Auctioning Dynamic Procurement Contracts*

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Abstract

This paper considers the problem faced by a welfare maximizing government agency seeking, at each of $T \geq 2$ periods, to procure an indivisible good from one of N firms. The firms are privately informed about a time varying cost parameter and can exert unobservable effort toward cost reduction. In the benchmark case in which costs are observed by the government agency, the optimal mechanism calls, in every period $t \in 1, ..., T$, for the selection of the firm with the lowest cost parameter in that period, who is then offered contracts that induce first best levels of efforts. Under private information, *given* a selection procedure, the government distorts downward the recommended effort levels in all periods so to reduce the informational rents left to the firms. Such distortions are more pronounced if the firms' privately observed cost parameters display a larger degree of persistence, but decrease over time. For all periods $t \geq 2$, the optimal selection procedure is biased in favor of firms that drew lower cost parameters in the *first* period, since it is cheaper (in terms of informational rents) to provide more powerful incentives toward cost reduction to those firms. In particular, the firm who produces in period 1 is given an advantage over competitors in *all* future periods. We also show, by means of an example, that such ex-post bias may induce more investments from all firms.

Keywords: Dynamic Procurement, Time-Varying Private Information, Selection Procedure, Dynamic Incentives

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1 Introduction

In a seminal paper, Demsetz (1968) argued that the competition induced by auctions for the right to produce a good/service (or to explore a resource) can perfectly substitute ex-post regulation of a monopolist. Also

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seminally, Laffont and Tirole (1987) show that, although powerful, the reasoning behind this substitution is, to say the least, incomplete. Indeed, in a (one-period) model in which potential producers are privately informed about their costs to produce the relevant good, the optimal mechanism still calls for non-trivial regulation policies ex-post and leaves rents for the firm who ultimately produce. More surprisingly, they prove a dichotomy/separation result: once one firm – the most efficient in their mechanism – is selected, the procurement contract between the government agency and this firm is exactly the same as in a single firm case.

One important aspect that is not considered by both Demsetz (1968) and Laffont and Tirole (1987) is the fact that the "right to be a monopolist" may be auctioned-off in each of multiple periods. A time dimension may allow for somewhat more sophisticated mechanisms that could, at least in principle – and as opposed to the separation result of Laffont and Tirole (1987) – explicitly link the competition and regulation stages and, as a consequence, get us closer to Demsetz' conjecture.

We introduce such time dimension by building a dynamic variant of Laffont and Tirole's (1987) paper, in which, for each of $T > 1$ periods, a benevolent government agency, who is able to commit to long-term contracts, procures an indivisible good from one of N ex-ante symmetric firms. At each period, a firm's cost to produce the good depends on (i) the amount of effort it exerts toward cost reduction, and (ii) a cost parameter that it privately observes. The privately known cost parameter evolves according to an AR(1) process. Except for this time dependence in the firms cost-structure, there are no other parameters linking periods in the model. Hence, all the dynamics produced by the optimal procurement mechanism stem from the evolution of the firms' cost parameters.

Much as happens in Laffont and Tirole's (1987) static setting, our dynamic model is such that ex-post regulation can be perfectly substituted by ex-ante competition for the benchmark case in which cost parameters are publicly observed. In terms of allocations, this amounts to an outcome in which, period by period, (i) the firm with the lowest cost parameter is selected to produce the good, and this firm (ii) exerts first best levels of effort toward cost reduction and (iii) gets zero profits.

When cost parameters are privately observed matters are slightly more complicated: the government agency faces a dynamic mechanism design problem, as it seeks to maximize total welfare subject to a sequence of incentive compatibility constraints: at any period, (given truthful past announcements) each firm must find it optimal to report truthfully its current cost parameter given truthful announcements by its competitors. As each firm learns its cost parameter privately over time, the set of possible deviations maybe large. Hence, as opposed to standard static contracting models (e.g., Myerson (1981) and Laffont and Tirole, 1986), a full characterization of the firms' incentive compatibility constraints is not feasible. Therefore, to solve the government's problem, we rely on a first order approach (Kapicka (2013) and Pavan, Segal and Toikka (forthcoming)). More precisely, instead of considering the whole set of incentive compatible constraints, we only impose a first order (necessary) condition for truthtelling on the government agency's problem, and solve a "relaxed" problem (and, of course, later verify that the solution to such relaxed problem satisfies all incentive compatibility constraints).

The relaxed problem is extremely simple. Similarly to Laffont and Tirole (1987), it calls for the government agency to maximize – by choice of a selection procedure and recommended effort levels – ,the present value of the social surplus generated by the production of the good deducted from the social cost of the informational rents that must be left to the firms in any incentive compatible mechanism. In our dynamic context, however, the informational rents that any firm of a given "type" (i.e., a cost parameter) derives at the moment the optimal mechanism is designed depends on the *whole sequence* of effort levels exerted by less efficient types. The intuitive reason is that, due to the degree of persistence of the stochastic process that describes the evolution of its cost parameters, the piece that is private information to the each firm at the contracting stage (period 1) is informative about future cost parameters. Therefore, upon drawing a low cost parameter at the contracting stage, a firm expects to draw lower cost parameters – and, as a consequence, be able to exert lower effort – in the future. This, in turn, allows a firm who draws a low type/cost parameter in the *first* period to lie upward (i.e., pretend it has a higher cost parameter) and save on current *and* future (expected) effort levels. Hence, through the informal rents component of the government agency's objective, the cost to induce larger effort levels from the firm who was selected to produce in t depends on the cost parameter it drew in period 1.

The costs of leaving (dynamic) informational rents to the firms fully shape the government agency's choice of the selection procedure and the sequence of recommended effort levels. As it is easier to convey the main interpretations of the results by considering the effect of informational rents on the selection procedure and recommended effort levels separately, we start with recommended effort levels. For a *fixed* selection procedure, the costs of leaving informational rents for a firm are larger: (i) the higher its likelihood of drawing low cost parameters in the first period and (ii) the larger the degree of persistence of the AR (1) process that describes the evolution of the its cost parameter – since a high degree of persistence makes a firm who drew a lower cost parameter in period 1 more likely to draw lower cost parameters in the future. Hence, at any given period and whatever firm selected to produce, effort levels are smaller (i.e., distorted downward in comparison to the first best) for technologies that display larger persistence. Last, as cost parameters drawn at the first period are less informative about cost parameters in the far future, recommended effort levels increase (and approach first best levels) over time.

As for the selection procedure, the government agency faces a non-trivial trade-off regarding its decision of whom to pick to produce in period t. On the one hand, holding *fixed* the amount of effort to be exerted toward cost reduction, picking the firm with the lowest cost parameter in period t obviously lowers the government agency's total costs to procure the good. On the other hand, the informational rent component of the cost to induce higher effort levels from any given firm depends on the cost parameter it drew in period 1. Hence, its cheaper to provide incentives for effort toward cost reduction for firms that drew lower cost parameters in period 1. This latter effect is a force towards tilting the selection procedure in favor of firms who drew lower cost parameters in period 1. In fact, at the optimal mechanism, the selection procedure implies, for any period $t \geq 2$, a bias in favor of the firm who drew the lowest cost parameter (and was selected) in the first period, in the following sense: if firm i was selected to produce in the first period, to be selected to produce in period $t \geq 2$, firm j must have, in that period, a cost parameter that is smaller than firm i's by a strictly positive amount. We name this strictly positive amount "bias function", and we fully derive the optimal mechanism's selection procedure in terms of such bias functions (as well as derive their dependence on time and the main parameters of the model).

Curiously, the (apparent) distortion introduced by the optimal selection procedure serves the purpose of making recommended effort levels closer to first best. Alternatively, the bias in favor of firms who drew lower cost parameters plays the role of reducing distortions in the amount of effort provided by firms who are ultimately selected to produce the good. Much as in Laffont and Tirole's (1987) static model, effort levels are distorted in our model to reduce the amount of informational rents the must be left to the firms. As, for an AR1, there is weak statistical link between a firm's cost parameter in the first period and the cost parameter it will draw in the far future, there is no value in neither distorting effort levels – since effort levels in the far future are not a source of informational rents for the firms in period 1 –, nor the selection procedure – since the selection procedure is only distorted to allow for higher effort – in the far future. Hence, as time goes by, the allocation induced by the optimal mechanism converges to first best levels, on the one hand, and, from a first period perspective, the firms collect no rents associated with those future allocations.[1] Put differently, in the very far future, it as if Demsetz' (1968) conjecture were to hold exactly true: at the time the mechanism is offered, firms compete aggressively for the right to produce the good in the far future, and the government agency is able to collect up-front the rents associated with ex-post monopoly. For intermediate periods, the government agency uses the fact that it will repeatedly auction off the right to produce the good to induce more competition ex-ante, when compared to Laffont and Tirole (1987). In fact, in period 1, firms compete for a bundled good: they compete for the right to produce in that period *and* to be granted a form of favoritism in later periods.

On top of its role in enhancing competition in period 1, the bias introduced at the optimal selection procedure may have yet and additional benefit. Indeed, by considering an extension of the model in which firms, before entering the contracting stage, may perform non-contractible investments that reduce the expected values for the cost parameters in all periods, we show, by means of a two-firm example, that the biased selection procedure induces more investments from both firms in any symmetric equilibrium when compared to a selection rule that, period by period, picks the firm with the lowest cost parameter. In the process of establishing this result, we also derive the optimal procurement mechanism for the case in which firms are ex-ante asymmetric.

Related Literature. We now discuss how our findings compare to the existing literature. In our motivation, we have already discussed Laffont and Tirole (1987). As we have pointed out, our model is a T-period version of theirs, in which the firms' cost parameters evolve stochastically. Among other things, by considering a full blown dynamic model, we show that their dichotomy result ceases to prevail. In particular, the selection procedure in periods $t \geq 2$ is tilted towards the firm that produced in the first period in order to allow for more powerful incentives at the procurement stage.

The fact that we consider the possibility of competition among N firms (rather than just a single one) to produce the good is the main difference between our work and Baron and Besanko (1984) – the first paper to consider time-varying private information in a two-period regulatory design problem –, and Besanko (1985), who considers an infinitely long relationship between a principal and an informed agent, and derive how allocative distortions evolve over time as a result of the agent's private information. In contrast to these papers, the fact that the principal in our model can pick from N possible agents makes the selection procedure something of great importance in our setting. Cisternas and Figueroa (2009) consider a two-period, pure private information, procurement model in which the firm who wins the right to produce the good in the first period can invest in cost reduction for the *second* period. In their model, cost parameters are iid over time. For their commitment benchmark (which is closest to our model), the profit-maximizing buyer grants, to the firm who wins the right to produce the good, an advantage in the selection procedure for the second period. While in their model, such advantage comes solely from the incentives that the buyer (a monopsonist) has to distort the allocation in order to induce more aggressive bids in the procurement auction (as in Myerson, 1981), in ours the distortion is intrinsically linked to (i) the time-dependence of the firms' cost parameters

[1] In fact, the government agency does not (in the limit) distort effort levels and the selection procedure in the far future precisely because the firms do not enjoy informational rents related to those allocations.

– in fact, there would be no bias in the selection procedure if the firms' cost parameter were to evolve in an iid fashion in our model –, and (ii) the fact that, by reducing the cost of leaving informational rents, the bias in the selection procedure allows for more powerful incentives for effort provision.[2]

Methodologically, we adopt the first order approach for dynamic design problems developed by Kapicka (2013) and Pavan, Segal and Toikka (forthcoming). Also, our interpretation of how the power of the procurement contract varies over time as a function the statistical linkage between the firm's period-t information and the one held by it at the period it signs the contract is borrowed from Pavan, Segal and Toikka (forthcoming).

Our work is one of many recent applied papers on Dynamic Mechanism Design in environments with evolving private information[3]. Among those, the closest to ours is Garrett and Pavan (2012a). While we are interested in understanding how a government agency's selection procedure evolves over time as a function of the suppliers' private information, Garrett and Pavan (2012a) are interested in understanding how CEOs' retention and turnover interact with their private information. In their model, upon firing an incumbent CEO in period t, the firm is randomly matched with a new CEO and designs a new compensation mechanism. From an ex-ante (period 0) perspective, the informational rents that have to be left for a replacement CEO in period t are larger than the ones for an incumbent CEO. This leads to excessive (and inefficient) retention in their model as time goes by. In contrast, in our model, from an ex-ante perspective, period-t informational rents become less costly for all firms that might deliver the good to the government agency. Hence, the selection procedure becomes closer to the first best as time goes by.

Organization. Section 2 lays down the model and derives the optimal mechanism for the benchmark case in which the firms' cost parameters are publicly observed. In section 3, we derive the optimal mechanism for the case in which firms have private information regarding their costs. Section 4 considers the case in which firms can perform non-contractible investments prior to the mechanism stage. In such section, we derive the optimal mechanism for the case in which firms are asymmetric and, by means of a two-firm example, establish that the bias introduced by the optimal selection procedure, may induce more ex-ante investments from the firms. We draw our concluding remarks in Section 5. Proofs that are not presented in the text can be found in the Appendix (Section 6).

2 The Model

There are N firms and a government agency who interact over T periods ($2 \leq T \leq \infty$). At each period t, $t = 1, ..., T$, the government agency seeks to procure, from (at most) one of the firms, a single indivisible good that yields $S_t > 0$ in value to the consumers.

Firm i's contractible cost to deliver the good at period t is

$$c_{it} = \theta_{it} - e_{it},$$

where $\theta_{it} \in \Theta_t = \left[\underline{\theta}_t, \overline{\theta}_t\right]$ is a cost parameter that is privately observed by the firm at the beginning of period

[2] Also in a pure procurement auction setting, Rezende (2009) considers the effect of an *exogenous* preference for a given firm on the selection procedure of a profit-maximizing monopsonist. He shows that, at the optimal mechanism, the monopsonist grants an advantage to its favorite firm that is smaller than its true preference differential.

[3] Some examples are Courty and Li (2000), Battaglini (2005) and Board (2008) for dynamic sales problems, Eso and Szentes (2007) for dynamic auctions and Pavan and Garret (2012b) for seniority based remuneration schemes.

t, and $e_{it} \in [0, \overline{e}]$ – where $\overline{e}$ is a large, but finite, number[4] – is the unobservable amount effort it exerts to reduce the cost of the project after learning θ_{it}.

We denote by

$$C_{it} \equiv \{c_{it} \in \mathbb{R} \,|\, c_{it} = \theta_{it} - e_t, \theta_{it} \in \Theta_t, e_{it} \in [0, \overline{e}]\}$$

the set of all possible cost realizations of firm i in period t. To exert effort level e_{it}, firm i incurs in a disutility given by $\varphi(e_{it}) \in \mathbb{R}$, which satisfies $\varphi(0) = \varphi'(0) = 0$, $\varphi'(e_{it}) > 0$ for $e_{it} > 0$, and $\varphi''(e_{it}) > 0$ and $\varphi'''(e_{it}) \geq 0$ for all e_{it} in $[0, \overline{e}]$.

We adopt the accounting convention that, at any period t, if firm i is in charge of delivering the good, the cost c_{it} is paid by the government agency, who then makes a net transfer of p_{it} to the firm. Letting δ be firm i's discount factor, $p_i^T = (p_{i1}, ..., p_{iT})$ streams of pay made to firm i, $e_i^T = (e_{i1}, ..., e_{iT})$ sequences of effort levels toward cost reduction chosen by the firm, and sequences $x_i^T = (x_{i1}, ..., x_{iT})$ – where $x_{it} \in \{0, 1\}$ denotes whether firm i is in charge of delivering the good in period t –, firm i's (Bernoulli) utility is:

$$U_i\left(p_i^T, e_i^T, x_i^T\right) = \sum_{t=1}^{T} \delta^{t-1} x_{it}\left[p_{it} - \varphi\left(e_{it}\right)\right].$$

In words, the firm's payoff is the discounted value of the transfers it receives net of effort costs over the periods the good is procured from it.

The government agency faces a shadow cost $\lambda > 0$ of public funds. As a result, the net surplus the consumers enjoy if the projects is provided at period t by firm i is:

$$S_t - (1 + \lambda)\left[p_{it} + c_{it}\right].$$

Throughout, we assume that the consumers' have the same discount rate as the firms. Hence, the discounted value of the consumers' net surplus when the project is provided by firms according to $\left\{x_i^T\right\}_{i=1}^N$, costs are $\left\{c_i^T\right\}_{i=1}^N$ and net transfers are $\left\{p_i^T\right\}_{i=1}^N$

$$U^C(\left\{x_i^T\right\}_{i=1}^N, \left\{c_i^T\right\}_{i=1}^N, \left\{p_i^T\right\}_{i=1}^N) = \sum_{t=1}^{T} \sum_{i=1}^{N} \delta^{t-1} x_{it}\left[S_t - (1 + \lambda)\left[p_{it} + c_{it}\right]\right]$$

The government agency is benevolent and contracts with the firm in $t = 1$ to maximize the expected sum of the firm's payoff and the discounted value of the consumer's net surplus.

$$U^P = U^C(\left\{x_i^T\right\}_{i=1}^N, \left\{c_i^T\right\}_{i=1}^N, \left\{p_i^T\right\}_{i=1}^N) + \sum_{i=1}^{N} U_i\left(p_i^T, e_i^T, x_i^T\right)$$

The evolution of the cost parameters. We assume that firms are ex-ante symmetric in their cost structure.[5] Firm i's first period cost, θ_{i1}, is drawn from a log-concave distribution $F(\theta_{i1})$, with density $f(\theta_{i1})$. For $t \geq 2$, the cost parameter θ_{it} evolves according to the following AR(1) process:

$$\theta_{it} = \alpha + \beta\theta_{it-1} + \epsilon_{it} \tag{AR1}$$

[4] More precisely, we assume that $\overline{e}$ is such that

$$1 < \varphi'(\overline{e}),$$

where $\varphi(.)$ is the cost of exerting productive effort for the firm as will be defined below.

[5] We assume a symmetric cost structure so to make our result that the optimal procurement mechanism is asymmetric starker. In Section 4, we deal with the case in which firms are asymmetric at the contracting stage.

where $\alpha > 0$, $\epsilon_{it} \in [\underline{\epsilon}_t, \overline{\epsilon}_t]$ is a zero mean random shock statistically independent of θ_{it-1} and iid across firms, with density $g(\epsilon_{it}) > 0$, and $0 < \beta < 1$ is the parameter that captures the degree of persistence in the firms' technologies.

2.1 Benchmark: Full Information

To understand better the forces at play in the model and, in particular, the difficulties faced by the government agency in designing an incentive scheme that, at each t, selects the firm with the lowest cost parameter and induces high effort toward cost reduction over time, it is worth analyzing, as a benchmark, the model for the case in which the government agency fully observes the cost parameters of the firms, $\{\theta_{it}\}_{it=1}^{T}$.

Toward that, we start by noticing that, since

$$\sum_{t=1}^{T} \delta^{t-1} x_{it} p_t = U^A \left(p_i^T, e_i^T, x_i^T\right) + \sum_{t=1}^{T} \delta^{t-1} x_{it} \varphi\left(e_t\right),$$

one can write the government agency's payoff as

$$\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it} \left[S_t - (1+\lambda)\left[\theta_{it} - e_{it} + \varphi\left(e_{it}\right)\right] - \lambda U_i \left(p_i^T, e_i^T, x_i^T\right)\right] \qquad \text{(Gov. Payoff)}$$

Hence, assuming that, for all t, S_t is large, the government agency's problem under complete information amounts to minimizing the present value of the (social) costs to procure the good over time and the sum of the rents left to the firms, $U_i(p^T, e^T)$. Leaving rents to the firms is perceived by the government agency as socially costly because of the shadow cost of public funds, $\lambda > 0$. The next result follows from these observations:

Proposition 1 *When $\{\theta_{it}\}_{it}$ is observed by the government agency, the optimal selects, at each t, the most efficient to deliver the good, i.e.:*

$$x_{it} = 1 \textit{ if, and only if, } \theta_{it} \leq \theta_{jt} \textit{ for all } j$$

Furthermore, the selected firm is induced to exert an effort level defined by

$$\varphi'(e_{it}^{FB}) = 1.$$

This effort level can be achieved by fixed-price contracts that also leave no rents to the firms.

Under complete information, the government agency's problem can be broken in two separate parts: (i) selection stage: identify, at any given period, the firm with the lowest cost parameter, (ii) procurement stage: induce the selected firm to choose first best levels of effort toward cost reduction.

A setting in which $\{\theta_t\}_t$ is privately observed by the firm introduces two new effects. First, as the government agency is forced to rely on the selected firm's information to set cost targets $\{c_t\}_t$, there will be a conflict between inducing the firm to exert high effort toward cost reduction and the goal of minimizing the rents left to the firms. Second, there will be a non-trivial link between contract offered at the procurement stage and the selection criterion utilized by the government agency.

3 Incomplete Information: The Mechanism Design Problem:

Basic Definitions:

By the Revelation Principle, we can, without loss of generality, restrict attention to *direct* mechanisms for which the firms find it optimal to be truthful about their costs and obedient regarding the government agency's effort recommendation.

A direct mechanism $\Omega = \left\langle \{x_{it}, e_{it}, p_{it}\}_{i=1}^{N} \right\rangle_{t=1}^{T}$ consists of a collection of functions

$$\begin{aligned} x_{it}(.) &: X_1^t \times ... \times X_N^{t-1} \times \Theta_1^t \times ... \times \Theta_N^t \times C_1^{t-1} \times ... \times C_N^{t-1} \to \Re \\ e_{it}(.) &: X_1^t \times ... \times X_N^t \times \Theta_1^t \times ... \times \Theta_N^t \times C_1^{t-1} \times ... \times C_N^{t-1} \to \Re, \\ p_{it}(.; I) &: X_1^t \times ... \times X_N^t \times \Theta_1^t \times ... \times \Theta_N^t \times C_1^{t-1} \times ... \times C_N^{t-1} \to \Re, \end{aligned}$$

where, as functions of past allocations $x^{t-1} = \{(x_{i1}, ..., x_{it-1})\}_{i=1}^{N}$, the vector of realized past costs $c^{t-1} = \{(c_{i1}, ..., c_{it-1})\}_{i=1}^{N}$, and current and past reports $\theta^t = \left\{ \left(\widehat{\theta}_{i1}, ..., \widehat{\theta}_{it} \right) \right\}_{i=1}^{N}$,

1. $x_{it}\left(x_1^t, ..., x_N^t, \widehat{\theta}_1^t, ..., \widehat{\theta}_N^t, c_1^{t-1}, ..., c_N^{t-1} \right)$ denotes whether, at period t, the mechanism prescribes that firm i will deliver the good
2. $e_{it}\left(x_1^t, ..., x_N^t, \widehat{\theta}_1^t, ..., \widehat{\theta}_N^t, c_1^{t-1}, ..., c_N^{t-1} \right)$ is the amount of effort toward cost reduction recommended to firm i by the regulator in period t, and
3. $p_{it}\left(x_1^t, ..., x_N^t, \widehat{\theta}_1^t, ..., \widehat{\theta}_N^t, c_1^{t-1}, ..., c_N^{t-1} \right)$ is the net transfer the regulator makes to the firm i by the end of period t.

Throughout, with a slight abuse of notation, we will denote the allocation, equilibrium effort levels and net transfers induced by the mechanism $\left\langle \{x_{it}, e_{it}, p_{it}\}_{i=1}^{N} \right\rangle_{t=1}^{T}$ by $x_{it}(\theta^t) \equiv x_{it}(\theta^t, x^{t-1}(\theta^{t-1}), c^{t-1}(\theta^{t-1}))$, $e_{it}(\theta^t) \equiv e_{it}(\theta^t, x^t(\theta^t), c^{t-1}(\theta^{t-1}))$, and $p_t(\theta^t) \equiv p_t(\theta^t, x^t(\theta^t), c^t(\theta^t))$, respectively.

In a Direct Mechanism, firms choose, at each period t, the announcement of the current cost parameter $\widehat{\theta}_{it}$, given (a) current and past realizations of cost parameters they have observed, (b) the past efforts that were recommended, (c) past payments received, and (d) past allocations. Formally, a (pure) reporting strategy for the firm in direct mechanism is a collection of messages $\mathbf{m}_i = \{m_{it}\}_{t=1}^{T}$, where

$$m_{i1} \in \left[\underline{\theta}_1, \overline{\theta}_1 \right]$$

and, for $t \geq 2$,

$$m_{it} : \Theta_i^t \times \Theta_i^{t-1} \times [0, \overline{e}]^{tN} \times \Re^{tN} \times \{0, 1\}^{T(N-1)} \to \Theta_{it}$$

Incentive Compatibility and Participation:

A reporting strategy is said to be truthful if, for all $\left(\left(\theta_t, \theta^{t-1} \right), m^{t-1}, e^{t-1}, p^{t-1}, x^{t-1} \right)$,

$$m_t\left(\left(\theta_t, \theta^{t-1} \right), m^{t-1}, e^{t-1}, p^{t-1}, x^{t-1} \right) = \theta_t.$$

Denoting firm i's expected utility when it adopts reporting strategy $\mathbf{m}_i$, and its opponents adopt reporting strategies $\mathbf{m}_{-i}$ in the direct mechanism Ω by

$$\mathbb{E}^{\Omega, \mathbf{m}_i, \mathbf{m}_{-i}} \left[U_i^A \left(p_i^T, e_i^T, x_i^T \right) \right]$$

and letting $\mathbf{m}_i = \boldsymbol{\theta}_i^T = (\theta_{i1}, \theta_{i2}, ..., \theta_{iT})$ be a truthful strategy, the mechanism Ω is (Bayesian) incentive compatible if

$$\mathbb{E}^{\Omega, \boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right] \geq \mathbb{E}^{\Omega, \mathbf{m}_i, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right], \text{ for all } \mathbf{m}_i.$$

We assume that each firm has an outside option that yields an expected utility of zero. Hence, firm i will be willing to participate in the procurement mechanism as long as

$$\mathbb{E}^{\Omega, \boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right] \geq 0.$$

The timing in the (Direct) Procurement Mechanism:

At period $t = 1$, after firms learn θ_{i1}, the government agency proposes a direct mechanism Ω. Firms, then, send messages $\widehat{\theta}_{i1}$ to the government agency, who then decide on the allocation for the first period, $\left\{ x_{i1} \left(\widehat{\theta}_{i1}, \widehat{\theta}_{-i1} \right) \right\}_i$, recommends effort levels $e_{i1} \left(\widehat{\theta}_1 \right)$ to the firms, and proposes net transfers of $p_{i1} \left(\widehat{\theta}_1 \right) : C_t \rightarrow \Re$.

For periods $t \geq 2$, firms learn θ_{it} and send messages $\widehat{\theta}_{it}$ to the government agency, who then decides on the allocation for period t, $\{x_{it}(.)\}_i$, recommends effort $e_{it}(.)$ and proposes net transfers $p_{it}(.)$ to the firms.

3.1 The Government Agency's Problem:

Using the expression (Gov. Payoff) for the government agency's payoff, its problem of designing a dynamic procurement scheme can be written as

$$\max_{\Omega} E^{\Omega, \theta_i^T, \theta_{-i}^T} \left[\sum_{t=1}^{T} \delta^{t-1} \left(\sum_{i=1}^{N} x_{it} \left[S_t - (1+\lambda) \left[c_{it} \left(\theta^{t-1} \right) + \varphi(e_{it} \left(\theta^{t-1} \right)) \right] \right] - \lambda U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right) \right]$$

subject to a set of Incentive Compatibility constraints

$$\mathbb{E}^{\Omega, \boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right] \geq \mathbb{E}^{\Omega, \mathbf{m}_i, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right], \text{ for all } \mathbf{m}_i, i = 1, ..., N. \tag{IC}$$

and participation constraints

$$\mathbb{E}^{\Omega, \boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) \right] \geq 0, i = 1, ..., N. \tag{IR}$$

Since it learns the cost parameters $\{\theta_t\}_{t=1}^T$ privately over time, the set of deviations available for a firm in a given mechanism may be large: at any point in time and for a given realization of past cost parameters, as well as announcements regarding such parameters, the firm may decide to lie about its current cost parameter *conditional* on such past information. Hence, the set of constraints described by (IC) is large and a full characterization of such set is hard to obtain.

Instead of trying to fully characterize the set of IC constraints, we solve the government agency's problem by adopting the first order approach developed by Kapicka (2013) and Pavan, Segal and Toikka (forthcoming). This approach consists of replacing the constraints in (IC) by a first order (necessary) condition for truthtelling – which summarizes local incentive constraints – and solving a "relaxed" problem. We then check that the solution to the relaxed problem is in fact a solution for our problem of interest.

To characterize the government agency's relaxed problem, it is useful to define, for a given mechanism Ω,

$$V_i^{\Omega} (\theta_{i1}) \equiv \mathbb{E}^{\Omega, \boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T} \left[U_i^A \left(\left(p_i^T, e_i^T, x_i^T \right) \right) | \theta_{i1} \right] \tag{1}$$

as firm i's expected utility in period $t = 1$ when it observes θ_{i1}, and all firms (including firm i) adopt truthful strategies. If Ω is incentive compatible, $V_i^{\Omega}(\theta_{i1})$ represents firm i's value function when his initial type is θ_{1i}.

The following result, which is an application of Pavan, Segal and Toikka's (2012) Dynamic Envelope Theorem to our setting, establishes a key necessary condition that an Incentive Compatible mechanism must satisfy in terms of $V_i^{\Omega}(\theta_{i1})$.

Lemma 1 *If Ω is Incentive Compatible, then $V^{\Omega}(\theta_{i1})$ is absolutely continuous (and, therefore, differentiable almost everywhere) and satisfies the following formula*

$$V_i^{\Omega}(\theta_{i1}) = V_i^{\Omega}(\overline{\theta}_{i1}) + \int_{\theta_1}^{\overline{\theta}_1} \left[\mathbb{E}^{\boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T} \sum_{t=1}^{T} \delta^{t-1} x_{it}\left(\tau, \theta_{2i}, ..., \theta_{iT}, \boldsymbol{\theta}_{-i}^T\right) \beta^{t-1} \varphi'\left(e_{it}\left(\tau, \theta_{2i}, ..., \theta_{iT}, \boldsymbol{\theta}_{-i}^T\right)\right) |\tau \right] d\tau \tag{Envelope}$$

The interpretation for the above result is simple. For $\eta > 0$, a "type" θ_{i1} firm is $\eta > 0$ more efficient than a firm of type $\theta_{i1} + \eta$ in period $t = 1$. Moreover, for a common set of shocks $\{\epsilon_{it}\}_{t=2}^{T}$ in periods $t = 2, ..., T$, the type θ_{i1} firm will be $\beta^t \eta$ more efficient in period $t+1$ than type $\theta_{i1} + \eta$ firm. Hence, a firm with cost θ_{i1} can always behave as type $\theta_{i1} + \eta$, exert the effort levels recommended by the regulator to such type, call them $\{e_t\}$, and save

$$\sum_{t=1}^{T} \delta^{t-1} \mathbb{E}^{\theta^T} \left[x_{it} \left(\varphi(e_{it}) - \varphi\left(e_{it} - \beta^t \eta\right)\right)\right]$$

in terms of expected disutility of effort.

It follows that, for small $\eta > 0$, in any Incentive Compatible Mechanism, the expected utility of a firm with cost parameter θ_{i1} must be at least the expected utility of a firm with cost parameter $\theta_{i1} + \eta$ plus an (approximate) amount of[6]

$$\sum_{t=1}^{T} \delta^{t-1} \beta^{t-1} \mathbb{E}^{\theta^T} \left[x_{it} \varphi'(e_t) \right] \eta,$$

which captures the informational rents type θ_{i1} earns in excess of type $\theta_{i1} + \eta$'s payoff. Summing up the informational rents that type θ_{i1} collects in addition to payoff of all types τ larger than θ_{i1}, one obtains equation (Envelope).

The Relaxed Program:

The relaxed program maximizes the regulator's expected utility subject to the IR constraints and the necessary condition derived in Lemma 1, which, through equation (Envelope), pins down the value that $V_i^{\Omega}(\theta_{i1}; I)$ must have in an Incentive Compatible mechanism.

Plugging equation (Envelope) in the regulator's objective function, the relaxed problem can be written,

[6] We use the fact that a First Order Taylor expansion of $\varphi\left(e_t - \beta^{t-1}\eta\right)$ around e_t yields

$$\varphi\left(e_t - \beta^t \eta\right) \simeq \varphi(e_t) - \beta^{t-1} \varphi'(e_t).$$

after some integration by parts, as:

$$\max_{V_i^{\Omega}(\overline{\theta}_1),\{x_{it}(.),e_{it}(.)\}_{i,t}} \mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T}\delta^{t-1}\sum_{i=1}^{N}x_{it}\left(\theta^t\right)\left(S_t-(1+\lambda)\left[\theta_{it}-e_t\left(\theta^t\right)+\varphi\left(e_{it}\left(\theta^t\right)\right)\right]-\lambda\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\left(\theta^t\right)\right)\right)\right]$$
$$-\sum_{i=1}^{N}V_i^{\Omega}\left(\overline{\theta}_{i1}\right) \tag{2}$$

subject to the IR constraints[7]:

$$V_i^{\Omega}\left(\overline{\theta}_{i1}\right)\geq 0,\ i=1,...,N.$$

Clearly, it is optimal to set $V_i^{\Omega}\left(\overline{\theta}_{i1}\right)=0$ for all i. As for the other term of the objective, the following useful Lemma guides us on how to maximize it.

Lemma 2 *The sequences* $\left\{x_{it}\left(\theta^t\right),e_{it}\left(\theta^t\right)\right\}_{i,t,\theta^t}$ *maximize*

$$\mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T}\delta^{t-1}\sum_{i=1}^{N}x_{it}\left(\theta^t\right)\left(S_t-(1+\lambda)\left[\theta_{it}-e_t\left(\theta^t\right)+\varphi\left(e_{it}\left(\theta^t\right)\right)\right]-\lambda\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\left(\theta^t\right)\right)\right)\right]$$

if, and only, they maximize

$$\max_{\{x_{it}(\theta^t),e_{it}(\theta^t)\}_{i,t,\theta^t}}\sum_{t=1}^{T}\delta^{t-1}\sum_{i=1}^{N}x_{it}\left[e_{it}-\theta_{it}-\varphi\left(e_{it}\right)-\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\right)\right]$$

for (almost) all $\{\theta_{it}\}_{it}$ *..*

Lemma (2) states that the solution to the relaxed program must coincide with the solution of the following (simpler) program:

$$\max_{\{x_{it}(\theta^t),e_{it}(\theta^t)\}_{i,t,\theta^t}}\sum_{t=1}^{T}\delta^{t-1}\sum_{i=1}^{N}x_{it}\left[S_t+e_{it}-\theta_{it}-\varphi\left(e_{it}\right)-\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\right)\right] \tag{PointwiseProgram}$$

To solve (PointwiseProgram), it is convenient to define the effort-related value of having firm i delivering the good in period t as

$$S_{it}\left(\theta_{i1};\beta\right)=\max_{e_{it}}\left[e_{it}-\varphi\left(e_{it}\right)-\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\right)\right]$$

and, for $\theta_{i1}<\theta_{k1}$ the effort-related *incremental* value of having firm i rather than firm k delivering the good in period t as

$$\Delta_{ik}^{t}\left(\beta\right)=S_{it}\left(\theta_{i1};\beta\right)-S_{kt}\left(\theta_{j1};\beta\right)>0$$

Having defined those objects, we are in shape to establish our main result:

[7] As

$$V_i^{\Omega}\left(\theta_{i1}\right)=V_i^{\Omega}\left(\overline{\theta}_{i1}\right)+\int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[\mathbb{E}^{\boldsymbol{\theta}_i^T,\boldsymbol{\theta}_{-i}^T}\sum_{t=1}^{T}\delta^{t-1}x_{it}\left(\tau,\theta_{2i},...,\theta_{iT},\boldsymbol{\theta}_{-i}^T\right)\beta^{t-1}\varphi'\left(e_{it}\left(\tau,\theta_{2i},...,\theta_{iT},\boldsymbol{\theta}_{-i}^T\right)\right)|\tau\right]$$

in any Incentive Compatible mechanism, as long as $V_i^{\Omega}\left(\overline{\theta}_{i1}\right)$ is non-negative, firm i will have a non-negative expected payoff.

Proposition 2 *Assume that, for all $t = 1, ..., T$, S_t is large enough so to guarantee that*

$$S_t - \theta_{it} + S_{it}(\theta_{i1}; \beta) > 0 \text{ for some firm } i.$$

Denote by $j^* = \arg\min_k \{\theta_{k1}\}$ the firm who draws the lowest cost parameter in period 1. Then, the solution to the government agency's relaxed problem is given by $V^{\Omega}(\overline{\theta}_1) = 0$,

$$x_{j^*1}(\theta_{11}, ..., \theta_{N1}) = 1, x_{i1}(\theta_{11}, ..., \theta_{N1}) = 0 \text{ for all } i,$$

for $t \geq 2$,

$$\begin{aligned} x_{it}(\theta^t) &= \begin{cases} 1 \text{if } \theta_{it} + \Delta^t_{j^*i}(\beta) < \min_{k \neq j^*} \left\{\theta_{j^*t}, \theta_{kt} + \Delta^t_{j^*k}(\beta)\right\} \\ 0 \text{ otherwise} \end{cases}, i \neq j^* \\ x_{j^*t}(\theta^t) &= \begin{cases} 1 \text{if } \theta_{j^*t} \leq \min_k \left\{\theta_{kt} + \Delta^t_{j^*k}(\beta)\right\} \\ 0 \text{ otherwise} \end{cases} \end{aligned}$$

and $\left\{e_{it}(\theta^t)\right\}_{it}$ so that $e_{it}(\theta^t) = 0$ whenever $x_{it}(\theta^t) = 0$ and

$$e_{it}(\theta^t) = e^*_{it}(\theta_{i1}) = \arg\max_{e_{it}} e_{it} - \varphi(e_{it}) - \frac{\lambda}{(1+\lambda)} \frac{F(\theta_{i1})}{f(\theta_{i1})} \beta^{t-1} \varphi'(e_{it})$$

if $x_{it}(\theta^t) = 1$.

Before proceeding with a discussion of Proposition 2, we notice that we just assume that S_t is large to make sure that the government agency will find it optimal to buy the good in all t. If the assumption were not to hold – i.e., for some t,

$$S_t - \theta_{it} + S_{it}(\theta_{i1}; \beta) < 0 \text{ for all } i$$

– one would only need to Proposition 2 by making $x_{it}(\theta^t) = 0$ for all i.

Now, the mechanism that solves the Relaxed Program works as follows. In any period t, if firm i is selected to provide the good, its recommended effort level maximizes the period-t surplus net of the impact that period-t effort has on the informational rents cost that the government agency faces in period 1:

$$\underbrace{S_t + e_{it} - \theta_{it} - \varphi(e_{it})}_{\text{period-t total surplus}} - \underbrace{\frac{\lambda}{(1+\lambda)} \frac{F(\theta_{i1})}{f(\theta_{i1})} \beta^{t-1} \varphi'(e_{it})}_{\text{Impact of } e_{it} \text{ on Info Rents Cost}}.$$

Regarding the selection procedure, in the very first period, the firm with the lowest cost parameter, j^*, is chosen. For periods $t \geq 2$, the government agency computes the effort-related incremental value of having firm j^* rather than firm i delivering the good in period t :

$$\Delta^t_{j^*,i}(\theta_{i1}, \theta_{j1}; \beta) = S_{j^*t}(\theta_{j^*1}; \beta) - S_{it}(\theta_{i1}; \beta) > 0,$$

then (i) treats firm i as having (adjusted) costs parameters

$$\widetilde{\theta}_{it} = \theta_{it} + \Delta^t_{j^*,i}(\theta_{i1}, \theta_{j1}; \beta)$$

and (ii) selects the firm with the smallest $\widetilde{\theta}_{it}$.[8]

Some features of the mechanism in Proposition 2 deserve longer discussion. First, period-t the recommended effort level if firm i is selected is

$$e_{it}^{*}\left(\theta_{i1}\right)=\arg\max_{e_{it}} e_{it}-\varphi\left(e_{it}\right)-\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\right), \tag{Effort}$$

which only depends on θ_{i1}, the cost parameter drawn by the firm i in the *first* period. This is a joint implication of (i) the fact that the piece of information that is private to firm i at the contracting stage is θ_{i1} and (ii) the fact that cost parameters follow an AR1 process. Due to the persistence of the cost process, if firm i draws a low cost parameter in period 1, it is able to collect informational rents stemming from the effort levels recommended by the government agency in all periods that it is selected to provide the good. The reason is simple: a firm that draws a low θ_{i1} is likely to draw a low θ_{it} in any period t. Hence, it is feasible for a firm that draws a low θ_{i1} to report to have higher cost parameters in all future periods and, by doing so, economize (in expected value) on the amount of effort levels the mechanism prescribes for each t when it is selected to produce. Therefore, to report truthfully in period 1, the firm demands up-front rents that relate to the amount of effort it could economize by reporting to be less efficient than what it really is in all future periods. As our discussion of Lemma 1 suggests, the amount of effort the firm expects to economize in period t by reporting a higher type in period 1 depends on the statistical "linkage" between θ_{i1} and θ_{it} (see Pavan, Segal and Toikka (2013)). For an AR1 process, this linkage is fully described by θ_{i1} and the impulse response function, β^{t-1} : this is why only θ_{i1} affects the effort level in equation (Effort).

Second, and stemming from the feature described above, the period-1 draw of the firms cost parameter impact the mechanism's selection procedure in *all* later periods. In fact, for any firm i selected to produce in period t, a lower cost parameter drawn in period 1 increases

$$S_{it}\left(\theta_{i1};\beta\right)\equiv e_{it}^{*}\left(\theta_{i1}\right)-\varphi\left(e_{it}^{*}\left(\theta_{i1}\right)\right)-\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}^{*}\left(\theta_{i1}\right)\right),$$

which is the component of the period t surplus that depends on effort. Indeed, by the Envelope Theorem,

$$\frac{dS_{it}\left(\theta_{i1};\beta\right)}{d\theta_{i1}}=-\frac{\lambda}{(1+\lambda)}\frac{d\frac{F(\theta_{i1})}{f(\theta_{i1})}}{d\theta_{i1}}\beta^{t-1}\varphi'\left(e_{it}^{*}\left(\theta_{i1}\right)\right)<0$$

In particular, if firm i draws θ_{i1} while firm k draws $\theta_{k1}>\theta_{i1}$ *in period 1*, the effort-related incremental value of having firm i rather than firm k providing the good *in period* t is

$$\Delta_{i,k}^{t}\left(\theta_{i1},\theta_{j1};\beta\right)=S_{it}\left(\theta_{i1};\beta\right)-S_{kt}\left(\theta_{j1};\beta\right)>0.$$

This, in turn, implies that the government agency will only select firm k over firm i to deliver the good in period t if firm $k's$ cost advantage in that period outweighs the effort-related incremental value that firm i can deliver:

$$\theta_{kt}\leq\theta_{it}-\Delta_{i,k}^{t}\left(\theta_{i1},\theta_{j1};\beta\right).$$

[8] Notice that

$$\widetilde{\theta}_{j^*t}=\theta_{j^*t}$$

for

$$\Delta_{j^*j^*}^{t}\left(\beta\right)=0.$$

and this holds true if, and only if,

$$\widetilde{\theta}_{kt} \leq \widetilde{\theta}_{it}$$

as prescribed by the selection criterion in Proposition 2. Alternatively, as

$$\Delta^t_{i,k}\left(\theta_{i1}, \theta_{j1}; \beta\right) = \Delta^t_{j^*k}\left(\beta\right) - \Delta^t_{j^*i}\left(\beta\right),$$

in order to select whether firm i or k should produce in period t, all that the government agency needs to know is their true cost parameters, θ_{it} and θ_{kt}, and their cost disadvantage in relation to j^*, the firm that was selected in the first period.

The above discussion shows that the selection criterion in Proposition 2 treats (ex-ante) symmetric firms asymmetrically in all periods $t \geq 2$. In fact, for any two firms that have the *same* cost parameter in period t, the one who had a lower cost in period 1 will be favored in period t.

The solution to the government agency's relaxed problem differs substantially from the (first best) allocation attained under complete information. When the government agency observers the firms' cost parameters $\{\theta_{it}\}_{it}$ over time, the optimal mechanism (as described in Proposition 1) is time-independent: at each t, the firm with the lowest period-t cost parameter is selected and is induced to exert efficient amounts of effort. In contrast, when cost parameters are privately observed by the firms, the government agency must leave informational rents to the firms, and those rents depend on the amount of recommended effort. To reduce the amount of informational rents, the government agency distorts downward the amount of effort recommended to the firms in any period t, and such distortion is more pronounced for larger cost parameters drawn in the *first* period, $\{\theta_{i1}\}_i$. Therefore, to move closer toward first best level of efforts, the selection procedure in period t is biased in favor of firms that had lower cost parameters in period 1.

The link between the selection and the procurement stages described above is a joint implication of the asymmetry of information and the repeated interaction between the government agency and the firms.[9] Indeed, from Proposition 1, one sees that there is full separation between selection and the contract offered at the procurement stage under complete information. Moreover, in a static procurement model with asymmetric information such as the one in Laffont and Tirole (1986) – which corresponds to our model with $T = 1$ –, a separation/dichotomy result also holds: the most efficient firm is selected to produce and, at the procurement stage, the contract between the government agency and this firm is exactly the same as in a single firm case.

Overall Incentive Compatibility and Indirect Implementation:

Clearly, as the set of constraints the government agency faces is larger than the set of constraints implied by the relaxed problem, the value attainable by the latter is an upper bound to the value attainable by government agency's true problem. We now show that the government agency's true maximization problem attains such upper bound.

Proposition 3 *Coupled with properly designed transfers, the solution to the government agency's relaxed problem satisfies all Incentive Compatibility constraints. Therefore, the mechanism in Proposition 2 is the optimal procurement mechanism.*

[9] As Proposition 4 below shows, the persistence in the firms' techonology, as captured by β, is also key for the result.

3.2 Comparative Statics:

As we have discussed, given our assumption that $\{\theta_{it}\}$ follow and AR1 process, both the effort levels recommended by the firms in case they are selected to produce, $\{e_{it}(\theta_{i1};\beta)\}_{it}$, and $\{\Delta^t_{i,j}(\theta_{i1},\theta_{j1};\beta)\}$, the effort-related incremental value of having firm i rather than firm j delivering the good in period t – which is the source of the bias in favor of firms that drew lower cost parameters in period 1 implied by the optimal selection procedure –, will only depend on the first period draw $\{\theta_{i1}\}$ and the persistence parameter of the firms' technology, β. Our next result derives how effort levels and the "bias" functions change with those parameters:

Proposition 4 *For fixed $(\theta_{1i};\beta)$, with $\beta \in (0,1)$, the effort level $e_{it}(\theta_1;\beta,I)$ increases over time:*

$$e_{it+1}(\theta_{i1};\beta) > e_{it}(\theta_{i1};\beta), ...t = 1,...,T-1.$$

Moreover, for any given t,

$$\frac{\partial e_{it}(\theta_{i1};\beta)}{\partial\beta} < 0 \text{ and } \frac{\partial e_{it}(\theta_{i1};\beta)}{\partial\theta_{i1}} < 0.$$

As $t \to \infty$, recommended effort levels converge to first best levels.
$\Delta^t_{i,j}(\theta_{i1},\theta_{j1};1)$ is constant over time. For $\theta_{i1} < \theta_{j1}$,

$$\frac{\partial\Delta^t_{i,j}(\theta_{i1},\theta_{j1};\beta)}{\partial\theta_{i1}} < 0 < \frac{\partial\Delta^t_{i,j}(\theta_{i1},\theta_{j1};\beta)}{\partial\theta_{j1}}.$$

Moreover, for $\beta < 1$, $\Delta^t_{i,j}(\theta_{i1},\theta_{j1};\beta) \to 0$, as t grows large.

Recall from Lemma 2 that the optimal mechanism maximizes

$$\sum_{t=1}^{T}\delta^{t-1}\sum_{i=1}^{N}x_{it}\left(\theta^t\right)\left[S_t + e_{it} - \theta_{it} - \varphi\left(e_{it}\right) - \frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\right)\right].$$

All the properties established in the above Proposition stem from the properties of the period-t informational rents cost component of the above objective:

$$\sum_{i=1}^{N}x_{it}\left(\theta^t\right)\left[\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)}\beta^{t-1}\varphi'\left(e_{it}\right)\right].$$

Firstly, such cost decreases over time for any $\beta \in (0,1)$. Therefore, recommended effort levels are non-decreasing over time. Second, for any given period t, the largest (i) the persistence of the firms' cost parameters, β, and (ii) the cost drawn by the firms in the first period, $\{\theta_{i1}\}$, the higher the period-t informational rents cost (and, therefore, the smaller the recommended effort level). Third, the governments agency only finds it optimal to distort the selection decision in favor of firms that had lower period-1 cost parameters to reduce the informational rents it has to leave to the firms at an optimal mechanism. Therefore, as informational rents decrease over time, the "bias" functions $\Delta^{t+1}_{i,j}(\theta_{i1},\theta_{j1};\beta)$ go to zero as time goes by. Alternatively, the only reason why the government agency distorts the selections procedure is to be able to induce, through more powerful incentives, higher effort levels for the firm that is selected. As the costs to induce higher efforts decrease, the benefits of distorting the selections procedure are reduced. Last, for any two firms, say i and j, with $\theta_{i1} < \theta_{j1}$, the incremental effort-related value of having i rather than j producing in t is decreasing in θ_{i1} and increasing in θ_{j1} and this explains the last comparative statics result.

4 Investments:

Assume now that, *before* interacting with the government agency, say, in period 0, each firm i can make a non-contractible investment I_i at a cost $g(I_i)$ – with $g(0) = g'(0) = 0$, $g'(I_i) > 0$ and $g''(I_i) > 0$ for strictly positive I_i. The investment is interpreted as capital expenditure that will determine firm $i's$ cost structure over time as follows: I_i determines a log-concave distribution $F(\theta_{i1}|I_i)$, with density $f(\theta_{i1}|I_i)$, from which firm i's the cost parameter $\theta_{i1} \in [\underline{\theta}_1, \overline{\theta}_1]$ is drawn.[10] The density $f(\theta_{i1}|I_i)$ is assumed to be differentiable in I_i. Moreover, larger investments make lower cost parameters more likely: if $I_i' > I_i$

$$\frac{f(\theta_{i1}|I_i')}{f(\theta_{i1}|I_i)} \tag{MonotoneLR}$$

is strictly decreasing in θ_{i1}. We also assume that $f(\theta_{i1}|I_i)$ is differentiable in I_i.

For $t \geq 2$, the cost parameter θ_{it} evolves as before

$$\theta_{it} = \alpha + \beta\theta_{it-1} + \epsilon_{it}$$

Notice that the investment I_i reduces firm i's expected costs for all periods. In fact, a larger investment makes lower θ_{i1} more likely. This, in turn, reduces the expected costs for periods $t \geq 2$ through the process (AR1). Clearly, such an expected reduction will be larger for technologies that display larger persistence (as captured by a larger β).

In period 1, a vector of investment levels (I_i, I_{-i}) induce a subgame among the firms and the government agency. It can be easily seen[11] that, once one redefines

1. the *effort-related* value of having firm i delivering the good in period t as

$$S_{it}(\theta_{i1}; \beta, I_i) = \max_{e_{it}} (1+\lambda) \left[e_{it} - \varphi(e_{it}) - \frac{\lambda}{(1+\lambda)} \frac{F(\theta_{i1}|I_i)}{f(\theta_{i1}|I_i)} \beta^{t-1} \varphi'(e_{it}) \right]$$

   to incorporate firm i's investment, and

2. for $\theta_{i1} < \theta_{k1}$ the effort-related *incremental* value of having firm i rather than firm k delivering the good in period t as

$$\Delta_{ik}^t(\theta_{i1}, \theta_{j1}; \beta, I_i, I_k) = S_{it}(\theta_{i1}; \beta, I_i) - S_{kt}(\theta_{j1}; \beta, I_k)$$

   to also incorporate the investment levels of both firms[12]

all the results we derived go through with slight changes.

In fact, we can establish the following analogous of Proposition 3 for the case in which the firms investment levels are $I = (I_i, I_{-i})$

[10] This is the way Laffont and Tirole (1993, Chapter 1) model investment in their one-firm procurement model.

[11] For the sake of brevity we omit the full derivation, which can be obtained from the authors upon request.

[12] Notice that the effort-related *incremental* value of having firm i rather than firm k delivering the good in period t might be negative even if $\theta_{i1} < \theta_{k1}$. In fact, if $I_k > I_i$, one can have $\Delta_{ik}^t(\theta_{i1}, \theta_{j1}; \beta, I_i, I_k) < 0$ even if $\theta_{i1} < \theta_{k1}$. This is, however, immaterial for the analysis to come.

Proposition 5 *Fix a profile of investments $I = (I_i, I_{-i})$ and assume that, for all $t = 1, ..., T$, S_t is large enough so to guarantee that*

$$S_t - \theta_{it} + S_{it}(\theta_{i1}; \beta, I_i) > 0 \text{ for some firm } i.$$

Denote by $j^* = \arg\min_k \{\theta_{k1} + S_{k1}(\theta_{k1}; \beta, I_k)\}$ the firm with the lowest "total cost" to provide the service in $t = 1$. Then, at the procurement stage, the optimal procurement mechanism has

$$x_{j^*1}(\theta_{11}, ..., \theta_{N1}|I) = 1, x_{i1}(\theta_{11}, ..., \theta_{N1}|I) = 0 \;\; i \neq j,$$

for $t \geq 2$,

$$\begin{aligned} x_{it}\left(\theta^t|I\right) &= \begin{cases} 1 \text{if } \theta_{it} + \Delta^t_{j^*i}(\theta_{j^*1}, \theta_{i1}; \beta, I_{j^*}, I_i) < \min_{k \neq j^*}\left\{\theta_{j^*t}, \theta_{kt} + \Delta^t_{,j^*k}(\theta_{j^*1}, \theta_{k1}\beta, I_{j^*}, I_k)\right\} \\ 0 \text{ otherwise} \end{cases} , i \neq j^* \\ x_{j^*t}\left(\theta^t|I\right) &= \begin{cases} 1 \text{if } \theta_{j^*t} \leq \min_k\left\{\theta_{kt} + \Delta^t_{,j^*k}(\theta_{j^*1}, \theta_{k1}, \beta, I_{j^*}, I_k)\right\} \\ 0 \text{ otherwise} \end{cases} \end{aligned}$$

and $\left\{e_{it}\left(\theta^t|I\right)\right\}_{it}$ so that $e_{it}\left(\theta^t|I\right) = 0$ whenever $x_{it}\left(\theta^t|I\right) = 0$ and

$$e_{it}\left(\theta^t|I\right) = e^*_{it}(\theta_{i1}|I_i) = \arg\max_{e_{it}} e_{it} - \varphi(e_{it}) - \frac{\lambda}{(1+\lambda)}\frac{F(\theta_{i1}|I_i)}{f(\theta_{i1}|I_i)}\beta^{t-1}\varphi'(e_{it})$$

if $x_{it}\left(\theta^t|I\right) = 1$.

In period 0, firm i anticipates that, upon investing I_i (and when other firms' investment vector is I_{-i}), the procurement mechanism will evolve as described by Proposition 5. One can compute firm i's expected payoff at the investment stage as:

$$\begin{aligned} u_i(I_i, I_{-i}) &= \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[\int_{\theta_{1i}}^{\overline{\theta}_1}\left[\mathbb{E}^{\boldsymbol{\theta}_i^T, \boldsymbol{\theta}_j^T}\sum_{t=1}^{T}\delta^{t-1}x_{it}\left(\tau, \theta_{i2}, ..., \theta_{iT}, \boldsymbol{\theta}_j^T|I\right)\beta^{t-1}\varphi'\left(e_{it}\left(\tau, \theta_{2i}, ..., \theta_{iT}, \boldsymbol{\theta}_j^T|I\right)\right)|\tau, I\right]f(\theta_{i1}|I_i)\,d\theta_{i1}\right] \\ &= E_{\theta_1}\left[V_i^{\Omega}(\theta_{i1}, I)|I_i\right]. \end{aligned}$$

It follows that the investment stage can, then, be seeing as a normal-form game in which firm i's payoff is

$$u_i(I_i, I_{-i}) - g(I_i)$$

The following result establishes that an equilibrium for this game exists.

Proposition 6 *An equilibrium (possibly in mixed strategies) for the investment game exists*

What is of special interest for us in this extension that considers investments is how the optimal procurement – in particular, the ex-post bias favoring the firm who is picked to produce the service in period 1 – may affect the firms' ex-ante investments from the firms.

We tackle this issue by comparing the firms' ex-ante incentives to invest when the selection criterion is the one in Proposition 5 vis à vis when the criterion is such that the firm with the lowest cost parameter is selected period by period:

$$x^{FB}_{it}\left(\theta^t\right) = 1 \Leftrightarrow \theta_{it} \leq \theta_{jt} \text{ for all } j \qquad \text{(Eff. Selection)}$$

We perform such comparison (i) for the case of two firms, (ii) assuming that the effort levels are fixed at the levels defined in Proposition 5 (so that all that is varying is the selection criterion) and (iii) imposing that a symmetric equilibrium exists.

Proposition 7 *Assume there are two firms and that a symmetric pure strategy equilibrium exists for the normal-form game induced by the investment stage. Then, investments are larger under the selection procedure of the mechanism in Proposition 5 when compared to the case in which the selection procedure is as in condition (Eff. Selection)*

Under the conditions of Proposition, one has that the bias in the selection procedure introduced by the optimal procurement mechanism, on top of its role of reducing the firms' informational rents, has the beneficial (although involuntary) role of inducing more investments of the type considered by Laffont and Tirole (1993). The economic interpretation is simple: higher investments make lower first period cost parameters more likely, enhancing a firm's chance to be selected in the first period. As the optimal procurement mechanism favors, in periods $t \geq 2$, the firm that is selected in the first period, an additional marginal benefit to invest arises (when compared to a selection procedure with such bias), and the result ensues.

5 Concluding Remarks

We have considered the problem faced by a welfare maximizing government agency that seeks to procure goods over time from firms who are privately informed about a time varying cost parameter, and can exert unobservable effort toward cost reduction. The results were summarized, and their economic interpretations extensively discussed, in the Introduction. We, therefore, conclude with some avenues for future research.

It has been assumed that the firms' cost parameters follow an AR(1) process. While our model may represent a good first step in understanding the effects of evolving private information in a setting as Laffont and Tirole (1987), it would be interesting to extend the analysis for more general stochastic processes. Also, we have assumed that a firm's effort has only contemporaneous effect on its costs to produce the good. In many applications, it might be natural to assume that some sort of learning by doing takes place, so that effort exerted in period t reduces production costs for all periods $\tau \geq t$. Last, we have assumed that the set of firms that can produce the good remains fixed over time. If one assumes that new firms might arrive over a time (as buyers in Fuchs and Skrzypacz' (2010) model or CEOs in Garrett and Pavan's (2012), it is not a priori clear how the optimal selection procedure might change. We hope future research addresses these topics.

6 Appendix:

In this Appendix, we provide the proofs which cannot be found in the text.

Proof of Lemma 1: The assumptions on the processes $\{\theta_{it}\}_{t=1}^{T}$, along with $e_{it} \in [0, \overline{e}]$, guarantee that all the conditions of Pavan, Segal and Toikka's (2013) Theorem 1 holds. Their example 3 for the case of an AR1 process implies that

$$\frac{dV_i^{\Omega}(\theta_{i1})}{d\theta_{i1}} = -\mathbb{E}^{\boldsymbol{\theta}_i^T, \boldsymbol{\theta}_{-i}^T}\left[\sum_{t=1}^{T} \delta^{t-1} x_{it}\left(\theta_{i1}, \theta_{2i}, ..., \theta_{iT}, \boldsymbol{\theta}_{-i}^T\right) \beta^{t-1} \varphi'\left(e_{it}\left(\tau, \theta_{2i}, ..., \theta_{iT}, \boldsymbol{\theta}_{-i}^T\right)\right) |\theta_{i1}\right]$$

Integrating both sides from $\overline{\theta}_{i1}$ to θ_{i1}, the result follows.■

Proof of Lemma 2: Assume that $\left\{x_{it}^*\left(\theta^t\right), e_{it}^*\left(\theta^t\right)\right\}_{i,t,\theta^t}$ is the solution of

$$\max_{\left\{x_{it}\left(\theta^t\right), e_{it}\left(\theta^t\right)\right\}_{i,t,\theta^t}} \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it} \left[e_{it} - \theta_{it} - \varphi\left(e_{it}\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}\right)\right]$$

for almost all $\{\theta_{it}\}_{it}$. Then, by definition, for almost all $\{\theta_{it}\}_{it}$,

$$\begin{aligned} & \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it}^*\left(\theta^t\right) \left[e_{it}^*\left(\theta^t\right) - \theta_{it} - \varphi\left(e_{it}^*\left(\theta^t\right)\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}^*\left(\theta^t\right)\right)\right] \\ \geq & \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} \widetilde{x}_{it}\left(\theta^t\right) \left[\widetilde{e}_{it}\left(\theta^t\right) - \theta_{it} - \varphi\left(\widetilde{e}_{it}\left(\theta^t\right)\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(\widetilde{e}_{it}\left(\theta^t\right)\right)\right] \end{aligned}$$

for any alternative $\left\{\widetilde{x}_{it}\left(\theta^t\right), \widetilde{e}_{it}\left(\theta^t\right)\right\}$. Taking expectations (with respect to processes $\{\theta_{it}\}_{it}$) from both sides, sufficiency is shown.

Now, assume that $\left\{x_{it}^*\left(\theta^t\right), e_{it}^*\left(\theta^t\right)\right\}_{i,t,\theta^t}$ is the solution of

$$\max_{\left\{x_{it}\left(\theta^t\right), e_{it}\left(\theta^t\right)\right\}_{i,t,\theta^t}} \mathbb{E}^{\theta^T} \left[\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it}\left(\theta^t\right) \left(S_t - (1+\lambda)\left[\theta_{it} - e_t\left(\theta^t\right) + \varphi\left(e_{it}\left(\theta^t\right)\right)\right] - \lambda \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}\left(\theta^t\right)\right)\right)\right]$$

and that there is a positive probability set A of sequences $\{\theta_{it}\}_{it}$ for which

$$\begin{aligned} & \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} \widetilde{x}_{it}\left(\theta^t\right) \left[\widetilde{e}_{it}\left(\theta^t\right) - \theta_{it} - \varphi\left(\widetilde{e}_{it}\left(\theta^t\right)\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(\widetilde{e}_{it}\left(\theta^t\right)\right)\right] \quad \text{(Inequality)} \\ > & \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it}^*\left(\theta^t\right) \left[e_{it}^*\left(\theta^t\right) - \theta_{it} - \varphi\left(e_{it}^*\left(\theta^t\right)\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}^*\left(\theta^t\right)\right)\right] \end{aligned}$$

Let

$$\left\{\widehat{x}_{it}\left(.\right), \widehat{e}_{it}\left(.\right)\right\}_{i,t} = \begin{cases} \left\{\widetilde{x}_{it}\left(\theta^t\right), \widetilde{e}_{it}\left(\theta^t\right)\right\} & \text{if } \theta^T \in A \\ \left\{x_{it}^*\left(\theta^t\right), e_{it}^*\left(\theta^t\right)\right\}_{i,t,\theta^t} & \text{if } \theta^T \notin A \end{cases}$$

Then:

$$\begin{aligned} & \mathbb{E}^{\theta^T} \left[\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it}^*\left(\theta^t\right) \left(S_t - (1+\lambda)\left[\theta_{it} - e_t^*\left(\theta^t\right) + \varphi\left(e_{it}^*\left(\theta^t\right)\right)\right] - \lambda \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}^*\left(\theta^t\right)\right)\right)\right] \\ & -\mathbb{E}^{\theta^T} \left[\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} \widehat{x}_{it}\left(\theta^t\right) \left(S_t - (1+\lambda)\left[\theta_{it} - \widehat{e}_t\left(\theta^t\right) + \varphi\left(\widehat{e}_{it}\left(\theta^t\right)\right)\right] - \lambda \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(\widehat{e}_{it}\left(\theta^t\right)\right)\right)\right] \end{aligned}$$

is equal to $\Pr\left(\theta^T \in A\right)$ times

$$\left[\begin{array}{c} \mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it}^*\left(\theta^t\right) \left(S_t - (1+\lambda)\left[\theta_{it} - e_t^*\left(\theta^t\right) + \varphi\left(e_{it}^*\left(\theta^t\right)\right)\right] - \lambda \frac{F(\theta_{i1})}{f(\theta_{i1})} \beta^{t-1} \varphi'\left(e_{it}^*\left(\theta^t\right)\right)\right) | \theta^T \in A\right] \\ -\mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} \widehat{x}_{it}\left(\theta^t\right) \left(S_t - (1+\lambda)\left[\theta_{it} - \widehat{e}_t\left(\theta^t\right) + \varphi\left(\widehat{e}_{it}\left(\theta^t\right)\right)\right] - \lambda \frac{F(\theta_{i1})}{f(\theta_{i1})} \beta^{t-1} \varphi'\left(\widehat{e}_{it}\left(\theta^t\right)\right)\right) | \theta^T \in A\right] \end{array}\right]$$

which is strictly smaller than zero due to (Inequality), contradicting the optimality of $\left\{x_{it}^*\left(\theta^t\right), e_{it}^*\left(\theta^t\right)\right\}_{i,t,\theta^t}$. ■

Proof of Proposition 2: Follows from the discussion in the text.■

Proof of Proposition 3: It suffices to establish that the solution of PointwiseProgram described in Proposition 2, when coupled with the Dynamic Envelope Formula in Lemma 1, satisfies the overall incentive compatibility constraints. Noticing that both $x^*_{it}\left(\theta^t\right)$ and $e^*_{it}\left(\theta^t\right)$ are weakly decreasing in θ^t_i, the integral monotonicity condition of Pavan, Segal, and Toikka's (2013) Theorem 2 holds, and $\left\{x^*_{it}\left(\theta^t\right), e^*_{it}\left(\theta^t\right)\right\}_{i,t,\theta^t}$ is incentive compatible. .

Proof of Proposition 4: We first notice that solving

$$\max_{\{x_{it}(\theta^t),e_{it}(\theta^t)\}_{i,t,\theta^t}} \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} x_{it}\left[S_t + e_{it} - \theta_{it} - \varphi\left(e_{it}\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}\right)\right]$$

is equivalent to solving, for all $t \in \{1, ..., T\}$,

$$\max_{x_{it},e_{it}} \sum_{i=1}^{N} x_{it}\left[S_t + e_{it} - \theta_{it} - \varphi\left(e_{it}\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}\right)\right]$$

For the latter program, whenever $x_{it} = 1$, the optimal effort level solves

$$\max_{e_{it}} \underbrace{e_{it} - \varphi\left(e_{it}\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e_{it}\right)}_{\Omega(e_{it};\beta,\theta_{i1},t)}$$

Notice that

$$\frac{\partial \Omega\left(e_{it};\beta,\theta_{i1}\right)}{\partial e_{it}} = 1 - \varphi'\left(e_{it}\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi''\left(e_{it}\right)$$

is, for non-increasing in β and θ_{i1} and non-decreasing in t (this latter property whenever $\beta \in [0,1]$ as we have assumed).

It follows that $\Omega\left(e_{it};\beta,\theta_{i1},t\right)$ is supermodular in e_{it} and has increasing differences in $\left(e_{it}, -\beta, -\theta_{i1}, t\right)$. By Topkis' Theorem (Topkis (1998)),

$$\begin{aligned} e^*_{it+1}(\theta_{i1};\beta) &> e^*_{it}(\theta_{i1};\beta), ...t = 1, ..., T-1, \\ \frac{\partial e^*_{it}(\theta_{i1};\beta)}{\partial \beta} &< 0 \text{ and } \frac{\partial e^*_{it}(\theta_{i1};\beta)}{\partial \theta_{i1}} < 0 \end{aligned}$$

As $t \to \infty$, $\Omega\left(e_{it};\beta,\theta_{i1},t\right) \to e_{it} - \varphi\left(e_{it}\right)$, if $\beta < 1$, and recommended effort levels approach first best levels.

Now, for $\theta_{i1} < \theta_{k1}$,

$$\Delta^t_{i,k}\left(\theta_{i1},\theta_{k1};\beta\right) = S_{it}\left(\theta_{i1};\beta\right) - S_{kt}\left(\theta_{k1};\beta\right) > 0$$

where

$$S_{it}\left(\theta_{i1};\beta\right) \equiv e^*_{it}\left(\theta_{i1}\right) - \varphi\left(e^*_{it}\left(\theta_{i1}\right)\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{i1}\right)}{f\left(\theta_{i1}\right)} \beta^{t-1} \varphi'\left(e^*_{it}\left(\theta_{i1}\right)\right),$$

and

$$S_{kt}\left(\theta_{k1};\beta\right) \equiv e^*_{kt}\left(\theta_{k1}\right) - \varphi\left(e^*_{kt}\left(\theta_{k1}\right)\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_{k1}\right)}{f\left(\theta_{k1}\right)} \beta^{t-1} \varphi'\left(e^*_{kt}\left(\theta_{k1}\right)\right)$$

When $\beta = 1$, neither $S_{it}\left(\theta_{i1};\beta\right)$ nor $S_{kt}\left(\theta_{i1};\beta\right)$ depend on t. Hence, $\Delta^t_{i,k}\left(\theta_{i1},\theta_{k1};\beta\right)$ does not depend on t when $\beta = 1$. The second claim follows because, by the Envelope Theorem

$$\frac{dS_{it}\left(\theta_{i1};\beta\right)}{d\theta_{i1}} = -\frac{\lambda}{(1+\lambda)} \frac{d\frac{F(\theta_{i1})}{f(\theta_{i1})}}{d\theta_{i1}} \beta^{t-1} \varphi'\left(e^*_{it}\left(\theta_{i1}\right)\right) < 0$$

and

$$-\frac{dS_{kt}(\theta_{k1};\beta)}{d\theta_{k1}}=\frac{\lambda}{(1+\lambda)}\frac{d\frac{F(\theta_{k1})}{f(\theta_{k1})}}{d\theta_{k1}}\beta^{t-1}\varphi'\left(e_{kt}^{*}(\theta_{k1})\right).$$

By inspection, $\Delta_{i,k}^{t}(\theta_{i1},\theta_{k1};\beta)\to 0$ as $t\to\infty$, if $\beta<1$.■

Proof of Proposition 6: Notice that $u_i(.,I_j)$ is bounded for all I_j. In fact,

$$\begin{aligned}u_i(I_i,I_j) &= \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[\int_{\theta_{1i}}^{\overline{\theta}_1}\left[\mathbb{E}^{\boldsymbol{\theta}_i^T,\boldsymbol{\theta}_j^T}\sum_{t=1}^{T}\delta^{t-1}x_{it}\left(\tau,\theta_{i2},...,\theta_{iT},\boldsymbol{\theta}_j^T|I\right)\beta^{t-1}\varphi'\left(e_{it}\left(\tau,\theta_{2i},...,\theta_{iT},\boldsymbol{\theta}_j^T|I\right)\right)|\tau,I\right]f(\theta_{i1}|I_i)\,d\theta_{i1}\right]\\ &< \varphi'(\overline{e})\sum_{t=1}^{T}(\beta\delta)^{t-1}<\infty\end{aligned}$$

Moreover, since $g'(.)>0$, there is a finite $\overline{I}$ such that, for all I_j,

$$\arg\max_{I\geq 0}u_i(I_i,I_j)-g(I_i)=\arg\max_{I\in\left[0,\overline{I}\right]}u_i(I_i,I_j)-g(I_i)$$

Hence, without any loss of generality, we can assume that, at the investment stage, the firms' choice set is $\left[0,\overline{I}\right]$.

By Lebesgue's Dominated Convergence Theorem, the continuity (and boundness) of

$$\begin{aligned}&\int_{\theta_{1i}}^{\overline{\theta}_1}\left[\mathbb{E}^{\boldsymbol{\theta}_i^T,\boldsymbol{\theta}_j^T}\sum_{t=1}^{T}\delta^{t-1}x_{it}\left(\tau,\theta_{i2},...,\theta_{iT},\boldsymbol{\theta}_j^T|I\right)\beta^{t-1}\varphi'\left(e_{it}\left(\tau,\theta_{2i},...,\theta_{iT},\boldsymbol{\theta}_j^T|I\right)\right)|\tau,I_{i1}\right]\\ &= V_i^{\Omega}(\theta_{i1},I)\end{aligned}$$

in I[13] (along with the fact that $f(\theta_{i1}|I_i)$ is bounded in I_i) implies that

$$\begin{aligned}&\int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[\int_{\theta_{1i}}^{\overline{\theta}_1}\left[\mathbb{E}^{\boldsymbol{\theta}_i^T,\boldsymbol{\theta}_j^T}\sum_{t=1}^{T}\delta^{t-1}x_{it}\left(\tau,\theta_{i2},...,\theta_{iT},\boldsymbol{\theta}_j^T|I\right)\beta^{t-1}\varphi'\left(e_{it}\left(\tau,\theta_{2i},...,\theta_{iT},\boldsymbol{\theta}_j^T|I\right)\right)|\tau,I\right]f(\theta_{i1}|I_i)\,d\theta_{i1}\right]\\ &= E_{\theta_1}\left[V_i^{\Omega}(\theta_{i1},I)\,|I_i\right]\end{aligned}$$

is continuous in I.

By invoking Corollary 2.4 of Reny (2005), the result follows.

Proof of Proposition 7:

We wish to establish that the optimal procurement mechanism devised (that biases the selection procedure in periods $t\geq 2$ in favor of the firm who was selected in period 1) induces, in a *symmetric equilibrium*, more investment from the firms than a mechanism that, keeping the amount of effort by the selected firm the same as the one prescribed by the mechanism in Proposition 5, uses the first best selection procedure

$$x_{it}^{FB}=1\Leftrightarrow\theta_{it}\leq\theta_{jt},$$

in which the most efficient firm is always selected.

[13] The Envelope Theorem guarantees that

$$V_i^{\Omega}(\theta_{i1},I)$$

is continuous.

To do that, by Topkis' (1998) Theorem it suffices to show that, for any given firm, say, 1, and given firm 2's investment levels, $\overline{I}_2$, firm 1's marginal benefit to invest at $I_1 = \overline{I}_2$ is larger under the optimal procurement mechanism.

Toward establishing the above fact, let x_{it}^{FB}, $i = 1,2$ and $t = 1,2,...,T$, be the first best selection procedure, x_{it}^{*}, $i = 1,2$ and $t = 1,2,...,T$ be the selection procedure at the procurement stage (as described in Proposition 5). Clearly, when both firms invest the same amount (as it is the case in any symmetric equilibrium) $x_{i1}^{FB} = x_{i1}^{*}$.[14].

Also, we can define by Δ_{12}^t the incremental effort-related value of having firm 1 producing at period t, when the vector of investment levels is $I = \left(I_1, \overline{I}_2\right)$:

$$\begin{aligned}\Delta_{12}^t(\theta_{11},\theta_{21},I) &= \max_e \left[e - \psi(e) - \beta^{t-1}\frac{\lambda}{(1+\lambda)}\frac{F(\theta_{11}|I)}{f(\theta_{11}|I)}\psi'(e)\right] \\ &\quad - \max_e \left[e - \psi(e) - \beta^{t-1}\frac{\lambda}{(1+\lambda)}\frac{F(\theta_{21}|I)}{f(\theta_{21}|I)}\psi'(e)\right],\end{aligned}$$

and note that, by the Envelope Theorem, whenever $I_1 = \overline{I}_2$, Δ is positive when $\theta_{11} < \theta_{21}$ and negative otherwise.

Fixing firm 2´s investment level as $\overline{I}_2$, the difference between firm 1's expected investment benefit under the procurement's selection procedure and under the first-best selection procedure, at period t, is

$$E_{\theta_{11}}\left[\left(\int_{\underline{\theta}_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\ldots\theta_{1Tt}\theta_{21},\ldots,\theta_{2T}}\left[\left(x_{1t}^{*}(.) - x_{1t}^{FB}(.)\right)\varphi'\left(e_{1t}^{*}\left(\theta_1^t\right)I\right)|\tau\right]d\tau\right)|I\right] \tag{DI}$$

$$\begin{aligned}&= E_{\theta_{11}}\left[\left(\int_{\underline{\theta}_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\ldots\theta_{1Tt}\theta_{21},\ldots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t>0\right)\int_{\theta_{1t}-\Delta_{12}^t}^{\theta_{1t}}\varphi'\left(e_{1t}^{*}\left(\theta_1^t\right)\right)f\left(\theta_{2t}|\tau\right)d\theta_{2t}|\tau;\Delta>0\right)d\tau\right)|I\right] \\ &\quad - E_{\theta_{11}}\left[\left(\int_{\underline{\theta}_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\ldots\theta_{1Tt}\theta_{21},\ldots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t<0\right)\int_{\theta_{1t}}^{\theta_{1t}+\Delta_{12}^t}\varphi'\left(e_{1t}^{*}\left(\theta_1^t\right)\right)f\left(\theta_{2t}|\tau\right)d\theta_{2t}|\tau;\Delta<0\right)d\tau\right)|I\right]\end{aligned}$$

As the cost parameters are i.i.d. over time and firms, we have that $f(\theta_{2t}|\tau) = f(\theta_{2t})$, and so the equation above can be rewritten as

$$\begin{aligned}&E_{\theta_{11}}\left[\left(\int_{\underline{\theta}_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\ldots\theta_{1Tt}\theta_{21},\ldots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t>0\right)\int_{\theta_{1t}-\Delta_{12}^t}^{\theta_{1t}}\varphi'\left(e_{1t}^{*}\left(\theta_1^t\right)\right)f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t>0\right)d\tau\right)|I\right] \\ &- E_{\theta_{11}}\left[\left(\int_{\underline{\theta}_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\ldots\theta_{1Tt}\theta_{21},\ldots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t<0\right)\int_{\theta_{1t}}^{\theta_{1t}+\Delta_{12}^t}\varphi'\left(e_{1t}^{*}\left(\theta_1^t\right)\right)f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t<0\right)d\tau\right)|I\right].\end{aligned} \tag{DI1}$$

[14]Formally, $j^* = \arg\min_k \{\theta_{k1} + S_{k1}(\theta_{k1};\beta,I_k)\}$ if, and only if,

$$j^* = \arg\min_k \{\theta_{k1}\}.$$

Note that Δ_{12}^t is a random variable that depends on τ (that is, θ_{11}), θ_{21} and I_1,and, when I_1 is equal to $\overline{I}_2$, it is a zero-mean random variable with symmetric density.

As we argued above, by Topkis' Theorem (1998), if we show that the derivative of equation DI with respect to I_1 at $I_1 = \overline{I}_2$ is positive, the result is shown.

Now, the derivative of equation DI1 with respect to I_1 at $I_1 = \overline{I}_2$ is:

$$\underbrace{E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t>0\right)\varphi'\left(e_{1t}^*\left(\theta_1^t\right)\right)\int_{\theta_{1t}-\Delta_{12}^t}^{\theta_{1t}} f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t>0\right)d\tau\right)\frac{f_I\left(\theta_{11}|\overline{I}_2\right)}{f\left(\theta_{11}|\overline{I}_2\right)}|I\right]}_{A}$$

$$\underbrace{-E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t<0\right)\varphi'\left(e_{1t}^*\left(\theta_1^t\right)\right)\int_{\theta_{1t}}^{\theta_{1t}+\Delta_{12}^t} f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t<0\right)d\tau\right)\frac{f_I\left(\theta_{11}|\overline{I}_2\right)}{f\left(\theta_{11}|\overline{I}_2\right)}|I\right]}_{B}$$

$$\underbrace{E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\frac{\partial\Pr\left(\Delta_{12}^t>0\right)}{\partial I}\varphi'\left(e_{1t}^*\left(\theta_1^t\right)\right)\int_{\theta_{1t}-\Delta_{12}^t}^{\theta_{1t}} f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t>0\right)d\tau\right)|I\right]}_{C}$$

$$\underbrace{-E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\frac{\partial\Pr\left(\Delta_{12}^t<0\right)}{\partial I}\varphi'\left(e_{12}^*\left(\theta_{12}\right)\right)\int_{\theta_{1t}}^{\theta_{1t}+\Delta_{12}^t} f\left(\theta_{22}\right)d\theta_{22}|\tau,\Delta_{12}^t<0\right)d\tau\right)|I\right]}_{D}$$

$$\underbrace{+E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t>0\right)\varphi'\left(e_{1t}^*\left(\theta_1^t\right)\right)f\left(\theta_{12}-\Delta\right)\frac{\partial\Delta_{12}^t}{\partial I}\rfloor_{I=\overline{I}_2}|\tau,\Delta_{12}^t>0\right)d\tau\right)|I\right]}_{E}$$

$$\underbrace{-E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t<0\right)\varphi'\left(e_{12}^*\left(\theta_{12}\right)\right)f\left(\theta_{12}+\Delta\right)\frac{\partial\Delta_{12}^t}{\partial I}\rfloor_{I=\overline{I}_2}|\tau,\Delta_{12}^t<0\right)d\tau\right)|I\right]}_{G}$$

$$\underbrace{+E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t>0\right)\varphi''\left(e_{1t}^*\left(\theta_1^t\right)\right)\frac{\partial e_{1t}^*\left(\theta_1^t\right)}{\partial I}\int_{\theta_{1t}-\Delta_{12}^t}^{\theta_{1t}} f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t>0\right)d\tau\right)|I\right]}_{H}$$

$$\underbrace{-E_{\theta_{11}}\left[\left(\int_{\theta_{11}}^{\overline{\theta}_{11}} E_{\theta_{12},\dots\theta_{1Tt}\theta_{21},\dots,\theta_{2T}}\left(\Pr\left(\Delta_{12}^t<0\right)\varphi''\left(e_{1t}^*\left(\theta_1^t\right)\right)\frac{\partial e_{1t}^*\left(\theta_1^t\right)}{\partial I}\int_{\theta_{1t}}^{\theta_{1t}+\Delta_{12}^t} f\left(\theta_{2t}\right)d\theta_{2t}|\tau,\Delta_{12}^t<0\right)d\tau\right)|I\right]}_{I}$$

Now, since Δ_{12}^t is a zero-mean random variable with symmetric density and since firms are ex-ante symmetric, $H + I = 0$. Since $\frac{\partial\Pr\left(\Delta_{12}^t<0\right)}{\partial I} < 0 < \frac{\partial\Pr\left(\Delta_{12}^t>0\right)}{\partial I}$, $C + D > 0$. Moreover, as the expectation is taken on all the cost parameters θ_1^t, θ_2^t, we must have that $E + G = 0$. Finally, since the density of Δ_{12}^t is symmetric, we get that $A + B = 0$.

Thus, it follows that, fixing the amount invested by the other firm, starting from that same amount of investment, firm 1's incentive to increase it investment is larger under the selection procedure of the optimal procurement scheme than under a first-best selection procedure, and this proves the result. ■.

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