Optimal Dynamic Procurement and Investment

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Abstract

This paper considers the problem faced a regulator in providing dynamic incentives to a regulated firm who is privately informed about a varying cost parameter. In the model, before entering the regulation stage, the firm makes a non-contractible investment that reduces its expected costs to provide a public good over all periods of interaction with the regulator. We first derive the optimal regulatory scheme for a given amount of investment made by the firm. We establish that, any given period of its interaction with the regulator, the firm will faces more powerful incentives (i) the smaller the degree of persistence of the firm's privately known parameter, (ii) the smaller the initial investment. Moreover, contracts become more powerful over time. After deriving the optimal regulatory scheme for a given amount of investment, we characterize properties of the firm's optimal investment. Since the regulatory scheme is such that the firm does not internalize all its benefits, the firm underinvests relatively to the first best. More surprisingly, despite the fact that the social benefits of investments (measured as the reduction in the firm's expected costs to deliver the project over time) are increasing in the persistence of the firm's private information, the firm's optimal investment may be non-monotone in the degree of persistence of its private information.

Keywords: Regulation, Dynamic Private Information, Ex-Ante Investments.

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1 Introduction

Governments and firms often engage in long-term procurement contracts with their suppliers. In such relationships, not only the contracting parties may be unequally informed about, say, the cost of production of procured good/service at the outset, but also the private information held by the supplier is likely to evolve over time as a result of contingencies that he privately observes. Also, anticipating its long-term interaction with a supplier, a buyer may be willing to commit to relationship-specific investments that reduce the costs at which the supplier will be able to deliver the good throughout the relationship. As a consequence, a well designed contract must take into account the dynamics of the supplier's private information and the impact of

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ex-ante investments on the procurement phase. In particular, questions such as "should contracts be history-dependent?", "should contracts become more or less powerful over time?", or "should the procurement stage vary with the amount invested?" are extremely important. By considering the problem faced by a buyer who can invest to reduce the expected cost of a long-term supplier and who needs to provide dynamic incentives to such supplier, this paper tackles these (and other related) questions.

In the model, a benevolent government agency, who is able to commit to long-term contracts, procures an indivisible good from a supplier over a given period of time. At each period, the production cost depends on (i) the amount of effort the supplier exerts toward cost reduction, and (ii) a cost parameter that is his private information. The parameter cost evolves according to an AR (1) process. Before signing-up the contract, the government is allowed to make (or, alternatively, fully reimburse the supplier for) an investment that makes lower values for the cost parameters more likely. Hence, the government can induce lower costs of production through (i) an investment which affects the supplier's cost structure/technology and (ii) the incentives for effort toward cost reduction implicit in the contract it offers to the supplier.

For the benchmark case in which the cost parameters are publicly observed, efficient levels of both investment and effort towards cost reduction prevail at the optimal contract. To obtain the latter, all that the government needs to do is to make, period by period, the supplier residual claimant of any cost reduction it attains: this can be achieved through a sequence of fixed-price contracts. As for the investment, given that its choice does not interact with the contracting stage when information about the cost structure is symmetric, the best the government can do is to invest as much as prescribed by the first best.

For the case in which the firm has private information about its cost parameters, matters are slightly more complicated. In particular, there is a non-trivial interaction between the government's investment decision and the contracting stage. Such interaction is better understood if the analysis is split in two. Indeed, we first analyze the government's optimal contract for a *given* level of investment. Then, we analyze the government's investment problem.

Given a certain investment level, the government's problem is to design a dynamic contract that maximizes total welfare subject to a sequence of incentive compatibility constraints: at any period, (and given truthful past announcements) the supplier's must find it optimal to report truthfully its current cost parameter. As he learns his cost parameter privately over time, the set of deviations available to the supplier at any given moment may be large. Hence, as opposed to standard static contracting models (e.g., Myerson (1981) and Laffont and Tirole, 1986), a full characterization of his incentive compatibility constraints is not feasible. Therefore, to solve the government's problem, we rely on a first order approach (Kapicka (2010) and Pavan, Segal and Toikka (2010)). More precisely, instead of considering the whole set of incentive compatible constraints, we only impose a first order (necessary) condition for truthtelling on the government's problem, and solve a "relaxed" problem (and later verify that the solution to such relaxed problem turns out to satisfy all incentive compatibility constraints).

The relaxed problem is extremely simple. Similarly to static procurement models such as Laffont and Tirole (1986), it calls for the government to maximize the (present value of the) social surplus generated by the procured good deducted from the social cost of the informational rents that must be left to the supplier in any incentive compatible mechanism. In our dynamic context, however, the informational rents that the supplier of a given type derives at the moment it signs the contract depends on the *whole sequence* of effort levels exerted by less efficient types throughout their relationship with the government. The reason is that, due to the degree of persistence of the stochastic process that describes the evolution of its cost parameters, the piece that is private information to the supplier at the contracting stage is informative about future cost parameters. Therefore, upon drawing a low cost parameter at the contracting stage, the supplier expects to draw lower cost parameters (and, as a consequence, be able to exert lower effort) in the future. This, in turn, allows a supplier who draws a low cost parameter at the contracting stage to lie upward (i.e., pretend it has a higher cost parameter) and save on current *and* future (expected) effort levels.

The cost of leaving (dynamic) informational rents to the supplier fully shapes the government's decision of which sequence of effort levels to recommend. Such cost is larger (i) the higher the likelihood of the supplier drawing low cost parameters at the contracting stage and (ii) the degree of persistence of the AR (1) process that describes the evolution of the supplier's cost parameter. Hence, the sequence of recommended effort levels will be smaller (distorted downward in comparison to the first best) for technologies that display larger persistence. Also, since investments increase the likelihood of the supplier drawing lower cost parameters, the sequence of recommended effort levels decreases with investments. Finally, as cost parameters drawn at the contracting stage are less informative about cost parameters in the far future, recommended effort levels increase (and approach first best levels) over time.

The recommended effort levels can be implemented through a sequence of linear contracts. Those contracts lie in between the fixed-price contracts that are used by the government when information is symmetric (our benchmark case) and cost-plus contracts, in the sense that, at a period t, they make the supplier claimant of a fraction of any cost reduction it implements in t. As time goes by, the firm claims a larger fractions of any cost reduction it implements. Moreover, at a given period t, the supplier claims a larger fractions of any cost reduction it implements when the amount invested is smaller and for technologies (cost parameter processes) that display lower persistence. Put differently, contracts become closer to fixed-price contracts (i) as time goes by, and, at any given period, (ii) for smaller amounts of investments and (iii) the smaller the persistence in the evolution of the cost parameters.

The derivation of the optimal procurement contract for a given amount of investment highlights the government's trade-off when choosing how much to invest. On the one hand, by investing more, the government reduces (probabilistically) the values of the supplier's cost parameters throughout their interaction. This *direct* effect of investments reduces the (expected present value of the) costs the government faces at the procurement stage. On the other hand, larger investments induce less powerful contracts (and, therefore, a lower sequence of efforts towards cost reductions) at the procurement stage. This *indirect* effect of investments increases the (expected present value of the) costs the government faces at the procurement stage. As a consequence of the indirect effect, the amount invested is smaller than what prevails in the first best. More surprisingly, despite the fact that, in the model, the marginal social benefit of investing is increasing in the degree of persistence of the technology, the amount invested by the government may be non-monotone in the persistence of the technology. In fact, we present a numerical example in which, for a wide range of parameters, the government invests less when the degree of persistence increases.

Related Literature. [to be completed]

Organization. The paper is organized as follows. In section , the set-up of the model and the timing of events are described. In section **??**, the model is solved for the benchmark in which the quality of the project is known by the regulators. Section solves the model for the single-regulator case under asymmetric information. Section **??** considers a two-regulator arrangement for the case in which they fully coordinate. In section , we consider the case in which the regulators choose payments independently, whereas, in section , we consider the case in which payments and monitoring levels are chosen independently. Section draws the concluding remarks. All proofs are relegated to the Appendix.

2 The Model

There is a firm and a government agency who interact over T periods ($2 \leq T < \infty$). The firm, through its manager, can implement an indivisible, long-term, project. At period t, $t = 1, ..., T$,.the project yields $S_t > 0$ in value to the consumers and has a (contractible) cost of

$$c_t = \theta_t - e_t,$$

where $\theta_t \in \left[\underline{\theta}_t, \overline{\theta}_t\right]$ is a cost parameter that is privately observed by the firm at the beginning of period t, and $e_t \in [0, \overline{e}]$ – where $\overline{e}$ is a large, but finite, number[1] – is the unobservable amount effort it exerts to reduce the cost of the project after learning θ_t.

We denote by

$$C_t \equiv \{c_t \in \mathbb{R} \,|\, c_t = \theta_t - e_t, \theta_t \in \Theta_t, e_t \in [0, \overline{e}]\}$$

the set of all possible cost realizations in period t.

When the agent chooses an effort level e_t, it incurs in a disutility given by $\varphi(e_t) \in \mathbb{R}$, which satisfies $\varphi(0) = \varphi'(0) = 0$, and $\varphi''(e_t) > 0$ and $\varphi'''(e_t) \geq 0$ for all e_t in $[0, \overline{e}]$.

We adopt the accounting convention that, at any period t, the cost c_t is paid by the government agency, who then makes a net transfer of p_t to the firm. Letting δ be the firm's discount factor, $p^T = (p_1, ..., p_T)$ and $e^T = (e_1, ..., e_T)$, the firm's (Bernoulli) utility for streams of pay p^T and sequences of effort levels e^T is:

$$U^A\left(p^T, e^T\right) = \sum_{t=1}^{T} \delta^{t-1}\left[p_t - \varphi(e_t)\right].$$

In words, the firm's payoff is the discounted value of the transfers it receives net of effort costs.

There is shadow cost $\lambda > 0$ of public funds. As a result, the net surplus the consumers enjoy if the projects is provided at period t is:

$$S_t - (1 + \lambda)\left[p_t + c_t\right].$$

Throughout, we assume that the consumers' have the same discount rate as the firm. Hence, the discounted value of the consumers' net surplus when the project is provided in all periods (at a cost c_t,in period t) and the stream of pay made to the firm is p^T is:

$$U^C(c^T, p^T) = \sum_{t=1}^{T} \delta^{t-1}\left[S_t - (1 + \lambda)\left[p_t + c_t\right]\right]$$

The government agency is benevolent and contracts with the firm in $t = 1$ to maximize the expected sum of the firm's payoff and the discounted value of the consumer's net surplus.

$$U^P = U^C(c^T, p^T) + U^A(p^T, e^T)$$

[1] More precisely, we assume that $\overline{e}$ is such that

$$1 < \varphi'(\overline{e}),$$

where $\varphi(.)$ is the cost of exerting productive effort for the firm as will be defined below.

The evolution of the cost parameters. Right before contracting with the firm at $t = 1$, the government agency can make, at a cost $g(I)$, an investment I. The function $g(.)$ satisfies $g(0) = 0 < g'(I)$ for all $I > 0$.

One may interpret this investment as capital expenditure that will determine the project's cost structure over time. In fact, we take that the investment determines a log-concave distribution $F(\theta_1|I)$, with density $f(\theta_1|I)$, from which firm's the cost parameter θ_1 is drawn.

Larger investments make lower cost parameters more likely: if $I' > I$

$$\frac{f(\theta_1|I')}{f(\theta_1|I)} \tag{MonotoneLR}$$

is decreasing in θ_1.

For $t \geq 2$, the cost parameter θ_t evolves according to the following AR(1) process:

$$\theta_t = \alpha + \beta\theta_{t-1} + \epsilon_t \tag{AR1}$$

where $\alpha > 0$, $\epsilon_t \in [\underline{\epsilon}_t, \overline{\epsilon}_t]$ is a zero mean random shock (independent of θ_{t-1}), with density $g(\epsilon_t) > 0$, and $0 < \beta < 1$ is the parameter that captures the persistence of costs.

Under (AR1), a given investment will reduce (expected) costs over all periods. Clearly, for $t \geq 2$, such an expected reduction will be larger for technologies that display larger persistence (as captured by a larger β).

Timing of events and the contract space. First, the government agency chooses its investment level I. After the investment is made, the firm learns θ_1 in the beginning of $t = 1$, and then contracts with the government agency, who can fully commit to the offered contract.

Denoting by $h^t = \{I, (p_1, c_1), (p_2, c_2), ..., (p_t, c_t)\}$ a public history (indexed by the level of investment I made) of length t of net transfers and realized costs up until t, and by H_t be the set of all such histories. In $t = 1$, the government agency offers a complete contingent contract to the firm of the form

$$\{(p_t : H_{t-1} \times C_t \to \Re, c_t : H_{t-1} \to \Re)\}_{t=1,}^{T}$$

where $p_t(h_t, c_t)$ specifies the net transfer to be made to the firm following history h_{t-1} in case the project is delivered at cost c_t in period t, and $c_t(h_{t-1})$ prescribes the cost at which the project must be delivered in t following history h_{t-1}.

The firm's outside option grants it a payoff of zero regardless of θ_1. Hence, the contract offered by the government agency will only be accepted for a firm with cost parameter $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$ if it yields non-negative expected payoffs.

3 The Complete Information Benchmark

To understand better the forces at play in the model and, in particular, the difficulties faced by the government agency in designing a contract that induces high effort toward cost reduction over time, it is worth analyzing, as a benchmark, the model for the case in which the government agency fully observes the cost parameters $\{\theta_t\}_{t=1}^{T}$.

Toward that, we start by noticing that, since

$$\sum_{t=1}^{T} \delta^{t-1} p_t = U^A\left(p^T, e^T\right) + \sum_{t=1}^{T} \delta^{t-1} \varphi(e_t),$$

one can write the government agency's payoff at the procurement stage as:

$$U^P(c^T, p^T) = \sum_{t=1}^{T} \delta^{t-1} \left(S_t - (1+\lambda)\left[c_t + \varphi(e_t)\right]\right) - \lambda U^A(p^T, e^T) \qquad \text{(PayoffReg)}$$
$$= \sum_{t=1}^{T} \delta^{t-1} \left(S_t - (1+\lambda)\left[\theta_t - e_t + \varphi(e_t)\right]\right) - \lambda U^A(p^T, e^T).$$

The government's agency problem under complete information can then be written as

$$\max_{I, \{e_t\}_{t=1}^T, U^A(p^T, e^T) \geq 0} \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} \left(S_t - (1+\lambda)\left[\theta_t - e_t + \varphi(e_t)\right]\right) - \lambda U^A(p^T, e^T) | I\right] - g(I)$$

Hence, the government agency wishes to minimize the sum of rents left to the entrepreneur, $\mathbb{E}\left[U^A(p^T, e^T)\right]$, and the total expected present value of the costs to implement the project. The reason why the government agency finds costly to leave rents to the entrepreneur is simple: by leaving one (monetary) unit of rent to the firm, the regulator needs to collect public funds that cost $1 + \lambda$.

The next lemma follows from these observations.

Lemma 1 *In a complete information environment, the optimal contract must implement the (unique) effort level implicitly defined by $\varphi'(e_t^{FB}) = 1, t = 1, .., T$ and leave no rents for the firm, $U^A(p^T, e^T) = 0$. Furthermore, investment $I^*(\beta)$ must be chosen to minimize the total expected costs to implement the project:*

$$I^*(\beta) = \arg\min_I g(I) + \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1}\theta_t | I\right].$$

Lemma 1 establishes that, under complete information, the optimal contract must implement first best levels of investment and effort toward cost reduction. We now establish that a sequence of a fixed-price contracts implement such allocation and, the same time, can be designed in a way that no rents are left to the firm.

Toward that, consider the a sequence of net transfers from the government agency to the firm of the following form:

$$p_t(c_t) = a_t - c_t$$

with

$$a_t = \begin{cases} \mathbb{E}\left[\theta_1 | I^*\right] - e_1^{FB} + \varphi(e_1^{FB}) + g(I^*), & \text{if } t = 1, \\ \mathbb{E}\left[\theta_t | I^*\right] - e_t^{FB} + \varphi(e_t^{FB}), & \text{if } t \geq 2. \end{cases}$$

Clearly, the net transfers implied by $\{p_t(c_t)\}_{t=1}^T$ make the firm the residual claimant of any reduction in the cost c_t. Indeed, when facing $\{p_t(c_t)\}_{t=1}^T$ and deciding how much effort to exert at period t, the firm solves:

$$\max_{e_t} \underbrace{a_t - \theta_t + e_t}_{p_t(c_t)} - \varphi(e_t)$$

which yields first-best amount of effort toward cost reduction in period t :

$$\varphi'(e_t^{FB}) = 1$$

Moreover, when deciding how much to invest in period zero (and anticipating the amount of effort to be chosen in all later periods), the firm solves

$$\max_{I} \ \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t}\left(a_{t}-\theta_{t}+e_{t}^{FB}\right)-g\left(I\right) \mid I\right],$$

which is clearly equivalent to choosing an investment level to minimize

$$\arg\min_{I} \ g\left(I\right)+\mathbb{E}\left[\left(\sum_{t=1}^{T} \delta^{t}\theta_{t}\right) \mid I\right],$$

and therefore leads to $I^{*}\left(\beta\right)$

It follows that, by inducing an efficient choice of investment and effort levels by the firm, the net transfers $\{p_{t}\left(c_{t}\right)\}$ minimize the expected cost at which the project is delivered. Moreover, it is easy to see (we show this formally in the appendix) that the net transfers $\{p_{t}\left(c_{t}\right)\}$ were constructed in such a way that the firm earns zero rents: $U^{A}(p^{T}, e^{T}) = 0$. Therefore, we have:

Proposition 1 *When $\{\theta_{t}\}_{t}$ is observed by the government agency, a set of fixed-price contracts yields an optimal regulatory mechanism in which first-best levels of effort towards cost reduction and ex-ante investment are attained. .*

A setting in which $\{\theta_{t}\}_{t}$ is privately observed by the firm introduces two new effects. First, as the government agency is forced to rely on the firm's information to set cost targets $\{c_{t}\}_{t}$, there will be a conflict between the goals of minimizing the expected present value of the cost to implement the project and leaving no rents to the firm. Second, the amount that the government agency invests ex-ante interacts, in a non-trivial way, with the procurement mechanism the government agency proposes at the procurement stage.

4 Incomplete Information :

In order to understand the interplay between the government's agency investment decision and the procurement contract it offers to the firm, we first take as given the amount I of ex-ante investment and derive the optimal procurement mechanism as a function of I. We then analyze the government agency's investment problem.

4.1 The Mechanism Design Problem for a Given Level of Investment

Basic Definitions:

By the Revelation Principle (cf. [14]), we can, without loss, restrict attention to *direct* mechanisms for which the firm finds it optimal to be truthful and obedient.

A direct mechanism $\Omega = \langle e_{t}, p_{t}|I\rangle_{t=1}^{T}$, indexed by the amount I of investment made by the government agency, consists of a collection of functions

$$e_{t}\left(.;I\right):\Theta^{t}\times C^{t-1}\rightarrow\Re,\ p_{t}\left(.;I\right):\Theta^{t}\times C^{t}\rightarrow\Re,$$

where (i) $e_t\left(\widehat{\theta}^t, c^{t-1}; I\right)$ is the amount of effort toward cost reduction recommended by the government agency in period t when the firm reports $\widehat{\theta}^t = \left(\widehat{\theta}_1, ..., \widehat{\theta}_1\right)$ and the vector of realized past costs is $c^{t-1} = (c_1, ..., c_{t-1})$, and (ii) $p_t\left(\widehat{\theta}^t, c^{t-1}; I\right)$ is the net transfer the government agency makes to the firm by the end of period t when the firm reports $\widehat{\theta}^t = \left(\widehat{\theta}_1, ..., \widehat{\theta}_1\right)$ and the whole vector of realized costs is $c^t = (c_1, ..., c_t)$. Throughout, with a slight abuse of notation, we will denote the equilibrium effort levels and net transfers induced by the mechanism $\langle e_t, p_t\rangle_{t=1}^T$ by $e_t(\theta^t; I) \equiv e_t(\theta^t, c^{t-1}(\theta^{t-1}); I)$ and $p_t(\theta^t) \equiv p_t(\theta^t, c^t(\theta^t); I)$, respectively, where $c^t(\theta^t) = (c_s(\theta^s))_{s=1}^t$.

In a Direct Mechanism, the firm chooses, at each period t, its announcement of its current cost parameter $\widehat{\theta}_t$, given current and past realizations of the cost parameters it has observed, the past messages it has sent, the past efforts that were recommended to it and the past payments it received. Formally, for a given amount I of ex-ante investment performed in $t = 0$, a (pure) reporting strategy for the firm in direct mechanism is a collection of messages $\mathbf{m} = \{m_t\}_{t=1}^T$, where

$$m_1 \in \left[\underline{\theta}_1, \overline{\theta}_1\right]$$

and, for $t \geq 2$,

$$m_t : \Theta^t \times \Theta^{t-1} \times [0, \overline{e}]^t \times \Re^t.$$

Incentive Compatibility and Participation:

A reporting strategy is said to be truthful if, for all $\left(\left(\theta_t, \theta^{t-1}\right), m^{t-1}, e^{t-1}, p^{t-1}\right)$,

$$m_t\left(\left(\theta_t, \theta^{t-1}\right), m^{t-1}, e^{t-1}, p^{t-1}\right) = \theta_t.$$

In words, a truthful reporting strategy prescribes that, at any period t, the firm reports its true current costs in all possible contingencies.

Denoting by

$$\mathbb{E}^{\Omega,\mathbf{m}}\left[U^A\left(\left(p^T, e^T; I\right)\right)\right]$$

the firm's expected utility when I has been chosen in the investment stage and when it adopts a reporting strategy $\mathbf{m}$ in the direct mechanism Ω, and letting $\boldsymbol{\theta}^T = (\theta_1, \theta_2, ..., \theta_T)$ be a truthful strategy, the mechanism Ω is incentive compatible if

$$\mathbb{E}^{\Omega,\boldsymbol{\theta}^T}\left[U^A\left(\left(p^T, e^T; I\right)\right)\right] \geq \mathbb{E}^{\Omega,\mathbf{m}}\left[U^A\left(\left(p^T, e^T; I\right)\right)\right], \text{ for all } \mathbf{m}.$$

Given ts outside option, the firm will be willing to participate in the procurement mechanism as long as

$$\mathbb{E}^{\Omega,\boldsymbol{\theta}^T}\left[U^A\left(\left(p^T, e^T; I\right)\right)\right] \geq 0.$$

The timing in the (Direct) Procurement Mechanism:

At period $t = 1$, after the firm learns θ_1, the government agency proposes a direct mechanism Ω. The firm, then, sends a message $\widehat{\theta}_1$ to the government agency, who then recommends effort $e_1\left(\widehat{\theta}_1\right)$ to the firm and proposes a net transfer of $p_1\left(\widehat{\theta}_1, .; I\right) : C_t \to \Re$. For periods $t \geq 2$, the timing is analogous: the firm learns θ_t and sends a message $\widehat{\theta}_t$ to the government agency who then recommends effort $e_t\left(\widehat{\theta}^t, c^{t-1}\left(\widehat{\theta}^{t-1}\right); I\right)$ and proposes a net transfer of

$$p_t\left(\widehat{\theta}^t, c^{t-1}\left(\widehat{\theta}^{t-1}\right), .; I\right) : C_t \to \Re$$

to the firm.

4.2 The Government Agency's Problem:

Using the expression (PayoffReg) for the government agency's payoff, its problem of designing a procurement scheme for a given level of investment I can be written as

$$\max_{\Omega=\langle e_t,p_t\rangle_{t=1}^{T}} E^{\Omega,\theta^T}\left[\sum_{t=1}^{T}\delta^{t-1}\left[S_t-(1+\lambda)\left[c_t\left(\theta^{t-1};I\right)+\varphi(e_t\left(\theta^{t-1},c_t\left(\theta^{t-1};I\right);I\right)\right]\right]-\lambda U^A(p^T,e^T,I)|I\right]$$

subject to a set of Incentive Compatibility constraints

$$\mathbb{E}^{\Omega,\theta^T}\left[U^A\left(\left(p^T,e^T;I\right)\right)|I\right]\geq\mathbb{E}^{\Omega,\mathbf{m}}\left[U^A\left(\left(p^T,e^T;I\right)\right)|I\right],\ \text{for all } \mathbf{m}. \tag{IC}$$

and a participation constraint

$$\mathbb{E}^{\Omega,\theta^T}\left[U^A\left(\left(p^T,e^T;I\right)\right)|I\right]\geq 0 \tag{IR}$$

Since it learns the cost parameters $\{\theta_t\}_{t=1}^{T}$ privately over time, the set of possible deviations available for the firm in a given mechanism may be large: at any point in time and for a given realization of past cost parameters (as well as announcements regarding such parameters), the firm may decide to lie about its current cost parameter *conditional* on such past information. Hence, the set of constraints described by (IC) is large and a full characterization is hard to obtain.

Instead of trying to fully characterize the IC constraints, we solve the government agency's problem by adopting the first order approach developed by Kapicka (2010) and Pavan, Segal and Toikka (2010). This approach consists of replacing the constraints in (IC) by a first order (necessary) condition for truthtelling – which summarizes local incentive constraints – and solving a "relaxed" problem. We then check that the solution to the relaxed problem is in fact a solution for our problem of interest.

To characterize the government agency's relaxed problem, it is useful to define, for a given mechanism Ω,

$$V^{\Omega}\left(\theta_1;I\right)\equiv\mathbb{E}^{\Omega,\theta^T}\left[U^A\left(\left(p^T,e^T;I\right)\right)|\theta_1;I\right] \tag{1}$$

as the firm's expected utility in period $t=1$ when it adopts a truthful strategy and observes θ_1. If Ω is incentive compatible, $V^{\Omega}\left(\theta_1|I\right)$ represents the firm's value function when his initial type is θ_1.

The following result, which is an application of Pavan, Segal and Tokkia's (2012) Dynamic Envelope Theorem to our setting, establishes a key necessary condition that an Incentive Compatible mechanism must satisfy in terms of $V^{\Omega}\left(\theta_1;I\right)$.

Lemma 2 *If Ω is Incentive Compatible, then, for a given I, $V^{\Omega}\left(\theta_1;I\right)$ is absolutely continuous (and, therefore, differentiable almost everywhere) and satisfies the following formula*

$$V^{\Omega}\left(\theta_1;I\right)=V^{\Omega}\left(\overline{\theta}_1;I\right)+\int_{\theta_1}^{\overline{\theta}_1}\mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T}\delta^{t-1}\beta^{t-1}\varphi'\left(e_t\left(\tau,\theta_2,...,\theta_T;I\right)\right)|\tau;I\right]d\tau \tag{Envelope}$$

The interpretation for the above result is simple. A firm which, for $\eta>0$, has cost parameter θ_1 is $\eta>0$ more efficient than a firm with cost parameter $\theta_1+\eta$ in period $t=1$. Moreover, for a common set of shocks $\{\epsilon_t\}_{t=2}^{T}$ in periods $t=2,...,T$, the firm that drew θ_1 in period 1 will be $\beta^t\eta$ more efficient in period $t+1$ than the firm with cost parameter $\theta_1+\eta$. Hence, a firm with cost θ_1 can always pretend to be type "$\theta_1+\eta$", exert the effort levels recommended by the regulator to such type, call them $\{e_t\}$, and expect to save, at the moment it signs the contract with save

$$\sum_{t=1}^{T} \delta^{t-1} \mathbb{E}^{\theta^T} \left[\varphi\left(e_t\right) - \varphi\left(e_t - \beta^t \eta\right)\right]$$

in terms of expected disutility of effort.

It follows that, for small $\eta > 0$, in any Incentive Compatible Mechanism, the expected utility of a firm with cost parameter θ_1 must be at least the expected utility of a firm with cost parameter $\theta_1 + \eta$ plus an (approximate) amount of[2]

$$\sum_{t=1}^{T} \delta^{t-1} \beta^{t-1} \mathbb{E}^{\theta^T} \left[\varphi'\left(e_t\right)\right] \eta,$$

which captures the informational rents type θ_1 earns in excess of type $\theta_1 + \eta$'s payoff. Summing up the informational rents that type θ_1 collects in addition to payoff of all types τ larger than θ_1, one obtains equation (Envelope).

The Relaxed Program:

The relaxed program maximizes the government agency's expected utility subject to the IR constraint and the necessary condition for incentive compatibility derived in Lemma 2, which, through equation (Envelope), pins down the value that $V^{\Omega}\left(\theta_1; I\right)$ must have in any Incentive Compatible mechanism.

Plugging equation (Envelope) in the government agency's objective function, the relaxed problem can be written, after some integration by parts, as:

$$\max_{V^{\Omega}\left(\overline{\theta}_1; I\right), \left\{e_t\left(\theta^t\right)\right\}_{t,\theta^t}} \mathbb{E}^{\theta^T} \left[\sum_{t=1}^{T} \delta^{t-1} \left(S_t - (1+\lambda)\left[\theta_t - e_t\left(\theta^t; I\right) + \varphi\left(e_t\left(\theta^t; I\right)\right)\right] - \lambda \frac{F\left(\theta_1 | I\right)}{f\left(\theta_1 | I\right)} \beta^{t-1} \varphi'\left(e_t\left(\theta^t; I\right)\right)\right) | I\right] - V^{\Omega}\left(\overline{\theta}_1; I\right) \tag{Relaxed Program}$$

subject to

$$V^{\Omega}\left(\overline{\theta}_1; I\right) \geq 0.$$

Clearly, it is optimal to set $V^{\Omega}\left(\overline{\theta}_1; I\right) = 0$. Moreover, as we show in the Appendix, a sequence of efforts $\left\{\widetilde{e}_t\left(\theta^t, I\right)\right\}_{t,\theta^t}$ maximize

$$\mathbb{E}^{\theta^T} \left[\sum_{t=1}^{T} \delta^{t-1} \left(S_t - (1+\lambda)\left[\theta_t - e_t\left(\theta^t; I\right) + \varphi\left(e_t\left(\theta^t; I\right)\right)\right] - \lambda \frac{F\left(\theta_1 | I\right)}{f\left(\theta_1 | I\right)} \beta^{t-1} \varphi'\left(e_t\left(\theta^t; I\right)\right)\right) | I\right]$$

if, and only, it solves

$$\max_{\left\{e_t\left(\theta^t\right)\right\}_{t,\theta^t}} \sum_{t=1}^{T} \delta^{t-1} \left[e_t - \varphi\left(e_t\right) - \frac{\lambda}{(1+\lambda)} \frac{F\left(\theta_1 | I\right)}{f\left(\theta_1 | I\right)} \beta^{t-1} \varphi'\left(e_t\right)\right]$$

for (almost) all θ_1. These observations allow us to establish

[2] We use the fact that a First Order Taylor expansion of $\varphi\left(e_t - \beta^{t-1}\eta\right)$ around e_t yields

$$\varphi\left(e_t - \beta^t \eta\right) \simeq \varphi\left(e_t\right) - \beta^{t-1} \varphi'\left(e_t\right).$$

Proposition 2 *The solution to the governmet agency's relaxed problem is given by* $V^{\Omega}\left(\overline{\theta}_1\right)=0$ *and effort levels* $\left\{e_t\left(\theta_1,\beta,I\right)\right\}_t$ *implicitly defined by*

$$1=\varphi'\left(e_t\left(\theta_1,\beta,I\right)\right)+\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_1|I\right)}{f\left(\theta_1|I\right)}\beta^{t-1}\varphi''\left(e_t\left(\theta_1,\beta,I\right)\right),\qquad t=1,...,T \tag{FOCEffort}$$

The interpretation of the conditions for the optimal effort levels $\left\{e_t\left(\theta_1,\beta,I\right)\right\}_t$ implicitly defined by equations (FOCEffort) is as follows. When choosing the effort level in period t to recommend to the firm, the government agency equates the marginal social benefit of effort – the marginal reduction in the cost c_t –, 1, with the marginal cost of effort, as expressed by the sum of the firm's marginal cost to provide effort, $\varphi'(e)$, and the marginal effect of higher a effort in period t on the amount of informational rents that must be left to firm at the *contracting stage*.

The latter component – related to the informational rents left to the firm – of the marginal cost of effort as perceived by the government agency is captured by the term

$$\frac{\lambda}{(1+\lambda)}\frac{F\left(\theta_1|I\right)}{f\left(\theta_1|I\right)}\beta^{t-1}\varphi''\left(e_t\left(\theta_1,\beta,I\right)\right) \tag{InfoRents}$$

and is the source of two features that are worth noticing. First, as opposed to what prevails under complete information, the effort levels that solve the regulator's relaxed program are time-dependent. Second, the only component of the history of announcements $\theta^t=(\theta_1,\theta_2,...,\theta_t)$ that affects the effort level recommended to the firm in period t is θ_1.

To understand both of these features, notice that, at the contracting stage, the piece of information that is private to the firm is θ_1. As such, $\theta_1$is the only source of informational rents that the firm may collect at that stage. However, a firm that draws a low type in period 1 is able to collect information rents stemming from the effort levels recommended by the regulator in *all* periods. Indeed, a firm that draws a low θ_1 in period 1 is likely to draw a low θ_t in any period t. Therefore, a firm with a low θ_1 expects to be able to report to have higher cost parameters in all future periods and economize on the amount of effort levels the mechanism prescribes for each t. Therefore, to report truthfully in period 1, the firm demands upfront rents that relate to the amount of effort it could economize by reporting to be less efficient than what it really is in all future periods. As our discussion of Lemma 2 suggests, the amount of effort the firm expects to economize in period t by reporting a higher type in period 1 depends on the statistical "linkage" between θ_1 and θ_t. For an AR1 process, this linkage is fully described by θ_1 and the impulse response function, β^{t-1}, and this is why the history of announcements only affect effort levels through θ_1.

The following result establishes some interesting properties of the effort levels $\left\{e_t\left(\theta_1,\beta,I\right)\right\}_t$ implicitly defined by (FOCEffort)

Proposition 3 *For fixed* $\left(\theta_1;\beta,I\right)$, $e_t(\theta_1;\beta,I)$ *increases over time:*

$$e_{t+1}(\theta_1;\beta,I)>e_t(\theta_1;\beta,I),...t=1,...,T-1.$$

For any given t,

$$\frac{\partial e_t(\theta_1;\beta,I)}{\partial\beta}<0,\frac{\partial e_t(\theta_1;\beta,I)}{\partial I}<0\frac{\partial e_t(\theta_1;\beta,I)}{\partial\theta_1}<0.$$

If, in addition, we have that $\frac{\varphi''(e_t^{SB})}{\varphi'''(e_t^{SB})} \geq \frac{\varphi'''(e_t^{SB})}{\varphi^{(4)}\left(e_t^{SB}\right)}$[3], *then*

$$\frac{\partial^2 e_t^{SB}(\theta_1, \beta, I)}{\partial \beta \, \partial I} \leq 0.$$

In words, Proposition 3 states that the efforts levels that solve the government agency's relaxed problem increase over time and, for any given period, will be smaller (i) the higher the degree of persistence of the technology (β), (ii) the larger the investment made (I) and (iii) the larger the first cost parameter drawn by the firm (θ_1). Moreover, if the marginal cost of effort $\varphi'(.)$ is not "too convex", the negative effect of investment on the effort level in period t will be magnified when β is large.

The interpretation is straightforward. Since the statistical "linkage" between θ_1 and θ_t, as captured by β^{t-1}, decreases over time and does so more rapidly the smaller β, the cost component of the government agency's objective related to informational rents is, at the margin, lower over time and as a function of β. In turn, a larger investment I makes it more likely that the firm draws a lower θ_1.[4] This raises the amount of informational rents the government agency has to leave for the firm. Last, much as static models of optimal procurement, the amount of distortion imposed by the government agency at the optimal mechanism will be smaller for firms that are more efficient at the contracting stage (i.e., firms with lower θ_1). What is new to our dynamic setting is this will hold true for the effort levels chosen for all periods: a firm that is more efficient at the contracting stage will be called to exert effort levels that are closer to the first best for *all* periods.

Overall Incentive Compatibility and Indirect Implementation:

Clearly, as the set of constraints the government ageny faces is larger than the set of constraints implied by the relaxed problem, the value attainable by the latter is an upper bound to the value attainable by the former. We now argue that the government agency's true maximization problem attains such upper bound. We do so by arguing that the effort levels $\{e_t(\theta_1, \beta, I)\}_t$ described in Proposition 2 satisfy *all* Incentive Compatibility constraints.

Toward this goal, we first observe that, since the effort levels $\{e_t(\theta_1, \beta, I)\}_t$ in Proposition (2) only depend on θ_1, the government agency just needs to provide incentives for truthful revelation of θ_1.[5] In other words, given our candidate to an optimum, the only (relevant) Incentive Compatibility constraint in our dynamic environment resembles the constraint a mechanism designer faces in a *static* setting. It is widely known that, in static mechanism design problems in which the agent's payoff satisfy a single-crossing, Incentive Compatibility is equivalent to an Envelope Condition such as the one in equation (Envelope) and a monotonicity constraint. Since $F(\theta_1|I)$ is logconcave, the effort levels $\{e_t(\theta_1, \beta, I)\}_t$ are decreasing in θ_1. In the Appendix, we use such fact, along with the Envelope Condition (Envelope), to establish that the mechanism described in Proposition 2 in fact induces truthtelling.

Once truthtelling is assured, all that is left is to establish that the firm will have incentives to exert the prescribed effort levels. The next result shows that a sequence of linear contracts induces the firm to exert

[3] As an example, this condition for quadratic cost functions.

[4] We show in the Appendix that the monotone likelihood ratio property in (MonotoneLR) implies that

$$\frac{F(\theta_1|I)}{f(\theta_1|I)}$$

is increasing in I.

[5] In fact, if the regulator induces truthfulness in period $t = 1$, any set of transfers to the firm in periods $t \geq 2$ that do not depend on $\{\theta_t\}_{t\geq 2}$ will induce truthtelling.

the effort levels $\{e_t(\theta_1, \beta, I)\}_t$.

Proposition 4 *The solution to the Relaxed Problem is the optimal procurement mechanism. The optimal mechanism can be implemented by the following sequence of (menus of) linear contracts:*

$$p_t^{SB}(c_t; \theta_1; \beta, I) = \gamma_t(\theta_1; \beta, I) - \zeta_t(\theta_1; \beta, I) \cdot c_t, \quad t = 1, ..., T$$

where

$$\begin{aligned} \gamma_t(\theta_1; \beta, I)) &= \varphi(e_t(\theta_1, \beta, I)) + \int_{\theta_1}^{\overline{\theta}_1} \beta^{t-1} \varphi'(e_t\left(\widetilde{\theta}_1, \beta, I\right)) d\tilde{\theta}_1 + \mathbb{E}\left[\varphi'(e_t(\theta_1, \beta, I))(\theta_t - e_t(\theta_1, \beta, I) | \theta_1\right] \\ \zeta_t(\theta_1; \beta, I)) &= \varphi'(e_t(\theta_1; \beta, I)) \end{aligned}$$

The implementation part of Proposition 4 states that, in order to induce the firm to choose $e_t(\theta_1, \beta, I)$ in period t, all the government agency needs to do is to pay to the firm a share $\zeta_t(\theta_1; I) = \varphi'(e_t(\theta_1; I))$ of any reduction it implements in the accounting cost c_t.[6]

To understand why this is the case, notice that, when confronted with a sequence of contracts of the form

$$p_t(c_t) = a_t - b_t \cdot c_t,$$

the firm will choose its effort level in t to solve

$$\max_{e_t} a_t - b_t \cdot [\theta_t - e_t] - \varphi(e_t)$$

Clearly, the firm will find it optimal to choose efforts so to equate the marginal monetary benefit it derives from reducing accounting costs, b_t, to the marginal cost of exerting effort, $\varphi'(e_t)$:

$$b_t = \varphi'(e_t^*).$$

Hence, to induce the firm to the firm to choose $e_t^* = e_t(\theta_1, \beta, I)$, the government agency needs to set $b_t = \zeta_t(\theta_1; \beta, I))$.

Comparative Statistics:

The variable component of the firm's pay in period t implied by the menu of contracts described in Proposition 4, $\zeta_t(\theta_1; \beta, I)$, is a measure of how powerful the incentives provided to the firm are in the optimal procurement mechanism. One key feature of such variable component is that the fact it does not depend on the firm's cost parameters for $t \geq 2$. In fact, at the contracting stage, all that the government agency needs to know to decide on the contract the firm will face in period t is the realization of the firm's cost parameter in period 1, θ_1, and the "impulse response" parameter β, that captures the statistical link between θ_1 and the (expected) cost parameters θ_t. An immediate consequence of the sole dependence of $\{\zeta_t(\theta_1; \beta, I)\}_t$ on θ_1 is that the contracts in Proposition 4 display a substantial amount of persistence. As an extreme example, one has

$$\zeta_t(\underline{\theta}_1; \beta, I) = 1, \qquad t = 1, ..., T.$$

[6] The fixed component of the linear contracts in Proposition 4 is designed so to guarantee that, at the contracting stage, the firm's expected payment induces truthtelling.

Hence, much as what happens in static models of optimal procurement (e.g., Laffont and Tirole (1986)), the firm is confronted with a fixed-price contract in period 1 (i.e., $\zeta_1(\underline{\theta}_1; I) = 1$).when it reports to be the most efficient one at the contracting stage (i.e., its cost parameter at the contracting stage is $\underline{\theta}_1$). What is new to our model of optimal dynamic procurement is the fact a firm that reports $\underline{\theta}_1$ in the first period will be offered fixed-price contracts in *all* future periods, *regardless* of its future cost parameters $\{\theta_t\}_{t\geq 2}$.

A fixed price contract corresponds to a contract

$$p_t(c_t) = a_t - b_t \cdot c_t,$$

with $b_t = 1$. As the firm is residual claimant of whatever reduction in accounting costs it implements, it will choose first best levels of effort when confronted with fixed-prices. Hence, a firm that reports $\underline{\theta}_1$ at the contracting stage will exert efficient amounts of effort toward cost reduction in all periods.

On the other side of the spectrum, a cost-plus contract implies $b_t = 0$. Clearly, as it does not appropriate any of gains of a reduction in accounting costs c_t, the firm does not exert effort when facing cost-plus contracts.

Except for $\underline{\theta}_1$, the contracts in Proposition 4 trade-off the dynamic provision of incentives to the firm – which is a force toward fixed-price contracts – with reducing the amount of informational rents left to the firm – which is a force toward cost-plus contracts. Hence, much as in virtually all literature of optimal procurement with asymmetric information, the optimal mechanism in our setting is implemented by sets of contracts whose power, as captured by the sequence $\{\zeta_t(\theta_1; \beta, I)\}_t$, lies half-way between fixed-price and cost-plus contracts:

$$\zeta_t(\theta_1; I) \in (0,1) \text{ for all } \theta_1 \in \left(\underline{\theta}_1, \overline{\theta}_1\right],\ I \text{ and } t = 1, ..., T,$$

where, the larger (lower) $\zeta_t(\theta_1; I)$, the closer the contract to a fixed-price (cost-plus) contract.

Our dynamic setting, however, allows for even further insights regarding the behavior of the power of the optimal contracts over time and as a function of some key parameters of the model, such as degree of persistence of technology, i.e., the parameter β in equation (AR1), and the investment level I – which is viewed by the government agency as a parameter at the procurement stage.

The next result, which is the counterpart of Proposition 3 for the contracts offered to the firm, establishes some interesting comparative static results for $\{\zeta_t(\theta_1; \beta, I)\}_t$.

Proposition 5 *$\zeta_t(\theta_1; \beta, I)$ is decreasing in all arguments. Moreover, for fixed $(\theta_1; \beta, I)$, one has:*

$$\zeta_{t+1}(\theta_1; I) > \zeta_t(\theta_1; I), \text{ for all } \theta_1 \in \left[\underline{\theta}_1, \overline{\theta}_1\right], t = 1, ..., T-1.$$

It follows that, at a given period t, the firm will face more powerful incentives (i) the smaller the degree of persistence of technology (β), (ii) the smaller the initial investment I, and (iii) the lower its first period cost parameter. Moreover, regardless of the firm's characteristics, contracts become more powerful over time.

Although there is a one-to-one correspondence between Proposition 5 and Proposition 3, the former, as it refers to contracts which, as opposed to effort levels, are observable, gives us a set of potentially testable predictions. In fact, our dynamic model suggests the following empirical predictions: controlling for all other characteristics that may affect the contracts that are offered to the firm, in industries with a larger amount of initial investment (as measured by Capital Expenditures) and with a higher degree of the persistence of the technology, the procurement contracts will be closer to cost-plus. Also, controlling for all other features, the model predicts that one should observe more powerful procurement contracts as time goes by.

4.3 The Investment Stage

Having fully derived the optimal procurement mechanism for a *given* amount of I , we now turn to the government agency's problem of choosing its ex-ante amount of investment. When deciding to invest, the govenment agency firm maximizes its expected payoff anticipating the effect of such choice on the procurement stage to come.

One can write the government agency's problem at the investment stage as

$$\max_{I\geq 0}\mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T}\delta^{t-1}\left(S_t-(1+\lambda)\left[\theta_t-e_t\left(\theta_1,\beta,I\right)+\varphi\left(e_t\left(\theta_1,\beta,I\right)\right)\right]-\lambda\frac{F\left(\theta_1|I\right)}{f\left(\theta_1|I\right)}\beta^{t-1}\varphi'\left(e_t\left(\theta_1,\beta,I\right)\right)\right)|I\right]-g\left(I\right),$$

where $\{e_t(\theta_1,\beta,I)\}_t$ is the the sequence of effort levels derived in Proposition 2.

As we show in Propositions 3 and 5, larger investments reduce the power of incentives ath the procurent stage. This effect, in turn, is perceived by the government agency as an additional cost of investing. Not surprisingly, the amount invested under incomplete information is smaller than the first-best level of investments:

Proposition 6 *A solution for government agency's investment problem exists. In any solution, the amount invested is smaller than the first best level of investment* $I^*(\beta)$ *.*

A bit less obvious is the effect of the degree of persistence of the technology on the government agency's incentives to invest. Again as a benchmark, it is useful to understand how the first best level of investments depend on β. Toward that, it is worth noticing that a larger ex-ante investment makes it more likely that a low cost parameter θ_1 is drawn in period 1. Given the law of motion for $\{\theta_t\}$ in (AR1), a lower θ_1 induces lower expected cost parameters for all periods. Hence, larger investments induce, on average, lower cost parameters θ_t. Moreover, the larger the degree of persistence of the technology, the larger the impact of a lower θ_1 (and, therefore, larger investments) on all future cost parameters. Hence, when β is large, the benefits of the investment made in period zero will accrue more intensely over all periods, so the following results holds true.

Proposition 7 *The first best level of investments,* $I^*(\beta)$*, is increasing in* β*.*

Under incomplete information, there are two opposing effects of β on its incentives to invest. On the one hand, as can be seen by the objective function in program (4.3) the larger β, the smaller the present value of the expected costs of the prject, as measured by $\mathbb{E}^{\theta^T}\left[\sum_{t=1}^{T}\delta^{t-1}\theta_t|I\right]$, for a *given* regulatory mechanism. On the other hand, as shown by Proposition 5, the larger β, the less powerful the incentives provided to the firm to exert effort toward cost reduction in any given period t, and, therefore, the larger the cost at which the project is ultimately delivered.

The discussion suggests that the effect of β on investments may be ambiguous. In fact, the following example describes a set of parameters and probability distributions for which the relationship between investment and β is non-monotone.

Example 1 *Let us consider an example in which the investment is a non-monotone function of* β*. We will assume that there are 3 periods,* $\theta_t\in[0,5]$*,* $I\in[0,5]$ *and* $\lambda=0.3$*. The distribution of the cost parameters* θ_t *is a truncated normal distribution with mean* $5-I$ *and variance 2. We also take the disutility of effort to* [scale=.85]utility

Figure 1: Regulator's utility function.

[scale=.45]investment

Figure 2: Optimal Investment Levels.

be quadratic, $\varphi(e) = \frac{e^2}{2}$*, and the investment cost function to be linear,* $g(I) = \frac{I}{100}$*. These parameters imply that, in the first-best case, the regulator invests as much as possible.*

In the incomplete information, non-contractible investment case, the regulator's utility graph as a function of the investment I *and the persistence parameter* β *is shown in Figure 1. It suggest some non-linearity in the optimal investment choice* $I^*(\beta)$ *when* β *is sufficiently large. Indeed, Figure 2 shows that the optimal investment increases with* β *only until* $\beta = 0.5$*. Soon after that mark, the optimal investment* decreases *with* β*.*

Hence, in this example, the negative effect of β *on investments is more pronounced for higher degrees of persistence, even when those investments are desirable from a social perspective.*

EXAMPLE:Non-monotone Investment as a function of β

As Figure 1 shows, for low levels of persistence, the optimal investment rises with β. For a sufficiently large degree of persistence, when β rises, the amount invested decreases. Hence, in the example, the negative effect of β on investments is more pronounced for degrees of persistence for which those investments are *more* desirable from a social perspective.

5 Concluding Remarks

We have considered a setting in which a regulator designs an optimal regulatory scheme for a firm with private information about a persistent cost parameter. In the model, before contracting with the firm at the procurement stage, the government agency performs a non-contractible investment that reduces the expected costs at which an indivisible project can be provided. The unobservability of the firm's cost parameters by the government agency creates a non-trivial link between the firm's investment decision and the procurement mechanism it proposes to the firm.

Since all the results and their economic interpretations were summarized in the Introduction, we conclude with a few words about additional issues that can be tackled with a model such as the one we laid out in this paper. Allowing for a risk averse firm is a natural first step toward a more general theory of optimal procurement with a firm that is privately informed about a persistent parameter. Indeed, risk aversion is likely to affect the power of the contracts that will be offered to the firm, and, as consequence, the government agency's ex-ante incentives to invest. Also, the contracts derived in Proposition 4 are not renegotiation in proof. Hence, following what Skreta (2012) does in a dynamic auction setting, one might, then, be interested in understanding what contracts would be offered by a government agency who cannot commit to contracts that prescribes inefficiencies ex-post. It would also be interesting to analyze what is the impact of having the firm choosing the amount to invest ex-ante when the regulator lacks commitment at the investment stage.

We believe these and other extensions are interesting avenues for future research.

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