Pontifícia Universidade Católica
do Rio de Janeiro

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Inflation Targeting with a Fiscal Taylor Rule

Dissertação de Mestrado

Thesis presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Yvan Bécard

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To my wife Cintia and to my newborn son Victor. I hope I will be able to spend more time with you from now on.

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Abstract

Leitner, Eduardo Henrique; Bécard, Yvan (Advisor). **Inflation Targeting with a Fiscal Taylor Rule**. Rio de Janeiro, 2020. 58p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This study proposes and tests an alternative inflation targeting regime which we call the fiscal Taylor rule (FTR). In this regime, the government keeps the nominal interest rate constant and uses the consumption tax rate as an instrument to stabilize inflation and the output gap. We estimate a standard business cycle model on US data from the Great Moderation period (1985-2007) and compare the observed outcomes to those of a counterfactual simulation where we apply the estimated shocks to the same business cycle model replacing the standard Taylor rule by the FTR. We find that compared to the standard Taylor rule, the FTR may be capable of providing similar performance in terms of economic stabilization and thus constitutes a theoretically viable option of policy framework.

Keywords

Inflation targeting; Fiscal policy; Monetary policy; Simple rules.

Resumo

Leitner, Eduardo Henrique; Bécard, Yvan. **Metas de Inflação com uma Regra de Taylor Fiscal**. Rio de Janeiro, 2020. 58p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Este estudo propõe e testa um regime de metas de inflação alternativo que nós chamamos de Regra de Taylor Fiscal (FTR). Nesse regime, o governo mantém a taxa de juros nominal constante e usa a alíquota de imposto sobre o consumo como instrumento para estabilizar a inflação e o hiato do produto. Nós estimamos um modelo padrão de ciclo de negócios a partir de dados dos EUA do período da Grande Moderação (1985-2007) e comparamos os resultados observados aos resultados de uma simulação contrafactual em que aplicamos os choques estimados ao mesmo modelo substituindo a regra de Taylor padrão pela FTR. Nós verificamos que, comparada a uma regra de Taylor padrão, a FTR pode ser capaz de prover uma performance similar em termos de estabilização econômica e portanto constitui uma opção teoricamente viável de política de estabilização econômica.

Palavras-chave

Metas de inflação; Política fiscal; Política monetária; Regras simples.

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List of Abreviations

ELB – Effective Lower Bound

FTR – Fiscal Taylor Rule

FTPL – Fiscal Theory of the Price Level

IRF – Impulse Response Function

LSAPs – Large-Scale Asset Purchase programs

QE – Quantitative Easing

The oldest and strongest emotion of mankind is fear, and the oldest and strongest kind of fear is fear of the unknown.

H. P. Lovecraft, *Supernatural Horror in Literature.*

1
Introduction

Most national governments delegate to their central banks the responsibility of keeping inflation low and stable, and give them control of the short term interest rate as the main tool to achieve this objective. Among macroeconomists, it has become almost a consensus that inflation targeting with the interest rate as the stabilizing tool is the best available regime to keep inflation under control and promote economic stability. Indeed, it has been in use for decades and inflation has been low and fairly stable almost everywhere this regime was adopted.

But there are some limitations to this framework. Since the 2008 financial crisis, most developed countries found themselves constrained by the effective lower bound (ELB) and were unable to bring the interest rate down to the levels prescribed by theory to the current economic situation. In the United States, Blanchard et al. (2010) estimate that when aggregate demand collapsed in 2008 the Fed would have decreased the rate 3 to 5 percent further if it could, and Williams (2009) calculates that not reducing the interest rate by an additional 4 percentage points would cost \$1.8 trillion in forgone U.S. output over four years.

Unconventional measures have been adopted by central banks to mitigate the problem, such as forward guidance and large-scale asset purchase programs (LSAPs) – also known as quantitative easing (QE) –, which have been the subject of much controversy. A number of policymakers believe that promising lower interest rates for longer than necessary may not be very effective because of its inherent time-inconsistency problem[1]. LSAPs have been criticized on the grounds that they distort asset markets and extrapolate the role central banks should play in a market economy.[2] A positive assessment of these new policies is presented in Bernanke (2020), which estimates that, as long as the nominal neutral interest rate does not fall below the range of 2–3 percent, a combination of QE and forward guidance can provide the equivalent of roughly

[1]See Nakata (2015) for quotes from various policymakers.

[2]As an example, in 2016 Bloomberg reported that the Bank of Japan (BoJ) was already a top-10 shareholder in about 90% of the Nikkei 225 companies. At the end of March 2018, Harada and Okimoto (2019) estimate that the BoJ was the top shareholder of more than 55 Nikkei 225 companies.

3 percentage points of additional policy space. At the time of writing, because of the coronavirus crisis, all major developed economies are back at the ELB and we have yet to see whether this extra policy space will be enough to counter the severity of the unfolding crisis.

This means that even if the standard approach may be the right choice of policy in many occasions, countries could benefit from having viable alternative instruments at their disposal. This work proposes and tests an alternative inflation targeting regime where the government keeps the nominal interest rate constant and uses the consumption tax rate as an instrument to stabilize inflation and the output gap, raising the rate to contain upward pressures on inflation or output and lowering it when the opposite is needed.[3] We call this regime a fiscal Taylor rule, as the authority reaction function is very similar to that of a standard Taylor rule, except that the tool is the consumption tax rate instead of the nominal interest rate.

To study this framework, we begin with the basic New Keynesian model. We do this to verify the behavior of output and inflation[4] following a consumption tax rate shock when a FTR is in place. We find that a tax rate shock that increases the rate by 0.8 p.p. in a FTR regime causes the same inflation reduction as a monetary shock that increases the interest rate by 1 p.p. in a standard regime. For these specific shocks, output responds 42% more to the tax rate shock in the FTR regime than to the monetary policy shock in the standard regime. In summary, the responses of output and inflation to shocks to these different tools under the different regimes are similar in shape and of the same order of magnitude.

Motivated by this preliminary evidence, we set up and estimate a medium-scale DSGE model in order to verify whether this finding holds in a more realistic framework that accounts for capital, investment, different tax rates etc. Using US data from the Great Moderation period (1985-2007) and assuming a Taylor rule was in place for the entire period, we compare the observed outcomes to those of counterfactual simulations where we apply the estimated shocks to the same model, but replace the standard Taylor rule by the FTR.

When using only output and inflation as observable variables and only

[3]Just as it has been established in most inflation targeting countries, it would be advisable that the institution responsible for these decisions was somewhat shielded from short-term political pressures. In fact, the central bank seems an appropriate candidate to assume this responsibility.

[4]The variances of the output gap and of inflation are the variables usually considered relevant for social welfare, figuring with varying weights both in theoretically derived welfare loss functions and in central banks' mandates. See Debortoli et al. (2019) for a recent survey of theoretical loss functions and central banks' mandates.

a pair of shocks (price markup and investment) to estimate the model, we find that over that period a FTR could have delivered 25% lower output volatility and similar inflation volatility, while keeping the consumption tax rate movements inside the range 4.4%-5.4%, but we show in Appendix C that this result is somewhat sensitive to the choice of shocks. When using four observable variables and six types of shocks to estimate the model, we find similar output volatility for both models and a 22.5% higher inflation volatility in the FTR model, where the consumption tax rate is kept inside the range 3.6%-7.4%. This suggests that the FTR may constitute a theoretically viable option of policy framework that is able to provide economic stability without the need for undesirably large swings in the instrument rate, but how much better or worse it performs in terms of welfare compared to the standard Taylor rule depends on the specificities of the model.

Related Literature This work builds on the literature on policy rules and their welfare implications. Most of this literature is concerned with how monetary policy is conducted in a context where fiscal policy is assumed to be passive[5] and carried out using non-distortionary taxes.[6] A few papers do pay attention to fiscal policy. For example, Benigno and Woodford (2003) derive targeting rules through which the monetary and fiscal authorities may implement the optimal equilibrium in a setting where the only available sources of government revenue are distorting taxes. They arrive at a system of two target relations between endogenous variables that could in theory be achieved by various combinations of monetary and fiscal policy, all of which are model dependent and may be very complex to implement. Here we focus on a simple and practical rule that only requires the policymaker to know the values of the current endogenous variables. Schmitt-Grohé and Uribe (2007) also study simple and implementable monetary and fiscal rules, allowing for fiscal policy to be active, in a model with distortionary taxation, but in all their specifications, fiscal policy reacts only to the level of government debt and not to other endogenous variables of interest. In the FTR that we study here, fiscal policy ignores the level of government debt, as it is not directly relevant to households' welfare, and reacts only to inflation and to the output gap.

Because in our proposed framework fiscal policy is active and monetary policy is passive, this study is also related to the fiscal theory of the price level (FTPL).[7] The debate about whether fiscal policy in the past has been active

[5]in the sense defined by Leeper (1991).
[6]Two seminal examples are Clarida et al. (2000) and Woodford (2003).
[7]See Cochrane (2011) for a good account of the FTPL.

or passive and about whether prices have been determined by the government debt valuation equation or not doesn't appear to be settled yet, but if the authorities were to follow the proposed FTR, fiscal policy would be active. While the FTPL emphasizes that the price level is determined exclusively by the government debt equation when fiscal policy is active, we show that in our model this equation cannot determine the price level on its own. When a FTR is in place, the price level can only be determined by solving the entire system, of which the government debt equation is a fundamental piece.

Discussion To the best of our knowledge, little research has been devoted to finding a satisfactory alternative instrument to the interest rate. One reason commonly cited in the literature for the preference for monetary policy over fiscal policy, as argued by Taylor (2000), is that monetary policy decisions come into force immediately, while changes in fiscal policy would have to pass through the budgetary process and have to be approved by the legislative body to come into effect in the following year. But not all fiscal measures need to go through this lengthy process. In many countries, some tax rates can be changed directly by the executive body and come into effect at a fairly short period of time. For example, in Brazil, the president can change the IPI rate (Tax on Industrialized Products) by decree and the change comes into force ninety days later. In the United Kingdom, the December 2008 VAT temporary tax reduction from 17.5% to 15% came into effect seven days after being announced at the HM Treasury Pre-budget report of November 2008.[8]

It is important to highlight some assumptions to clarify what exactly is the policy that we are studying. A consumption tax introduces a gap between prices paid by consumers and prices received by producers. It is to changes in producer prices that the authority reacts. According to Bernanke and Mishkin (1997), it is typical of inflation targeting regimes that the inflation target would exclude first-round effects of the consumption tax (value added tax). Was that not the case, the very policy decision would represent a change in the price level that would prompt a reinforcement of the decision just taken, which would be a highly destabilizing feature. Since consumer demand is not entirely inelastic for most goods, the first order effect of a consumption tax change on producer prices tends to be counter cyclical, as firms have to share some of the cost of the tax increase and of the benefit of a tax decrease.

[8]In a survey commissioned by the HMRC, Myant and Hawkins (2010) found that businesses spent a median 2.7 hours on compliance activities due to the VAT rate change and that, overall, businesses felt confident that they were fully compliant (98%) and that they had been given enough time to comply (90%), but 80% of businesses felt that more than a week's notice would have been preferable, with the majority suggesting four weeks.

In our models, we also consider that only producer prices are sticky, so that changes in the tax rates will affect the final prices of all goods the moment they come into force. We can imagine something like a sticky gondola price, but a flexible factor equal to the tax rate – and equal to all firms – that is added to the price at the cashier. This separation between gondola prices and the sales tax/VAT is very common in the USA and in Canada. In some states in the USA such as New York and Florida sellers are even prohibited by law from including the sales tax in the prices displayed.

Accepting the arguments favoring rules based policy over fully discretionary policy put forward in Taylor (2018) and understanding they also apply to the alternative policy tool presented in this study, policy is simulated in all cases as an automatic mechanism reacting to observable variables without further discussion about this issue. Nevertheless, we should keep in mind that, as is the case with monetary policy, some degree of discretion may be warranted and preferred by policymakers in practical situations.

Layout Chapter 2 presents the basic model and the IRFs under the different regimes. It also includes a discussion about price determinacy. Chapter 3 describes the medium-scale model and presents the simulations results. Chapter 4 concludes.

2
The Fiscal Taylor Rule in the Basic Model

To start our analysis, we incorporate a consumption tax rate to the textbook New Keynesian model. This basic model has only three fundamental equations, which are the well-known New Keynesian Phillips Curve, Dynamic IS curve, and the government debt accumulation equation:

$$\begin{aligned}
&\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + \Theta \hat{\tau}^c_t. \\
&\hat{y}_t = E_t \hat{y}_{t+1} - \lambda \left[\hat{r}_t - E_t \hat{\pi}_{t+1} - \tfrac{\tau^c}{1+\tau^c} E_t \Delta \hat{\tau}^c_{t+1} \right]. \\
&\hat{b}_t = R \left(\hat{r}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t \right) - \tau^c \tfrac{Y}{B} \hat{y}_t - \tau^c \tfrac{C}{B} \hat{\tau}^c_t - \tfrac{\tau}{B} \hat{\tau}_t.
\end{aligned}$$

The hat signs are used to indicate that we are writing the variables as log-deviations from the steady state. In this notation, $\hat{\pi}$ is inflation, $\hat{y}_t$ is output, $\hat{\tau}^c_t$ is the consumption tax rate, $\hat{r}_t$ is the nominal interest rate, $\hat{b}$ is government debt, and $\hat{\tau}_t$ is lump-sum taxes. The parameters κ, Θ and λ are calculated from the model's structural parameters (see Appendix A), and β is the discount factor. The other parameters τ^c, R, Y, B, C and τ are the steady-state values of the consumption tax rate, gross nominal interest rate, output, government debt, consumption and lump-sum tax, respectively.

The derivation of these equations is very well-known in the literature and we present it in Appendix A.

To close the model, we add the policy equations. In the standard version, the interest rate reacts to output and inflation, the lump-sum tax is changed in order to guarantee a non-exploding level of debt and the consumption tax rate is kept constant. Specifically, the policy equations are

$$\begin{aligned}
&\hat{r}_t = \rho_m \hat{r}_{t-1} + (1-\rho_m)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) + \epsilon^m_t. \\
&\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\gamma \left(\hat{b}_t - \hat{y}_t \right). \\
&\tau^c_t = 0.
\end{aligned}$$

In the Fiscal Taylor Rule model, the basic interest rate and the lump-sum tax are kept constant at their respective steady-state values at all times, while the consumption tax rate reacts to output and inflation in a similar way to how interest rates behave under a standard Taylor rule, as described below

$$\hat{r}_t = 0.$$
$$\hat{\tau}_t = 0.$$
$$\hat{\tau}_t^c = \rho_\tau^c \hat{\tau}_{t-1}^c + (1-\rho_\tau^c)\,\tfrac{1+\tau^c}{\tau^c}\left(\phi_\pi^c \hat{\pi}_t + \phi_y^c \hat{y}_t\right) + \epsilon_t^c.$$

The parameter values used in this chapter are displayed in Table 2.1 and reflect values commonly found in the literature.

Table 2.1: Basic model parameters

Structural parameters		Policy parameters	
β	0.9951	ρ_m	0.8
κ	0.4212	ϕ_π	1.5
Θ	0.0088	ϕ_y	0.5
λ	0.7939	ρ_τ	0.8
τ^c	0.0497	γ	0.1
R	1.0049	ρ_τ^c	0.8
$B/(4Y)$	0.5723	ϕ_π^c	1.5
C/Y	0.7939	ϕ_y^c	0.5
τ/Y	0.1779		

2.1 Price determinacy

In the standard formulation, the conditions under which the model presents a unique solution are well established in the literature. The same cannot be said regarding the fiscal Taylor rule formulation. In order to get a glimpse at those conditions, we will simplify a bit the model at hand by removing the lag term and the output gap reaction term in the consumption tax policy equation and substituting the policy equations in the three fundamental equations:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + \Theta\left(\tfrac{1+\tau^c}{\tau^c}\phi_\pi^c \hat{\pi}_t + \epsilon_t^c\right).$$
$$\hat{y}_t = E_t \hat{y}_{t+1} - \lambda\left[-E_t \hat{\pi}_{t+1} - \tfrac{\tau^c}{1+\tau^c} E_t\left(\tfrac{1+\tau^c}{\tau^c}\phi_\pi^c \hat{\pi}_{t+1} + \epsilon_{t+1}^c - \tfrac{1+\tau^c}{\tau^c}\phi_\pi^c \hat{\pi}_t - \epsilon_t^c\right)\right].$$
$$\hat{b}_t = R\left(\hat{b}_{t-1} - \hat{\pi}_t\right) - \tau^c \tfrac{Y}{B}\hat{y}_t - \tau^c \tfrac{C}{B}\left(\tfrac{1+\tau^c}{\tau^c}\phi_\pi^c \hat{\pi}_t + \epsilon_t^c\right).$$

Rearranging the terms, we obtain:

$$\hat{b}_t = R\hat{b}_{t-1} - \left[R + \tfrac{C}{B}\left(1+\tau^c\right)\phi_\pi^c\right]\hat{\pi}_t - \tau^c \tfrac{Y}{B}\hat{y}_t - \tau^c \tfrac{C}{B}\epsilon_t^c$$
$$E_t \hat{\pi}_{t+1} = \frac{1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c}{\beta}\hat{\pi}_t - \tfrac{\kappa}{\beta}\hat{y}_t - \tfrac{\Theta}{\beta}\epsilon_t^c$$

$$E_t\hat{y}_{t+1}=\lambda\left[\phi_\pi^c-\frac{(1+\phi_\pi^c)\left(1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c\right)}{\beta}\right]\hat{\pi}_t+\left[1+\frac{\lambda\kappa(1+\phi_\pi^c)}{\beta}\right]\hat{y}_t+\lambda\left[\frac{\tau^c}{1+\tau^c}+\frac{\Theta(1+\phi_\pi^c)}{\beta}\right]\epsilon_t^c$$

An important observation to be drawn from the system above is that the price level is not determined exclusively by the debt equation, as in the simplest applications of the fiscal theory of the price level. Under a fiscal Taylor rule, which does not determine directly the fiscal surplus, the debt equation only gives us one relation between inflation and output that must hold each period. In order to determine the price level at period t, one still needs to solve the system as a whole.

To be clear, the equilibrium price level still equates the real value of government debt to the present value of fiscal surpluses, as emphasized in the FTPL, but this present value of fiscal surpluses depends on all current and future values of consumption and consumption tax rates, which in turn depend on current and future values of output and inflation.

To see if the system above has a unique solution, we write the system above in matrix format:

$$\begin{bmatrix}\hat{b}_t\\E_t\hat{\pi}_{t+1}\\E_t\hat{y}_{t+1}\end{bmatrix}=A\begin{bmatrix}\hat{b}_{t-1}\\\hat{\pi}_t\\\hat{y}_t\end{bmatrix}+H\epsilon_t^c$$

where the transition matrix A is:

$$A\equiv\begin{bmatrix}R & -\left[R+\frac{C}{B}\left(1+\tau^c\right)\phi_\pi^c\right] & -\tau^c\frac{Y}{B}\\0 & \frac{1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c}{\beta} & -\frac{\kappa}{\beta}\\0 & \lambda\left[\phi_\pi^c-\frac{(1+\phi_\pi^c)\left(1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c\right)}{\beta}\right] & 1+\frac{\lambda\kappa(1+\phi_\pi^c)}{\beta}\end{bmatrix}$$

and H is:

$$H=\begin{bmatrix}-\tau^c\frac{C}{B}\\-\frac{\Theta}{\beta}\\\lambda\left[\frac{\tau^c}{1+\tau^c}+\frac{\Theta(1+\phi_\pi^c)}{\beta}\right]\end{bmatrix}$$

According to Blanchard and Kahn (1980), this system has a unique non-explosive solution if, and only if, the number of eigenvalues of A outside the unit circle is equal to the number of non-predetermined variables. In the present case, the non-predetermined variables are two: output and inflation.

The eigenvalues of A are the values of ω that satisfy:

$$\begin{vmatrix} R-\omega & -\left[R+\frac{C}{B}\left(1+\tau^c\right)\phi_\pi^c\right] & -\tau^c\frac{Y}{B} \\ 0 & \frac{1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c}{\beta}-\omega & -\frac{\kappa}{\beta} \\ 0 & \lambda\left[\phi_\pi^c-\frac{(1+\phi_\pi^c)\left(1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c\right)}{\beta}\right] & 1+\frac{\lambda\kappa(1+\phi_\pi^c)}{\beta}-\omega \end{vmatrix}=0$$

or, solving the determinant:

$$(R-\omega)\left\{\left(\frac{1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c}{\beta}-\omega\right)\left[1+\frac{\lambda\kappa(1+\phi_\pi^c)}{\beta}-\omega\right]+\frac{\lambda\kappa}{\beta}\left[\phi_\pi^c-\frac{(1+\phi_\pi^c)\left(1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c\right)}{\beta}\right]\right\}=0$$

The eigenvalue R determined by the debt equation gives us one eigenvalue outside the unit circle. The other two eigenvalues are given by the following equation:

$$\left(\frac{1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c}{\beta}-\omega\right)\left[1+\frac{\lambda\kappa(1+\phi_\pi^c)}{\beta}-\omega\right]+\frac{\lambda\kappa}{\beta}\left[\phi_\pi^c-\frac{(1+\phi_\pi^c)\left(1-\Theta\frac{1+\tau^c}{\tau^c}\phi_\pi^c\right)}{\beta}\right]=0$$

which is a quadratic equation that, after some algebra, can be written in the following canonical form:

$$\beta\omega^2-\left[1+\beta+\lambda\kappa+\left(\lambda\kappa-\Theta\frac{1+\tau^c}{\tau^c}\right)\phi_\pi^c\right]\omega+1+\left(\lambda\kappa-\Theta\frac{1+\tau^c}{\tau^c}\right)\phi_\pi^c=0$$

The solutions to this equation are given by:

$$\omega=\frac{1+\beta+\lambda\kappa+\left(\lambda\kappa-\Theta\frac{1+\tau^c}{\tau^c}\right)\phi_\pi^c\pm\sqrt{\left[1+\beta+\lambda\kappa+\left(\lambda\kappa-\Theta\frac{1+\tau^c}{\tau^c}\right)\phi_\pi^c\right]^2-4\beta\left[1+\left(\lambda\kappa-\Theta\frac{1+\tau^c}{\tau^c}\right)\phi_\pi^c\right]}}{2\beta}$$

For the basic model parameters shown in Table 2.1, the plus solution lies outside the unit circle and the minus solution lies inside the unit circle, implying that the FTR is able to implement a unique non-explosive solution. To assess the robustness of this fortunate result, we conducted a sensitivity analysis in which we determined the lowest and the largest values each parameter in this last equation could assume in order to maintain price determinacy when the other parameters are kept constant. The boundaries for price determinacy are shown in Table 2.2.

Therefore, at least in a simple model such as this, the fiscal Taylor rule is able to implement a unique non-explosive solution for a wide range of reasonable parameters.[1]

[1]We may not be able to judge immediately whether the upper bound for Θ is too tight

Table 2.2: Parameter boundaries for price determinacy

Parameter	Lower bound	Upper bound
β	0.0001	$> 10^6$
κ	0.0001	$> 10^6$
Θ	$< -10^6$	0.0840
λ	0.0001	$> 10^6$
τ^c	0.0050	$> 10^6$
ϕ_π^c	-14.5581	$> 10^6$

2.2 Simulations

We simulate the IRFs of the FTR model to a consumption tax rate shock and the IRFs of the standard model to an interest rate shock. Figure 2.1 shows that a consumption tax rate shock in the FTR model causes a decline in output and inflation. This is expected, since a higher consumption tax rate encourages households to postpone consumption, making firms produce less in equilibrium, which means that output declines. To produce less, firms reduce the demand for labor, so wages tend to decrease. Faced with lower demand and lower marginal costs, firms find it optimal to reduce prices, which explains the reduction in inflation.

Quantitatively, a tax rate shock that increases the rate by 0.8 p.p. in a FTR regime causes the same inflation reduction as a monetary shock that increases the interest rate by 1 p.p. in a standard regime. For these specific shocks, output responds 42% more to the tax rate shock in the FTR regime than to the monetary policy shock in the standard regime. This is encouraging, because it indicates that both policy frameworks may have similar "stabilization power", i.e., both frameworks may be able to achieve similar inflation and output volatility with comparable policy tool volatility.

But this is a very basic model that doesn't take into account many important characteristics of the real world, such as capital, decreasing marginal productivity of factors, consumption habits, differentiated wages across different sectors, economic growth, etc. In the next chapter we will see how the fiscal Taylor rule performance compares with that of the standard Taylor rule in a medium-scale DSGE model.

or not. The definition of Θ in terms of deep parameters is at the end of Appendix A and it depends on ξ, β and τ^c. Using the same type of sensitivity analysis as the one presented in Table 2.2 we find that for Θ to be larger than 0.0840, we would need either $\xi < 0.29$ or $\tau^c > 0.82$, while β could assume any non-negative value.

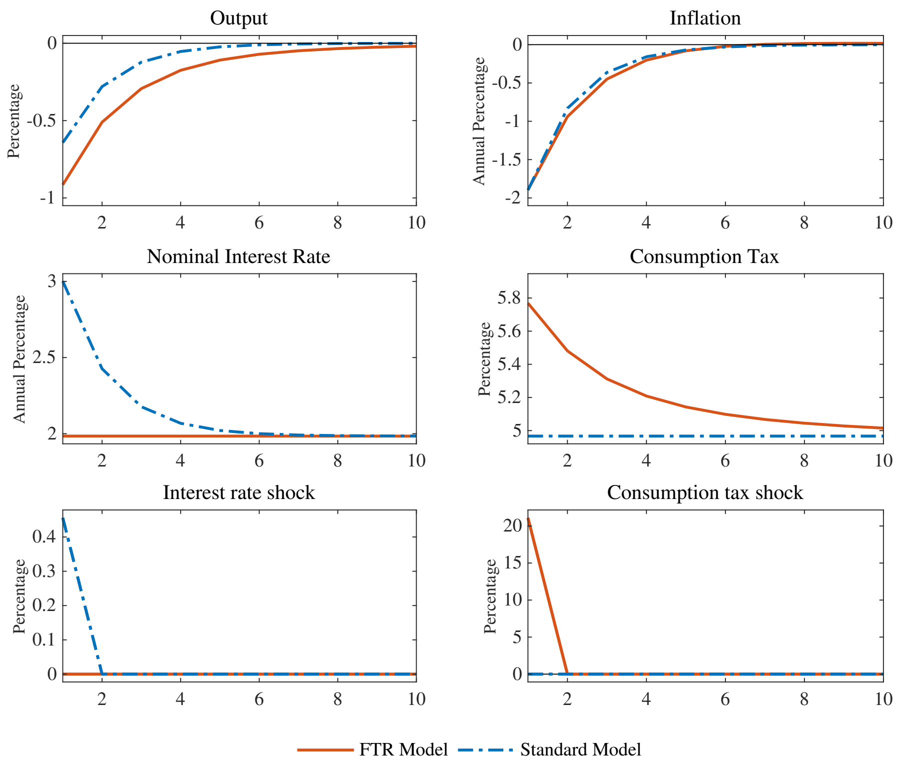

Figure 2.1: IRFs – Basic Models

3
The Fiscal Taylor Rule in a Medium-Scale Model

We move now to a medium-scale DSGE model, which is based on Christiano et al. (2014), itself a variant of Smets and Wouters (2007). The model takes into account many features that the basic model does not, such as habit formation in consumption, differentiated labor and wages, sticky wages, price and wage indexation, capital, investment, investment adjustment costs, variable capital utilization, positive economic growth, positive steady-state inflation, and labor and capital income taxes. Because its microfounded derivation is long and well-known in the literature, we defer it to Appendix B. The final log-linearized equations can also be seen at the end of that appendix.

In the standard version of this medium-scale model, as in the previous chapter, we assume the government changes the interest rate $\hat{r}_t$ in reaction to inflation $\hat{\pi}_t$ and to the output gap $\hat{\tilde{y}}_t$, following a standard Taylor rule, and changes the level of its expenditures $\hat{g}_t$ to guarantee a non-exploding level of debt $\hat{b}_t$. The consumption tax rate τ_t^c and the lump-sum tax τ_t do not react to any macroeconomic variable. Specifically, the policy equations are

$$
\begin{aligned}
&\hat{r}_t = \rho_m \hat{r}_{t-1} + (1-\rho_m)[\phi_\pi \hat{\pi}_t + \phi_y \hat{\tilde{y}}_t] + \epsilon_t^m.\\
&\hat{g}_t - \hat{y}_t = \rho_g(\hat{g}_{t-1} - \hat{y}_{t-1}) - (1-\rho_g)\gamma(\hat{b}_t - \hat{y}_t) + \epsilon_t^g.\\
&\hat{\tau}_t^c = \rho_{\tau^c}\hat{\tau}_{t-1}^c + \epsilon_t^c.\\
&\hat{\tau}_t = \rho_\tau \tau_{t-1} + \epsilon_t^\tau.
\end{aligned}
$$

In the FTR version, we have again the government changing the consumption tax rate in reaction to inflation and to the output gap. The interest rate, the lump-sum tax and the level of its expenditures also do not react to any macroeconomic variable. The policy equations are

$$
\begin{aligned}
&\hat{r}_t = \rho_m \hat{r}_{t-1} + \epsilon_t^m.\\
&\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_t^g.\\
&\hat{\tau}_t^c = \rho^f \hat{\tau}_{t-1}^c + (1-\rho^f)\tfrac{1+\tau^c}{\tau^c}\left[\phi_\pi^f \hat{\pi}_t + \phi_y^f \hat{\tilde{y}}_t\right] + \epsilon_t^c.\\
&\hat{\tau}_t = \rho_\tau \tau_{t-1} + \epsilon_t^\tau.
\end{aligned}
$$

3.1 Data and Calibration

We partition the parameters in two sets, one that we calibrate and the other that we estimate. Table 3.1 presents the calibrated parameters of the model, which we further divide in two panels. In panel A we have the parameters calibrated using USA data from the great moderation period of 1985-2007, and in panel B we have the parameters that were calibrated with values commonly found in the literature.

Table 3.1: Parameters calibrated with US data

Panel A. Parameters calibrated with US data		
Discount rate	β	0.9984
Growth rate of the economy	μ_z^*	1.0048
Inflation rate	π	1.0060
Power on capital in production function	α	0.3841
Government spending / GDP	η	0.1685
Government debt / annual GDP	$B/(4Y)$	0.5723
Investment-specific technological change	Υ	1.0025
Consumption tax rate	τ^c	0.0496
Labor income tax rate	τ^l	0.2017
Capital income tax rate	τ^k	0.2323
Panel B. Other calibrated parameters		
Depreciation rate on capital	δ	0.025
Govt spending reaction to the level of debt	γ	0.1
Price markup, intermediate goods	λ_p	1.2
Wage markup	λ_w	1.2
Curvature on disutility from labor	σ_l	1

The great moderation period was chosen to calibrate the parameters in panel A so as to exclude the period where the Federal Reserve was targeting monetary aggregates and to avoid the complications brought about by the Zero Lower Bound in the years that followed the global financial crisis of 2008. To obtain the average effective consumption, labor income and capital income tax rates for the period, we applied the methodology proposed by Mendoza et al. (1994) using annual data from the OECD.Stat database for different types of tax revenues of the US government and for the different types of households' income and expenditures. The other parameters in panel A were calibrated using quarterly data from the Federal Reserve Economic Data (FRED, Federal Reserve Bank of St. Louis) for macroeconomic variables such as GDP, inflation (as measured from the GDP implicit price deflator), population, fed funds rate, personal consumption expenditures, investments, and share of labor compensation in GDP.

All other parameters and shocks were estimated at each comparative simulation using Bayesian techniques.

3.2 Comparative Simulations of the Two Policy Frameworks

To compare the ability of each policy framework to stabilize the economy and thus promote welfare for the households, we estimate the remaining parameters and shocks assuming that the standard policy framework was in place during the whole period. We then feed these shocks to the FTR model and compare the outcomes for the main macroeconomic variables.

In the first simulation, we only require the standard model to replicate the historical values of output and inflation and we assume the economy was driven by price markup shocks and investment shocks. Under these assumptions, the Bayesian estimation results are presented in Table 3.2. The FTR policy parameters used in this simulation, shown in Table 3.3, were chosen so as to deliver similar inflation volatility to the one obtained in the standard model while also keeping output and consumption tax volatility low. Simulating both models subjected to the same shocks, the macroeconomic outcomes we obtain are shown in Figure 3.1.

Table 3.2: Simulation 1 – Model Priors and Posteriors

		Prior distribution			Posterior distribution	
Parameter name	Parameter	Prior dist	Mean	SD	Mode	SD
Panel A. Economic parameters						
Calvo price stickiness	ξ_p	beta	0.7	0.15	0.7173	0.0877
Calvo wage stickiness	ξ_w	beta	0.7	0.15	0.8056	0.1323
Habit parameter	b_c	beta	0.75	0.1	0.7936	0.1090
Price indexation	ι_p	beta	0.5	0.2	0.1047	0.0765
Wage indexation	ι_w	beta	0.5	0.2	0.5040	0.2950
Mon. policy weight on inflation	ϕ_π	normal	1.5	0.25	1.4524	0.2512
Mon. policy weight on output gap	ϕ_y	normal	0.12	0.1	0.0059	0.0396
Mon. policy smoothing parameter	ρ_m	beta	0.7	0.1	0.6517	0.1037
Govt. spending smoothing parameter	ρ_g	beta	0.5	0.2	0.5203	0.2768
Investment adjustment cost	S''	normal	3	2	4.3578	1.4982
Utilization cost	σ_a	normal	1	0.25	0.9633	0.2544
Panel B. Shocks						
Autocorrelation, price markup	ρ_{λ_p}	beta	0.5	0.2	0.8764	0.0680
Autocorrelation, investment	ρ_{ζ_i}	beta	0.5	0.2	0.3291	0.1118
Std dev, price markup	$\sigma_{\epsilon^{\lambda_p}}$	invg2	0.01	1	0.0065	0.0031
Std dev, investment	$\sigma_{\epsilon^{\zeta_i}}$	invg2	0.01	1	0.0133	0.0019

Table 3.3: Simulation 1 – FTR policy parameters

FTR policy weight on inflation	ϕ^f_π	2.0
FTR policy weight on output	ϕ^f_y	0.1
FTR policy smoothing parameter	ρ^f	0.8

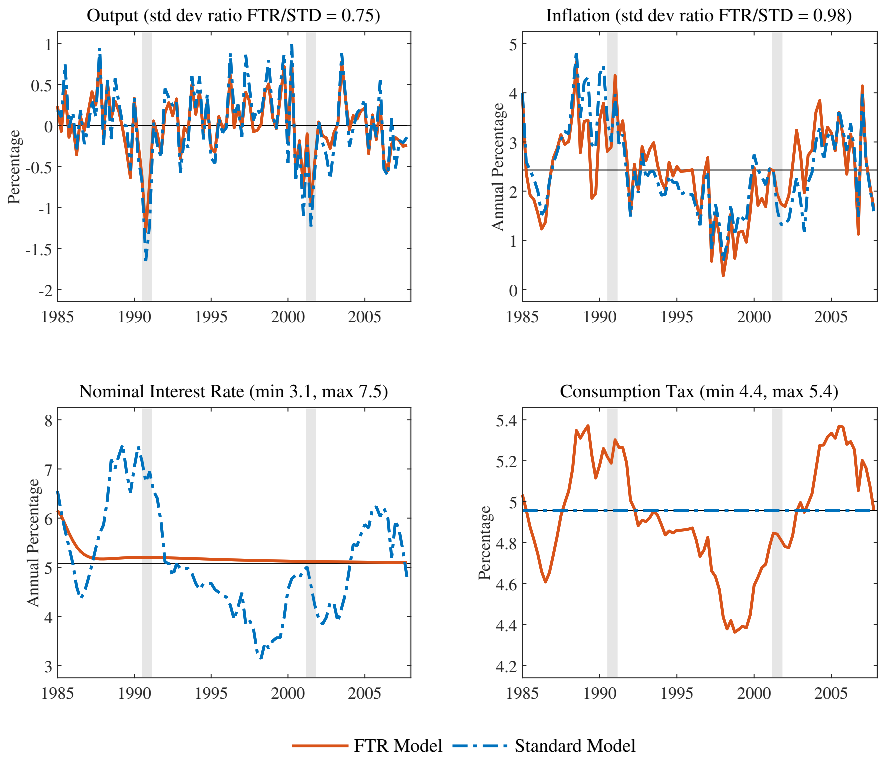

Figure 3.1: Simulation 1 – Macroeconomic outcomes

We display on top of the graphs of output and inflation the ratio between the standard deviation of the respective macroeconomic variable under the FTR model and that under the standard model. We see that in this simulation, compared to the standard Taylor rule, the FTR delivers a 25% lower output volatility and similar inflation volatility while maintaining the changes in the consumption tax rate inside the range 4.4%-5.4%.

In Appendix C we present additional exercises considering different pairs of supply/demand shocks using the same FTR policy parameters. The results obtained in those exercises indicate that the performance of the FTR, relative to the standard model, in delivering low volatility for output and inflation depends on the choice of shocks. When consumption preference is one of the shocks considered, then having a FTR in place instead of a standard Taylor rule results in higher output volatility and slightly lower inflation volatility. When consumption preference shocks are not considered, the FTR implies both lower output volatility and lower inflation volatility relative to the standard Taylor rule.

Next we perform a more complete simulation, where we set the observable variables, i.e. the macroeconomic variables that we match with their historical values, to be inflation, output, government spending and government debt, and we allow the economy to be driven by more shocks – namely, the shocks allowed are price markup shocks, investment shocks, government spending shocks, consumption preference shocks, utility preference shocks, and technology shocks.

The estimated parameters are presented in Table 3.4. The FTR policy parameters used in this simulation, shown in Table 3.5, were chosen so as to deliver low inflation volatility, respecting the condition that the range of motion of the consumption tax rate in the FTR model was not larger than the range of motion of the interest rate in the standard model. Simulating both models under the same shocks, the macroeconomic outcomes we obtain are shown in Figure 3.2.

Table 3.4: Simulation 2 – Model Priors and Posteriors

		Prior distribution			Posterior distribution	
Parameter name	Parameter	Prior dist	Mean	SD	Mode	SD
Panel A. Economic parameters						
Panel A. Economic parameters						
Calvo price stickiness	ξ_p	beta	0.7	0.15	0.9549	0.0087
Calvo wage stickiness	ξ_w	beta	0.7	0.15	0.5332	0.1397
Habit parameter	b_c	beta	0.75	0.1	0.8404	0.0693
Price indexation	ι_p	beta	0.5	0.2	0.0992	0.0749
Wage indexation	ι_w	beta	0.5	0.2	0.4915	0.2773
Mon. policy weight on inflation	ϕ_π	normal	1.5	0.25	1.6114	0.1774
Mon. policy weight on output gap	ϕ_y	normal	0.12	0.1	0.0218	0.0106
Mon. policy smoothing parameter	ρ_m	beta	0.7	0.1	0.7626	0.1071
Govt. spending smoothing parameter	ρ_g	beta	0.5	0.2	0.9649	0.0139
Investment adjustment cost	S''	normal	3	2	5.9969	1.3568
Utilization cost	σ_a	normal	1	0.25	1.0024	0.2525
Panel B. Shocks						
Autocorrelation, price markup	ρ_{λ_p}	beta	0.5	0.2	0.6370	0.0815
Autocorrelation, investment	ρ_{ζ_i}	beta	0.5	0.2	0.4908	0.0919
Autocorrelation, consumption preference	ρ_{ζ_c}	beta	0.5	0.2	0.5006	0.2792
Autocorrelation, utility preference	ρ_{ζ_u}	beta	0.5	0.2	0.5008	0.2794
Autocorrelation, technology	ρ_ε	beta	0.5	0.2	0.1655	0.0787
Std dev, price markup	$\sigma_{\epsilon^{\lambda_p}}$	invg2	0.01	1	0.2754	0.1215
Std dev, investment	$\sigma_{\epsilon^{\zeta_i}}$	invg2	0.01	1	0.0152	0.0014
Std dev, govt. spending	σ_{ϵ^g}	invg2	0.01	1	0.0162	0.0012
Std dev, consumption preference	$\sigma_{\epsilon^{\zeta_c}}$	invg2	0.01	1	0.0033	0.0019
Std dev, utility preference	$\sigma_{\epsilon^{\zeta_u}}$	invg2	0.01	1	0.0033	0.0019
Std dev, technology	$\sigma_{\epsilon^\varepsilon}$	invg2	0.01	1	0.0876	0.0080

Table 3.5: Simulation 2 – FTR policy parameters

FTR policy weight on inflation	ϕ_π^f	9.0
FTR policy weight on output	ϕ_y^f	0.2
FTR policy smoothing parameter	ρ^f	0.95

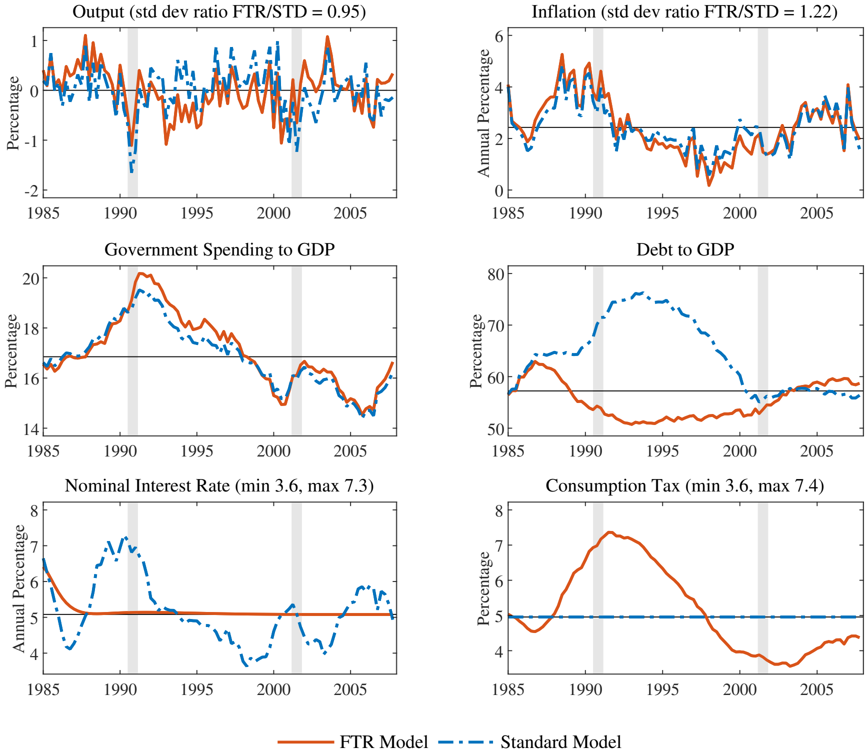

Figure 3.2: Simulation 2 – Macroeconomic outcomes

In this simulation, keeping the variations of the consumption tax rate inside the interval 3.6%-7.4%, the FTR model delivers standard deviations 5% lower for output and 22% higher for inflation than the standard model.

It is clear from the simulations above that the comparative results are somewhat sensitive to the choice of observable variables and to the shocks allowed to drive the economy. Also, the performance of both models depends on the policy parameters and a valid point of concern is that the most suitable choice of parameters may not be known beforehand by the policymakers as there is a lot of uncertainty regarding the future structural parameters and shock volatilities.

4
Conclusions

We have shown that, in a medium-scale DSGE model of the sort that is widely used in the literature to study business cycles estimated to resemble the US economy during the great moderation period, the performance of a FTR in terms of achieving economic stability, as measured by inflation and output volatility, can be comparable to that of a standard Taylor rule. Therefore, the FTR may be an alternative policy framework for countries that for some reason cannot or do not want to rely on monetary policy to do the job of steering the economy back to its steady state. Specially interested could be countries that find themselves constrained by the effective lower bound on interest rates, which at the time of writing include most developed countries, or countries where inflation may not be very responsive to interest rate movements, which potentially include some developing countries.

An interesting extension to this study would be to compare the outcomes of the FTR with that of a standard regime during the period where the effective lower bound was binding. Notice that, because the average consumption tax rate in the US was approximately 5%, a recessionary or deflationary shock strong enough to drive the interest rate to the effective lower bound would very likely drive the consumption tax rate to zero in case a FTR were in place, so the policy instrument would again be incapable of providing the appropriate stimulus. But the steady-state consumption tax rate is a political choice, and governments could very well set it at a level much higher, say 15%, without necessarily increasing the overall tax burden on citizens. This way governments could dramatically reduce the likelihood of zero lower bound events. It is difficult to do a similar trick with the nominal interest rate because the real interest rate depends on structural parameters of the economy. Some authors have suggested a higher inflation target[1] so that, keeping the real interest rate constant, the nominal interest rate would fluctuate around a higher steady-state level. It is evident, though, that there is much less space to increase the steady-state interest rate this way, as few economists would advocate for an annual inflation target as high as 10%.

Another interesting extension to this study would be to compare the

[1] See, for example, Blanchard et al. (2010).

outcomes of both policy frameworks in the context of an emerging economy, that would be modeled as an open economy subjected to shocks to the exchange rate, commodity prices, foreign investments, etc.

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A
Basic Model Derivation

In this appendix we present the basic model used in Chapter 2.

A.1
Model Setup

Households We assume a representative household with the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right\}.$$

where C_t is a consumption index and L_t is the aggregate labor given by:

$$C_t = \left(\int_0^1 C_{i,t}^{\frac{1}{\lambda_p}} di \right)^{\lambda_p}.$$

$$L_t = \int_0^1 L_{i,t} di.$$

A well-know results in the literature is that, being $P_{i,t}$ is the price of the differentiated good i, consumers will buy

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_p}{1-\lambda_p}} C_t.$$

where

$$P_t = \left(\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_p}} di \right)^{1-\lambda_p}.$$

We assume that households pay consumption taxes and lump-sum taxes, earn wages and profits from the firms and can buy government bonds to save for next period. So the representative household aims to maximize its utility function subject to the following budget constraint

$$(1+\tau_t^c) P_t C_t + P_t B_t + P_t \tau_t = P_t W_t L_t + R_{t-1} P_{t-1} B_{t-1} + P_t \Omega_t.$$

The Lagrangian of its maximization problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right.$$
$$\left. + \Lambda_t \left[P_t W_t L_t + R_{t-1} P_{t-1} B_{t-1} + P_t \Omega_t - (1+\tau_t^c) P_t C_t - P_t B_t - P_t \tau_t \right] \right\}.$$

Taking the FOCs

$$\begin{aligned} C_t: &\quad C_t^{-\sigma} = (1+\tau_t^c)\Lambda_t P_t. \\ L_t: &\quad L_t^{\varphi} = \Lambda_t P_t W_t. \\ B_t: &\quad \Lambda_t = \beta R_t E_t \Lambda_{t+1}. \end{aligned}$$

Firms We assume each firm i has a linear production function given by

$$Y_{i,t} = L_{i,t}.$$

The firm's nominal profit is

$$P_t \Omega_{i,t} = P_{i,t} Y_{i,t} - P_t W_t L_{i,t}.$$

At every period, given its total production, the problem of choosing the amount of labor to hire can be used to obtain the firm's marginal cost. The Lagrangian of the cost minimization problem is

$$\mathcal{L} = -P_t W_t L_{i,t} + S_{i,t}[L_{i,t} - Y_{i,t}].$$

Taking the FOC with respect to labor, we have the nominal marginal cost $S_{i,t}$

$$S_{i,t} = P_t W_t.$$

So the marginal cost is the same for all firms, but we keep the notation $S_{i,t}$ to differentiate firms' marginal cost from the aggregate marginal cost S_t, which we define as the cost to produce an additional unit of the Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{1}{\lambda_p}} di \right)^{\lambda_p}.$$

So the firm's nominal profit can now be written as

$$P_t \Omega_{i,t} = [P_{i,t} - S_{i,t}] Y_{i,t}.$$

We assume Calvo pricing, where at every period a fraction ξ of firms cannot change its price. So the price setting problem becomes to maximize its expected profit considering only the stories where it can't readjust it's price:

$$\max_{P_{i,t}} E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{1+\tau_t^c}{1+\tau_{t+s}^c} \frac{C_t^\sigma}{C_{t+s}^\sigma} \frac{P_t}{P_{t+s}} P_{t+s} \Omega_{i,t+s} \text{ s.t. } Y_{i,t+s} = \left(\frac{P_{i,t}}{P_{t+s}} \right)^{\frac{\lambda_p}{1-\lambda_p}} Y_{t+s}.$$

Taking the FOC with respect to $P_{i,t}$

$$P_{i,t} \equiv P_t^* = \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{1}{1+\tau_{t+s}^c} C_{t+s}^{-\sigma} Y_{t+s} P_{t+s}^{\frac{1}{\lambda_p - 1}} S_{i,t+s} \frac{\lambda_p}{1-\lambda_p}}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{1}{1+\tau_{t+s}^c} C_{t+s}^{-\sigma} Y_{t+s} P_{t+s}^{\frac{1}{\lambda_p - 1}} \frac{1}{1-\lambda_p}} \equiv \frac{X_{1i,t}}{X_{2,t}}.$$

We can write the infinite sums in recursive form

$$X_{1i,t} = (1+\tau_t^c)^{-1} C_t^{-\sigma} Y_t P_t^{\frac{\lambda_p}{\lambda_p - 1}} \frac{S_{i,t}}{P_t} \frac{\lambda_p}{1-\lambda_p} + \xi \beta E_t X_{1i,t+1}.$$

$$X_{2,t} = (1+\tau_t^c)^{-1} C_t^{-\sigma} Y_t P_t^{\frac{1}{\lambda_p - 1}} \frac{1}{1-\lambda_p} + \xi \beta E_t X_{2,t+1}.$$

Government The law of motion of government debt is given by

$$P_t G + R_{t-1} P_{t-1} B_{t-1} = \tau_t^c P_t C_t + P_t B_t + P_t \tau_t.$$

where we assume constant government spending G

Aggregation The following equations have to hold in equilibrium.
Resource constraint

$$Y_t = C_t + G.$$

Production index will be given by

$$Y_t = L_t D_t^{-1}, \quad D_t \equiv \int_0^1 (P_{i,t}/P_t)^{\frac{\lambda_p}{1-\lambda_p}} \, di.$$

The aggregate marginal cost, i.e., the cost to produce an additional using of the production index Y_t is such that

$$\frac{S_{i,t}}{P_t} = \frac{S_t}{P_t} D_t^{-1}.$$

If we plug the last expression into the price FOC, we obtain

$$P_t^* = \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{1}{1+\tau_{t+s}^c} C_{t+s}^{-\sigma} Y_{t+s} P_{t+s}^{\frac{\lambda_p}{\lambda_p-1}} \frac{S_{t+s}}{P_{t+s}} D_{t+s}^{-1} \frac{\lambda_p}{1-\lambda_p}}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{1}{1+\tau_{t+s}^c} C_{t+s}^{-\sigma} Y_{t+s} P_{t+s}^{\frac{1}{\lambda_p-1}} \frac{1}{1-\lambda_p}}.$$

So the optimal price will be given by

$$P_t^* = \frac{X_{1,t}}{X_{2,t}},\ X_{1,t} = \frac{1}{1+\tau_t^c} C_t^{-\sigma} Y_t P_t^{\frac{\lambda_p}{\lambda_p-1}} \frac{S_t}{P_t} D_t^{-1} \frac{\lambda_p}{1-\lambda_p} + \xi\beta E_t X_{1,t+1}.$$

Because at every period only a fraction $(1-\xi)$ of firms optimize their price, the price index evolves according to

$$P_t = [(1-\xi) P_t^{*,\frac{1}{1-\lambda_p}} + \xi P_{t-1}^{\frac{1}{1-\lambda_p}}]^{1-\lambda_p}.$$

If we raise to the power $(1-\lambda_p)^{-1}$ and divide by $P_{t-1}^{\frac{1}{1-\lambda_p}}$ on both sides, and define $\pi_t \equiv P_t/P_{t-1}$, and $\pi_t^* \equiv P_t^*/P_{t-1}$, we can write

$$\pi_t^{\frac{1}{1-\lambda_p}} = (1-\xi)\pi_t^{*,\frac{1}{1-\lambda_p}} + \xi.$$

Flexible-Price Equilibrium In this basic model, we define the flexible-price equilibrium as the equilibrium that would prevail if prices were flexible and if the consumption tax were constant, i.e., if $\xi = 0$ and $\tau_t^c = \tau^c$.[1]

The equations for labor demand, labor supply, production function, and resource constraint, would be

$$\begin{aligned} W_t^f &= \frac{S_t}{P_t} = \lambda_p^{-1}. \\ W_t^f &= (1+\tau^c) L_t^{f,\varphi} C_t^{f,\sigma}. \\ Y_t^f &= L_t^f. \\ Y_t^f &= C_t^f + G. \end{aligned}$$

Combining these four equations, we obtain the flexible-price output

$$(1+\tau^c) Y_t^{f,\varphi} (Y_t^f - G)^{\sigma} = \lambda_p^{-1}.$$

[1]The consumption tax rate is considered constant in the flexible-price equilibrium of the basic model for simplicity, since it will imply output equals the output gap and so we can eliminate one equation from the model.

A.2
Equilibrium Conditions

Defining $s_t \equiv S_t/P_t$, $x_{1,t} \equiv X_{1,t}/P_t^{\frac{\lambda_p}{\lambda_p-1}}$, $x_{2,t} \equiv X_{2,t}/P_t^{\frac{1}{\lambda_p-1}}$, we summarize below the equilibrium conditions.

$$\pi_t^{\frac{1}{1-\lambda_p}} = (1-\xi)\pi_t^{*,\frac{1}{1-\lambda_p}} + \xi.$$
$$x_{1,t} = (1+\tau_t^c)^{-1} Y_t C_t^{-\sigma} s_t \lambda_p + \xi\beta E_t \pi_{t+1}^{\frac{\lambda_p}{\lambda_p-1}} x_{1,t+1}.$$
$$x_{2,t} = (1+\tau_t^c)^{-1} Y_t C_t^{-\sigma} + \xi\beta E_t \pi_{t+1}^{\frac{1}{\lambda_p-1}} x_{2,t+1}.$$
$$\pi_t^* = \frac{x_{1,t}}{x_{2,t}}\pi_t.$$
$$W_t = s_t Y_t L_t^{-1}.$$
$$Y_t = L_t D_t^{-1}.$$
$$Y_t = C_t + G.$$
$$R_t = R_t^r E_t \pi_{t+1}.$$
$$W_t = (1+\tau_t^c) L_t^{\varphi} C_t^{\sigma}.$$
$$1 = \beta R_t E_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}}\frac{1}{\pi_{t+1}}.$$
$$G + \frac{1}{\pi_t} R_{t-1} B_{t-1} = \tau_t^c C_t + B_t + \tau_t.$$
$$(1+\tau^c) Y_t^{f,\varphi} (Y_t^f - G)^{\sigma} = \lambda_p^{-1}.$$
$$\tilde{Y}_t = Y_t / Y_t^f.$$
$$D_t \equiv \int_0^1 (P_{i,t}/P_t)^{\frac{\lambda_p}{1-\lambda_p}}\, di.$$

A.3
Steady State

Writing the above equations in the steady-state:

$$R = \frac{1}{\beta}.$$
$$R^r = R.$$
$$s = \frac{1}{\lambda_p}.$$
$$x_1 = x_2 = \frac{Y C^{-\sigma}}{(1-\xi\beta)(1+\tau^c)}.$$
$$L = Y.$$
$$W = sYL^{-1}.$$
$$\eta \equiv \frac{G}{Y}.$$
$$C = Y - G.$$
$$W = (1+\tau^c) L^{\varphi} C_t^{\sigma}.$$
$$\frac{\tau}{Y} = (R-1)\frac{B}{Y} + \frac{G}{Y} - \tau^c \frac{C}{Y}.$$
$$Y = Y^f = \left[\frac{1}{1+\tau^c} s (1-\eta)^{-\sigma}\right]^{\frac{1}{\sigma+\varphi}}.$$
$$\tilde{Y} = Y/Y^f.$$

A.4
Loglinear Equilibrium

For any variable X_t, define $\hat{x}_t \equiv \ln(X_t/X)$. Also define $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^f$. Using standard first order Taylor expansion, the log-linear equilibrium is given by the following equations

$$\hat{\pi}_t = (1-\xi)\hat{\pi}_t^*.$$

$$\hat{x}_{1,t} = (1-\xi\beta)(\hat{y}_t - \sigma\hat{c}_t + \hat{s}_t - \tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c) + \xi\beta E_t[\tfrac{\lambda_p}{\lambda_p - 1}\hat{\pi}_{t+1} + \hat{x}_{1,t+1}].$$

$$\hat{x}_{2,t} = (1-\xi\beta)(\hat{y}_t - \sigma\hat{c}_t - \tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c) + \xi\beta E_t(\tfrac{1}{\lambda_p - 1}\hat{\pi}_{t+1} + \hat{x}_{2,t+1}).$$

$$\hat{\pi}_t^* = \hat{\pi}_t + \hat{x}_{1,t} - \hat{x}_{2,t}.$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \tfrac{(1-\xi\beta)(1-\xi)}{\xi}\hat{s}_t.$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \tfrac{(1-\xi\beta)(1-\xi)}{\xi}\left[(\varphi + \sigma\tfrac{Y}{C})\tilde{y}_t + \tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c\right]. \tag{A-1}$$

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{l}_t. \tag{A-2}$$

$$\hat{y}_t = \hat{l}_t. \tag{A-3}$$

$$\hat{y}_t = \tfrac{C}{Y}\hat{c}_t. \tag{A-4}$$

$$\hat{r}_t^r = \hat{r}_t - E_t\hat{\pi}_{t+1}. \tag{A-5}$$

$$\hat{w}_t = \varphi\hat{l}_t + \sigma\hat{c}_t + \tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c. \tag{A-6}$$

$$\hat{c}_t = E_t\hat{c}_{t+1} - \tfrac{1}{\sigma}[\hat{r}_t - E_t\hat{\pi}_{t+1} - \tfrac{\tau^c}{1+\tau^c}(E_t\hat{\tau}_{t+1}^c - \hat{\tau}_t^c)]. \tag{A-7}$$

$$\tfrac{B}{Y}\hat{b}_t = R\tfrac{B}{Y}(\hat{r}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t) - \tau^c\tfrac{C}{Y}(\hat{c}_t + \hat{\tau}_t^c) - \tfrac{\tau}{Y}\hat{\tau}_t. \tag{A-8}$$

$$\hat{y}_t^f = 0. \tag{A-9}$$

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^f. \tag{A-10}$$

If we are only interested in studying output, inflation and government debt[2], we can omit most of the above equations. Using (A-9) and (A-10) in (A-1), and (A-4) in (A-7) and in (A-8), we obtain the basic log-linearized equilibrium conditions:

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \tfrac{(1-\xi\beta)(1-\xi)}{\xi}\left[\left(\varphi + \sigma\tfrac{Y}{C}\right)\hat{y}_t + \tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c\right].$$

$$\hat{y}_t = E_t\hat{y}_{t+1} - \tfrac{C}{Y}\tfrac{1}{\sigma}[\hat{r}_t - E_t\hat{\pi}_{t+1} - \tfrac{\tau^c}{1+\tau^c}(E_t\hat{\tau}_{t+1}^c - \hat{\tau}_t^c)].$$

$$\tfrac{B}{Y}\hat{b}_t = R\tfrac{B}{Y}(\hat{r}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t) - \tau^c(\hat{y}_t + \tfrac{C}{Y}\hat{\tau}_t^c) - \tfrac{\tau}{Y}\hat{\tau}_t.$$

We can define

$$\kappa \equiv \tfrac{(1-\xi\beta)(1-\xi)}{\xi}\left(\varphi + \sigma\tfrac{Y}{C}\right)$$

[2]At first we would only be interested in output and inflation, but the law of motion of government debt is important because if we want to obtain internal solutions, we need to impose that government debt does not go to infinity.

$$
\begin{aligned}
\Theta \equiv& \tfrac{(1-\xi\beta)(1-\xi)}{\xi}\tfrac{\tau^c}{1+\tau^c} \\
\lambda \equiv& \tfrac{C}{Y}\tfrac{1}{\sigma}
\end{aligned}
$$

and write the basic log-linearized equilibrium conditions as

$$
\begin{aligned}
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \kappa\hat{y}_t + \Theta\hat{\tau}^c_t. \\
\hat{y}_t &= E_t\hat{y}_{t+1} - \lambda\left[\hat{r}_t - E_t\hat{\pi}_{t+1} - \tfrac{\tau^c}{1+\tau^c}E_t\Delta\hat{\tau}^c_{t+1}\right]. \\
\hat{b}_t &= R\left(\hat{r}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t\right) - \tau^c\tfrac{Y}{B}\hat{y}_t - \tau^c\tfrac{C}{B}\hat{\tau}^c_t - \tfrac{\tau}{B}\hat{\tau}_t.
\end{aligned}
$$

B
Medium-Scale Model Derivation

In this appendix we present the model used in Chapter 3, which is a standard medium-scale DSGE model based on Christiano et al. (2014), with the important difference that we removed the financial accelerator mechanism, which is not the object of the present study. The model takes into account habit formation in consumption, differentiated labor and wages, sticky wages, price and wage indexation, capital, investment, investment adjustment costs, variable capital utilization, positive economic growth, positive steady-state inflation, and labor and capital income taxes.

B.1
Model Setup

Households A representative household contains a large number of workers who supply differentiated labor $l_{k,t}$, $k \in [0,1]$. The household derives utility from consumption C_t and disutility from labor $l_{k,t}$ according to

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{u,t} \left\{ \zeta_{c,t} \ln(C_t - b_c C_{t-1}) - \psi_l \int_0^1 \frac{l_{k,t}^{1+\sigma_l}}{1+\sigma_l} dk \right\}, \quad b_c, \psi_l, \sigma_l > 0,$$

where $\beta \in (0,1)$ is a discount factor and $\zeta_{u,t}$ and $\zeta_{c,t}$ are preference shocks. The budget constraint of the household writes

$$\begin{aligned}
&(1+\tau_t^c) P_t C_t + P_t B_t + P_t \Upsilon^{-t} \mu_{\Upsilon,t}^{-1} I_t + P_t \tau_t \\
&\quad \leq (1-\tau^l) \int_0^1 W_{k,t} l_{k,t} dk + R_{t-1} P_{t-1} B_{t-1} \\
&\qquad + (1-\tau^k) P_t \left[u_t \tilde{r}_t^k - \Upsilon^{-t} a(u_t) \right] \bar{K}_{t-1} + \tau^k P_t \Upsilon^{-t} \delta \bar{K}_{t-1} + \Delta_t,
\end{aligned}$$

where τ_t^c and τ^l are consumption and labor tax rates, P_t is the price index, $W_{k,t}$ is the nominal wage of worker k, $P_t \tilde{r}_t^k$ is the nominal rental rate of capital, $\Upsilon > 1$ is the constant rate at which the relative price of capital goods with respect to consumption goods falls over time, $\mu_{\Upsilon,t}$ is an investment-specific shock that changes the rate at which final goods are converted into $\Upsilon^t \mu_{\Upsilon,t}$ investment goods, u_t is the utilization rate of capital, and $a(u_t)$ is a standard utilization adjustment cost function given by

$$a(u_t) = r^k(\exp[\sigma_a(u_t - 1)] - 1)/\sigma_a, \quad \sigma_a > 0,$$

where r^k is the steady-state rental rate of capital.

The household has to pay a lump-sum tax τ_t and allocates the rest of its budget on consumption, bonds B_t, and capital investment. Its revenues come from labor income, previous-period bonds, capital income and dividends from firms Δ_t. The household builds raw capital according to a standard technology

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + [1 - S(\zeta_{i,t} I_t / I_{t-1})] I_t, \quad \delta \in (0,1),$$

where I_t is investment, $\zeta_{i,t}$ is a shock to the marginal efficiency of investment and S is a standard investment adjustment cost function given by

$$S(x_t) = \exp\left[\sqrt{S''/2}(x_t - x)\right] + \exp\left[-\sqrt{S''/2}(x_t - x)\right] - 2,$$

where $x_t \equiv \zeta_{i,t} I_t / I_{t-1}$. Note that $S(x) = S'(x) = 0$ and S'' is a parameter.

The Lagrangian of the household utility maximizing problem writes

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{u,t} \Bigg\{ & \zeta_{c,t} \ln(C_t - b_c C_{t-1}) - \psi_l \int_0^1 \frac{l_{k,t}^{1+\sigma_l}}{1+\sigma_l} dk \\ & + \Lambda_{z,t} \left[(1-\tau^l) \int_0^1 W_{k,t} l_{k,t} dk + R_{t-1} P_{t-1} B_{t-1} \right. \\ & + (1-\tau^k) P_t \left[u_t \tilde{r}_t^k - \Upsilon^{-t} a(u_t) \right] \bar{K}_{t-1} + \tau^k P_t \Upsilon^{-t} \delta \bar{K}_{t-1} + \Delta_t - (1+\tau_t^c) P_t C_t \\ & \left. - P_t B_t - P_t \Upsilon^{-t} \mu_{\Upsilon,t}^{-1} I_t - P_t \tau_t \right] \\ & + \Lambda_{2z,t} \left[(1-\delta)\bar{K}_{t-1} + [1 - S(\zeta_{i,t} I_t / I_{t-1})] I_t - \bar{K}_t \right] \Bigg\}.\end{aligned}$$

The FOCs are

$$\begin{aligned}
C_t :\ 0 =& \zeta_{u,t} \Lambda_{z,t} (1+\tau_t^c) P_t - \zeta_{u,t} \zeta_{c,t} / (C_t - b_c C_{t-1}) \\
& + \beta b_c E_t \zeta_{u,t+1} \zeta_{c,t+1} / (C_{t+1} - b_c C_t) \\
B_t :\ 0 =& \zeta_{u,t} \Lambda_{z,t} P_t - \beta P_t E_t \zeta_{u,t+1} \Lambda_{z,t+1} R_t \\
I_t :\ 0 =& \zeta_{u,t} \Lambda_{2z,t} \left[1 - S\left(\zeta_{i,t} I_t / I_{t-1}\right) - \zeta_{i,t} I_t / I_{t-1} S'\left(\zeta_{i,t} I_t / I_{t-1}\right)\right] \\
& - \zeta_{u,t} \Lambda_{z,t} P_t \Upsilon^{-t} \mu_{\Upsilon,t}^{-1} + \beta E_t \zeta_{u,t+1} \Lambda_{2z,t+1} \zeta_{i,t+1} \left(I_{t+1}/I_t\right)^2 S'\left(\zeta_{i,t+1} I_{t+1}/I_t\right) \\
\bar{K}_t :\ 0 =& \zeta_{u,t} \Lambda_{2z,t} - \beta E_t \zeta_{u,t+1} \Lambda_{2z,t+1} (1-\delta) \\
& - \beta E_t \zeta_{u,t+1} \Lambda_{z,t+1} \left\{ (1-\tau^k) P_{t+1} \left[u_{t+1} \tilde{r}_{t+1}^k - \Upsilon^{-t-1} a(u_{t+1}) \right] + \tau^k P_{t+1} \Upsilon^{-t-1} \delta \right\} \\
u_t :\ 0 =& a'(u_t) - \Upsilon^t \tilde{r}_t^k = r^k \exp(\sigma_a [u_t - 1]) - \Upsilon^t \tilde{r}_t^k
\end{aligned}$$

Define Tobin's Q as the relative price of capital, $Q_t \equiv \Lambda_{2z,t}/\Lambda_{z,t}$

$$\begin{aligned} I_t: \quad 0 =& \zeta_{u,t}\Lambda_{z,t}Q_t\left[1 - S\left(\zeta_{i,t}I_t/I_{t-1}\right) - \zeta_{i,t}I_t/I_{t-1}S'\left(\zeta_{i,t}I_t/I_{t-1}\right)\right] \\ & -\zeta_{u,t}\Lambda_{z,t}P_t\Upsilon^{-t}\mu_{\Upsilon,t}^{-1} + \beta E_t\zeta_{u,t+1}\Lambda_{z,t+1}Q_{t+1}\zeta_{i,t+1}\left(I_{t+1}/I_t\right)^2 S'\left(\zeta_{i,t+1}I_{t+1}/I_t\right). \end{aligned}$$

$$\begin{aligned} \bar{K}_t: \quad \zeta_{u,t}\Lambda_{z,t}Q_t = \beta E_t\zeta_{u,t+1}\Lambda_{z,t+1}\Big\{&(1-\tau^k)P_{t+1}[u_{t+1}\tilde{r}^k_{t+1} - \Upsilon^{-(t+1)}a(u_{t+1})] \\ &+ \tau^k P_{t+1}\Upsilon^{-t-1}\delta + (1-\delta)Q_{t+1}\Big\}. \end{aligned}$$

Final Good Producers A representative, competitive final good firm combines intermediate goods $Y_{j,t}$, $j \in [0,1]$, to produce final output Y_t using the technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{\lambda_{p,t}}} dj\right]^{\lambda_{p,t}},$$

where $\lambda_{p,t} \geq 1$ is a markup shock. The budget constraint is

$$\int_0^1 P_{j,t}Y_{j,t}dj = P_tY_t.$$

The FOC with respect to intermediate good $Y_{j,t}$ is

$$\left[\int_0^1 Y_{j,t}^{\frac{1}{\lambda_{p,t}}} dj\right]^{\lambda_{p,t}-1} Y_{j,t}^{\frac{1-\lambda_{p,t}}{\lambda_{p,t}}} = xP_{j,t},$$

where x is the multiplier on the budget constraint. Integrate over all goods, solve for x, rearrange, and obtain the demand function for a generic intermediate good

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t.$$

Plug the demand function into the aggregator and obtain the aggregate price index

$$P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{1-\lambda_{p,t}}} dj\right]^{1-\lambda_{p,t}}.$$

Intermediate Firms Each intermediate good j is produced by a monopolist according to the production function

$$Y_{j,t} = \max\left\{\varepsilon_t(u_tK_{j,t-1})^\alpha(z_tl_{j,t})^{1-\alpha} - \theta z_t^* \ ; \ 0\right\}, \quad \alpha \in (0,1),$$

where $K_{j,t-1}$ denotes capital services, $l_{j,t}$ is a homogeneous labor input, u_t is the aggregate utilization rate of capital, ε_t is a covariance stationary technology shock, and θ is a fixed cost. There are two sources of growth in the model. The first one is z_t, a shock to the growth rate of technology. The second one is an investment-specific shock $\mu_{\Upsilon,t}$ that changes the rate at which final goods are converted into $\Upsilon^t \mu_{\Upsilon,t}$ investment goods, with $\Upsilon > 1$. In equilibrium the price of investment goods is $P_t/(\Upsilon^t \mu_{\Upsilon,t})$. As in Christiano et al. (2010), the fixed cost θ is proportional to z_t^*, which combines the two trends, $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$. The intermediate good producer faces standard Calvo frictions. Every period, a fraction $1 - \xi_p$ of intermediate firms sets their price $P_{j,t}$ optimally. The remaining fraction follows an indexation rule $P_{j,t} = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{j,t-1}$, where $\iota_p \in (0, 1)$ and $\pi_t \equiv P_t/P_{t-1}$ is inflation. Throughout this appendix, a variable without the subscript t denotes its steady-state value. Intermediate good producer j makes the following profit

$$P_{j,t} Y_{j,t} - W_t l_{j,t} - P_t \tilde{r}_t^k u_t \bar{K}_{j,t-1},$$

where $P_t \tilde{r}_t^k$ represents the nominal rental rate of capital. The firm minimizes cost subject to the production function. The FOCs with respect to capital services $u_t \bar{K}_{j,t-1}$ and labor $l_{j,t}$ are

$$\begin{aligned} P_t \tilde{r}_t^k &= \alpha S_{j,t} \varepsilon_t (u_t \bar{K}_{j,t-1})^{\alpha-1} (z_t l_{j,t})^{1-\alpha}, \\ W_t &= (1-\alpha) S_{j,t} \varepsilon_t (u_t \bar{K}_{j,t-1})^{\alpha} z_t^{1-\alpha} l_{j,t}^{-\alpha}, \end{aligned}$$

where $S_{j,t}$ is the multiplier on the production function and is interpreted as the marginal cost. Combine the two FOCs

$$\frac{u_t \bar{K}_{j,t-1}}{l_{j,t}} = \frac{\alpha}{(1-\alpha)} \frac{W_t}{P_t \tilde{r}_t^k}.$$

The capital-to-labor ratio depends only on aggregate quantities and is therefore common to all intermediate producers. If firms pay the same factor prices, receive the same aggregate shocks, and choose the same proportion of inputs, then they have the same marginal cost $S_t = S_{j,t}$

$$S_t = \frac{1}{\varepsilon_t} \left(\frac{P_t \tilde{r}_t^k}{\alpha} \right)^{\alpha} \left(\frac{W_t}{(1-\alpha) z_t} \right)^{1-\alpha}.$$

Intermediate good producer j chooses a price $P_{j,t}$ to maximize the sum of expected future discounted profits

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} \Lambda_{z,t+s} \left[P_{j,t} \Pi_{t,t+s} Y_{j,t+s} - W_{t+s} l_{j,t+s} - P_{t+s} \tilde{r}_{t+s}^k u_{t+s} \bar{K}_{j,t-1+s} \right],$$

subject to a demand function. In this equation, $P_{t+s} = \pi_{t+s} \ldots \pi_{t+1} P_t$, $\Pi_{t,t+s} \equiv$

$\prod_{k=1}^{s} \tilde{\pi}_{t+k} = \tilde{\pi}_{t+s} \ldots \tilde{\pi}_{t+1}$, $\Pi_{t,t+s} = 1$ for $s = 0$, and $\tilde{\pi}_t = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}$ is an indexation term. The firm discounts the future in the same way as the household it belongs to. Since the marginal cost equals the average variable cost we rewrite the problem as

$$\max_{P_{j,t}} E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} \Lambda_{z,t+s} Y_{j,t+s} (P_{j,t}\Pi_{t,t+s} - S_{t+s}),$$

subject to the demand function. The FOC with respect to price $P_{j,t}$ is

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} \Lambda_{z,t+s} Y_{t+s} \left(\frac{\tilde{\pi}_{t+s} \ldots \tilde{\pi}_{t+1}}{(\pi_{t+s} \ldots \pi_{t+1})^{\lambda_{p,t+s}}} \right)^{\frac{1}{1-\lambda_{p,t+s}}} \left(\frac{\tilde{P}_t}{P_t} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \frac{1}{1-\lambda_{p,t+s}}$$
$$= E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} \Lambda_{z,t+s} Y_{t+s} \left(\frac{\tilde{\pi}_{t+s} \ldots \tilde{\pi}_{t+1}}{\pi_{t+s} \ldots \pi_{t+1}} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \left(\frac{\tilde{P}_t}{P_t} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}} \frac{S_{t+s}}{P_t},$$

where the optimal price $\tilde{P}_t \equiv P_{j,t}$ depends only on aggregate variables and is therefore common to all producers. Multiply by $P_t = P_{t+s}/(\pi_{t+s} \ldots \pi_{t+1})$ and rearrange

$$1 = \frac{E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} P_{t+s} \Lambda_{z,t+s} Y_{t+s} \left(\frac{\tilde{\pi}_{t+s} \ldots \tilde{\pi}_{t+1}}{\pi_{t+s} \ldots \pi_{t+1}} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \left(\frac{\tilde{P}_t}{P_t} \right)^{\frac{2\lambda_{p,t+s}-1}{1-\lambda_{p,t+s}}} \frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}} \frac{S_{t+s}}{P_{t+s}}}{E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} P_{t+s} \Lambda_{z,t+s} Y_{t+s} \left(\frac{\tilde{\pi}_{t+s} \ldots \tilde{\pi}_{t+1}}{\pi_{t+s} \ldots \pi_{t+1}} \right)^{\frac{1}{1-\lambda_{p,t+s}}} \left(\frac{\tilde{P}_t}{P_t} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \frac{1}{1-\lambda_{p,t+s}}}$$

The aggregate price level is given by

$$P_t = \left[(1-\xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_{p,t}}} + \xi_p \left(\tilde{\pi}_t P_{t-1} \right)^{\frac{1}{1-\lambda_{p,t}}} \right]^{1-\lambda_{p,t}}.$$

Labor Contractors A representative, competitive labor contractor aggregates specialized labor services $l_{k,t}$, $k \in [0,1]$, into homogeneous labor l_t using the technology

$$l_t = \left[\int_0^1 l_{k,t}^{\frac{1}{\lambda_w}} dk \right]^{\lambda_w}, \quad \lambda_w \geq 1.$$

The budget constraint is

$$\int_0^1 W_{k,t} l_{k,t} dk = W_t l_t.$$

The FOC with respect to differentiated labor $l_{k,t}$ is

$$\left[\int_0^1 l_{k,t}^{\frac{1}{\lambda_w}} dk \right]^{\lambda_w - 1} l_{k,t}^{\frac{1}{\lambda_w}} = x W_{k,t} l_{k,t},$$

where x is the multiplier on the budget constraint. Integrate over all inputs, solve for x, rearrange, and obtain the demand function for a generic labor input

$$l_{k,t} = \left(\frac{W_{k,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_t.$$

Plug the demand function into the Dixit-Stiglitz aggregator and obtain the aggregate wage index

$$W_t = \left[\int_0^1 W_{k,t}^{\frac{1}{1-\lambda_w}} dk\right]^{1-\lambda_w}.$$

Monopoly Unions Each worker of type k is represented by a monopoly union that sets its nominal wage rate $W_{k,t}$. All monopoly unions are subject to Calvo frictions in a similar fashion to intermediate firms. A fraction $1-\xi_w$ of monopoly unions chooses their wage optimally. The remaining fraction follows an indexation rule $W_{k,t} = \mu_{z^*}\pi_{t-1}^{\iota_w}\pi^{1-\iota_w}W_{k,t-1}$, where $\iota_w \in (0,1)$, $\mu_{z^*} \equiv z^*/z^*_{-1}$ is the steady-state growth rate of the economy, and $\mu_{z^*,t}$ is a shock. Worker union k maximizes the sum of future utilities

$$E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \zeta_{u,t+s}\left[-\psi_l \int_0^1 \frac{l_{k,t+s}^{1+\sigma_l}}{1+\sigma_l} dk + \Lambda_{z,t+s}(1-\tau^l)W_{k,t}\Pi^w_{t,t+s} l_{k,t+s}\right],$$

$$\text{subject to} \quad l_{k,t+s} = \left(\frac{W_{k,t}\Pi^w_{t,t+s}}{W_{t+s}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+s},$$

where $W_{t+s} = \pi_{w,t+s}\ldots\pi_{w,t+1}W_t$, $\Pi^w_{t,t+s} = \prod_{j=1}^s \mu_{z^*}\tilde{\pi}_{w,t+j}$, and $\tilde{\pi}_{w,t} = \pi_{t-1}^{\iota_w}\pi^{1-\iota_w}$ is an indexation term. The FOC with respect to wage $W_{k,t}$ is

$$0 = E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \zeta_{u,t+s}\left\{\Lambda_{z,t+s}(1-\tau^l)W_{k,t}^{\frac{1}{1-\lambda_w}}\left(\Pi^w_{t,t+s}\right)^{\frac{1}{1-\lambda_w}}\left(\frac{1}{W_{t+s}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+s} \right.$$
$$\left. - \psi_l \int_0^1 \frac{\left[\left(\frac{W_{k,t}\Pi^w_{t,t+s}}{W_{t+s}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+s}\right]^{1+\sigma_l}}{1+\sigma_l} dk\right\}$$

Rearranging this expression we have

$$0 = E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \zeta_{u,t+s} l_{t+s}\left(\frac{\Pi^w_{t,t+s}}{\pi_{w,t+s}\ldots\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}}\left\{\Lambda_{z,t+s}(1\right.$$
$$\left. -\tau^l)\frac{1}{1-\lambda_w}\Pi^w_{t,t+s} - \psi_l\frac{\lambda_w}{1-\lambda_w}\frac{1}{\tilde{W}_t}\left[\left(\frac{\tilde{W}_t\Pi^w_{t,t+s}}{W_{t+s}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+s}\right]^{\sigma_l}\right\}.$$

The optimal wage $\tilde{W}_t \equiv W_{j,t}$ depends only on aggregate variables and is therefore common to all worker unions. Divide by $W_t = W_{t+s}/(\pi_{w,t+s}\dots\pi_{w,t})$ and rearrange

$$\begin{aligned}\left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{1-\lambda_w(1+\sigma_l)}{1-\lambda_w}} & \frac{W_t}{P_t}\frac{1}{\psi_l} \\ &= \frac{E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\zeta_{u,t+s}\left(\frac{\Pi^w_{t,t+s}}{\pi_{w,t+s}\cdots\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} l_{t+s}^{1+\sigma_l}}{E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s\zeta_{u,t+s}\frac{(1-\tau^l)}{\lambda_w}l_{t+s}\left(\frac{\Pi^w_{t,t+s}}{\pi_{w,t+s}\cdots\pi_{w,t+1}}\right)^{\frac{1}{1-\lambda_w}}\left(\frac{\pi_{w,t+s}\cdots\pi_{w,t+1}}{\pi_{t+s}\cdots\pi_{t+1}}\right)\Lambda_{z,t+s}P_{t+s}} \\ &\equiv \frac{K_{w,t}}{F_{W,t}}.\end{aligned}$$

Express the infinite sums $K_{w,t}$ and $F_{W,t}$ in recursive form

$$\begin{aligned}K_{w,t} &= \zeta_{u,t}l_t^{1+\sigma_l} + \xi_w\beta E_t\left(\tilde{\pi}_{w,t+1}\pi_{w,t+1}^{-1}\mu_{z^*}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} K_{w,t+1}, \\ F_{W,t} &= (1-\tau^l)\lambda_w^{-1}\zeta_{u,t}l_tP_t\Lambda_{z,t} + \xi_w\beta E_t\left(\tilde{\pi}_{w,t+1}\mu_{z^*}\right)^{\frac{1}{1-\lambda_w}}\pi_{w,t+1}^{\frac{\lambda_w}{\lambda_w-1}}\pi_{t+1}^{-1}F_{W,t+1}.\end{aligned}$$

Therefore, the optimal wage writes

$$\frac{\tilde{W}_t}{W_t} = \left[\frac{\psi_l}{W_t/P_t}\frac{K_{w,t}}{F_{W,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_l)}}.$$

The aggregate wage level is given by

$$W_t = \left[(1-\xi_w)\tilde{W}_t^{\frac{1}{1-\lambda_w}} + \xi_w(\tilde{\pi}_{w,t}\mu_{z^*}W_{t-1})^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_{w,t}}.$$

Divide by W_t and plug the expression into the optimal wage equation

$$K_{w,t} = \frac{1}{\psi_l}\left[\frac{1-\xi_w(\tilde{\pi}_{w,t}\pi_{w,t}^{-1}\mu_{z^*})^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_l)}\frac{W_t}{P_t}F_{W,t}.$$

Government Government debt accumulation is given by

$$\begin{aligned}P_tG_t + R_{t-1}P_{t-1}B_{t-1} = &\left\{u_t\tilde{r}_t^k - \Upsilon^{-t}\left[a(u_t)+\delta\right]\right\}P_t\bar{K}_{t-1}\tau^k \\ &+ W_tl_t\tau^l + P_tC_t\tau_t^c + P_t\tau_t + P_tB_t.\end{aligned}$$

where G_t is government spending

Aggregation and Market Clearing All intermediate goods producers have the same capital to labor ratio and the same marginal cost. Therefore, aggregate output writes

$$Y_t = \varepsilon_t(u_t\bar{K}_{t-1})^{\alpha}(z_tl_t)^{1-\alpha} - \theta z_t^*.$$

Clearing in the goods market imposes

$$Y_t = G_t + C_t + \Upsilon^{-t}\mu_{\Upsilon,t}^{-t} I_t + a(u_t)\Upsilon^{-t}\bar{K}_{t-1}.$$

The GDP we read read from the data does not include the capital utilization cost:

$$Y_t^{\mathrm{gdp}} = G_t + C_t + \Upsilon^{-t}\mu_{\Upsilon,t}^{-t} I_t.$$

B.2 Equilibrium Conditions

In order to solve our model, we need to stationarize it. Scaled variables are as follows

$$\begin{aligned}
b_t &= B_t/z_t^*, & i_t &= I_t/(z_t^*\Upsilon^t), & r_t^k &= \Upsilon^t \tilde{r}_t^k, & y_{z,t} &= Y_t/z_t^*,\\
c_t &= C_t/z_t^*, & k_t &= \bar{K}_t/(z_t^*\Upsilon^t), & s_t &= S_t/P_t, & \mu_{z^*,t} &= z_t^*/z_{t-1}^*,\\
F_{w,t} &= F_{W,t} z_t^*, & \lambda_{z,t} &= \Lambda_{z,t} P_t z_t^*, & w_t &= W_t/(z_t^* P_t), & z_t^* &= z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}.\\
g_t &= G_t/z_t^*, & q_t &= Q_t \Upsilon^t / P_t, & y_t &= Y_t^{\mathrm{gdp}}/z_t^*,
\end{aligned}$$

Prices and Wages Optimal price and wage equations

$$\begin{aligned}
1 &= \frac{E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} \lambda_{z,t+s} y_{z,t+s} \left(\frac{\tilde{\pi}_{t+s}\cdots\tilde{\pi}_{t+1}}{\pi_{t+s}\cdots\pi_{t+1}}\right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{2\lambda_{p,t+s}-1}{1-\lambda_{p,t+s}}} \frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}} s_{t+s}}{E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \zeta_{u,t+s} \lambda_{z,t+s} y_{z,t+s} \left(\frac{\tilde{\pi}_{t+s}\cdots\tilde{\pi}_{t+1}}{\pi_{t+s}\cdots\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{p,t+s}}} \left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \frac{1}{1-\lambda_{p,t+s}}}\\
P_t &= \left[(1-\xi_p)\tilde{P}_t^{\frac{1}{1-\lambda_{p,t}}} + \xi_p \left(\tilde{\pi}_t P_{t-1}\right)^{\frac{1}{1-\lambda_{p,t}}}\right]^{1-\lambda_{p,t}}\\
F_{w,t} &= (1-\tau^l)\lambda_w^{-1}\zeta_{u,t}\lambda_{z,t} l_t + \xi_w \beta \mu_{z^*}^{\frac{1}{1-\lambda_w}} E_t \mu_{z^*,t+1}^{-1} \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}} \pi_{w,t+1}^{\frac{-\lambda_w}{1-\lambda_w}} \pi_{t+1}^{-1} F_{w,t+1}\\
K_{w,t} &= \zeta_{u,t} l_t^{1+\sigma_l} + \xi_w \beta E_t (\tilde{\pi}_{w,t+1}\pi_{w,t+1}^{-1}\mu_{z^*})^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} K_{w,t+1}\\
K_{w,t} &= \psi_l^{-1}\left\{\left[1-\xi_w(\tilde{\pi}_{w,t}\pi_{w,t}^{-1}\mu_{z^*})^{\frac{1}{1-\lambda_w}}\right](1-\xi_w)^{-1}\right\}^{1-\lambda_w(1+\sigma_l)} w_t F_{w,t}
\end{aligned}$$

Production Labor demand, capital demand, capital utilization, capital accumulation, return to capital, production function, resource constraint, GDP, and definition of real interest rate

$$\begin{aligned}
w_t &= (1-\alpha) s_t \varepsilon_t \left(u_t k_{t-1}\right)^{\alpha} \left(\mu_{z^*,t}\Upsilon l_t\right)^{-\alpha}.\\
r_t^k &= \alpha s_t \varepsilon_t (u_t k_{t-1})^{\alpha-1} (\mu_{z^*,t}\Upsilon l_t)^{1-\alpha}.\\
r_t^k &= r^k \exp(\sigma_a[u_t-1]).\\
k_t &= (1-\delta)\Upsilon^{-1}\mu_{z^*,t}^{-1} k_{t-1} + \left[1 - S\left(\zeta_{i,t}\Upsilon\mu_{z^*,t} i_t/i_{t-1}\right)\right] i_t.
\end{aligned}$$

$$
\begin{aligned}
R_t^k =& \left[(1-\tau^k)[u_t r_t^k - a(u_t)] + \tau^k\delta + (1-\delta)q_t\right]\Upsilon^{-1}q_{t-1}^{-1}\pi_t.\\
y_{z,t} =& \varepsilon_t(\Upsilon^{-1}\mu_{z^*,t}^{-1}u_t k_{t-1})^{\alpha} l_t^{1-\alpha} - \theta.\\
y_{z,t} =& g_t + c_t + \mu_{\Upsilon,t}^{-1}i_t + \Upsilon^{-1}\mu_{z^*,t}^{-1}a(u_t)k_{t-1}.\\
y_t =& g_t + c_t + \mu_{\Upsilon,t}^{-1}i_t.\\
R_t^r =& R_t/E_t\pi_{t+1}.
\end{aligned}
$$

Households Optimal consumption, bonds, capital, and investment

$$
\begin{aligned}
0=&\zeta_{u,t}\lambda_{z,t}(1+\tau_t^c) - \zeta_{u,t}\zeta_{c,t}\mu_{z^*,t}/(\mu_{z^*,t}c_t - b_c c_{t-1})\\
&+\beta b_c E_t\zeta_{u,t+1}\zeta_{c,t+1}/(\mu_{z^*,t+1}c_{t+1} - b_c c_t)\\
0=&\zeta_{u,t}\lambda_{z,t} - \beta E_t\zeta_{u,t+1}(\pi_{t+1}\mu_{z^*,t+1})^{-1}\lambda_{z,t+1}R_t\\
0=&\zeta_{u,t}\lambda_{z,t} - \beta E_t\zeta_{u,t+1}(\pi_{t+1}\mu_{z^*,t+1})^{-1}\lambda_{z,t+1}R_{t+1}^k\\
0=&\zeta_{u,t}\lambda_{z,t}q_t\left[1-S\left(\zeta_{i,t}\mu_{z^*,t}\Upsilon\frac{i_t}{i_{t-1}}\right)-\zeta_{i,t}\mu_{z^*,t}\Upsilon\frac{i_t}{i_{t-1}}S'\left(\zeta_{i,t}\mu_{z^*,t}\Upsilon\frac{i_t}{i_{t-1}}\right)\right]-\zeta_{u,t}\mu_{\Upsilon,t}^{-1}\lambda_{z,t}\\
&+\beta E_t\zeta_{u,t+1}(\mu_{z^*,t+1}\Upsilon)^{-1}\lambda_{z,t+1}q_{t+1}\zeta_{i,t+1}\left(\mu_{z^*,t+1}\Upsilon\frac{i_{t+1}}{i_t}\right)^2 S'\left(\zeta_{i,t+1}\mu_{z^*,t+1}\Upsilon\frac{i_{t+1}}{i_t}\right)
\end{aligned}
$$

Government Government debt accumulation

$$
g_t+(\pi_t\mu_{z^*,t})^{-1}R_{t-1}b_{t-1}=[u_t r_t^k - a(u_t)-\delta]\Upsilon^{-1}\mu_{z^*,t}^{-1}k_{t-1}\tau^k + w_t l_t\tau^l + c_t\tau_t^c + \tau_t + b_t.
$$

Flexible-Price Equilibrium For the medium-scale model, we define the flexible-price equilibrium as the equilibrium that would prevail if prices were flexible ($\xi_p = 0$), wages were flexible ($\xi_w = 0$), and if policy variables were set as in the non-flexible equilibrium, i.e., $\tau_t^{c,f} = \tau_t^c$ and $g_t^f = g_t$. Following (Smets and Wouters (2007)), there are also no markup shocks ($\varepsilon_{p,t} = \varepsilon_p$ in our definition of flexible-price equilibrium. The flexible-price equilibrium equations are:

$$
\begin{aligned}
s_t^f =& \lambda_p^{-1}\\
w_t^f =& \frac{\lambda_w\psi_l}{(1-\tau^l)}\frac{l_t^{f,\sigma_l}}{\lambda_{z,t}^f}\\
w_t^f =& (1-\alpha)\lambda_p^{-1}\varepsilon_t\left(u_t^f k_{t-1}^f\right)^{\alpha}\left(\mu_{z^*,t}\Upsilon l_t^f\right)^{-\alpha}\\
r_t^{k,f} =& \alpha\lambda_p^{-1}\varepsilon_t(u_t^f k_{t-1}^f)^{\alpha-1}(\mu_{z^*,t}\Upsilon l_t^f)^{1-\alpha}\\
r_t^{k,f} =& r^k\exp(\sigma_a[u_t^f - 1])\\
k_t^f =& (1-\delta)\Upsilon^{-1}\mu_{z^*,t}^{-1}k_{t-1}^f + [1 - S(\zeta_{i,t}\Upsilon\mu_{z^*,t}i_t^f/i_{t-1}^f)]i_t^f
\end{aligned}
$$

$$
\begin{aligned}
R_t^{k,f} =& \left[(1-\tau^k)[u_t^f r_t^{k,f} - a(u_t^f)] + \tau^k\delta + (1-\delta)q_t^f\right]\Upsilon^{-1}q_{t-1}^{f,-1}\pi \\
y_{z,t}^f =& \varepsilon_t(\Upsilon^{-1}\mu_{z^*,t}^{-1}u_t^f k_{t-1}^f)^\alpha l_t^{f,1-\alpha} - \theta \\
y_{z,t}^f =& g_t + c_t^f + \mu_{\Upsilon,t}^{-1}i_t^f + \Upsilon^{-1}\mu_{z^*,t}^{-1}a(u_t^f)k_{t-1}^f \\
y_t^f =& g_t + c_t^f + \mu_{\Upsilon,t}^{-1}i_t^f \\
0 =& \zeta_{u,t}\lambda_{z,t}^f(1+\tau_t^c) - \zeta_{u,t}\zeta_{c,t}\mu_{z^*,t}/(\mu_{z^*,t}c_t^f - b_c c_{t-1}^f) \\
& + \beta b_c E_t\zeta_{u,t+1}\zeta_{c,t+1}/(\mu_{z^*,t+1}c_{t+1}^f - b_c c_t^f) \\
0 =& \zeta_{u,t}\lambda_{z,t}^f - \beta E_t\zeta_{u,t+1}(\pi\mu_{z^*,t+1})^{-1}\lambda_{z,t+1}^f R_{t+1}^{k,f} \\
\zeta_{u,t}\mu_{\Upsilon,t}^{-1}\lambda_{z,t}^f =& \zeta_{u,t}\lambda_{z,t}^f q_t^f\left[1 - S\left(\zeta_{i,t}\mu_{z^*,t}\Upsilon\frac{i_t^f}{i_{t-1}^f}\right) - \zeta_{i,t}\mu_{z^*,t}\Upsilon\frac{i_t^f}{i_{t-1}^f}S'\left(\zeta_{i,t}\mu_{z^*,t}\Upsilon\frac{i_t^f}{i_{t-1}^f}\right)\right] \\
& + \beta E_t\zeta_{u,t+1}\left(\mu_{z^*,t+1}\Upsilon\right)^{-1}\lambda_{z,t+1}^f q_{t+1}^f\zeta_{i,t+1}\left(\mu_{z^*,t+1}\Upsilon\frac{i_{t+1}^f}{i_t^f}\right)^2 S'\left(\zeta_{i,t+1}\mu_{z^*,t+1}\Upsilon\frac{i_{t+1}^f}{i_t^f}\right) \\
\tilde{y}_t =& y_t/y_t^f
\end{aligned}
$$

Auxiliary Expressions Price and wage indexation, wage inflation, utilization cost, and investment adjustment cost

$$
\begin{aligned}
\tilde{\pi}_t &= \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}. \\
\tilde{\pi}_{w,t} &= \pi_{t-1}^{\iota_w}\pi^{1-\iota_w}. \\
\pi_{w,t} &= \pi_t\mu_{z^*,t}w_t/w_{t-1}. \\
a(u_t) &= r^k(\exp[\sigma_a(u_t-1)]-1)\frac{1}{\sigma_a}. \\
S(\zeta_{i,t}\mu_{z^*,t}\Upsilon i_t/i_{t-1}) &= e^{\sqrt{\frac{S''}{2}}\Upsilon\left(\zeta_{i,t}\mu_{z^*,t}\frac{i_t}{i_{t-1}}-\mu_{z^*}\right)} + e^{-\sqrt{\frac{S''}{2}}\Upsilon\left(\zeta_{i,t}\mu_{z^*,t}\frac{i_t}{i_{t-1}}-\mu_{z^*}\right)} - 2.
\end{aligned}
$$

B.3 Steady State

In steady state, $\pi^* = \tilde{\pi} = \pi$ and $u = 1$. Normalize $l = 1$.

$$
\begin{aligned}
s =& \frac{1}{\lambda_p}. \\
q =& 1. \\
R =& \frac{\pi\mu_{z^*}}{\beta}. \\
R^r =& \frac{R}{\pi}. \\
R^k =& R.
\end{aligned}
$$

$$
\begin{aligned}
r^k =& \frac{\frac{R^k\Upsilon}{\pi} - 1}{1-\tau^k} + \delta. \\
k =& \left(\frac{r^k}{\alpha s}\right)^{\frac{1}{\alpha-1}}\Upsilon\mu_{z^*}. \\
i =& \left(1 - \frac{1-\delta}{\Upsilon\mu_{z^*}}\right)k. \\
\theta =& (1-s)\left(\frac{k}{\Upsilon\mu_{z^*}}\right)^\alpha.
\end{aligned}
$$

$$b = \frac{\frac{(r_t^k-\delta)k\tau^k}{\Upsilon\mu_{z^*}} + w\tau^l + c\tau^c + \tau - g}{\frac{R}{\pi\mu_{z^*}} - 1}.$$

$$y_z = \left(\frac{k}{\Upsilon\mu_{z^*}}\right)^{\alpha} - \theta.$$
$$y = y_z.$$
$$g = \eta y.$$
$$c = y - g - i.$$

$$w = (1-\alpha)sk^{\alpha}\left(\mu_{z^*}\Upsilon\right)^{-\alpha}.$$
$$\lambda_z = \frac{\mu_{z^*} - \beta b_c}{(\mu_{z^*} - b_c)\,c}\frac{1}{1+\tau^c}.$$
$$F_w = \frac{\lambda_z(1-\tau^l)}{\lambda_w(1-\xi_w\beta)}.$$
$$K_w = \frac{1}{1-\xi_w\beta}.$$
$$\psi_l = w\frac{F_w}{K_w} = \frac{\lambda_z(1-\tau^l)w}{\lambda_w}.$$

B.4 Loglinear Equilibrium

Prices and wages

$$\hat{\pi}_t = \hat{\bar{\pi}}_t + \beta\left(E_t\hat{\pi}_{t+1} - E_t\hat{\bar{\pi}}_{t+1}\right) + \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p}\left(\hat{s}_t + \hat{\lambda}_{p,t}\right). \tag{B-1}$$

$$\begin{aligned}
\hat{F}_{w,t} =& (1-\xi_w\beta)\left(\hat{l}_t + \hat{\lambda}_{z,t} + \hat{\zeta}_{u,t}\right) \\
&+ \xi_w\beta E_t\left(\hat{F}_{w,t+1} - \hat{\pi}_{t+1} + \tfrac{1}{1-\lambda_w}\hat{\bar{\pi}}_{w,t+1} - \tfrac{\lambda_w}{1-\lambda_w}\hat{\pi}_{w,t+1} - \hat{\mu}_{z^*,t+1}\right). \\
\hat{K}_{w,t} =& (1-\xi_w\beta)\Big[(1+\sigma_l)\hat{l}_t + \hat{\zeta}_{u,t}\Big] + \xi_w\beta E_t\Big[\hat{K}_{w,t+1} + \tfrac{\lambda_w(1+\sigma_l)}{1-\lambda_w}\left(\hat{\bar{\pi}}_{w,t+1} - \hat{\pi}_{w,t+1}\right)\Big]. \\
\hat{K}_{w,t} =& \hat{w}_t + \hat{F}_{w,t} - \tfrac{1-\lambda_w(1+\sigma_l)}{1-\lambda_w}\tfrac{\xi_w}{1-\xi_w}(\hat{\bar{\pi}}_{w,t} - \hat{\pi}_{w,t}). \\
\hat{w}_t =& (1-\xi_w\beta)\left(\sigma\hat{l}_t - \hat{\lambda}_{z,t}\right) + \tfrac{\xi_w}{1-\xi_w}\tfrac{1-\lambda_w(1+\sigma_l)}{1-\lambda_w}\left(\hat{\bar{\pi}}_{w,t} - \hat{\pi}_{w,t}\right) \\
&+ \xi_w\beta E_t\left(\hat{w}_{t+1} + \hat{\pi}_{t+1} + \hat{\mu}_{z^*,t+1}\right) - \xi_w\beta\tfrac{1-\lambda_w(1+\sigma_l)}{(1-\lambda_w)(1-\xi_w)}E_t\hat{\bar{\pi}}_{w,t+1} \\
&+ \xi_w\beta\tfrac{\xi_w - \lambda_w\sigma_l - \xi_w\lambda_w}{(1-\lambda_w)(1-\xi_w)}E_t\hat{\pi}_{w,t+1}.
\end{aligned} \tag{B-2}$$

Production

$$\hat{w}_t = \alpha\left(\hat{u}_t + \hat{k}_{t-1} - \hat{l}_t - \hat{\mu}_{z^*,t}\right) + \hat{s}_t + \hat{\varepsilon}_t. \tag{B-3}$$
$$\hat{r}_t^k = (1-\alpha)\left(\hat{l}_t - \hat{u}_t - \hat{k}_{t-1} + \hat{\mu}_{z^*,t}\right) + \hat{s}_t + \hat{\varepsilon}_t. \tag{B-4}$$
$$\hat{r}_t^k = \sigma_a\hat{u}_t. \tag{B-5}$$
$$\hat{k}_t = \tfrac{1-\delta}{\Upsilon\mu_{z^*}}\left(\hat{k}_{t-1} - \hat{\mu}_{z^*,t}\right) + \left(1 - \tfrac{1-\delta}{\Upsilon\mu_{z^*}}\right)\hat{i}_t. \tag{B-6}$$
$$\hat{R}_t^k = \tfrac{(1-\tau^k)r^k}{(1-\tau^k)(r^k-\delta)+1}\hat{r}_t^k + \tfrac{1-\delta}{(1-\tau^k)(r^k-\delta)+1}\hat{q}_t + \hat{\pi}_t - \hat{q}_{t-1}. \tag{B-7}$$
$$\hat{y}_{z,t} = \tfrac{y_z+\theta}{y_z}\left[\alpha\hat{u}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{l}_t - \alpha\hat{\mu}_{z^*,t} + \hat{\varepsilon}_t\right]. \tag{B-8}$$
$$\hat{y}_{z,t} = \tfrac{g}{y_z}\hat{g}_t + \tfrac{c}{y_z}\hat{c}_t + \tfrac{i}{y_z}\left(\hat{i}_t - \hat{\mu}_{\Upsilon,t}\right) + \tfrac{r^k k}{\Upsilon\mu_{z^*}y_z}\hat{u}_t. \tag{B-9}$$
$$\hat{y}_t = \tfrac{g}{y}\hat{g}_t + \tfrac{c}{y}\hat{c}_t + \tfrac{i}{y}(\hat{i}_t - \hat{\mu}_{\Upsilon,t}). \tag{B-10}$$
$$\hat{r}_t = \hat{r}_t^r + E_t\hat{\pi}_{t+1}. \tag{B-11}$$

Households

$$0 =(\mu_{z^*}-b_c)(\mu_{z^*}-\beta b_c)\left(\hat{\lambda}_{z,t}+\tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c\right)-b_c\mu_{z^*}\hat{c}_{t-1}+\left(\mu_{z^*}^2+\beta b_c^2\right)\hat{c}_t-\beta b_c\mu_{z^*}E_t\hat{c}_{t+1}$$
$$+b_c\mu_{z^*}\hat{\mu}_{z^*,t}-\beta b_c\mu_{z^*}E_t\hat{\mu}_{z^*,t+1}-\beta b_c(\mu_{z^*}-b_c)\left(\hat{\zeta}_{u,t}-E_t\hat{\zeta}_{u,t+1}+\tfrac{\mu_{z^*}}{\beta b_c}\hat{\zeta}_{c,t}-E_t\hat{\zeta}_{c,t+1}\right) \quad \text{(B-12)}$$

$$0 =\hat{\lambda}_{z,t} - \hat{R}_t + E_t\hat{\pi}_{t+1} - E_t\hat{\lambda}_{z,t+1} + \hat{\zeta}_{u,t} - E_t\hat{\zeta}_{u,t+1} + E_t\hat{\mu}_{z^*,t+1} \quad \text{(B-13)}$$

$$0 =\hat{\lambda}_{z,t} - E_t\hat{\lambda}_{z,t+1} + E_t\hat{\pi}_{t+1} - E_t\hat{R}_{t+1}^k + \hat{\zeta}_{u,t} - E_t\hat{\zeta}_{u,t+1} + E_t\hat{\mu}_{z^*,t+1} \quad \text{(B-14)}$$

$$\hat{q}_t+\hat{\mu}_{\Upsilon,t}=(\Upsilon\mu_{z^*})^2S''\left[\hat{\mu}_{z^*,t}-\beta E_t\hat{\mu}_{z^*,t+1}-\hat{i}_{t-1}+(1+\beta)\hat{i}_t-\beta E_t\hat{i}_{t+1}+\hat{\zeta}_{i,t}-\beta E_t\hat{\zeta}_{i,t+1}\right] \quad \text{(B-15)}$$

Government

$$g\hat{g}_t + \tfrac{Rb}{\pi\mu_{z^*}}\left(\hat{r}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t - \hat{\mu}_{z^*,t}\right) = \tfrac{\tau^k(r^k-\delta)k}{\Upsilon\mu_{z^*}}\left(\tfrac{r^k}{r^k-\delta}\hat{r}_t^k + \hat{k}_{t-1} - \hat{\mu}_{z^*,t}\right)$$
$$+\tau^l w\left(\hat{w}_t+\hat{l}_t\right)+\tau^c c\left(\hat{c}_t+\hat{\tau}_t^c\right)+\tau\hat{\tau}_t+b\hat{b}_t. \quad \text{(B-16)}$$

Flexible-price equilibrium

$$\hat{w}_t^f = \sigma_l\hat{l}_t^f - \hat{\lambda}_{z,t}^f \quad \text{(B-17)}$$

$$\hat{w}_t^f = \alpha\left(\hat{u}_t^f + \hat{k}_{t-1}^f - \hat{l}_t^f - \hat{\mu}_{z^*,t}\right) + \hat{\varepsilon}_t \quad \text{(B-18)}$$

$$\hat{r}_t^{k,f} = (1-\alpha)\left(\hat{l}_t^f - \hat{u}_t^f - \hat{k}_{t-1}^f + \hat{\mu}_{z^*,t}\right) + \hat{\varepsilon}_t \quad \text{(B-19)}$$

$$\hat{r}_t^{k,f} = \sigma_a\hat{u}_t^f. \quad \text{(B-20)}$$

$$\hat{k}_t^f = \tfrac{1-\delta}{\Upsilon\mu_{z^*}}\left(\hat{k}_{t-1}^f - \hat{\mu}_{z^*,t}\right) + \left(1-\tfrac{1-\delta}{\Upsilon\mu_{z^*}}\right)\hat{i}_t^f \quad \text{(B-21)}$$

$$\hat{R}_t^{k,f} = \tfrac{(1-\tau^k)r^k}{(1-\tau^k)(r^k-\delta)+1}\hat{r}_t^{k,f} + \tfrac{1-\delta}{(1-\tau^k)(r^k-\delta)+1}\hat{q}_t^f - \hat{q}_{t-1}^f \quad \text{(B-22)}$$

$$\hat{y}_{z,t}^f = \tfrac{y_z+\theta}{y_z}\left[\alpha\hat{u}_t^f + \alpha\hat{k}_{t-1}^f + (1-\alpha)\hat{l}_t^f - \alpha\hat{\mu}_{z^*,t} + \hat{\varepsilon}_t\right] \quad \text{(B-23)}$$

$$\hat{y}_{z,t}^f = \tfrac{g}{y_z}\hat{g}_t + \tfrac{c}{y_z}\hat{c}_t^f + \tfrac{i}{y_z}\left(\hat{i}_t^f - \hat{\mu}_{\Upsilon,t}\right) + \tfrac{r^k k}{\Upsilon\mu_{z^*}y_z}\hat{u}_t^f \quad \text{(B-24)}$$

$$\hat{y}_t^f = \tfrac{g}{y}\hat{g}_t + \tfrac{c}{y}\hat{c}_t^f + \tfrac{i}{y}\left(\hat{i}_t^f - \hat{\mu}_{\Upsilon,t}\right) \quad \text{(B-25)}$$

$$0=(\mu_{z^*}-b_c)(\mu_{z^*}-\beta b_c)\left(\hat{\lambda}_{z,t}^f+\tfrac{\tau^c}{1+\tau^c}\hat{\tau}_t^c\right)-b_c\mu_{z^*}\hat{c}_{t-1}^f+\left(\mu_{z^*}^2+\beta b_c^2\right)\hat{c}_t^f-\beta b_c\mu_{z^*}E_t\hat{c}_{t+1}^f$$
$$+b_c\mu_{z^*}\hat{\mu}_{z^*,t}-\beta b_c\mu_{z^*}E_t\hat{\mu}_{z^*,t+1}-\beta b_c(\mu_{z^*}-b_c)\left(\hat{\zeta}_{u,t}-E_t\hat{\zeta}_{u,t+1}+\tfrac{\mu_{z^*}}{\beta b_c}\hat{\zeta}_{c,t}-E_t\hat{\zeta}_{c,t+1}\right) \quad \text{(B-26)}$$

$$0 = \hat{\lambda}_{z,t}^f - E_t\hat{\lambda}_{z,t+1}^f - E_t\hat{R}_{t+1}^{k,f} + \hat{\zeta}_{u,t} - E_t\hat{\zeta}_{u,t+1} + E_t\hat{\mu}_{z^*,t+1} \quad \text{(B-27)}$$

$$\hat{q}_t^f+\hat{\mu}_{\Upsilon,t}=(\Upsilon\mu_{z^*})^2S''\left[\hat{\mu}_{z^*,t}-\beta E_t\hat{\mu}_{z^*,t+1}-\hat{i}_{t-1}^f+(1+\beta)\hat{i}_t^f-\beta E_t\hat{i}_{t+1}^f+\hat{\zeta}_{i,t}-\beta E_t\hat{\zeta}_{i,t+1}\right] \quad \text{(B-28)}$$

$$\hat{\hat{y}}_t = \hat{y}_t - \hat{y}_t^f. \quad \text{(B-29)}$$

Auxiliary expressions

$$\hat{\bar{\pi}}_t = \iota_p\hat{\pi}_{t-1}. \quad \text{(B-30)}$$

$$\hat{\tilde{\pi}}_{w,t} = \iota_w \hat{\pi}_{t-1}. \tag{B-31}$$

$$\hat{\pi}_{w,t} = \hat{\pi}_t + \hat{w}_t - \hat{w}_{t-1} + \hat{\mu}_{z^*,t}. \tag{B-32}$$

C
Additional Comparative Simulations

In this appendix we repeat the first simulation presented in Chapter 3, still requiring the model to replicate the historical values for output and inflation and using the same FTR policy parameters, but assuming the economy was driven by different pairs of demand/supply shocks in order to get an idea of the robustness of that result to the choice of shocks.

Figure C.1 shows the macroeconomic outcomes for both models when technology and investment shocks are considered. In this case, the FTR results in 27% lower output volatility and 35% lower inflation volatility relative to the standard model, while maintaining the consumption tax rate inside a fairly small range.

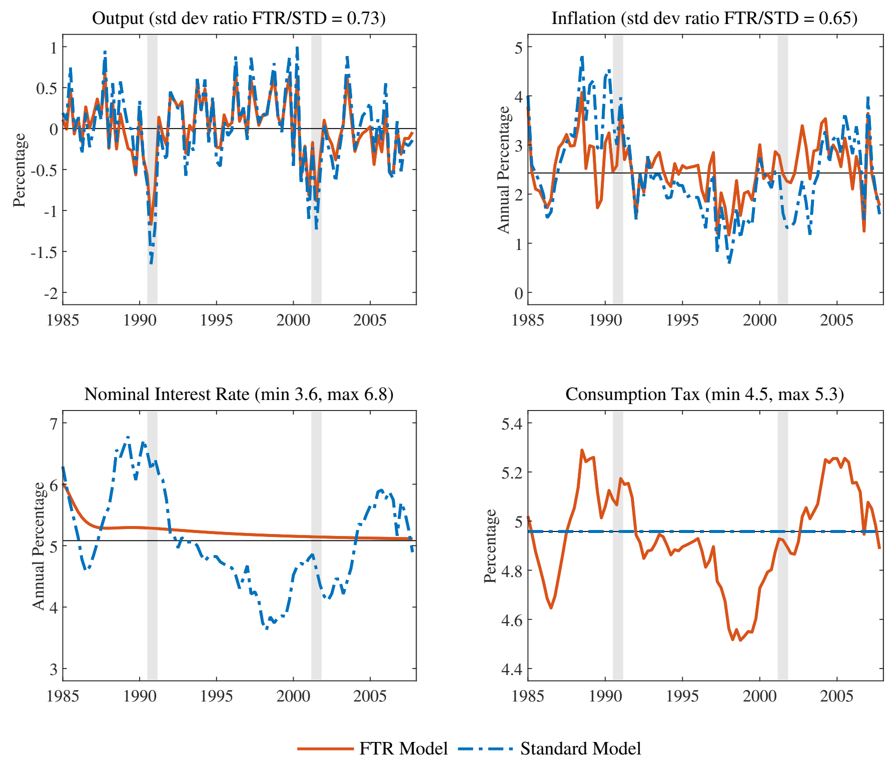

Figure C.1: Technology and Investment Shocks

Figure C.2 repeats the exercise with price markup and consumption preference shocks. In this case, the FTR results in 39% higher output volatility and 6% lower inflation volatility relative to the standard model.

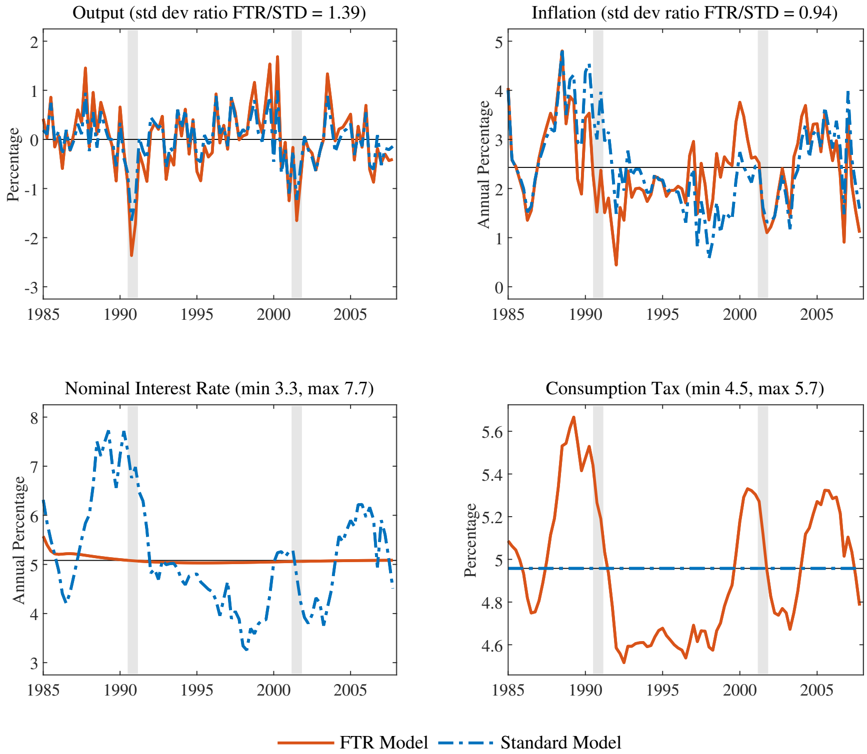

Figure C.2: Price Markup and Consumption Preference Shocks

Figure C.3 presents the results using technology and consumption preference shocks. In this case, the FTR results in 37% higher output volatility and 5% lower inflation volatility relative to the standard model.

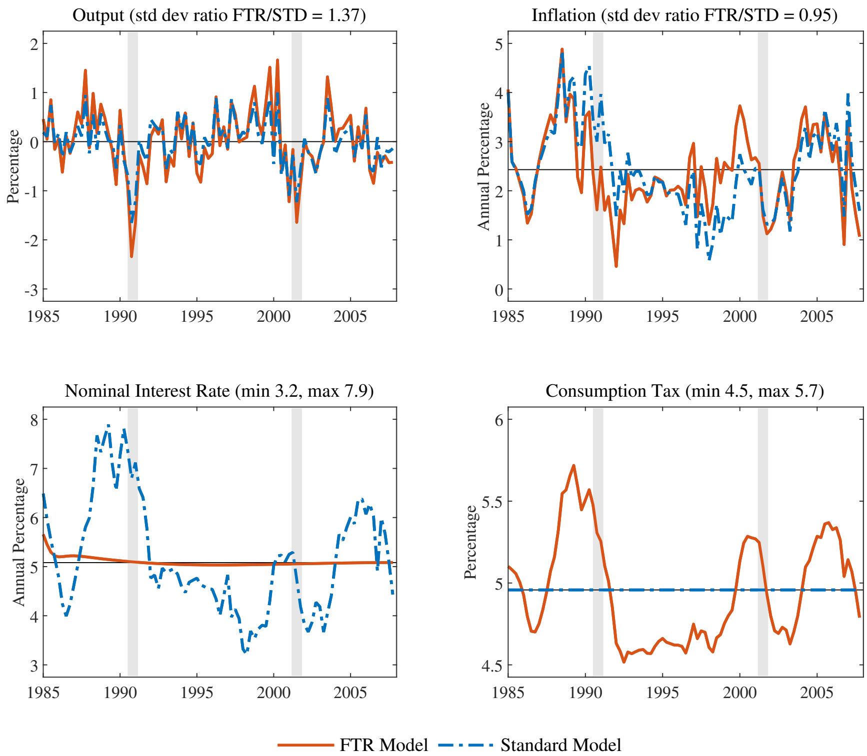

Figure C.3: Technology and Consumption Preference Shocks

Figure C.4 presents the results using technology and government spending shocks. In this case, the FTR results in 39% lower output volatility and 38% lower inflation volatility relative to the standard model.

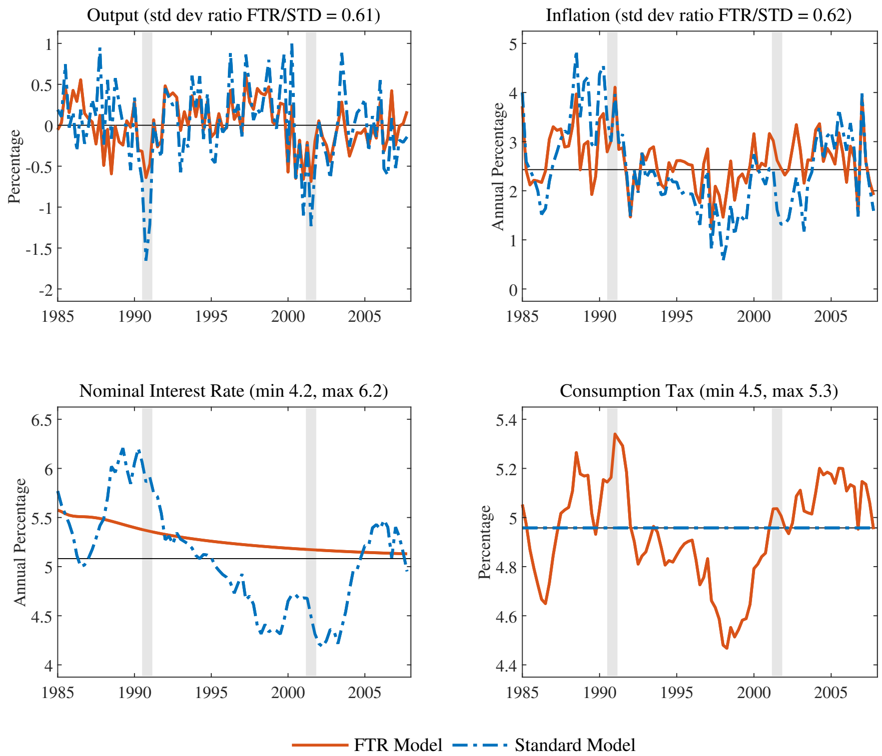

Figure C.4: Technology and Government Spending Shocks

Figure C.5 presents the results using price markup and government spending shocks. In this case, the FTR results in 34% lower output volatility and 25% lower inflation volatility relative to the standard model.

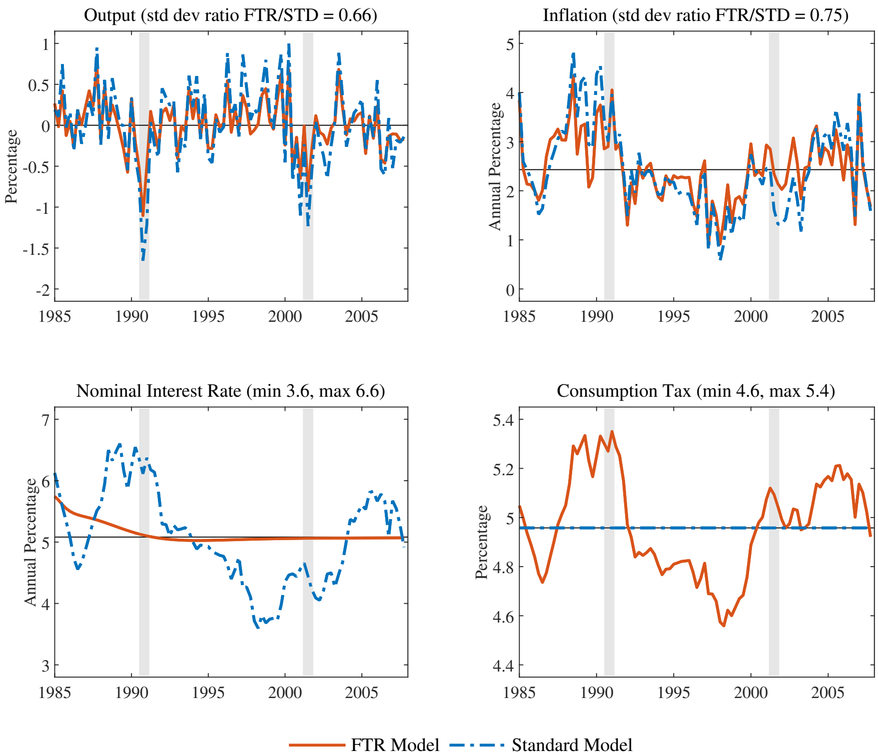

Figure C.5: Price Markup and Government Spending Shocks