Coordinated Strategic Defaults and Financial Fragility in a Costly State Verification Model[1]

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Abstract

Diversification through a financial intermediary has the benefit of transforming loans that need costly monitoring into bank deposits that do not. We show, however, that financial intermediation in a costly state verification model has a cost not yet analyzed: it allows for the existence of multiple equilibria, some of which are characterized by borrowers defaulting on their loans because they expect other borrowers to do the same (i.e. bad equilibria arise due to strategic complementarities in entrepreneurs' actions). We propose two mechanisms that fully implement the desired equilibrium allocation.

Keywords: Costly State Verification, Multiple Equilibria, Coordinated Default, Full Implementation.

J.E.L. Classifications: D82 (Asymmetric and Private Information), D86 (Economics of Contract Theory), G21 (Banks; Other Depository Institutions), M42 (Auditing).

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1 Introduction

A run on a bank takes place when a large number of its clients simultaneously renege on its services, promoting its disintermediation and occasionally its demise. The most common form of bank run occurs when depositors rush to withdraw their money because they fear the bank will be unable to honor all its liabilities at par. In this paper, we explore a different form of bank run, that which originates at the bank's asset side when a borrower defaults on his loan because he expects other borrowers to do the same. We refer to such a situation as a *coordinated strategic default*.[2]

There is evidence that coordinated strategic defaults occur across a variety of institutional arrangements. Krueger and Tornell (1999) document how the lack of transparent and effective bankruptcy procedures in Mexico during the 1995 crises led many borrowers to default, despite their full capacity to service their debt. Another case is Childreach, a microfinance program in Ecuador. According to Goering and Marx (1998) the program collapsed when "*the number of residents defaulting on their loans multiplied as the word spread that few people were paying*". Even the US, arguably the world's most financially developed country, has not gone unscathed. Guiso et al. (2012) document how households with underwater mortgages are more likely to strategically default on their loans if they are acquainted with someone who is also defaulting strategically.[3]

We analyze the issue of coordinated strategic defaults in a canonical model of entrepreneurial finance characterized by costly state verification (Townsend (1979); Gale and Hellwig (1985)).[4] In the model, a financial intermediary lends to a continuum of entrepreneurs at contractual

[2] A strategic default occurs when the borrower has the financial means to pay off his debt, but chooses not to. It is thus an issue of the borrower's willingness to pay, not of his capability to do so.

[3] Bond and Rai (2009) present more evidence on coordinated strategic defaults in microfinance programs, while Vlahu (2008) focuses on corporate credit in Eastern Europe and Asia.

[4] The main feature of costly state verification models is that the entrepreneur observes his project's return free of charge, while the financial intermediary must perform a costly audit to become informed.

terms endogenously chosen. To derive the optimal contract, we initially adopt the traditional mechanism design approach according to which the designer proposes a Bayesian game that has among its possibly many equilibria one that maximizes a predefined value criterion.[5] In this setting, we show that in the *good* equilibrium a standard debt contract provides entrepreneurs with incentives to repay their loan whenever they can while minimizing monitoring costs.

However, while repayment is *one* equilibrium of the optimal financial arrangement through which the bank finances projects (standard debt contract), it is not the only one. In the model the default by a group of debtors weakens the bank's financial position and hurts its monitoring capabilities, which ultimately makes the decision to default by any other entrepreneur more attractive. Such strategic complementarities in entrepreneurs' actions lead to multiplicity of equilibria. In some of them, a debtor declares default because he expects other debtors to do the same.

We establish that, apart from the good equilibrium, there is always an equilibrium in which *all* entrepreneurs default strategically. We refer to such an outcome as a *fully coordinated default*. One may argue that, due to communication and coordination costs, joint deviations by the whole set of entrepreneurs are not particularly worrisome. Nevertheless, we show that *partially coordinated default* equilibria always exist as well. In these equilibria, although some entrepreneurs repay their debt, a non-negligible subset of entrepreneurs default strategically.

In addition to establishing that banks may fall victim to coordinated defaults in a canonical model of financial contracting, the second goal of this paper is to consider alternatives banks may have to rule out these *bad* equilibria. We propose two main solutions, both of them sharing the following features: (i) to break the strategic complementarities among borrowers the bank must use what we call a *sequential audit strategy* and (ii) to be able to audit a given group that plays a special role in the sequential audit strategy the bank must secure a given

[5] Townsend (1979) and Gale and Hellwig (1985) also use the mechanism design approach, which implicitly assumes that when multiple equilibria are present the desired one is chosen.

amount of resources. The solutions differ mainly in the way the bank secures such resources.

The sequential audit strategy is implemented as follows. The bank first divides entrepreneurs into groups, which are then randomly ordered. Once the bank starts auditing, it does so sequentially, auditing entrepreneurs in group $n+1$ only after it has audited all defaulted projects in group n. *If* the bank can fully commit to audit entrepreneurs in the first group, such entrepreneurs find it optimal to report truthfully regardless of the announcements made by other entrepreneurs. With the payments collected from entrepreneurs belonging to the first group the bank's monitoring resources increase and it can credibly commit to audit entrepreneurs in the second group as well. Proceeding inductively, we show that coordinated strategic defaults unravel and the good equilibrium is restored.

Together, sequentiality and asymmetric treatment of ex-ante identical individuals are central features of some theoretical models of bank runs. In Diamond and Dybvig (1983), multiplicity of equilibria arises because ex-ante identical depositors are treated asymmetrically, according to a first-come first-serve basis (Jacklin (1987); Wallace (1996)). In this paper, however, multiplicity of equilibria is eliminated precisely when the bank starts to treat ex-ante identical borrowers asymmetrically and sequentially.

Finally, we propose two ways the bank can kick-start the sequential audit strategy by securing the needed resources to audit entrepreneurs in the first group with certainty. First, the bank can set aside a small amount of capital ex-ante. While we assume that the bank incurs in an opportunity cost of hoarding cash, we show that the bank's capital buffer can be arbitrarily small. Furthermore, capital hoarding guarantees truth-telling on the part of entrepreneurs through a process that resembles the iterative deletion of strictly dominated strategies proposed by Bergemann and Morris (2009). As a consequence, truth-telling is the only rationalizable strategy for the entrepreneurs and, as Bergemann and Morris (2009) call it, implementation is robust.

The second way for the bank to secure the initial necessary resources is by contracting with the entrepreneur through a debt contract coupled with a properly designed forgiveness clause. We show that no capital needs to be put aside to implement this solution, so it is less costly than the first one (in fact, it involves no cost whatsoever). The adding of a forgiveness clause has a drawback, however, in that the strategy adopted by each individual entrepreneur now depends on his correct beliefs about others' default intentions, so robustness in entrepreneurs' decision-making process is compromised.

The rest of this paper is structured as follows: We review the related literature in Section 2, lay down the model and establish the existence of coordinated strategic defaults in costly state verification models in Section 3. Section 4 introduces the sequential audit strategy and presents two possible solutions to the problem of bad equilibria. Section 5 discusses the validity of our results under alternative modeling assumptions and Section 6 concludes.

2 Related Literature

In this work, we draw on a diverse array of papers. We extend the costly state verification environment developed in Townsend (1979) and Gale and Hellwig (1985) to the case in which a single financial intermediary lending to a continuum of entrepreneurs has a limited monitoring capacity due to budgetary issues. Like Diamond (1984), we show that delegation reduces the costs of monitoring a set of fully diversified loans, but in our paper it also exposes the bank to the possibility of coordinated strategic defaults.

Other papers also consider the existence of runs on the asset side of a financial intermediary (Vlahu (2008); Bond and Rai (2009)). However, they differ with ours on many accounts. For example, Vlahu (2008) and Bond and Rai (2009) adopt a global games framework and prove multiplicity of equilibria in repayment behavior, but do not derive optimal financial contracts. In Bond and Rai (2009), repayment incentives stem from the prospect of receiving future credit. By contrast, our results do not rely on inter-temporal incentives, but on whether bankruptcy procedures are such that a bank must have a minimum of resources to collect its loans.[6]

An extensive theoretical literature focuses on strategic complementarity as a source of multiplicity of equilibria. In Farhi and Tirole (2012), the inability of authorities to commit not to bail out financial institutions after the realization of a negative shock creates, ex-ante, complementarities in their choice of leverage. On the other hand, in our paper, entrepreneurs' actions are complements because of the bank's potential inability to monitor all the projects it finances. We share with Silva and Kahn (1993), Bassetto and Phelan (2008), and Bond and Hagerty (2010) the idea that bad equilibria may result from the principal's limited resources to discipline agents. Silva and Kahn (1993) examine the optimal provision of a public good for which exclusion is possible, but imperfect. They show that if a sufficient number of agents in the economy free-ride the public good then it is desirable to free-ride as well, because the probability of being caught and punished is low. In a crime prevention setting, Bond and Hagerty (2010) analyze how the optimal punishment intensity varies along the various existing equilibria without addressing implementation issues. Bassetto and Phelan (2008) study optimal taxation and show that when the tax authority can only audit a fixed proportion of households due to a budget constraint the optimal mechanism also has equilibria in which households misreport. In our paper, the amount of resources the bank has for auditing purposes is endogenous, as proceeds collected from entrepreneurs who pay up can be used to audit other entrepreneurs. This feature, which is only present in our work, plays a crucial role in all solutions we propose to eliminate bad equilibria.[7]

[6] We have learned a great deal about the functioning of microfinance institutions from Bond and Rai (2009) and their article served as a useful guide to many episodes of coordinated strategic defaults.

[7] There is also empirical evidence supporting the existence of strategic complementarities in borrowers' default decisions. Using survey data on US households, Guiso et al. (2012) document that an agent who is acquainted with someone who has defaulted strategically is more likely to declare his intention to default strategically as well.

Many other papers propose modifications to the standard costly state verification model of Townsend (1979) and Gale and Hellwig (1985). Border and Sobel (1987) and Mookherjee and Png (1989) formally consider the possibility of random audits; Krasa and Villamil (2000) analyze the case in which audits must be sequentially rational; Lacker and Weinberg (1989) propose a model in which the agent can fabricate cash-flows; and Winton (1995) considers a single entrepreneur contracting with many investors that have different degrees of seniority. None of these papers address the implications of the principal's limited resources on its ability to audit and the resulting incentives for entrepreneurs to default.

Finally, our paper also relates to the mechanism design literature regarding full and robust implementation. The mechanism designer obtains full implementation when agents' behaviors lead to the desired outcome in *every* equilibria of the game induced by the mechanism. Our analysis shows that, with a continuum of entrepreneurs (and a symmetric audit strategy), a standard debt contract only *partially* implements the desired allocation. Another relevant issue is robustness, a measure of the complexity of the agents' decision process. Our sequential audit strategy induces truth-telling from entrepreneurs in a way that is related to the ideas of robust implementation put forth in Bergemann and Morris (2009).

3 The Model

We extend the model of Gale and Hellwig (1985) in two directions. First we assume that a unique investor lends to a continuum of entrepreneurs. Subsequently, we introduce a limit to the investor's monitoring capacity and analyze how entrepreneurs' repayment incentives are affected.

3.1 A Continuum of Entrepreneurs

There are two periods. At date 0 each of a continuum of identical entrepreneurs is endowed with a production technology which requires an initial investment of $I > 0$. Entrepreneurs have no wealth of their own, so they must borrow from a wealthy investor to undertake the project. Projects are risky and returns are i.i.d. across entrepreneurs, so the possibility of diversification makes it optimal for a unique agent to assume the role of a financial intermediary, as in Diamond (1984). This agent, who is delegated the task of monitoring the credit it extends, will be called the *bank* throughout.

At date 1 production is realized yielding a total output of $f(s)$ when the state s is realized. We assume that $f(0) = 0$ and $\partial f(s)/\partial s > 0$, so that states are ordered with higher states implying higher returns. The probability distribution of the states is given by an absolutely continuous cumulative distribution function H, with density h and support in a compact interval $[0, \overline{s}]$. Capital markets are perfectly competitive so the bank's expected profit is zero. Without loss of generality, the interest rate is normalized to zero and the mass of entrepreneurs normalized to 1.

We adopt the standard assumptions of the costly state verification model regarding the asymmetry of information between entrepreneurs and the bank. At date 1, while each entrepreneur observes the return of his own project free of charge, the bank must bear an audit cost of $c(s)$ to become informed, where $\partial c(s)/\partial s \geq 0$.[8] The audit is usually interpreted as the process of determining an inventory of the debtor's assets and liabilities, such as a bankruptcy procedure.

We follow Gale and Hellwig (1985) and assume that if the entrepreneur defaults on his loan the bank can impose on him a constant non pecuniary cost of c_0. We do not require that

[8] The existence of strategic defaults does not depend on the assumption that costs are weakly increasing in s. In particular, in Appendix B we argue that coordinated defaults continue to exist even under more general cost structures as long as $c(s) > 0$ for all s.

the bank audit a given entrepreneur's project for the imposition of the non-pecuniary cost to be possible. This penalty can be interpreted in a number of ways, one being as the bank's decision to inform a credit bureau of the entrepreneur's failure to comply with the agreed upon financial contract.

We now address the problem of establishing the optimal contractual arrangement between the parties. Initially, we follow the standard mechanism design approach and search for a Bayesian game that, while providing entrepreneurs with incentives for truthful reporting, has among its possibly many equilibria one that minimizes total expected audit costs. We also require that the bank break-even in expectation. This approach implicitly assumes that when multiple equilibria are present the desired equilibrium is chosen.

When signing a contract, a given entrepreneur and the bank must agree upon several issues. The first is on the audit region $B \subseteq [0, \overline{s}]$, which determines when the bank pays the observation costs. We initially restrict attention to deterministic audits on the part of the principal and analyze the case of random audits in Section 5. With a slight abuse of notation, let $B(s)$ be an indicator function defined in $[0, \overline{s}]$ and taking value 1 at states where audits occur and 0 otherwise. The second issue that must be agreed upon is on how parties share at each state the project's return net of observation costs, namely $f(s) - B(s)c(s)$. Let $R_b(s)$ and $R_e(s)$ be the return to the bank and the entrepreneur respectively when the state is s.

A contract can be represented by an array (R_b, R_e, B). An optimal contract provides the entrepreneur with incentives for truthful reporting while minimizing expected audit costs. One important feature of the optimal contract directly follows from the stated assumptions: the entrepreneur is fully expropriated when found out to have misreported after an audit. As opposed to partial expropriation, full expropriation is optimal because it loosens the bank's budget constraint, thus allowing for a reduction in the audit region.[9]

[9] The expropriation of the entrepreneur can also be interpreted as the seizure of collateral on the part of the bank. Under this interpretation, the non-pecuniary cost c_0 reflects the difference in relative valuations

An optimal contract is incentive compatible if and only if:

(i) there exists a constant D such that $R_b(s) = D$ whenever $B(s) = 0$;

(ii) for any states $s, \widehat{s}$ such that $B(s) = 1$ and $B(\widehat{s}) = 0,$ we have $D \geq R_b(s) + c(s)$.

Condition (i) specifies a constant repayment schedule for the entrepreneur in the no-audit region, while condition (ii) guarantees that it is never in the entrepreneur's interest to report a non-audit state when the true state specifies that an audit be realized.[10] Using the above characterization of the set of Incentive Compatible contracts, Gale and Hellwig (1985) show that in the case of a single entrepreneur-investor pairing, the optimal contract takes the form of what they call a *standard debt contract*, which is characterized by:

$$B = \begin{cases} 0 \text{ if } f \geq D \\ 1 \text{ if } f < D \end{cases} \qquad R_b = \begin{cases} D \text{ if } B = 0 \\ f - c \text{ if } B = 1 \end{cases} \tag{1}$$

In the above characterization, D is the face value of debt. According to the standard debt contract, when the entrepreneur fails to repay D he is instantly audited and fully expropriated. The face value of debt is chosen so as to guarantee that the bank breaks even in expectation.[11] Let s^D be such that $f(s^D) = D$, then D is implicitly defined by

$$D(1 - H(s^D)) + \int_0^{s^D} [f(s) - c(s)]h(s)ds = I. \tag{2}$$

In our setting, there is a continuum of entrepreneurs with i.i.d. projects and by the law

between bank and entrepreneur for the collateral good.

[10]Irrespective of who pays for the audit costs, the entrepreneur bears these costs in equilibrium because the bank must break-even.

[11]We restrict the analysis to the interesting situation in which $c_0 \leq D$ so that costly audits must be realized to provide incentives for the entrepreneur to pay off his debt.

of large numbers the bank knows the realized aggregate return. Hence, conditional on *all* entrepreneurs being truthful, it is as if the bank were dealing with a single representative borrower. Therefore, the following proposition holds:

Proposition 1. *When a single bank lends to a continuum of entrepreneurs with i.i.d. projects, the standard debt contract of Gale and Hellwig (1985) is an optimal contract.*

Proposition 1 shows that the results of Gale and Hellwig (1985) remain unaltered if we assume that a single agent lends to a continuum of entrepreneurs. The standard debt contract still provides each entrepreneur with incentives for truthful reporting while minimizing the bank's expected aggregate audit costs.

3.2 The Bank's Budget Constraint

We have been purposely silent as to whom - the bank or the entrepreneur - actually pays for the audit costs, but in the analysis that follows we assume explicitly that the bank must pay for these costs entirely.[12] More specifically, we make the following assumption:

Assumption 1. *Before an audit is realized, the bank must pay entirely for its costs. The bank can either set aside capital at* $t = 0$ *or use the proceeds collected from creditworthy entrepreneurs at* $t = 1$ *to pay for audits. Furthermore, conditional on having the necessary resources, the bank can credibly commit to audit entrepreneurs in default.*

In theory, the first part of Assumption 1 regarding who pays the audit costs is irrelevant, provided that the necessary resources are secured at the time the audit is to take place; since the bank only breaks even, audit costs are ultimately borne by the entrepreneur in equilibrium. However, the assumption of an unlimited budget to cover audit costs does not seem realistic

[12] This assumption is starker than necessary and made for expositional convenience. All that is needed is that the bank incur in any positive fraction of total costs of the audit.

and we wish to explore the implications of parting with it.[13] We will do so in the following section.

The second part of Assumption 1 regards the principal's commitment capabilities conditional on the availability of resources. It is instructive to be explicit about how our model compares to other papers in this regard. For example, Townsend (1979) and Gale and Hellwig (1985) assume the principal can fully commit to audit entrepreneurs in default, even if it is not optimal to spend resources on audits after the agent has revealed the true return from his project. By contrast, Krasa and Villamil (2000), in a variant of the costly state verification environment, impose the restriction that audits be sequentially rational (i.e. the principal has zero commitment capacity).[14] Our paper is thus an intermediate case; the principal can fully commit to use his resources, as long as they are available at $t = 1$.[15]

From now on, we incorporate Assumption 1 to the baseline model and analyze how debtors' repayment behavior changes. If entrepreneurs anticipate that audit resources are insufficient, the bank will be unable to provide them with repayment incentives. This situation can be amplified by opportunism on the part of debtors, since they benefit from actions that hurt the

[13]The case we study, in which the creditor bears the costs of collecting loans, might be specially applicable to countries with less developed bankruptcy laws, as was Mexico in the early nineties (Krueger and Tornell (1999); Luna-Martinez and Jose (2000)) or some developing countries today. For example, Morduch (1999) documents instances in which microfinance programs rely on the posting of collateral to grant credit, despite the absence of institutions that guarantee repossession by judicial means.

[14]More specifically, Krasa and Villamil (2000) restrict attention to sequentially-rational auditing policies in a very general (one borrower) model and show (Theorem 1) that the optimal mechanism takes the form of a simple debt contract coupled with a deterministic auditing policy. Our model would be the same as theirs if (i) we allowed entrepreneurs to retain a small amount κ of their project's return even after being audited and (ii) entrepreneurs incurred in a fraction of the auditing costs. All the results we derive would then continue to hold if we were to restrict attention to sequentially-rational equilibria of the game. However, since our goal is to understand how the principal's budget constraint interacts with the borrowers' repayment decisions, we assume that whenever endowed with resources the bank can fully commit to perform a costly audit.

[15]We assume the bank cannot raises resources when audits begin. This assumption may be justified on several grounds. According to Diamond and Rajan (2001) relationship lenders might be unable to raise resources when facing a liquidity shock because they cannot commit to use their specific ability to collect loans. Alternatively, the issue of new securities might be precluded given the existence of debt overhang on the part of the bank (Myers (1977)) or asymmetry of information between inside management and outside investors (Myers and Majluf (1984)).

lender's financial capability and make the collection of loans less likely.

With the introduction of the bank's budget constraint, we must be explicit as to how the bank's resources evolve over time. Timing is as follows. At date 0, the bank chooses an initial capital level E, which is common knowledge among every agent in the economy. We assume that equity capital is costly, so the bank chooses the lowest level of capital in accordance with equilibrium behavior.[16] The bank then signs a standard debt contract with all entrepreneurs, with face value D. Conditional on having the necessary resources, the bank fully commits to audit entrepreneurs in default. We will complete our description of the bank's audit strategy in a moment.

At date 1 each entrepreneur instantly observes his project's return and chooses whether to repay the loan. Let Λ denote the set of entrepreneurs who repay D to the bank and Λ^c the set of entrepreneurs who do not pay and declare default.[17] Entrepreneurs in default are either unable to repay their debt if project returns are lower than D, or are unwilling to do so. In the latter case, they default strategically.

After observing aggregate repayment behavior, the bank decides which projects to audit, subject to its budget constraint. The bank can only use its capital E and the proceeds from creditworthy entrepreneurs, given by $D \int_{\Lambda} dH$, to pay for audit costs. Because the bank signs with each individual entrepreneur a standard debt contract, it only audits entrepreneurs who do not pay D. Remember that we have also assumed that the bank always audits a project in default if it has the necessary resources to do so. The only remaining question then is how the bank conducts audits when its budget constraint is binding. When this occurs, we assume

[16] The source of funding the bank uses to pay for audits is immaterial. In particular, as suggested by a referee, the bank could use a fraction of its deposits to cover for the audit costs. In our model, E represents the total funds the bank has at its disposal to perform its audit activities, irrespective of their source. With this in mind, we will denote E by capital buffer, partly because deposits are short-term claims that must be readily available to depositors upon request, whereas capital is more of a longer-term source of funding that can be used in a committed way by the bank to audit entrepreneurs.

[17] More specifically, in a direct mechanism entrepreneurs in Λ report a state $\hat{s}$ that prescribes payment of D, while entrepreneurs in Λ^c report state $\hat{s}$ in the audit region.

the bank randomly chooses among projects in default until its resources are exhausted. This strategy can be argued to be rather arbitrary, but it seems to us as the most natural extension of the standard audit strategy of the costly state verification model to the case where there is a continuum of entrepreneurs.

We also adopt the following assumption:

Assumption 2. *The equilibrium notional value of debt D is such that the following inequality holds:*

$$D(1 - H(s^D)) > \int_0^{s^D} c\,(s)\,h\,(s)\,ds. \tag{3}$$

Assumption 2 guarantees that the amount the bank collects when all entrepreneurs report truthfully is more than enough to cover its audit costs. This appears to be a sensible assumption provided that the bank is willing to extend the standard debt contract to all entrepreneurs. In fact, using the bank's budget constraint given by equation (2), Assumption 2 is equivalent to

$$\int_0^{s^D} f(s)h(s)ds < I, \tag{4}$$

which states that the mean return in default states is insufficient to cover the initial investment.

3.3 Equilibrium

We now analyze the set of equilibria of the game induced by the standard debt contract and the bank's audit strategy. First, we characterize behavior on the part of entrepreneurs.

Proposition 2. *When confronted with a probability p of audit and a debt level D, there is a cutoff state s^* such that entrepreneur i declares default if and only if $s_i \leq s^*$.*

The intuition for the above result is clear. As the realized return of the project increases, so does the cost to the entrepreneur of being fully expropriated if found out to have reported untruthfully.

We now analyze how the probability p of each entrepreneur being audited is affected by the entrepreneurs' reports, given that the bank has fully committed to the audit strategy previously described. Suppose there's a cutoff state s^* such that every entrepreneur with $s_i \leq s^*$ declares default, while the remaining entrepreneurs pay off their debt. The bank must audit a total of $H(s^*)$ non-performing loans with $E + D(1 - H(s^*))$ in resources, which can either come from capital hoarded at the initial period or repayments from performing loans.

If $E + D(1 - H(s^*))$ is greater than total audit costs, given by $\int_0^{s^*} c(s)h(s)ds$, then all entrepreneurs in default are audited. On the other hand, if the bank does not collect enough resources from the creditworthy entrepreneurs to audit all projects in default, then it randomly chooses which projects to audit until it runs out of cash. The audit probability faced by each entrepreneur is given by

$$p(E, s^*) = \min \left\{ \frac{E + D(1 - H(s^*))}{\int_0^{s^*} c(s)h(s)ds}, 1 \right\}. \tag{5}$$

We are now ready to define an equilibrium of the entrepreneur game induced by the mechanism.

Definition 1. For a fixed capital buffer E, a *repayment equilibrium* is given by an ordered pair $\mathcal{E} = (s^*, p)$ such that:

(i) $p = p(E, s^*)$ as in Equation (5);

(ii) entrepreneur i defaults if and only if $s_i \leq s^*$.

The definition of a repayment equilibrium implies that entrepreneurs form beliefs about the cutoff state s^* and then choose actions that maximize profits given these beliefs. In addition, beliefs are correct in equilibrium.

We also have the following proposition:

Proposition 3. *Under Assumptions 1 and 2, for any given level of capital E chosen by the bank ex-ante (in particular, $E = 0$), the standard debt contract derived in Proposition 1 coupled with the symmetric audit strategy has a truth-telling equilibrium given by $\mathcal{E} = (s^D, 1)$.*

Initially, it seems that nothing is changed by the introduction of the bank's budget constraint, since in a truth-telling equilibrium the bank can set $E = 0$ at $t = 0$. When entrepreneurs tell the truth, the bank secures sufficient resources from creditworthy entrepreneurs to audit those who declare default and the budget constraint implicit in Assumption 1 is slack. The standard debt contract coupled with a symmetric audit strategy is a mechanism that at least partially implements the desired allocation.

However, while truth-telling is *one* possible equilibrium of the mechanism induced by the standard debt contract and the bank's audit strategy, it is not unique when the bank's capital level is below a certain threshold. Entrepreneurs can default strategically, impairing the bank's financial position and auditing capability. This is formally stated in the following result:

Proposition 4. *Consider the standard debt contract coupled with the symmetric audit strategy. Under Assumptions 1 and 2 and if $E < E_1 \equiv \frac{(D-c_0)\int_0^{\overline{s}} c(s)h(s)ds}{f(\overline{s})}$:*

(i) there is a fully coordinated default equilibrium, characterized by all entrepreneurs declaring default, that is $s^* = \overline{s}$;

(ii) there is a partially coordinated default equilibrium, characterized by a threshold $s^* \in (s^D, \overline{s})$.

The threshold capital level E_1 is decreasing in c_0, the non-pecuniary penalty that the bank can impose on entrepreneurs. Keeping all other parameters fixed, a larger non-pecuniary penalty reduces a borrower's incentives to default. Hence, as c_0 increases, the bank can lower the audit probability - and thus the capital set aside ex-ante - while still maintaining repayment incentives intact.

Jointly, Propositions 3 and 4 highlight how entrepreneurs' repayment incentives are affected by the bank's health. The results seem to be in line with the realities of microfinance programs in particular. For example, according to van Maanen (2004) *"If the (repayment) percentage sinks below - say 90% - a growing percentage of the clients is tempted to join the 10% that seems to get away with non-payment. Once the percentage sinks below 80% it is very difficult to reverse that trend, because the virus travels faster than any medicine: "Why should I repay to a MFI (microfinance institution) that is likely to go down? Let's wait and see what happens!""*

Proposition 4 also sheds some light on the potential limits of delegated monitoring. Diamond (1984) shows that, when financial intermediaries are fully diversified, delegation costs - given by the costs of providing the proper incentives to intermediaries, as opposed to entrepreneurs - are zero. In Diamond (1984), because the financial intermediary is fully diversified, the Law of Large Number eliminates the informational advantage that it may have over its depositors as a result of directly observing project returns. In this paper, however, the bank can always expropriate its depositors by claiming that it has suffered a full coordinated default, even if entrepreneurs report truthfully. Therefore, any potential benefit of delegated monitoring must be weighted against the costs of providing the intermediary with the proper incentives.

4 A General Solution to Rule Out Bad Equilibria

In this section we show how the bank can prevent entrepreneurs from coordinating on an undesirable equilibrium by adopting what we call a *sequential audit strategy.* This strategy can be implemented by the bank if, once auditing is to take place, it has any strictly positive level of resources. We first describe the sequential audit strategy assuming that an amount δ in capital can be secured. Subsequently, we show how the bank can secure this positive amount.[18]

4.1 The Sequential Audit Solution

Suppose that, at the moment of the signing of the contract, the bank divides entrepreneurs into $1/\varepsilon$ groups of mass ε, where

$$\varepsilon = \frac{\delta}{\int_0^{\overline{s}} c(s,k)h(s)ds}. \tag{6}$$

Groups are then randomly ordered as $(g_1, g_2, ..., g_{1/\varepsilon})$ and this order is common knowledge among bank and entrepreneurs. Note that $\varepsilon \int_0^{\overline{s}} c(s,k)h(s)ds$ is exactly the amount of resources that the bank must have to audit all entrepreneurs belonging to one given group, regardless of their reporting strategy.

Once the bank starts auditing, it does so sequentially, auditing entrepreneurs in group $n+1$ only after it has audited all defaulted projects in group n. Hence, even though entrepreneurs are ex-ante identical, the bank treats them asymmetrically when adopting a sequential audit strategy.

With δ in capital, the bank can credibly commit to audit all entrepreneurs in g_1. Therefore entrepreneurs belonging to the first group never defaults strategically, since doing so would automatically trigger an audit and the seizure of the entire project's return. We now show that if entrepreneurs in group n report truthfully, the bank collects enough resources to commit to audit all entrepreneurs belonging to group $n+1$ that have declared default.

[18]Note that we propose a modification to the symmetric audit strategy but keep the financial contract (standard debt contract) unaltered. What if the bank held, as opposed to debt, an alternative security issued against future proceeds from project returns? In the costly state verification environment that we study (Gale and Hellwig (1985); Townsend (1979)), this would be of no help. For example, by holding equity the bank would need to monitor the entrepreneur in all states of the world, rather than just in a subset of states as with debt. This would magnify the bank's exposure to a coordinated default by stretching its limited budget even more. Debt is optimal because it is the security that minimizes the bank's monitoring costs (and, consequently, its exposure to coordinated defaults).

Note that truthful reporting by a given group increases the bank's audit resources by

$$\varepsilon \left[(1 - H(s^D))D - \int_0^{s^D} c(s,k)h(s)ds \right]. \tag{7}$$

The first term inside the brackets is the amount raised from the creditworthy entrepreneurs. The second term is the total cost incurred when auditing projects in default. As a consequence of Assumption 2, the whole expression is positive. Therefore, if the bank has enough resources to provide incentives for truthful reporting from entrepreneurs in group n, it can also assure truthful reporting from entrepreneurs in group $n+1$. After all, the bank's financial strength is only improving and the pool of projects potentially subject to audit is decreasing. As this argument holds for an arbitrary n, no group will default strategically. The following proposition formalizes this argument.

Proposition 5. *For any $\delta > 0$, if the bank can secure δ of capital to audit and uses a sequential audit strategy, then truth-telling is the unique equilibrium of the standard debt contract. Moreover, truthful reporting is obtained through the process of iterative deletion of strictly dominated strategies.*

The intuition behind this simple solution is the following. Coordinated defaults exist because of the strategic complementarity between entrepreneurs' actions. The sequential audit strategy is successful precisely because it breaks this strategic complementarity. The default by a given group of entrepreneurs does not affect the probability of those in g_1 being audited. An entrepreneur in this group who declares default will be audited for sure, regardless of the other entrepreneurs' decisions. Therefore, his incentives to default are not affected by the default of others. Once g_n pays up, incentives for truth-telling by g_{n+1} are guaranteed and coordinated defaults unravel.

Furthermore, the unique equilibrium under a sequential audit strategy is robust (Bergemann and Morris

(2009)); it is the only strategy that survives a procedure of iterated deletion of strictly dominated strategies. To see this, note that the sequential audit strategy guarantees that truth-telling is dominant for entrepreneurs in g_1. Entrepreneurs in g_2, aware that those in g_1 will not lie, will also find it dominant to report truthfully and so on.[19]

In many instances, a bank naturally collects its non-performing loans in a given order. For example, a bank may prefer to begin auditing entrepreneurs who are geographically closer to its headquarters or are listed in jurisdictions with creditor friendly bankruptcy courts. Alternatively, audit costs might be reduced for those entrepreneurs with whom the bank has done business for a longer period of time, which would justify placing them first in line. In any case, these forms of heterogeneity among entrepreneurs may serve as an implicit ordering device, which in equilibrium may help prevent coordinated strategic defaults.

In the next sections, we will point out a few ways by which the bank can obtain the amount δ of Proposition 5.

4.2 Partially Coordinated Defaults

When *partially coordinated defaults* arise, the bank is able to raise resources form a group of creditworthy entrepreneurs who have chosen to report truthfully. Formally, suppose that all partially coordinated default equilibria are given by $\{(p_1, s_1^*)..., (p_K, s_K^*)\}$.[20] Let (p, s^{MAX}) be the one with the largest s_k^*.

The bank can apply the sequential audit strategy to prevent partially coordinated defaults,

[19]To provide truthtelling incentives, the bank need not announce how it orders the groups, but only that it will apply the sequential-audit solution. Irrespective of the beliefs regarding his order in the auditing sequence, an entrepreneur will anticipate that the bank will collect enough resources to audit every entrepreneur who defaults and therefore find it optimal to tell the truth.

[20]There can be either a finite number of equilibria featuring partially coordinated defaults or an infinite number of them depending on whether 0 is a regular value of the function Γ defined in the Appendix. If 0 is a regular value then the number of equilibria with partially coordinated defaults is finite and odd. We focus on regular equilibria, which are robust to small perturbations of the set of parameters.

taking

$$\delta = (1 - H(s^{MAX}))D. \tag{8}$$

This is the least amount of capital that the bank will raise in any of the partially coordinated defaults. The group size ε will then be chosen according to equation (6). Hence, the sequential audit solution is capable of eliminating all partially coordinated defaults, without any loss in efficiency since there is no need to hoard costly capital at date 0. This solution has its shortcomings, as it is ineffective in eliminating the fully coordinated default equilibrium. Indeed, if all entrepreneurs default, the bank will not collect the needed amount of resources to perform the sequential audit.

4.3 Solutions for Fully Coordinated Defaults

4.3.1 Positive Capital

The bank can guarantee the necessary audit resources by forming a capital cushion at the financing stage. More specifically, suppose that at date 0 the bank publicly announces that it is hoarding an amount of δ in capital, that is to be invested in risk-free securities.[21] This capital cushion, together with the sequential audit strategy, creates a mechanism that is (robust) incentive compatible. Furthermore, the inefficiency which arises from hoarding capital can be made arbitrarily small.

4.3.2 Debt Forgiveness

In this section, we study an alternative form the bank can raise the necessary capital to implement the sequential audit solution, which involves granting debt forgiveness to a group

[21] The bank must invest in risk-free securities to eliminate the possibility that an adverse shock to its securities portfolio reduces its ability to pay for the audit costs at $t = 1$. We are also assuming that the bank can costlessly and credibly disclose to entrepreneurs the amount of capital it has hoarded and its riskiness. We thank an anonymous referee for pointing this out.

of entrepreneurs. In the analysis that follows, we explicitly explore the fact that bad equilibria exist because entrepreneurs form beliefs that others will default strategically and these beliefs are correct in equilibrium.

We assume that at date 0 the bank randomly chooses a group Δ of mass δ of entrepreneurs and subsequently divides the remaining entrepreneurs in groups of size of at most ε. Entrepreneurs in Δ^c contract with the bank through the standard debt contract, whereas entrepreneurs in Δ sign a contract that is altered as follows. After the realization of the projects' returns, but before any payments are made, each entrepreneur in Δ is required to report a flag $f_i \in \{s^D, \overline{s}\}$ to the bank. This flag represents each entrepreneurs' belief about the behavior of the entrepreneurs in Δ^c. If every entrepreneur in Δ^c declares default, then those who reported $f_i = \overline{s}$, receive debt forgiveness of $D - c_0$. On the other hand, if a group of positive mass in Δ^c honors their debt, entrepreneurs in Δ who reported $f_i = \overline{s}$ are audited and fully expropriated.

Under this contract, when every entrepreneur in Δ^c declares default, entrepreneurs in Δ are indifferent between joining the coordinated default *and* suffering the non pecuniary penalty of c_0, or paying the bank that same amount. We suppose that when confronted with this situation, entrepreneurs in Δ who have at least c_0 always choose to pay the bank. As a result, the bank collects $\delta(1 - H(s^c))c_0$, where s^c is such that $f(s^c) = c_0$. Once the bank has raised this amount of capital, it can proceed with the sequential audits, provided that ε is chosen appropriately. Under this agreement, truth-telling by every entrepreneur is the unique repayment equilibrium and debt forgiveness only occurs off equilibrium.

Since no capital needs to be put aside for its implementation, the solution with debt forgiveness is less costly than hoarding capital ex-ante. However, when the bank uses the debt forgiveness solution, the strategy each entrepreneur adopts depends on his beliefs regarding other entrepreneurs' strategies. In this respect, it requires a complex decision making process on the part of entrepreneurs.

The non-pecuniary penalty plays a crucial role in the sequential audit solution with debt-forgiveness. Because the bank can impose this cost whatever the entrepreneurs' repayment decisions, it can always raise a positive amount of money through ex-post bargaining by monetizing the non-pecuniary penalty. This in turn, guarantees that, were entrepreneurs in Δ^c to coordinate on a strategic default, the bank would be able to raise a strictly positive amount of resources to kick-start the sequential audit strategy.

5 Robustness of Coordinated Strategic Defaults

In this section, we discuss the robustness of our previous results by modifying some of our modeling assumptions. We show that little is substantially changed if we assume that the bank only lends to a finite number of entrepreneurs or if we allow the bank to adopt more general mechanisms. We deal with these extensions one at a time.[22]

5.1 Finite Number of Creditors

Assume now that the bank lends to n entrepreneurs, where $n < \infty$. We keep the remaining features of the model unchanged. In particular, at $t = 0$ the bank can still credibly commit to use all its available resources to audit entrepreneurs who eventually declare default.

Proposition 6. *Consider the standard debt contract coupled with the symmetric audit strategy. Under Assumptions 1 and 2, for each n, there are two non-negative threshold capital levels $E_0(n)$ and $E_1(n)$, such that*

i) there exists a truthtelling equilibrium if and only if $E \geq E_0(n)$;

ii) if $E < E_0(n)$, there exists either a fully coordinated default equilibrium or a partially coordinated default equilibrium (or both);

[22]We gratefully acknowledge both referees for suggesting that we explore the issues presented in this section.

iii) if $E < E_1(n)$, there exists a fully coordinated default equilibrium;

iv) there exists a finite N such that $E_1(n) > E_0(n)$ for all $n > N$. If $E \in (E_0(n), E_1(n))$ for $n > N$, apart from the fully coordinated default equilibrium, there also exists at least one partially coordinated default equilibrium.

Proposition 6 is the analog of Proposition 4 to the case where the bank lends to a finite number of entrepreneurs. We therefore limit the following discussion to highlighting how both propositions differ. When $n < \infty$, an additional situation must be taken into account: if capital is below a threshold given by $E_0(n)$ then the standard debt contract does not provide incentives for truthful reporting. For these low capital levels an entrepreneur prefers, irrespective of other entrepreneurs' strategies, to occasionally default strategically. Because the bank is poorly diversified and many projects may simultaneously go sour, a given entrepreneur is still better off by misreporting in some states, even if he believes that other entrepreneurs always report truthfully.

The following proposition shows how threshold levels $E_0(n)$ and $E_1(n)$ vary with n.

Proposition 7. *When $n \to \infty$, then (after re-weighting the mass of each individual entrepreneur so that the total mass of entrepreneurs is always one) $E_0(n) \to 0$ and $E_1(n) \to E_1$, where $E_1 = \frac{(D-c_0)\int_0^{\overline{s}} c(s)h(s)ds}{f(\overline{s})}$ is as in Proposition 4.*

To capture the intuition behind the result of Proposition 7, consider the starkest case which occurs when $n = 1$. When lending to a single entrepreneur, the bank's capacity to engage in cross-subsidization once audits are to begin - using resources from creditworthy entrepreneurs to audit those in default - is eliminated. The bank must thus hoard enough capital ex-ante to guarantee the feasibility of any eventual audit. If the bank hoards insufficient capital, the single entrepreneur defaults strategically on his loan.

For $n > 1$, the bank may eventually raise resources at $t = 1$ from entrepreneurs who repay their loan. As n increases, the bank becomes more diversified and the fraction of projects that fail approaches their ex-ante probability of failure. For large n and if entrepreneurs report truthfully, Assumption 2 guarantees that the bank is likely to raises from creditworthy entrepreneurs the necessary resources to audit those in default. Any given entrepreneur is thus incentivized to tell the truth (even for a low E) provided that he believes other entrepreneurs will do the same. Truth-telling is therefore an equilibrium of the standard debt contract with symmetric audits. Proposition 7 shows that the case where there is a continuum of entrepreneurs serves as a good approximation to the case where n is finite but large.

Once again when capital is below a threshold, apart from the truth-telling equilibrium, coordinated default equilibria exist as well. The bank can employ the sequential audit strategy to reduce the amount of capital hoarded to provide incentives for truthful reporting. More specifically, irrespective of n, the bank must hoard sufficient capital to audit only one entrepreneur, precisely the one that was placed first in line in the sequential audit strategy.

5.2 General Mechanisms

So far, we have restricted the analysis to the case where the principal only uses deterministic mechanisms. This case is of special interest, since it is consistent with many features observed in financial markets (e.g. debt contracts and bankruptcy procedures). However, because the costly state verification environment has also been applied to the study of insurance and taxation - where stochastic audits are pervasive in real life - it is interesting to establish the validity of our results when the principal adopts more general mechanisms, in particular when he randomizes audits.[23]

When analyzing general mechanisms, we maintain Assumption 1 that introduces the principal's budget constraint into the mechanism design problem that we study. Therefore the principal must still secure beforehand the resources he spends in audits. In the Appendix, we

[23] Mookherjee and Png (1989) and Border and Sobel (1987) study optimal mechanisms in costly state verification environments when random audits are possible.

prove the following proposition:

Proposition 8. *Suppose that the principal adopts a stochastic mechanism coupled with a symmetric audit strategy. Then there is a threshold capital buffer E_S such that, if the bank sets capital buffer $E \leq E_S$ at $t = 0$, then the stochastic mechanism also has a fully coordinated default equilibrium and at least one partially coordinated default equilibrium*

The following intuition lies behind the existence of multiple equilibria even for general mechanisms. In a costly state verification environment, audits followed by a threat of (at least partial) expropriation of the agents returns' are the only disciplining device available to the principal. In particular, in the absence of audits an agent always reports that the return from his project is 0. Therefore truthful reporting only occurs if the bank commits to audit entrepreneurs with a positive probability at a set of states with positive measure. Because audits are costly, the principal must guarantee that he raises the necessary resources to realize them. If the principal can incentivize entrepreneurs to report truthfully, then creditworthy entrepreneurs provide the principal with the resources to pay for audits. Nevertheless, once the principal sets its capital buffer at $t = 0$ and commits to a symmetric audit strategy, agents play a game of strategic complementarity. It becomes more attractive for one agent to report an audit state when other agents are doing the same. For sufficiently low capital buffers, all agents prefer to misreport.

The bank can only be (at least weakly) better off by having the possibility of using more general mechanisms. It then follows that by adopting a (potentially random) sequential audit strategy the bank can induce truthtelling as the unique equilibrium whenever it hoards an arbitrarily small amount of capital at $t = 0$. Hence, all our previous results remain unaltered if one allows for more general mechanisms.

5.3 On the Impossibility of Raising Resources to Cover for Audit Costs

In the model we present, the existence of coordinated defaults results from the assumption that the bank is unable to raise resources at $t = 1$ to audit entrepreneurs. It is therefore important to discuss the validity of this assumption. In Appendix B we provide two examples that show that the bank may be unable to raise resources because it does not have enough pledgeble income, even if auditing projects in default has a positive net present value.

There is an extensive literature in corporate finance showing that financially constrained corporations often forgo profitable investment opportunities because they cannot pledge future income to financiers. Note that, in our model, the interaction between the bank and the entrepreneurs it finances is very similar to that between the bank and its potential financiers. It is therefore natural to assume that the bank's ability to raise resources at $t = 1$ may be affected by problems of asymmetry of information and/or other incentive problems as well.

Before moving to discuss the more interesting case in which those costs are not substantial, so that the NPV is positive, we briefly (and informally) discuss a source of costs, related to costly state verification settings as ours, that may render the NPV of the project negative. As argued by Diamond (1984), while delegating monitoring to a single intermediary saves on monitoring costs, it creates an additional cost: the monitor has to be monitored. A situation in which the bank is forced to raise additional resources in $t = 1$ leads to delegation costs that are potentially larger than I the net amount of revenues that auditing generates in $t = 1$. Moreover, the delegation costs in period $t = 1$ add up to those in $t = 0$, so, *de facto*, by raising additional costs at the auditing stage, overall delegations costs are duplicated and, therefore, from the bank's and all its investors' perspective net losses will ensue.

Now, while the impossibility of raising resources to conduct audits is obvious whenever such project has negative NPV, the bank might be unable to raise money in $t = 1$ even when the NPV is positive. Indeed, in Appendix B, we formalize this point using two examples based, respectively, on Myers and Majluf (1984) and Diamond and Rajan (2001).

6 Conclusion

In this paper, we revisit one of the most influential models of financial contracting, the costly state verification model first developed in Townsend (1979) and Gale and Hellwig (1985). We extend their analysis to the case of multiple borrowers and show that when a bank's resources to monitor projects are bounded, financial intermediation can lead to the existence of multiple equilibria in repayment behavior. In some of these equilibria, borrowers default because they expect other borrowers to do the same.

As opposed to what has been extensively analyzed in the academic literature, we study a bank run originating in the bank's asset side, rather than from its funding structure. The analysis suggests that coordinated strategic defaults are yet another source of financial fragility in the sense that small shocks have large effects (Allen and Gale (2000, 2004)).[24] We show that to prevent bad equilibria a bank needs to break the strategic complementarities in borrowers' default decisions, which can be done through the adoption of a sequential audit strategy.

While cast in terms of financial intermediation, the ideas we put forth in this paper can be applied to other settings in which a large number of agents have to be monitored or audited. One example is the deterrence of crime waves by a police force who faces a large population of criminals. Other examples that come to mind include the problem of a governmental agency that has to rely on income reports of individual tax payers and a CEO who relies on the reports about the profitability of a company's divisions by managers who can engage in self-dealing.

[24] We follow Allen and Gale (2004) in claiming that sunspot equilibria, where endogenous variables are influenced by variables that have no effect on fundamentals, constitute an extreme form of financial fragility.

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A Proofs

Proof of Proposition 2

When faced with a probability of audit p, entrepreneur i declares default if and only if

$$(1-p)f(s_i) - c_0 \geq f(s_i) - D. \tag{9}$$

The left hand side of Equation (9) is the entrepreneur's expected payoff when he declares default, while the right-hand side is the expected payoff when he honors the debt contract. Equation (9) is equivalent to $D - c_0 \geq p \cdot f(s_i)$. If $D - c_0 > p \cdot f(\overline{s})$, then $s^* = \overline{s}$ since even the entrepreneur with the highest possible return would rather default than honor the debt contract. If $D - c_0 = p \cdot f(s)$ for some s, define $s^* = s$. Uniqueness is due to the fact that $f(\cdot)$ is strictly increasing. The result then follows.

■

Proof of Proposition 3

If Assumption 2 holds, then $p(s^D) = 1$ and the bank can credibly commit to audit all projects in default. If default automatically triggers an audit, then the best response for each entrepreneur is to report truthfully. Therefore $s^* = s^D$. This holds for any capital level E.

■

Proof of Proposition 4

Define

$$\Gamma(s) \equiv p(E, s) \cdot f(s) - (D - c_0). \tag{10}$$

The function $\Gamma(s)$ is continuous. If $\Gamma(s) > 0$, then an entrepreneur that faces an audit probability of $p(E, s)$ prefers to repay his loan when the return from his project is $f(s)$. When $\Gamma(s) < 0$, the opposite holds.

To prove the existence of a full coordinated default, note that if $E < E_1$ and $s^* = \overline{s}$ then $p(E, \overline{s}) < \frac{D-c_0}{f(\overline{s})}$. Therefore, $\Gamma(\overline{s}) < 0$. From Proposition 2, it is optimal for every entrepreneur to declare default (irrespective of his project's return), if he expects all other entrepreneurs to do the same. Therefore $\mathcal{E} = (\overline{s}, p(E, \overline{s}))$ is a repayment equilibrium.

We now show the existence of a partially coordinated default. At a partially coordinated default with threshold s^*, an entrepreneur $s_i = s^*$ should be indifferent between repaying or defaulting, therefore $\Gamma(s^*) = 0$. Let $s^L = \sup\{s; p(E, s) = 1\}$. From Assumption 2, we have $s^L > s^D$. Since $\Gamma(s^L) > 0$ and $\Gamma(\overline{s}) < 0$, the Intermediate Value Theorem guarantees that there exists $s^* \in (s^L, \overline{s})$ such that $\Gamma(s^*) = 0$.

■

Proof of Proposition 6

To simplify notation, we assume that $c(s) = c$. Before proceeding, notice that when entrepreneur i declares default along with other $k < n$ entrepreneurs, the bank collects a total of $(n - (k + 1))D$ in resources to audit at most $(k + 1)$ projects. The *conditional* (on k other defaults) probability that entrepreneur i is subsequently audited is therefore given by

$$q(k, E, n) \equiv \min\left\{\frac{1}{k+1}\left\lfloor\frac{E + (n - (k + 1))D}{c}\right\rfloor, 1\right\}, \tag{11}$$

where $\lfloor x \rfloor$ is the highest integer smaller than x. $q(k, E, n)$ is weakly increasing in E and strictly decreasing in k.

Now, assume that all $j \neq i$ entrepreneurs adopt a threshold strategy given by $s < \overline{s}$. The probability that, among these $(n - 1)$ entrepreneurs, a total of k declare default is

$\binom{n-1}{k} H(s)^k (1-H(s))^{n-1-k}$. Therefore if we define

$$p(E,n,s) \equiv \begin{cases} \sum_{k=0}^{n-1} \binom{n-1}{k} H(s)^k (1-H(s))^{n-1-k} q(k,E,n) & \text{if } s < \overline{s} \\ q(n-1,E,n) = \frac{E}{nc} & \text{if } s = \overline{s} \end{cases} \tag{12}$$

then $p(E,n,s)$ is the *unconditional* probability that entrepreneur i is audited after declaring default if all other entrepreneurs use a threshold strategy s. Equation (12) is the equivalent of Equation (5) to the case of a finite number of entrepreneurs.

Taking $p(E,n,s)$ as given, entrepreneur i finds it optimal to repay his loan when $s_i = s$ if and only if $\Gamma(E,n,s) \geq 0$ where

$$\Gamma(E,n,s) \equiv p(E,n,s) \cdot f(s) - (D - c_0). \tag{13}$$

We are now ready to prove each claim of Proposition 6.

(i) Because $\Gamma(E,n,s)$ is left-continuous in E, we can define $E_0(n)$ as the minimum capital level such that $\Gamma(E,n,s^D) \geq 0$. Therefore if $E > E_0(n)$, entrepreneur i finds it optimal to tell the truth if he expects other entrepreneurs to do the same, and truthtelling is an equilibrium.

On the other hand, if $E < E_0(n)$, then $\Gamma(E,n,s^D) < 0$ and entrepreneur i does not have incentives to tell the truth for all $s \in (s^D, \overline{s}]$, even if he believes that all other entrepreneurs are reporting truthfully. Therefore, truth-telling cannot be an equilibrium.

To prove (ii), fix a capital level $E < E_0(n)$, which by claim (i) implies that $\Gamma(E,n,s^D) < 0$. If $\Gamma(E,n,s) < 0$ for all $s \in [s^D, \overline{s}]$ then a fully coordinated default equilibrium exists. Alternatively, if $\Gamma(E,n,s') \geq 0$ for some $s' \in [s^D, \overline{s})$ then by the Intermediate Value Theorem there exists a $s'' \in [s^D, \overline{s})$ such that $\Gamma(E,n,s'') = 0$ and a partially coordinated strategic default equilibrium exists.

To prove (iii), define

$$E_1(n) \equiv \frac{(D - c_0)nc}{f(\overline{s})}.$$

If $E < E_1(n)$, then $p(E, n, \overline{s}) = \frac{E}{nc} < \frac{D-c_0}{f(\overline{s})}$ which implies that $\Gamma(E, n, \overline{s}) < 0$, guaranteeing the existence of a fully coordinated default equilibrium.

(iv) The first part of the claim is an immediate consequence of Proposition 7, which states that, as $n \to \infty$, $E_0(n) \to 0$ and $E_1(n) \to E_1 > 0$. For the second part of the claim note that since $E \in (E_0(n), E_1(n))$,

$$\Gamma(E, n, s^D) > 0 > \Gamma(E, n, \overline{s}),$$

where the first inequality follows from Claim (i) and the second from Claim (iii). According to the Intermediate Value Theorem, there exists $s^* \in (s^D, s')$ such that $\Gamma(s^*) = 0$.

■

Proof of Proposition 7

Our goal is to analyze the convergence of an economy with $n < \infty$ entrepreneurs to an economy with a continuum of entrepreneurs with mass 1. To achieve this we must for every n weigh each entrepreneur by the mass $m(n) = \frac{1}{n}$ to guarantee that the total mass of the economy remains constant at 1 as n grows.

Recall that $E_0(n)$ is defined as the minimum amount of capital that will guarantee that truthtelling is an equilibrium when the bank lends to n entrepreneurs. To prove that $E_0(n) \to 0$ as $n \to \infty$, let X_n be a random variable representing the total weighted resources the bank collects from entrepreneurs that report truthfully when lending to n different entrepreneurs. By the Strong Law of Large Numbers,

$$X_n \to D(1 - H(s^D)) \ \ a.s. \quad \text{as } n \to \infty, \tag{14}$$

so that the total resources (already weighted by the mass of each individual entrepreneur) converges to its expected value.

Similarly, let total weighted audit costs when entrepreneurs use threshold strategy s be given by $C_n(s)$. Then

$$C_n(s^D) \to \int_0^{s^D} c(s)h(s)ds \;\; a.s. \;\; \text{as } n \to \infty. \tag{15}$$

A direct application of Slutsky's Theorem (Casella and Berger (1990) pg. 239) yields $X_n/C_n \to \frac{D(1-H(s^D))}{\int_0^{S^D} c(s)h(s)ds} > 1$. Therefore, for sufficiently large n the bank collects in a truthtelling equilibrium enough resources to audit all projects in default, even if the bank sets $E = 0$ at $t = 0$. But then truthtelling is a dominant strategy on the part of each individual entrepreneur. Therefore $E_0(n) \to 0$ as $n \to \infty$.

To prove the second part of Proposition 7, note that when audit costs are not constant, then the counterpart of $E_1(n)$ derived in Proposition 7 is given by

$$E_1(n) = \frac{(D - c_0)C_n(\overline{s})}{f(\overline{s})}. \tag{16}$$

The result then follows once again from the Strong Law of Large Numbers.

■

Proof that General Mechanism are also subject to Coordinated Defaults

A direct general mechanism can be fully characterized by an array (R_b, R_e, μ), where R_b and R_e are the returns to the bank and entrepreneur respectively.[25] Provided he has sufficient resources, the principal audits an agent who reports message $\hat{s}$ with probability $\mu = \mu(\hat{s}) \leq 1$. We assume that for each message the lottery that determines whether an audit takes place is

[25]In the following proof, we only assume that $c(s) > 0$ for all s.

independent of the distribution function of the states of nature.

A direct mechanism partitions the message space $\mathcal{M} = [0, \overline{s}]$ into the regions $\mathcal{A}$ and $\mathcal{A}^c$, such that

$$\mathcal{A} = \{\hat{s} \in \mathcal{M}; \mu(\hat{s}) > 0\} \text{ and } \mathcal{A}^c = \{\hat{s} \in \mathcal{M}; \mu(\hat{s}) = 0\}. \tag{17}$$

For the mechanism to be incentive compatible, the agent's transfer in the no-audit region $\mathcal{A}^c$ must be a constant given by D, which in turn is determined so as to guarantee that the bank breaks even in expectation. In the audit region $\mathcal{A}$, the entrepreneur's payment may depend on the message $\hat{s}$, the true state s, and whether the entrepreneur is found to have reported truthfully if audited. In the optimal general mechanism the principal fully expropriates an entrepreneur who is audited and discovered to have misreported the true state (Border and Sobel (1987); Mookherjee and Png (1989)).

Now the bank's budget constraint may eventually bind depending on the capital it hoards at $t = 0$ and on the entrepreneurs' reporting strategies. Before audits begin, the principal collects $D \int_{\mathcal{A}^c} h(s)ds$ in resources from agents that report a non-audit message to the principal. If the bank were to go through with its prescribed audit strategy given by $\mu(\cdot)$, total audit costs would be given by $\int_{\mathcal{A}} h(s)\mu(\hat{s}(s))c(s)ds$, where $\hat{s}(\cdot) : \mathcal{M} \to \mathcal{M}$ is the entrepreneurs' reporting function, which we assume without loss to be symmetric across entrepreneurs.

Now the bank has enough resources to implement its audit strategy if and only if

$$E + D \int_{\mathcal{A}^c} h(s)ds \geq \int_{\mathcal{A}} c(s)h(s)\mu(\hat{s}(s))ds. \tag{18}$$

For a given mechanism to be considered by the principal in the first place, it must at least partially implement the desired allocation; i.e. truthtelling must be an equilibrium. Therefore we assume that the equilibrium value of D is such that inequality in Equation (18) is strict even when $E = 0$.

We now turn to the entrepreneurs' behavior. Note that since p was arbitrary in the proof of Proposition 2, entrepreneurs still default according to a threshold strategy when the principal adopts stochastic mechanisms. It follows that in an optimal stochastic mechanism, the set $\mathcal{A}$ takes the form of an interval $[0, \widetilde{s}]$.

Let

$$s^o \in \arg\min_{\hat{s} \in \mathcal{A}} \mu(\hat{s})$$

be the state that entrepreneurs report when defaulting strategically and define

$$E_S \equiv \frac{(D - c_0)\mu(s^o) \int_0^{\overline{s}} c(s)h(s)ds}{f(\overline{s})}$$

and

$$\Gamma_g(s) = p(E, s, \mu)\mu(s^o)f(s) - (D - c_0),$$

where

$$p(E, s, \mu(\cdot)) = \min\left\{ \frac{E + D\int_{\mathcal{A}^c} h(s)ds}{\int_{\mathcal{A}} c(s)h(s)\mu(\hat{s}(s))ds}, 1 \right\}.$$

Because it is still optimal for the bank to fully expropriate an entrepreneur who is audited and found to have misreported, the zeros of the function $\Gamma_g(\cdot)$ are once again coordinated defaults of the game induced by the repayment mechanism. To prove that general mechanisms are also subject to full coordinated defaults, note that if $E < E_S$ then $p(E, \overline{s}, \mu)\mu(s^o) < \frac{D-c_0}{f(\overline{s})}$ so that $\Gamma_g(\overline{s}) < 0$. It is therefore optimal for every entrepreneur, irrespective of his project's return, to declare default if he expects all other entrepreneurs to do the same. To prove that there are partially coordinated defaults as well, note that $\Gamma_g(s^D) > 0$ since truthtelling is an equilibrium of the general mechanism. But then an application of the Intermediate Value Theorem once again guarantees that there is an $s^* \in (s^D, \overline{s})$ such that $\Gamma_g(s^*) = 0$, so a partially coordinated default exists.

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B The Impossibility of Raising Resources at $t = 1$

We now provide two examples that show that the bank may not be able (or willing) to raise resources to conduct audits in $t = 1$. In the first example, the bank suffers from adverse selection, which makes him unwilling to raise resources ex-post. In the second example, the bank cannot commit to use his skills to extract repayment from entrepreneurs on behalf of those that could provide him with additional finance in $t = 1$. In what follows, we assume the bank holds no capital at $t = 0$, an assumption that is without loss of generality and made for expositional convenience.

B.1 Example 1

Consider the following simplified version of Myers and Majluf (1984). The bank may be of two types $\gamma \in \{g, b\}$, where g stands for *good* and b for *bad*. Management is fully aware of the bank's type while outside investors believe the bank is good with probability p. Apart from the portfolio of loans that are considered throughout this essay, a good bank also has assets-in-place worth A, while a bad bank does not.[26] Assets-in-place are illiquid and only provide the good bank with cash-flow in the far future.

We follow Myers and Majluf (1984) and adopt the following assumptions: (i) financiers demand competitive rates of return (which we normalize to zero); (ii) banks can only raise equity; and (iii) the bank's management only cares about *old* stockholders.

[26] The assumption that $A = 0$ for a bad bank is without loss of generality. It is only necessary that the illiquid assets held by a good bank be strictly more valuable than the one held by the bad bank.

Fix a threshold default strategy characterized by s^* on the part of entrepreneurs. We wish to establish the conditions under which the bank can prevent entrepreneurs from coordinating on this given strategy by raising equity at at $t = 1$ to conduct audits. Let $S = D(1 - H(s^*))$ denote the amount of resources the bank collects from creditworthy entrepreneurs at $t = 1$; $E = \int_0^{s^*} c(s)h(s)ds - S$ denote the total stock issue that would be required for the bank to raise the necessary resources to audit all entrepreneurs in default; and $B = \int_0^{s^*} f(s)h(s)ds - \int_0^{s^*} c(s)h(s)ds$ denote the net present value of conducting audits after entrepreneurs have followed the prescribed default strategy.

If the bank, knowing the true value of its assets in place, does not issue equity, then the market value of the old stockholders' stake in the firm is $V^{old} = S + A$. If it does issue equity to pay for audit costs, then

$$V^{old} = \frac{P'}{P' + E} \left[E + S + A + B \right], \tag{19}$$

where $P' = D(1 - H(s^*)) + pA$ is the market value at $t = 1$ of old stockholder's shares if stock is issued. Old stockholders are better off if the bank issues equity if and only if

$$\frac{E}{E + P'}(S + A) \leq \frac{P'}{P' + E}(E + B). \tag{20}$$

As in Myers and Majluf (1984), the bank might refrain from issuing new equity to pay for audit costs when the share of existing cash $S = D(1 - H(s^*))$ and assets in place A going to new stockholders is greater than the share of increment to corporate value obtained by old stockholders. If the asymmetry of information regarding the bank's assets-in-place is sufficiently severe, the bank will refrain from issuing equity.

B.2 Example 2

Now, consider the following simplified version of Diamond and Rajan (2001). There are investors that have resources to loan to the bank, who has specific ability to collect the loans of the entrepreneurs it finances. The bank, however, cannot commit to any promises it makes to new investors. Events in the game unfold as following. The bank asks for a loan of $\int_0^{s^D} c(s)\,h(s)\,ds$ to conduct audits.

The potential lenders, who are willing to receive a competitive rate of return (that we normalize to zero), then decide whether to extend the loan or not. If the loan is not granted, the game ends. Otherwise, the bank, after collecting the loan, may propose to renegotiate the terms of the loan and, then, decides whether to audit the projects or not.

At this renegotiation stage, the bank makes a take it or leave it offer to the new investors. To simplify matters, any offer must be multiple of ϵ , where $\epsilon > 0$ is an arbitrarily small number (offers must be in cents of dollars).

Conditional on performing its auditing activities, the bank collects

$$D(1 - H(s^D)) + \int_0^{s^D} f(s)\,h(s)\,ds = I + \int_0^{s^D} c(s)\,h(s)\,ds \tag{21}$$

The amount $I + \int_0^{s^D} c(s)\,h(s)\,ds$ is then split between the bank and the new investors according to the terms of the renegotiation stage described above. In any subgame perfect equilibrium in the above game, the investors do not extend resources to the bank. The reason being that, having raised resources from the investors, it is optimal for the bank to offer at most ϵ to the investors. Anticipating that, investors do not provide resources. This discussion proves:

Proposition 9. *In any subgame perfect equilibrium of the game described above, the investors do not extend a loan to the bank.*

Much as in their paper, in our simplified version of Diamond and Rajan (2001), the bank is unable to raise resources when facing a liquidity shock (the need of resources to conduct audits in our model), because they lack the capacity to commit to use their specific ability to collect loans. In fact, as they argued, *conditional* on raising the resources to conduct the audits, a bank lacking commitment power towards its new financiers might threaten not to use its specific ability to audit projects unless it is given a lion's share of the collected loans."