock plans. As a special case, with $n = 1$, we have a unit root process with additive outliers that increase as the sample size increases. Hence, this differentiates our set-up from the one analysed in Franses and Haldrup (1994) and Vogelsang (forthcoming).

We first consider the limit of the autoregressive coefficient in the following AR(1) process estimated by least-squares:

$$y_t^{\mathrm{d}} = \hat{\alpha} y_{t-1}^{\mathrm{d}} + \hat{u}_t$$

Standard derivations show the following limit for $\alpha$:

$$\hat{\alpha} \Rightarrow \frac{\sigma^2 \int_0^1 W(r)^2 \, \mathrm{d}r + \mu^2 \left( \sum_{j=1}^{n} \lambda_j^2 \right)(n-1)}{\sigma^2 \int_0^1 W(r)^2 \, \mathrm{d}r + \mu^2 \left( \sum_{j=1}^{n} \lambda_j^2 \right) n} < 1$$

Hence, the presence of a plan biases $\hat{\alpha}$ below one even asymptotically. The extent of the bias is greatest for plans of short duration. If the plans are long-lasting the bias decreases. This is expected since long-lasting plans can be perceived as permanent changes in levels which, as shown in Perron (1990), bias the autoregressive coefficient towards one whether the noise function is stationary or integrated. This asymptotic bias helps to explain the strong rejections of the unit root when shock plans are present. The result is also qualitatively different from the standard additive outlier case analysed in Franses and Haldrup (1994) where the limit of $\hat{\alpha}$ is still one but nuisance parameters are introduced that produce distorted tests.

Consider now the limiting behaviour of the autocorrelation function of the first-differences of $y_t^{\mathrm{d}}$ given by:

$$\begin{aligned} \Delta y_t^{\mathrm{d}} &= u_t && t \neq T_{Bj}+1, \ldots, T_{Bj}+n+1 \\ &= -\mu T_{Bj} - S_{T_{Bj}} && t = T_{Bj}+1 \\ &= 0 && t = T_{Bj}+2, \ldots, T_{Bj}+n \\ &= S_{T_{Bj}+p+1} + \mu T_{Bj} && t = T_{Bj}+n+1 \end{aligned}$$

for $j = 1, \ldots, p$. It is easy to show that the following limits hold:

$$T^{-2} \sum_{t=q+1}^{T} (\Delta y_{t-q}^{\mathrm{d}})^2 \rightarrow 2\mu^2 \left( \sum_{j=1}^{n} \lambda_j^2 \right) \tag{13}$$

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and

$$
\begin{aligned}
T^{-2}\sum_{t=q+1}^{T}\Delta y_t^{\mathrm{d}}\Delta y_{t-q}^{\mathrm{d}} &\to 0 && q\neq n\\
T^{-2}\sum_{t=q+1}^{T}\Delta y_t^{\mathrm{d}}\Delta y_{t-q}^{\mathrm{d}} &\to -\mu^2\left(\sum_{j=1}^{n}\lambda_j^2\right) && q=n
\end{aligned}
\tag{14}
$$

Hence,

$$
\hat{\alpha}_q = T^{-2}\sum_{t=q+1}^{T}\Delta y_t^{\mathrm{d}}\Delta y_{t-q}^{\mathrm{d}}\bigg/ T^{-2}\sum_{t=q+1}^{T}(\Delta y_{t-q}^{\mathrm{d}})^2,
$$

the correlation coefficient of order $q$, converges to 0 unless the order $q$ is the same as the length of the plan $n$, in which case it converges to $-1/2$. This somewhat generalizes the result of Franses and Haldrup (1994). They showed that additive outliers induce a negative MA component in the level of the series. It is easy to see that if the magnitude of the outlier increases their result implies that the MA component converges to $-1$. This corresponds to the case where $n=1$ here since a correlation coefficient of $-1/2$ corresponds to that of an non-invertible MA process. Our result shows that, more generally, a large plan will induce a non-invertible MA($n$) process.

Consider finally, the limit behaviour of the persistence measure applied to $y_t^{\mathrm{d}}$ defined by equation (8) using the autoregression

$$
\Delta y_t^{\mathrm{d}} = \sum_{j=1}^{k} d_j \Delta y_{t-j}^{\mathrm{d}} + e_t
$$

We can use equations (13) and (14) to show that if $k<n$, the OLS estimates $\hat{d}_j$ ($j=1,\ldots,k$) all converge to 0. Also, $T^{-1}s_{ek}^2$ and

$$
T^{-1}\hat{\sigma}^2_{\Delta y^{\mathrm{d}}} = T^{-2}\sum_{t=1}^{T}(\Delta y_t^{\mathrm{d}})^2
$$

both converge to

$$
2\mu^2\left(\sum_{j=1}^{n}\lambda_j^2\right)
$$

and, hence $\hat{f}_{\Delta y^{\mathrm{d}}}(0)$ converges to 1. This explains the bias reported in the simulations showing that the limit of the persistence measure is 1 irrespective of the correlation structure of the errors if $k$ is chosen less than the length of the plan.

When $k$ is chosen at least as large as the length of the plan, things are different and depend on the specific values of $k$ and $n$. It is difficult to obtain a general closed-form solution valid for any pairs ($k$, $n$) but it is relatively easy to compute the limit of $\hat{f}_{\Delta y^{\mathrm{d}}}(0)$ numerically. As an example, consider the case where int($n/k$) $=1$ which corresponds to the simulation experiments reported in

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Table VII. Theoretical limit of $\hat{f}_{\Delta y}(0)$

| $k\backslash n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0·33 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 |
| 2 | 0·17 | 0·33 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 |
| 3 | 0·10 | 0·33 | 0·33 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 |
| 4 | 0·07 | 0·22 | 0·33 | 0·33 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 |
| 5 | 0·05 | 0·22 | 0·33 | 0·33 | 0·33 | 1·00 | 1·00 | 1·00 | 1·00 | 1·00 |
| 6 | 0·04 | 0·12 | 0·22 | 0·33 | 0·33 | 0·33 | 1·00 | 1·00 | 1·00 | 1·00 |
| 7 | 0·03 | 0·12 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 | 1·00 | 1·00 | 1·00 |
| 8 | 0·02 | 0·10 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 | 1·00 | 1·00 |
| 9 | 0·02 | 0·10 | 0·18 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 | 0·33 | 1·00 |
| 10 | 0·02 | 0·07 | 0·18 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 | 0·33 |
| 11 | 0·01 | 0·07 | 0·18 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 | 0·33 |
| 12 | 0·01 | 0·06 | 0·10 | 0·18 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 |
| 13 | 0·01 | 0·06 | 0·10 | 0·18 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 | 0·33 |
| 14 | 0·01 | 0·04 | 0·10 | 0·18 | 0·22 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 |
| 15 | 0·01 | 0·04 | 0·09 | 0·18 | 0·18 | 0·22 | 0·22 | 0·33 | 0·33 | 0·33 |

Table V when $k$ is between 6 and 10 (since $n = 6$). In this case, it is easy to show that $\hat{d}_j \to 0$ for $j \neq n$ and $\hat{d}_n \to -0{\cdot}5$. Hence, $\hat{d}(1) \to -0{\cdot}5$. Also,

$$T^{-1}s_{ek}^2 \to \left(\frac{3}{2}\right)\mu^2\left(\sum_{j=1}^{n}\lambda_j^2\right), \quad T^{-1}\hat{\sigma}^2_{\Delta y^d} \to 2\mu^2\left(\sum_{j=1}^{n}\lambda_j^2\right)$$

Hence, $\hat{f}_{\Delta y^d}(0) \to 1/3$. This corresponds, indeed, very closely to the values obtained by simulations for the various data-generating processes (see Table V). It is important to reiterate that this limiting value is valid irrespective of the true persistence of shocks, hence it is difficult to speak about biases *per se*.

Table VII presents the limiting value of $\hat{f}_{\Delta y^d}(0)$ for pairs $(k,\ n)$ with $k$ and $n$ ranging from 1 to 15 and 1 to 10, respectively. These show that the limit decreases as $k$ increases for a given $n$. Also, for a given $n$, the limit depends only on the value of $\text{int}(k/n)$. Since, the inflation rate is affected by plans that last between 3 and 8 periods, our theoretical results help explain why the persistence measures reported in Table III are decreasing as $k$ increases.

The simulation and theoretical results in this section explain the rejections reported in the previous section for the Brazilian inflation rate. Indeed, our experiments clearly show that shock plans induce a strong bias in unit root tests in concluding for stationarity whether the true noise component has a unit or explosive root. On the other hand, the measure of persistence $\hat{f}_{\Delta y}(0)$ is biased towards 1·0 (thereby suggesting persistence) when a small-order $k$ is selected. To verify the claim that the noise component of the inflation rate is not stationary, it remains to devise unit root tests that are immune to the presence of the shock plans. This is the object of the next section.

## 6. CORRECTED VERSIONS OF UNIT ROOT TESTS

We now present modifications to the unit root tests that take into account the presence of the shock plans. The strategy is similar to that used in Perron (1989, 1990) in the case of permanent changes in level or slope of the trend function. The idea is to take the shock plans from the noise function to the trend function. More precisely, it is the movements in and out of the periods called 'plans' that are isolated. This is not a statement about the deterministic nature of the timing

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and magnitude of the plans. Rather, it is to be viewed as a device to isolate their effect so that the tests can meaningfully assess the stochastic properties of the series when shock plans are not into effect. Since the timing of the plans is well documented and relates to governmental interventions, we treat the dates of their occurrence as known rather than as random variables to be estimated. In proposing alternative versions of the unit root tests, one must keep in mind that, in practical situations, the data-generating process may not exactly correspond to, say, the one described by process (10) used in the simulations. Some flexibility is warranted. To that effect, we present various versions that can capture more realistic situations.

It is useful to first define some notations. Let $da(j)_t$ denote a dummy variable taking value 1 when the time index $t$ corresponds to the first month when plan $j$ takes effect, and 0 otherwise. Similarly, let $db(j)_t$ be a dummy variable taking value 1 when the time index $t$ corresponds to the first month *after* the end of plan $j$, and 0 otherwise. Finally, let $D(j)_t$ be a dummy variable taking value 1 when the time index $t$ corresponds to one of the months when plan $j$ is in effect, and 0 otherwise.

The first modification to the ADF test is simply to include these dummies for each plan in autoregression (2). Hence, the relevant regression is:

$$y_t = \eta + \gamma t + \sum_{j=1}^{p}(\kappa_j da(j)_t + \lambda_j db(j)_t + \phi_j D(j)_t) + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + v_t \tag{15}$$

The test statistic, denoted ADF($C_A$), is again constructed as the $t$-statistic for testing that $\alpha$, the sum of the autoregressive coefficients, is unity. The number of lagged first-differences of the data, $k$, is again selected using the BIC criterion.

It is useful at this point to discuss the role played by the various dummies. First, note that $da(j)_t$ and $db(j)_t$ are used to allow removing the influence of the plans under the null hypothesis of a unit root. On the other hand, $D(j)_t$ is used to remove the influence of the plans under the alternative hypothesis of stationarity. This can be seen by noting that, with a unit root or an explosive process, $da(j)_t$ acts as a one-time blip that becomes a permanent decrease in level (the beginning of the plan). The dummy $db(j)_t$ also acts as a one-time blip that becomes permanent thereby allowing an increase in level that marks the end of the plan. When the series is stationary, $D(j)_t$ acts as a temporary level shift that marks the occurrence of the plan.

This first strategy for a modified test is akin to the 'innovational outlier' modelling device which implies that the movements in and out of plans depends on the correlation structure of the noise component. An alternative strategy is to adopt an 'additive outlier' modelling device where the movements in and out of plans are supposed not to be influenced by the noise component of the series. To that effect, we can posit a data-generating process of the form

$$y_t = a + bt + \sum_{j=1}^{p} \tau_j D(j)_t + z_t$$

where $z_t$ is defined by equation (1). Using an autoregressive approximation, the regression[6] is:

$$y_t = \eta + \gamma t + \sum_{i=0}^{k+1}\sum_{j=1}^{p} \phi_{j,i} D(j)_{t-i} + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + v_t \tag{16}$$

[6] In principle, one could use a two-step strategy by first eliminating the deterministic components and performing the unit root test on the detrended series. This, however, leads to substantial problems of inference when the deterministic components include step dummy variables (see Perron and Vogelsang, 1992).

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The relevant statistic, denoted ADF($C_B$), is again the $t$-statistic for testing that $\alpha = 1$. The modifications to the test $Z_\alpha$ are similar. For version A (akin to the 'innovational outlier' strategy), it involves using the following first-order autoregression:

$$y_t = \eta + \gamma t + \sum_{j=1}^{p} (\kappa_j da(j)_t + \lambda_j db(j)_t + \phi_j D(j)_t) + \alpha y_{t-1} + v_t \qquad (17)$$

Denote the OLS estimate of $\alpha$ by $\breve{\alpha}$ and the sample variance of the residuals, $\breve{v}_t$, by

$$\breve{s}_u^2 = T^{-1} \sum_{t=1}^{T} \breve{v}_t^2$$

Also, denote by $\breve{y}_{t-1}$ the residuals from the following regression:

$$y_{t-1} = \eta + \gamma t + \sum_{j=1}^{p} (\kappa_j da(j)_t + \lambda_j db(j)_t + \phi_j D(j)_t) + \xi_t \qquad (18)$$

The first modified version of the $Z_\alpha$ test can now be described as follows:

$$Z_\alpha(C_A) = T(\breve{\alpha} - 1) - (\breve{s}^2 - \breve{s}_u^2) \Bigg/ \left( 2T^{-2} \sum_{t=1}^{T} \breve{y}_{t-1}^2 \right)$$

where $\breve{s}^2$ is an estimate of the spectral density at frequency zero of the residuals $v_t$ described below. The modified test corresponding to the 'additive outlier' strategy, denoted $Z_\alpha(C_B)$, is similar except that regression (17) is replaced by the regression

$$y_t = \eta + \gamma t + \sum_{i=0}^{1} \sum_{j=1}^{p} \phi_{j,i} D(j)_{t-i} + \alpha y_{t-1} + v_t \qquad (19)$$

and regression (18) is replaced by the regression

$$y_{t-1} = \eta + \gamma t + \sum_{i=0}^{1} \sum_{j=1}^{p} \phi_{j,i} D(j)_{t-i} + \xi_t \qquad (20)$$

Since the test $MZ_\alpha$ is constructed directly from detrended series, only the 'additive outlier' strategy is entertained to construct a modified statistic given by:

$$MZ_\alpha(C) = (T^{-1} \ddot{y}_T^2 - \breve{s}^2) \Bigg/ \left( 2T^{-2} \sum_{t=1}^{T} \ddot{y}_{t-1}^2 \right)$$

where $\ddot{y}_t$ are the residuals from the following regression:

$$y_t = \eta + \gamma t + \sum_{j=1}^{p} \phi_j D(j)_t + \ddot{y}_t$$

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The modification to the spectral density estimator of the residuals $v_t$ is different. Here, we consider only the deterministic components that are relevant under the null hypothesis, namely the constant and the dummies $da(j)_t$ and $db(j)_t$. Denoting this estimator by $\breve{s}^2$, it is defined by equation (5) but with the autoregression (6) replaced by:

$$\Delta y_t = \eta + \sum_{i=0}^{k}\sum_{j=1}^{p}(\kappa_{j,i}da(j)_{t-i} + \lambda_{j,i}db(j)_{t-i}) + b_0 y_{t-1} + \sum_{i=1}^{k} b_i \Delta y_{t-i} + e_{tk} \tag{21}$$

Finally, the modified estimator of the persistence measure is given by $\breve{f}_{\Delta y}(0) = \breve{h}_{\Delta y}(0)/\breve{\sigma}^2_{\Delta y}$ where

$$\breve{\sigma}^2_{\Delta y} = T^{-1}\sum_{t=1}^{T}(\widetilde{\Delta y_t})^2$$

the sample variance of $\Delta y_t$ corrected for the plans, i.e. the sum of squared residuals from the following regression:

$$\Delta y_t = \eta + \sum_{j=1}^{p}(\kappa_j da(j)_t + \lambda_j db(j)_t) + e_t$$

The quantity $\breve{h}_{\Delta y}(0)$ is the autoregressive spectral density estimate at frequency 0 defined by $\breve{h}_{\Delta y}(0) = \breve{s}^2_{ek}/(1 - \breve{d}(1))^2$, with

$$\breve{s}^2_{ek} = T^{-1}\sum_{t=1}^{T}\breve{e}^2_{tk}, \quad \breve{d}(1) = \sum_{j=1}^{k}\breve{d}_j$$

where $\breve{d}_j$ and $\{\breve{e}_{tk}\}$ are obtained from the following $k$th-order augmented autoregression in $\Delta y_t$:

$$\Delta y_t = \eta + \sum_{i=0}^{k}\sum_{j=1}^{p}(\kappa_{j,i}da(j)_{t-i} + \lambda_{j,i}db(j)_{t-i}) + \sum_{i=1}^{k} b_i \Delta y_{t-i} + e_{tk}$$

### 6.1. Simulation Results

The modifications described above leave the asymptotic distributions of the unit root tests unchanged under the null hypothesis (compared to the unmodified statistics applied to series in the class described by equation (1)), provided the plans are treated as fixed in length as the sample size increases. However, for the asymptotic distributions to provide satisfactory approximations to the finite sample distributions, the shock plans must be of relatively short duration. In this section we present simulations whose aim is to verify whether the usual asymptotic distribution provides a satisfactory approximation and if the modifications are effective in making the test immune to the presence of the shock plans. To do this, we examine the exact size of the modified tests using the same experiments as in Section 5 (exactly the same generated series are used). We also performed additional experiments where the processes are generated under the alternative hypothesis of trend-stationarity interrupted by shock plans. In this case, the data are generated

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using the same specifications except that the process describing the behaviour of the series when no plans are in effect is given by:

$$y_t = a + \mu t + \alpha y_{t-1} + e_t \tag{22}$$

instead of the random walk with drift described by equation (10). To examine power against stationary fluctuations, we specify $\alpha = 0{\cdot}8$ and 0·9. We also investigated power against explosive alternatives using the process described by equation (11) with $\alpha = 1{\cdot}01$ and $\alpha = 1{\cdot}02$.

The results for size and power are presented in Tables VIII and IX. The first feature of interest is that the exact sizes of the tests are, in most cases, very close to the nominal 5% size.[7] Hence, the modifications are successful in providing tests that are immune to the presence of shock plans and the usual asymptotic distribution provides a good approximation to the finite sample distribution. The second feature to note is that the tests still have reasonable power. Furthermore, the power function appears little influenced by different rates of growth and it increases rapidly as the sample size increases. We, therefore, conclude that the modifications are adequate. Comparing the different tests, the corrected versions of ADF are more powerful than those of $Z_\alpha$ and $MZ_\alpha$ with trend-stationary alternatives. When the alternative is that of an explosive process, the reverse relation holds. The differences are, however, small.

Table X presents the mean of $\breve{f}_{\Delta y}(0)$, the persistence measure corrected for plans, for each case. We see that the means are very close to the true values, especially when the order $k$ is chosen using the BIC criterion.

### 6.2. Empirical Results

We applied the modified unit root tests to the Brazilian inflation rate series using the dates for the plans as specified in Table I. The results are presented in Table XI for the various subsamples that incorporate plans. For any subsample that includes the five plans, the results point to a strong rejection of the unit root but this time in favour of an explosive alternative. This is confirmed by the estimates of the persistence measure presented in Table XII, which show a rather erratic behaviour as $k$ varies. For example, with the full sample (74:1–93:6) the estimate is 1·67 for $k = 2$ (chosen by BIC) but 68·44 with $k = 4$. This should not be too surprising since, if the process is indeed explosive, the persistence measure is not well defined.

Consider now the two subsamples that only include the first three plans and exclude the period surrounding the Collor I plan, namely 1974:1 to 1989:8 and 1980:1 to 1989:8. For the latter, there is no evidence against a unit root (against either alternatives) and the persistence measure is fairly stable around 1·0 for different values of $k$. For 1974:1 to 1989:8, the conclusions are mixed; $Z_\alpha(C_A)$, ADF($C_A$) and ADF($C_B$) suggest an explosive process (though less strongly than with the full sample), and the other tests and the persistence measures suggest a unit root process.

Hence, there is strong evidence that the shock plans are responsible for the spurious finding of stationarity using standard tests and that once these are taken into account the evidence strongly supports an explosive path (interrupted by shock plans) when the whole sample is analysed. However, the explosive behaviour appears due to the presence of the very high inflation period surrounding the period of the Collor I plan. As one referee pointed out, a look at the graph of the

[7] One exception is the $MZ_\alpha(C)$ which is conservative in the left tail and liberal in the right tail for small-sample sizes but the discrepancies disappear with larger samples.

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Table VIII. Exact size of unit root tests corrected for shock plans. (5% nominal size)

| | $T=150$ | | $T=250$ | | $T=500$ | |
|---|---|---|---|---|---|---|
| | Left | Right | Left | Right | Left | Right |
| (a) $Z_\alpha(C_A)$ | | | | | | |
| $\mu=0{\cdot}1$ | 0·060 | 0·083 | 0·045 | 0·066 | 0·047 | 0·053 |
| $\mu=0{\cdot}2$ | 0·057 | 0·089 | 0·044 | 0·064 | 0·048 | 0·052 |
| $\mu=0{\cdot}4$ | 0·060 | 0·083 | 0·046 | 0·062 | 0·048 | 0·050 |
| $\mu=0{\cdot}8$ | 0·084 | 0·067 | 0·057 | 0·051 | 0·055 | 0·044 |
| (b) $Z_\alpha(C_B)$ | | | | | | |
| $\mu=0{\cdot}1$ | 0·056 | 0·080 | 0·043 | 0·061 | 0·045 | 0·050 |
| $\mu=0{\cdot}2$ | 0·053 | 0·081 | 0·044 | 0·060 | 0·046 | 0·050 |
| $\mu=0{\cdot}4$ | 0·058 | 0·079 | 0·047 | 0·055 | 0·046 | 0·048 |
| $\mu=0{\cdot}8$ | 0·082 | 0·060 | 0·060 | 0·049 | 0·049 | 0·046 |
| (c) $MZ_\alpha(C)$ | | | | | | |
| $\mu=0{\cdot}1$ | 0·027 | 0·126 | 0·024 | 0·102 | 0·033 | 0·090 |
| $\mu=0{\cdot}2$ | 0·023 | 0·131 | 0·024 | 0·101 | 0·033 | 0·091 |
| $\mu=0{\cdot}4$ | 0·022 | 0·123 | 0·024 | 0·098 | 0·034 | 0·088 |
| $\mu=0{\cdot}8$ | 0·037 | 0·093 | 0·033 | 0·082 | 0·040 | 0·076 |
| (d) ADF$(C_A)$ | | | | | | |
| $\mu=0{\cdot}1$ | 0·088 | 0·073 | 0·067 | 0·066 | 0·060 | 0·055 |
| $\mu=0{\cdot}2$ | 0·086 | 0·073 | 0·067 | 0·066 | 0·060 | 0·055 |
| $\mu=0{\cdot}4$ | 0·084 | 0·074 | 0·066 | 0·066 | 0·060 | 0·055 |
| $\mu=0{\cdot}8$ | 0·079 | 0·075 | 0·064 | 0·066 | 0·059 | 0·055 |
| (e) ADF$(C_B)$ | | | | | | |
| $\mu=0{\cdot}1$ | 0·094 | 0·055 | 0·065 | 0·051 | 0·055 | 0·043 |
| $\mu=0{\cdot}2$ | 0·098 | 0·055 | 0·066 | 0·046 | 0·058 | 0·043 |
| $\mu=0{\cdot}4$ | 0·107 | 0·052 | 0·072 | 0·046 | 0·059 | 0·043 |
| $\mu=0{\cdot}8$ | 0·154 | 0·043 | 0·101 | 0·043 | 0·065 | 0·040 |

inflation series might suggest the possibility that this period was an outlier that could be responsible for additional biases. This question is addressed in the next subsection.

### 6.3. Accounting for Possible Outliers

The problem at hand is the following. First, the results indicate a unit root for subperiods that do not include plans. Second, using uncorrected statistics show stationarity whether one uses subsamples that include all five plans or just the first three plans. Hence, the high spike in inflation around the end of 1989 is not solely responsible for the unit root rejections. Third, for subperiods that do not include the era around the Collor I plan, the results corrected for the first three plans show again a unit root behaviour. However, when considering the full sample, the corrected statistics indicate an explosive behaviour, contrary to subperiods with only the first three plans. Hence, it may be possible that the noise is a unit root process but that the era around the Collor I plan is an outlier that biases the corrected tests in favour of an explosive process. Therefore, to test for outliers we work under the maintained assumption that the noise structure is a unit root process and, accordingly, we adopt the methodology suggested by Vogelsang (forthcoming) for that case.

Table IX. Power of the unit root tests corrected for shock plans

| | $T = 150$ | | | | $T = 250$ | | | | $T = 500$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha =$ | 0·8 | 0·9 | 1·01 | 1·02 | 0·8 | 0·9 | 1·01 | 1·02 | 0·8 | 0·9 | 1·01 | 1·02 |
| (a) $Z_\alpha(C_A)$ | | | | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·90 | 0·40 | 0·12 | 0·74 | 1·00 | 0·84 | 0·48 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}2$ | 0·90 | 0·39 | 0·13 | 0·74 | 1·00 | 0·84 | 0·47 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}4$ | 0·91 | 0·41 | 0·12 | 0·74 | 1·00 | 0·85 | 0·47 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}8$ | 0·93 | 0·47 | 0·10 | 0·72 | 1·00 | 0·88 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| (b) $Z_\alpha(C_B)$ | | | | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·91 | 0·39 | 0·11 | 0·73 | 1·00 | 0·84 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}2$ | 0·91 | 0·39 | 0·11 | 0·73 | 1·00 | 0·84 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}4$ | 0·92 | 0·41 | 0·11 | 0·73 | 1·00 | 0·86 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}8$ | 0·94 | 0·47 | 0·09 | 0·72 | 1·00 | 0·89 | 0·44 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| (c) $MZ_\alpha(C)$ | | | | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·79 | 0·26 | 0·17 | 0·78 | 1·00 | 0·77 | 0·56 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}2$ | 0·79 | 0·26 | 0·17 | 0·78 | 1·00 | 0·76 | 0·56 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}4$ | 0·79 | 0·27 | 0·16 | 0·77 | 1·00 | 0·77 | 0·56 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}8$ | 0·83 | 0·31 | 0·14 | 0·76 | 1·00 | 0·83 | 0·54 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| (d) ADF$(C_A)$ | | | | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·93 | 0·43 | 0·12 | 0·72 | 1·00 | 0·82 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}2$ | 0·93 | 0·43 | 0·12 | 0·72 | 1·00 | 0·82 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}4$ | 0·93 | 0·43 | 0·12 | 0·72 | 1·00 | 0·82 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}8$ | 0·91 | 0·40 | 0·12 | 0·72 | 1·00 | 0·81 | 0·46 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| (e) ADF$(C_B)$ | | | | | | | | | | | | |
| $\mu = 0{\cdot}1$ | 0·95 | 0·46 | 0·08 | 0·70 | 1·00 | 0·86 | 0·43 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}2$ | 0·95 | 0·47 | 0·08 | 0·70 | 1·00 | 0·86 | 0·43 | 0·97 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}4$ | 0·96 | 0·52 | 0·08 | 0·69 | 1·00 | 0·88 | 0·43 | 0·96 | 1·00 | 1·00 | 0·97 | 1·00 |
| $\mu = 0{\cdot}8$ | 0·97 | 0·63 | 0·06 | 0·66 | 1·00 | 0·91 | 0·40 | 0·96 | 1·00 | 1·00 | 0·97 | 1·00 |

The method is quite simple and can be described as follows. First, consider running a regression of the form

$$y_t = \mu + \beta t + \theta D(T_{ao})_t + u_t \tag{23}$$

where $D(T_{ao})_t$ is an additive outlier at time $T_{ao}$, i.e. taking value 1 if $t = T_{ao}$ and 0 otherwise. Let $t_{\hat{\theta}}(T_{ao})$ be the $t$-statistic for testing that $\theta = 0$. Since the date at which the outlier occurs is unknown, the test is based on the statistic $\tau = \sup_{T_{ao}} |t_{\hat{\theta}}(T_{ao})|$. Vogelsang (forthcoming) derived the limiting distribution of $\tau$ under the assumption that the noise component $u_t$ in equation (23) is a unit root process and that the relative position of the date at which the outlier occurs is a fixed proportion of total sample, i.e. $T_{ao}/T = \lambda$. Then $\tau \Rightarrow \sup_\lambda |H(\lambda)|$ where $H(\lambda) = W^*(\lambda)/(\int_0^1 W^*(r)^2\, dr)^{1/2}$ with $W^*(r)$ the residuals from a projection of the Wiener process on the functions $\{1, r\}$. The critical values at the 1% and 5% significance levels are 3·73 and 3·31, respectively. If the test concludes in favour of a rejection, the date of the outlier is simply estimated as $\hat{T}_{ao} = \arg\max_{T_{ao}} |t_{\hat{\theta}}(T_{ao})|$. The corresponding observation is then dropped and the testing procedure is iterated until a non-rejection occurs.

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Table X. Mean of $\check{f}_{\Delta y}(0)$ corrected for shock plans

| | $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $kbic$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) $\rho = 0.0$ | | | | | | | | | | | | |
| $T = 150$ | $\mu = 0.1$ | 0·97 | 0·93 | 0·90 | 0·86 | 0·83 | 0·79 | 0·77 | 0·74 | 0·73 | 0·71 | 0·97 |
| | $\mu = 0.2$ | 0·98 | 0·94 | 0·91 | 0·86 | 0·82 | 0·79 | 0·76 | 0·74 | 0·73 | 0·71 | 0·98 |
| | $\mu = 0.4$ | 1·00 | 0·97 | 0·94 | 0·88 | 0·81 | 0·78 | 0·75 | 0·73 | 0·72 | 0·70 | 1·00 |
| | $\mu = 0.8$ | 1·07 | 1·08 | 1·05 | 0·93 | 0·78 | 0·74 | 0·72 | 0·70 | 0·68 | 0·67 | 1·07 |
| $T = 250$ | $\mu = 0.1$ | 0·99 | 0·96 | 0·94 | 0·92 | 0·90 | 0·88 | 0·87 | 0·85 | 0·84 | 0·83 | 0·99 |
| | $\mu = 0.2$ | 0·99 | 0·96 | 0·94 | 0·92 | 0·90 | 0·88 | 0·87 | 0·85 | 0·84 | 0·83 | 0·99 |
| | $\mu = 0.4$ | 1·00 | 0·98 | 0·96 | 0·93 | 0·89 | 0·87 | 0·86 | 0·85 | 0·83 | 0·82 | 1·00 |
| | $\mu = 0.8$ | 1·04 | 1·04 | 1·02 | 0·96 | 0·87 | 0·85 | 0·84 | 0·82 | 0·81 | 0·80 | 1·04 |
| $T = 500$ | $\mu = 0.1$ | 0·99 | 0·98 | 0·97 | 0·96 | 0·95 | 0·94 | 0·93 | 0·92 | 0·92 | 0·91 | 0·99 |
| | $\mu = 0.2$ | 0·99 | 0·98 | 0·97 | 0·96 | 0·94 | 0·94 | 0·93 | 0·92 | 0·92 | 0·91 | 0·99 |
| | $\mu = 0.4$ | 1·00 | 0·99 | 0·98 | 0·96 | 0·94 | 0·93 | 0·93 | 0·92 | 0·92 | 0·91 | 1·00 |
| | $\mu = 0.8$ | 1·02 | 1·02 | 1·01 | 0·97 | 0·93 | 0·92 | 0·91 | 0·91 | 0·90 | 0·89 | 1·02 |
| (b) $\rho = 0.5$ | | | | | | | | | | | | |
| $T = 150$ | $\mu = 0.1$ | 2·87 | 2·75 | 2·67 | 2·54 | 2·46 | 2·36 | 2·32 | 2·28 | 2·26 | 2·50 | 2·87 |
| | $\mu = 0.2$ | 2·88 | 2·75 | 2·67 | 2·54 | 2·45 | 2·36 | 2·32 | 2·27 | 2·25 | 2·49 | 2·88 |
| | $\mu = 0.4$ | 2·90 | 2·78 | 2·69 | 2·54 | 2·43 | 2·34 | 2·30 | 2·25 | 2·23 | 2·47 | 2·90 |
| | $\mu = 0.8$ | 3·01 | 2·89 | 2·77 | 2·56 | 2·34 | 2·25 | 2·21 | 2·17 | 2·15 | 2·37 | 3·01 |
| $T = 250$ | $\mu = 0.1$ | 2·92 | 2·84 | 2·78 | 2·73 | 2·68 | 2·64 | 2·61 | 2·56 | 2·53 | 2·48 | 2·92 |
| | $\mu = 0.2$ | 2·92 | 2·84 | 2·79 | 2·73 | 2·68 | 2·63 | 2·61 | 2·55 | 2·53 | 2·48 | 2·92 |
| | $\mu = 0.4$ | 2·93 | 2·85 | 2·80 | 2·73 | 2·66 | 2·62 | 2·60 | 2·54 | 2·51 | 2·47 | 2·93 |
| | $\mu = 0.8$ | 3·00 | 2·91 | 2·84 | 2·73 | 2·61 | 2·56 | 2·54 | 2·48 | 2·46 | 2·41 | 3·00 |
| $T = 500$ | $\mu = 0.1$ | 2·95 | 2·92 | 2·89 | 2·85 | 2·84 | 2·82 | 2·80 | 2·78 | 2·76 | 2·73 | 2·95 |
| | $\mu = 0.2$ | 2·96 | 2·92 | 2·89 | 2·85 | 2·84 | 2·82 | 2·80 | 2·78 | 2·76 | 2·73 | 2·96 |
| | $\mu = 0.4$ | 2·96 | 2·93 | 2·90 | 2·85 | 2·83 | 2·81 | 2·80 | 2·77 | 2·75 | 2·72 | 2·96 |
| | $\mu = 0.8$ | 2·99 | 2·96 | 2·91 | 2·85 | 2·80 | 2·78 | 2·76 | 2·74 | 2·72 | 2·69 | 2·99 |
| (c) $\rho = -0.5$ | | | | | | | | | | | | |
| $T = 150$ | $\mu = 0.1$ | 0·33 | 0·32 | 0·31 | 0·29 | 0·28 | 0·27 | 0·26 | 0·25 | 0·25 | 0·24 | 0·33 |
| | $\mu = 0.2$ | 0·33 | 0·32 | 0·31 | 0·30 | 0·28 | 0·27 | 0·26 | 0·25 | 0·25 | 0·24 | 0·33 |
| | $\mu = 0.4$ | 0·34 | 0·34 | 0·33 | 0·31 | 0·28 | 0·27 | 0·26 | 0·25 | 0·24 | 0·24 | 0·34 |
| | $\mu = 0.8$ | 0·39 | 0·43 | 0·42 | 0·37 | 0·27 | 0·26 | 0·25 | 0·24 | 0·23 | 0·23 | 0·39 |
| $T = 250$ | $\mu = 0.1$ | 0·33 | 0·33 | 0·32 | 0·31 | 0·30 | 0·30 | 0·29 | 0·29 | 0·28 | 0·28 | 0·33 |
| | $\mu = 0.2$ | 0·33 | 0·33 | 0·32 | 0·31 | 0·30 | 0·30 | 0·29 | 0·29 | 0·28 | 0·28 | 0·33 |
| | $\mu = 0.4$ | 0·34 | 0·34 | 0·33 | 0·32 | 0·30 | 0·30 | 0·29 | 0·29 | 0·28 | 0·28 | 0·34 |
| | $\mu = 0.8$ | 0·36 | 0·38 | 0·38 | 0·35 | 0·30 | 0·29 | 0·28 | 0·28 | 0·27 | 0·27 | 0·36 |
| $T = 500$ | $\mu = 0.1$ | 0·33 | 0·33 | 0·33 | 0·32 | 0·32 | 0·31 | 0·31 | 0·31 | 0·31 | 0·31 | 0·33 |
| | $\mu = 0.2$ | 0·33 | 0·33 | 0·33 | 0·32 | 0·32 | 0·31 | 0·31 | 0·31 | 0·31 | 0·31 | 0·33 |
| | $\mu = 0.4$ | 0·34 | 0·34 | 0·33 | 0·33 | 0·32 | 0·31 | 0·31 | 0·31 | 0·31 | 0·30 | 0·34 |
| | $\mu = 0.8$ | 0·35 | 0·36 | 0·35 | 0·34 | 0·31 | 0·31 | 0·31 | 0·30 | 0·30 | 0·30 | 0·35 |

Applying this procedure to the inflation rate series showed all the observations from 1989 : 8 to 1990 : 4 (inclusive) to be categorized as outliers. This is the period of very rapid growth in inflation just prior to the implementation of the Collor I plan.

Having identified the outliers, the unit root test proceeds by adding one-time dummy variables for each outlier and as many lagged values as there are lags of the inflation rate in the relevant

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Table XI. Empirical results for unit root tests corrected for shock plans

| | | | | | | | ADF($C_A$) | | | ADF($C_B$) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | $Z_\alpha(C_A)$ | $k$ | $Z_\alpha(C_B)$ | $k$ | $MZ_\alpha(C)$ | $k$ | $t_\alpha$ | $\hat{\alpha}$ | $k$ | $t_\alpha$ | $\hat{\alpha}$ | $k$ |
| 74:1–93:6 | 14·58[e] | 2 | 6·46[e] | 2 | −14·03 | 2 | 4·39[e] | 1·14 | 4 | 5·63[e] | 1·22 | 3 |
| 74:1–89:8 | 2·66[e] | 4 | −4·19 | 4 | −5·71 | 4 | −0·05[e] | 1·00 | 4 | 2·06[e] | 1·10 | 4 |
| 80:1–89:8 | −10·56 | 4 | −12·77 | 4 | −7·76 | 4 | −2·29 | 0·89 | 0 | 0·47[e] | 1·04 | 4 |
| 80:1–93:6 | 9·36[e] | 2 | 3·55[e] | 2 | −10·60 | 2 | 3·54[e] | 1·15 | 4 | 5·10[e] | 1·25 | 3 |
| 85:1–93:6 | 6·53[e] | 2 | 2·88[e] | 2 | −6·24 | 2 | 3·19[e] | 1·14 | 2 | 5·06[e] | 1·31 | 3 |

*Note*: The superscript e denotes significance at the 1% level in favour of explosive alternatives.

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Table XII. Empirical results for $\breve{f}_{\Delta y}(0)$ corrected for shock plans

| Sample\\$k$ | 1 | 2 | 3 | 4 | 5 | *kbic* ($k$) |
|---|---|---|---|---|---|---|
| 74:1–93:6 | 0·50 | 1·67 | 12·70 | 68·44 | 6·42 | 1·67 (2) |
| 74:1–89:8 | 1·07 | 0·75 | 0·97 | | | 1·07 (1) |
| 80:1–89:8 | 0·99 | 0·70 | 0·95 | | | 0·99 (1) |
| 80:1–93:6 | 0·46 | 1·64 | 18·53 | 14·67 | 2·78 | 0·46 (1) |
| 85:1–93:6 | 0·38 | 1·84 | 43·21 | 2·65 | 0·72 | 0·38 (1) |

regressions (understanding, of course, that we do not duplicate identical dummy regressors). The outcome of the unit root tests and persistence measure so-corrected for outliers are presented in Tables XIII and XIV. Consider first the unit root test correcting for outliers but not correcting for the four remaining plans (Table XIII, panel A). The conclusion is basically the same as before, namely a strong rejection in favour of a stationary process. Surprisingly, the persistence measure indicate a greater degree of non-stationarity with estimates close to 2·5 using any subsample (Table XIV, panel A). If one corrects for the presence of both the outliers and the plans, the unit root tests give mixed results (Table XIII, panel B); a borderline rejection in favour of an explosive process with the ADF versions and a borderline rejection in favour of a stationary process with the $Z_\alpha$ versions. The estimates of the persistence measures (Table XIV, Panel B) give a value close to 1·0 irrespective of the sub-sample used. We view these results as not casting strong enough evidence to reject the unit root hypothesis, consistent with the results from the other subsamples.

## 7. CONCLUSIONS

This paper has considered issues related to tests for a unit root and a measure of the persistence of shocks when a time series of data is contaminated by large level shifts that are of short duration. These temporary events are labelled as 'inliers' or 'shock plans' following our applications to the Brazilian inflation rate. We first showed that standard unit root tests are severely biased in favour of rejecting the unit root against stationary fluctuations when shock plans are present (whether the noise component be characterized by a unit or explosive root). On the other hand, the persistence measure considered shows a strong bias towards one when the order $k$ selected is small.

Hence, a practical recommendation is to complement the application of standard unit root tests with the calculation of measures of persistence. An outcome where the unit root tests reject in favour of stationary fluctuations and the measure of persistence is well above 0 can be a sign that the series is contaminated by 'inliers' or 'shock plans'. To avoid the bias present when applying standard unit root tests, our study proposed corrected versions of three unit root tests. These corrected versions are shown to be adequate in terms of size and power.

The application of our corrected tests showed that the noise function of the Brazilian inflation rate is an explosive process if the period around the end of 1989 is not treated as an outlier. If the latter is treated as an outlier (as the tests suggest), the noise component is better characterized as a unit root process.

The macroeconomic interpretation of our results is a support of the inflation inertia hypothesis which essentially states that shocks to inflation are highly persistent (see, among others, Arida and Lara-Resende, 1985; Bacha, 1988; Bresser Pereira and Nakano, 1986; Lopes, 1984; Modiano, 1988; Novaes, 1991; Pastore, 1994; Simonsen, 1988). This behaviour of the inflation process is

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Table XIII. Empirical results for unit root tests treating 1989:08–1990:04 as an outlier

(A) Not corrected for the presence of plans

| Sample | $Z_\alpha$ | $k$ | $MZ_\alpha$ | $k$ | ADF $t_\alpha$ | ADF $\hat{\alpha}$ | ADF $k$ |
|---|---|---|---|---|---|---|---|
| 74:1–93:6 | $-53\cdot01^{a}$ | 1 | $-45\cdot92^{a}$ | 1 | $-5\cdot62^{a}$ | 0·75 | 1 |
| 80:1–93:6 | $-40\cdot43^{a}$ | 1 | $-34\cdot59^{a}$ | 1 | $-4\cdot98^{a}$ | 0·73 | 1 |
| 85:1–93:6 | $-26\cdot33^{b}$ | 1 | $-22\cdot41^{c}$ | 1 | $-4\cdot03^{a}$ | 0·71 | 1 |

*Note*: The superscripts a, b and c denote significance in favour of stationary alternatives at the 1%, 2·5% and 5% levels, respectively.

(B) Corrected for the presence of plans

| Sample | $Z_\alpha(C_A)$ | $k$ | $Z_\alpha(C_B)$ | $k$ | $MZ_\alpha(C)$ | $k$ | ADF($C_A$) $t_\alpha$ | ADF($C_A$) $\hat{\alpha}$ | ADF($C_A$) $k$ | ADF($C_B$) $t_\alpha$ | ADF($C_B$) $\hat{\alpha}$ | ADF($C_B$) $k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 74:1–93:6 | $-21\cdot54^{d}$ | 1 | $-27\cdot47^{b}$ | 1 | $-19\cdot65^{d}$ | 1 | $-2\cdot81^{g}$ | 0·91 | 1 | $-0\cdot48^{e}$ | 0·98 | 3 |
| 80:1–93:6 | $-21\cdot14^{d}$ | 1 | $-25\cdot84^{b}$ | 1 | −16·47 | 1 | $-2\cdot98^{g}$ | 0·88 | 1 | $-0\cdot83^{e}$ | 0·95 | 3 |
| 85:1–93:6 | −10·15 | 1 | −13·60 | 1 | −7·75 | 1 | $-2\cdot28^{f}$ | 0·89 | 1 | $-0\cdot18^{e}$ | 0·99 | 3 |

*Note*: The superscripts b and d denote significance in favour of stationary alternatives at the 2·5% and 10% levels, respectively; and the superscripts e, f, and g denote significance in favour of explosive alternatives at the 1%, 5% and 10% levels, respectively.

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Table XIV. Empirical results for $\hat{f}_{\Delta y}(0)$ treating 1989:08 to 1990:04 as an outlier

(a) Not corrected for the presence of plans

| Sample\k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | *kbic* (*k*) |
|---|---|---|---|---|---|---|---|---|---|
| 74:1–93:6 | 3·90 | 2·39 | 2·12 | 1·62 | 1·46 | 1·26 | 1·02 | 0·87 | 2·39 (2) |
| 80:1–93:6 | 3·96 | 2·44 | 2·17 | 1·65 | 1·50 | 1·29 | 1·05 | 0·89 | 2·44 (2) |
| 85:1–93:6 | 4·15 | 2·57 | 2·26 | 1·74 | 1·59 | 1·37 | 1·10 | 0·95 | 2·57 (2) |

(b) Corrected for the presence of plans

| Sample\k | 1 | 2 | 3 | 4 | 5 | *kbic* (*k*) |
|---|---|---|---|---|---|---|
| 74:1–93:6 | 1·10 | 0·59 | 1·59 | 0·83 | 0·55 | 1·10 (1) |
| 80:1–93:6 | 1·02 | 0·53 | 1·55 | 0·81 | 0·53 | 1·02 (1) |
| 85:1–93:6 | 0·91 | 0·41 | 1·08 | 0·51 | 0·25 | 0·91 (1) |

mainly explained by the widespread indexation to lagged inflation (backward-looking indexation) and to a highly passive monetary policy that easily accommodates inflationary pressures while aiming at keeping unemployment low.

Note finally that, while the methodology developed in this paper is directly motivated by and applied to the Brazilian inflation rate, the tools developed will be of direct application to a wide variety of cases where a series is affected by temporary but important events (for example, wars, strikes, etc.).

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REFERENCES

Andrews, D. W. K. (1991), 'Heteroskedastic and autocorrelation consistent co-variance matrix estimation', *Econometrica*, **59**, 817–854.

Arida, P. and A. Lara-Resende (1985), 'Inertial inflation and monetary reform in Brazil', in J. Williamson (ed.), *Inflation and Indexation: Argentina, Brazil and Israel*, MIT Press, Cambridge, MA, 27–45.

Bacha, E. L. (1988), 'Moeda, inércia e conflito: reflexões sobre políticas de estabilização no Brasil', *Pesquisa e Planejamento Econômico*, **18**, 1–16.

Berk, K. N. (1974), 'Consistent autoregressive spectral estimates', *The Annals of Statistics*, **2**, 489–502.

Bresser Pereira, L. and Y. Nakano (1986), 'Inertial inflation and heterodox shocks in Brazil', in J. M. Rego (ed.), *Inertial Inflation, Theories of Inflation and the Cruzado Plan*, Editora Paz e Terra, Rio de Janeiro.

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Campbell, J. Y. and N. G. Mankiw (1987), 'Are output fluctuations transitory?' *Quarterly Journal of Economics*, **102**, 857–880.

Campbell, J. Y. and P. Perron (1991), 'Pitfalls and opportunities: what macroeconomists should know about unit roots', in O. J. Blanchard and S. Fisher (eds), *NBER Macroeconomics Annual*, MIT Press, Cambridge, MA, 141–201.

Cochrane, J. H. (1988), 'How big is the random walk in GNP?' *Journal of Political Economy*, **96**, 893–920.

Dickey, D. A. and W. A. Fuller (1979), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, **74**, 427–431.

Franses, P. H. and N. Haldrup (1994), 'The effect of additive outliers on tests for unit roots and cointegration', *Journal of Business and Economic Statistics*, **12**, 471–478.

Fuller, W. A. (1976), *Introduction to Statistical Time Series*, Wiley, New York.

Lopes, F. L. (1984), 'Inflação inertial, hiperinflação e desinflação: notas e conjecturas', *Revista Economia Politica*, **5**, 135–151.

Modiano, E. M. (1988), 'The Cruzado first attempt: the Brazilian stabilization program of 1986', in M. Bruno, R. Dornbush, S. Fisher and G. D. Tella (eds), *Inflation Stabilization*, MIT Press, Cambridge, MA.

Ng, S. and P. Perron (1995), 'Unit root tests in ARMA models with data dependent methods for the selection of the truncation lag', *Journal of the American Statistical Association*, **90**, 268–281.

Novaes, A. D. (1991), 'Um teste de hipotese da inflação inercial no Brasil', *Pesquisa e Planejamento Econômico*, **21**, 377–396.

Pastore, A. C. (1994), 'Déficit público, a sustentabilidade do crescimento das dívidas interna e externa, senhoriagem e inflação: uma análise do regime monetário Brasileiro', *Revista de Econometria*, **14**, 177–234.

Perron, P. (1989), 'The great crash, the oil price shock and the unit root hypothesis', *Econometrica*, **57**, 1361–1401.

Perron, P. (1990), 'Testing for a unit root in a time series with a changing mean', *Journal of Business and Economic Statistics*, **8**, 153–162.

Perron, P. and S. Ng (1996), 'Useful modifications to unit root tests with dependent errors and their local asymptotic properties', *Review of Economic Studies*, **63**, 435–463.

Perron, P. and T. J. Vogelsang (1992), 'Testing for a unit root in a time series with a changing mean: corrections and extensions', *Journal of Business and Economic Statistics*, **10**, 467–470.

Phillips, P. C. B. and P. Perron (1988), 'Testing for a unit root in time series regression', *Biometrika*, **75**, 335–346.

Said, S. E. and D. A. Dickey (1984), 'Testing for unit roots in autoregressive-moving average models of unknown order', *Biometrika*, **71**, 599–607.

Simonsen, M. H. (1988), 'Price stabilization and income policies: theory and the Brazilian case study', in M. Bruno, R. Dornbush, S. Fisher and G. D. Tella (eds), *Inflation Stabilization*, MIT Press, Cambridge, MA.

Stock, J. H. (1990), 'A class of tests for integration and cointegration', mimeo, Kennedy School of Government, Harvard University.

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