

Coordination and the Provision of Incentives to a Common Regulated Firm*

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This Version: January, 2011

Abstract

This paper considers the problem faced by two regulators in providing incentives to a common (privately informed) regulated firm under various degrees of coordination. In the model, the firm exerts effort toward cost reduction and self-dealing, and incentives can be input-based (monitoring) and output-based (demanded cost targets). Full coordination between the regulators leads to the second best allocation. A setting in which the regulators do not fully coordinate leads to (i) higher overall monitoring (more aggressive input-based incentives) and (ii) higher demanded cost targets (i.e., more lenience in terms of output-based incentives). As a consequence of (i), in all possible equilibria, effort toward cost reduction will be smaller when the agent reports to two regulators who do not coordinate. (i) and (ii) imply that the impact on effort toward self-dealing activities is ambiguous. In our leading example, self-dealing will be larger if the regulators coordinate on monitoring levels but smaller if they choose monitoring levels independently.

Keywords: Regulation, Common Agency, Coordination, Input and Output Based Incentives.

J.E.L. Classifications: D82 (Asymmetric and Private Information), D86 (Economics of Contract Theory), L51 (Economics of Regulation), M52 (Compensation and Compensation Methods and their Effects).

1 Introduction

In many industries, firms are subject to the oversight of more than one regulatory authority. More generally, economic agents are often subject to the oversight of multiple principals: taxpayers have to report to multiple tax authorities, policy-makers are overseen by legislators and the chief-executive, and a manager in a company

*We thank an anonymous referee for extremely useful comments and suggestions, which greatly improved the paper. We have also benefited from comments and suggestions by Carlos Da Costa, Ignacio Esponda, William Fuchs, Gustavo Manso, João De Mello, Pedro Miranda, Humberto Moreira, Leonardo Rezende, John Roberts, Arilton Teixeira, Jeffrey Zwiebel, and participants of the Stanford Theory Lunch. The second author is grateful to Susan Athey, Jonathan Levin and Ilya Segal for guidance on a closely related project that led to some of the ideas in this paper. He also thanks Financial support from a John M. Olin Summer Fellowship and a Siepr Dissertation Fellowship. All errors are ours.

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has to report to a Board of Directors, to other stakeholders (e.g., debt-holders) and, in some cases, to a regulatory agency. When an agent reports to multiple principals, the degree of coordination among them is likely to play a key role.¹ This paper studies the provision of incentives to a firm that is regulated by two regulators under varying assumptions regarding the coordination capabilities of the regulatory authorities.

In the model, a regulated firm, through its manager, can implement an indivisible socially beneficial project at a cost that depends on (i) the amount of effort the manager exerts toward cost reduction ("productive effort"), (ii) the amount of effort exerted toward activities that increase the cost at which the project can be implemented by an amount that can be privately appropriated by the firm (effort toward "self-dealing activities"),² and (iii) a cost parameter that is privately observed by the firm.

Productive effort is unobservable by the regulators. While the same is in principle true for effort toward self-dealing activities, upon monitoring the firm, the regulators may find hard evidence of it with some probability. In those states, the amount self-dealt can be fully recouped. In the model, if both regulators monitor the firm, the chances of hard evidence being found is larger. Put differently, in terms of monitoring, a two-regulator arrangement has a technological advantage over a single-regulator one.³ However, the exercise of such advantage calls for some coordination among the monitors.

The total cost at which the project is implemented is verifiable, so the regulators can demand from the firm the attainment of certain cost targets when implementing the project. Hence, incentives can be "input" based (monitoring) or "output" based (Lazear, 1995). Our main contribution is to analyze how the use of input based and output based incentives changes when one moves from a single-regulator arrangement to a two-regulator arrangement under various assumptions regarding the degree of coordination among the regulators.

For the benchmark case in which the regulators observe the cost parameter, we show that they can contract with the firm in a way that first best levels of both types of efforts prevail. Such allocation can be attained irrespective of whether the regulators coordinate or not. In fact, such allocation is attained even if the firm reports to a single regulator. This is so because, when the cost parameter is known, all that is needed to implement first best levels of efforts is the use of well designed output based incentives: the firm must be made the residual claimant of any reduction in the cost at which the project is implemented. In particular, being residual claimant of any reduction in costs, the firm will have no incentives to pursue self-dealing.

When the firm has private information regarding costs, matters are more complicated. Indeed, the interaction of information asymmetry with the non-observability of productive effort and the possibility of the pursuit of self-dealing by the firm makes the problem of designing an optimal contract non-trivial for the regulators. In a single regulator arrangement, as the technological gains stemming from monitoring by the two regulators are not fully exercised, the regulator will mainly rely on output based incentives. Moreover, the eliciting of information requires the provision (through a higher payments) of some rents to the firm. To reduce such rents, the regulator distorts upward the cost at which the project is implemented. This induces

¹The recent financial crisis has sparked discussions about the need for better regulation of the financial sector. In the bulk of such discussions, the question of whether or not regulators should coordinate among themselves has been prominent.

²When we lay down the model in section 2, we provide a couple of examples of such type of effort and argue that regulators should be concerned about self-dealing.

³This makes the comparison between a single principal arrangement with a two-principal arrangement meaningful. Indeed, without any type of technological advantage, a single-principal arrangement always (weakly) dominates a two-principal arrangement, as any outcome obtained by the latter could be reproduced by the former, whereas, due to strategic effects, the converse is not true.

the firm to exert an inefficient amount of efforts toward cost reduction and positive effort toward self-dealing activities.

A two-regulator arrangement has the benefit of allowing more effective monitoring of the firm. However, the extent to which this benefit results in better outcomes depends on the degree of coordination among the regulators. It is undeniable that, *fixing* the level of all other instruments available to provide incentives to the firm, the more monitoring performed by the regulators, the better. However, one cannot assume that all other incentives schemes are held the same when the regulators do not fully coordinate. In fact, it may be the case that, in *response* to more monitoring, the regulators decide to alter the intensity of other incentive instruments. Hence, the benefits of more monitoring have to be balanced against the possible strategic effects brought up by a two-regulator arrangement. Indeed, we establish that a key trade-off in our analysis is related to the benefits brought up by the possibility of more monitoring in a two-regulator arrangement *vis à vis* the costs of miscoordination by the principals.

We consider three different cases varying according to the degree of coordination among the regulators: (i) full coordination, (ii) independent choice of payments to the firm and full coordination on monitoring levels, and (iii) independent choice of payments *and* monitoring levels. In the first case the regulators jointly decide the levels of all relevant variables – demanded cost targets, monitoring levels and payments. In the second case, the regulators coordinate on the choices of cost targets and monitoring levels, but make payments to the firm in a non-coordinated fashion. In the third case, the regulators choose payments and monitoring levels independently.⁴

When there is full coordination among the regulators, the second best allocation is attained. In comparison to a single regulator arrangement, in which excessive reliance on output-based incentives prevail, with full coordination, the regulators increase the amount of monitoring and demand less aggressive cost targets from the firm. Put differently, the exploitation of the monitoring technology by the two (fully coordinated) regulators allow them to substitute output based incentives – saving, as consequence, on informational rents – for input based incentives. Therefore, a balanced choice of the incentive instruments ensues in such case.

Compared to the full coordination benchmark, a setting in which the regulators do not fully coordinate always leads to *higher* demanded cost targets and *higher* monitoring. In other words, when they do not coordinate, the regulators will always be *less* aggressive regarding output based incentives and *more* aggressive regarding input based incentives.

The reason is simple. Higher monitoring induces less effort toward self-dealing activities by the firm. Since in our model efforts are substitutes for the firm, this reduces its marginal cost to deliver projects more efficiently (i.e., at lower costs). When regulators do not coordinate, the reduction in the firm's marginal cost to deliver more efficient projects is perceived by each of them as an *additional* benefit of monitoring, so more monitoring takes place. This is the source of higher monitoring. Since (lower) cost targets and monitoring are *substitute* incentive instruments to preclude self-dealing, the regulators become more lenient with respect to the cost at which the project is implemented. Finally, if the project is implemented at a higher cost, the perception of an additional benefit of monitoring is just reinforced.

⁴When the regulators themselves are in charge of watching the firm, full coordination in monitoring would correspond to explicit communication between the regulators, perfect information sharing and transmission, avoidance of redundant efforts, and exploration of comparative advantages when executing the different activities that compose the monitoring process. A setting in which the regulators rely on a common third party, such as an auditor, to watch the firm would also fit our interpretation of full coordination.

The effect of the lack of coordination among the regulators on productive effort is unambiguous: in *all* possible equilibria, productive effort will decrease. The combination of more monitoring and higher cost targets have ambiguous consequences for self-dealing. On the one hand, for a fixed level of cost targets, the more monitored the manager is, the less self-dealing he will pursue. On the other hand, for a fixed level of monitoring, the higher the cost targets, the higher the incentives for the manager to exert effort towards self-dealing rather than toward cost reduction.

In our leading example, we show that, for the case in which regulators *coordinate* on monitoring (but not on payments), the amount self-dealt *increases* in comparison to a single-regulator arrangement despite the fact that more monitoring prevails in a two-regulator arrangement. However, when the regulators choose monitoring efforts independently, the amount self-dealt decreases. This is so because, somewhat surprisingly, compared to a setting in which they coordinate on monitoring (but set payments independently), the lack of coordination on monitoring leads to *higher* overall monitoring by the principals, with no effect on cost targets. The reason is that, without an explicit coordination on monitoring, the perception that monitoring has the additional benefit of reducing the agent's marginal cost to deliver more efficient project is *exacerbated*. This effect more than compensates the standard effect of free-riding in teams, which is a force toward less monitoring. These findings suggest that, whenever it is technologically advantageous to have more than one regulator monitoring, the best arrangement is one in which the regulated firm reports to all regulators, who then fully coordinate.

When full coordination among the regulators is not feasible, our leading example suggests that allowing them to choose monitoring levels independently may be beneficial. Indeed, a trade-off between a single regulator arrangement and a two-regulator arrangement in terms of their implications on the firm's choice of how to allocate its efforts arises. On the one hand, in a single regulator arrangement, the firm is forced to deliver lower cost targets but is monitored less. Lower cost targets naturally trigger more effort toward cost reduction. Less monitoring, however, by increasing the firm's marginal benefit to self-deal, leads to more self-dealing. In a two regulator setting in which monitoring levels are chosen independently, the regulators are more lenient with the firm regarding cost targets – and this induces lower efforts toward cost reduction – but exerts an overall amount of monitoring that leads to lower self-dealing.

Related Literature. We now discuss how our findings compare to the existing literature. Our modeling of the agent's efforts and private information is similar to the "cost-padding" model of regulation of Laffont and Tirole (1998, chapter 12). Their goal is to study the possibility of auditing in an optimal, single principal, regulatory design and collusion between auditors and those being audited, whereas we are interested in understanding how incentives are provided to a common privately informed agent under varying degrees of coordination among two *different* principals, who can monitor the agent and condition incentives on a measure of performance.

One of the first papers to consider regulation by more than one regulatory authority is Baron (1985). In his model, one of the regulators decide over abatement and the other establishes output. Hence, there is only room for full coordination (his cooperative equilibrium) or full separation (his non-cooperative equilibrium), Additionally, the regulators decide on their schemes sequentially rather than simultaneously. Consequently, as opposed to what happens in our model, one of the regulators is able to free-ride on the information obtained by the other, and this is what drives the (relative to the second best) distortions imposed by a dual structure in their paper. Bond and Gresik (1996) consider cost-based regulation of a multinational by multiple governments and show that non-cooperative behavior of the governments reduce both national

welfare and firms profits. In contrast to them, the regulators in our model can also monitor the firm and, as we assume that there are technological advantages in having two monitors, a dual arrangement can potentially lead to improvements when compared to a unitary arrangement.

Olsen and Torsvik (1995) and Martimort (1999) have pointed out the benefits of common agency arrangements when regulators cannot commit to contracts. Laffont and Tirole (1998, chapter 12) and Laffont and Martimort (1999) have shown that separating regulators may be helpful in avoiding collusion between the regulated firm and the regulators (capture). In analyzing the benefits of regulatory integration in a financial context, Dell' Arricia and Marquez (2008) highlight the trade-off between the loss of flexibility of a centralized regulatory arrangement (a single regulator arrangement) and the gains relatively to non-integration (a two regulator arrangement) stemming from coordination. We, instead, consider the case in which the regulators can commit to contracts, and are unable to collude with the firm. The potential benefit of a dual structure stems from enhanced monitoring in our setting, while the costs come from an unbalanced provision of incentives when regulators do not coordinate.

Other papers that analyze regulation by multiple regulators include, in the context of regulation of a multinational, Calzolari (2004), who compares national (independent) regulations with cooperative regulation by national regulatory authorities, and Wu (2004), who shows that, in a common agency setting, the rent extraction motive by regulators works against the "pollution haven hypothesis" is reverted. Dalen and (2002) and Dalen and Olsen (2003) analyze regulation by multiple regulators in the context of multi-national banking, and show that the lack of coordination among regulators reduce capital adequacy requirements. None of these papers consider the possibility of multiple incentive instruments.

There is a large literature on common agency games with asymmetric information. The basic methodology to solve such games was developed by Martimort (1996), Stole (1991), and Martimort and Stole (2002, 2003), and we make extensive use of the methodology they developed in those papers in this work. Those papers focus on non-linear pricing games, and aim to understand how direct and indirect externalities among principals affect the size and the direction of the distortions in the allocations when compared to monopolistic settings, whereas our paper aims to compare how the use of *two* – rather than just one (namely, prices or quantities) as in those papers – incentive instruments is affected by their control being in the hands of one or two principals.⁵

Khalil et al (2007) is the closest paper to ours. They consider the implications of monitoring by two principals on financial contracting (project-financing) with an agent who has private information about a project.⁶ Our paper differs from theirs in many respects. First of all, we focus on a different question. While they are interested in the implications of monitoring by two principals on the form by which an agent finances a project, we study how the use of output based incentives interact with monitoring when a firm is regulated by two regulators. Second, precisely because we tackle a different question, our model differs from theirs in two crucial points: (i) not only the principals can monitor the agent, but they can also provide incentives based on the costs at which the project is delivered, and (ii) on top of being better informed than the principals about a hidden characteristic, the agent can take hidden actions; in fact, it is the level of both incentive instruments set by the principals that determines how the agent chooses between productive effort

⁵There is a myriad of other applications of common agency games with asymmetric information. Two of them are: Biais et al (2000), who analyze competition of market makers in financial markets, and, recently, Martimort and Semenev (2007) who consider a model of interest group competition.

⁶They show that equity financing (which demands high levels of monitoring) ensues when principals coordinate, whereas debt financing (which is associated with lower levels of monitoring) ensues when principals do not coordinate.

and effort toward self-dealing in our model.

Regarding results, in contrast to Khalil et al, we find that more monitoring prevails when the principals do not coordinate on monitoring levels. This difference stems from two sources. The first one is related to our assumption that the regulators' monitoring levels are complements rather than substitutes. The second one is related to the strategic effects that prevail when, on top of monitoring, the regulators can also use output based incentives (which are ruled out in their paper). In fact, more monitoring reduces the amount of effort devoted by the firm to self-dealing activities, which, in turn, reduces, for a given level of demanded cost targets, the firm's marginal cost to exert productive effort. As the amount of (informational) rents left to the firm is proportional to the firm's marginal cost to exert productive effort, *ceteris paribus*, more monitoring by a regulator decreases the amount of rents left to the firm, which, in turn, reduces the regulator's perceived cost to demand more productive effort from the firm. Hence, an (unilateral) increase in monitoring is perceived by the regulator as having the additional benefit of reducing the firm's marginal cost to deliver projects more efficiently (i.e., at lower costs). The lack of coordination among the regulators exacerbate this effect and leads to more monitoring (and, in equilibrium, higher cost targets to justify the perception of the additional benefit of monitoring).⁷

Organization. The paper is organized as follows. In section 2, the set-up of the model and the timing of events are described. In section 3, the model is solved for the benchmark in which the quality of the project is known by the regulators. Section 4 solves the model for the single-regulator case under asymmetric information. Section 5 considers a two-regulator arrangement for the case in which they fully coordinate. In section 6, we consider the case in which the regulators choose payments independently, whereas, in section 7, we consider the case in which payments and monitoring levels are chosen independently. Section 8 draws the concluding remarks. All proofs are relegated to the Appendix.

2 The Model

There is a firm and two regulators in the model. The firm, through its manager, can implement an indivisible project that generates $v > 0$ in (social) value. The firm's manager can (i) exert productive effort toward cost reduction and, by virtue of his control over the activities related to its implementation, (ii) extract resources (self-deal) from the project. The cost of implementing the project is assumed to be contractible and is given by

$$c = \theta - e_1 + e_2,$$

where $\theta \in [\underline{\theta}, \bar{\theta}]$ is a cost parameter that is privately observed by the firm, e_1 is its unobservable effort towards cost reduction, and e_2 is effort toward self-dealing.

While the assumptions that the regulated firm is privately informed about its cost's structure and can exert effort to reduce the cost at which the project is implemented are standard in the literature (see Laffont and Tirole, 1998), the assumption that it can extract resources from the project is somewhat new and demands some justification.

By self-dealing in our regulatory context, we mean instances in which a regulated firm can exert effort to inflate its costs by an amount that can be privately appropriated. This may happen, for example, through

⁷As in Khalil et al (2007), regulators can commit to monitoring levels in our paper. For the implications of the lack of commitment of monitors in a mechanism design setting, see, for example, Khalil (1997).

the purchase by the manager/owner of the regulated firm of inputs above market prices from companies he (or an acquaintances) owns.⁸ Transfer pricing is a related – albeit more subtle – form by which self-dealing may take place in a regulatory setting. Indeed, a regulated firm may be charged above market prices for the use a resource provided by a subsidiary. Outright asset diversion is yet another (more direct) form by which self-dealing may take place in a regulated firm. Another simple form by which the manager of a regulated firm can inflate the firm’s cost by an amount he can pocket is by setting his own pay above what would be justifiable in terms of either performance or his outside options. In our model, e_2 captures these and other possibilities.

There is a clear reason why regulators want to preclude self-dealing in our model: it is purely wasteful and, if pursued, will increase the cost at which the project is produced. Therefore, if endowed with means to constraint the firm on the amount of self-dealt pursued, the regulators are able to keep costs down. We assume that, to prevent self-dealing, the regulators can monitor the firm. Hence, we have in mind settings for which alternative sources of monitoring either lack or are not fully effective in avoiding self-dealing, so that the regulators’ monitoring efforts can play a key role in avoiding (inefficient) malfeasance.⁹

We analyze two different arrangements. The first one has the firm reporting to a single regulator, while, in the second, the firm reports to both regulators. By monitoring the agent, the regulators can prevent some of the firm’s self-dealing. More specifically, if regulator i , $i = 1, 2$, monitors the agent at a rate $p_i \geq 0$, a fraction

$$q = z(p_1, p_2) \in [0, 1], \quad (1)$$

of the extracted resources can be recouped. Throughout, we call q the amount of monitoring exerted by the regulators.

The monitoring technology $z(\cdot, \cdot)$ satisfies the following properties: (1) $z(p_1, p_2) = z(p_2, p_1)$, (2) $z(0, 0) = 0$, (3) $z_{p_i}(p_1, p_2) > 0 \geq z_{p_i p_i}(p_1, p_2)$ for all p_i , (4) $z_{p_1 p_2}(p_1, p_2) > 0$ for all p_1 and p_2 . Condition (1) states that the regulators’ monitoring efforts enter symmetrically in the monitoring technology. Condition (2) says that, to recover extracted resources with positive probability, at least one of the regulators has to exert effort. Condition (3) says that each regulators’ marginal productivity of effort is strictly positive and non-increasing. Finally, condition (4) states that the regulators’ monitoring efforts are *complements*: the marginal benefit of monitoring by, say, regulator 1 strictly increases with the monitoring performed by regulator 2. The complementarities in the monitoring efforts imply that there is a technological advantage of having an arrangement in which the agent reports to both principals. However, the extent to which this benefit is exploited will depend on the of coordination among them.

Monitoring the firm is costly for the principals. The regulator i ’s cost of monitoring is $h(p_i)$, with $h'(p_i)$ and $h''(p_i) \geq 0$, $h(0) = h'(0) = 0$ and $\lim_{p_i \rightarrow \infty} h'(p_i) = \infty$.

We adopt the accounting convention that the regulators pay for the cost c to implement the project and make net payments to the firm. The firm’s utility function is

⁸McAfee and McMillan (1988) provide innumerable examples of such type of activities in the context of governmental procurement. When discussing the regulation of power transmission companies in the electricity sector, Joskow (1997) points out the incentive and the ability of a transmission operator to favor its own generators at the expense of competitors.

⁹In this respect, it is worth noticing that, in our setting, as they reimburse the firm for the project, the regulators will be the ones fully bearing the costs of positive self-dealing. Therefore, one would expect that other stakeholders – such as debtholders and shareholders, who are usually seen as key players in watching a firm’s management – would have low incentives to preclude self-dealing.

$$U(t_1, t_2, q, e_1, e_2) = t_1 + t_2 + (1 - q)e_2 - g(e_1, e_2),$$

where t_i is the payment made to the firm by regulator i , and

$$g(e_1, e_2)$$

is its total cost to exert efforts e_1 and e_2 .

The cost function $g(\cdot, \cdot)$ is strictly convex, and smooth with $g_{e_1}(0, e_2) = g_{e_2}(e_1, 0) = 0$, so that the marginal costs of both types of efforts are zero when no effort is exerted. The marginal cost of exerting effort e_i is non-decreasing in e_j : $g_{e_1 e_2}(e_1, e_2) \geq 0$ (i.e., efforts are substitutes). Additionally, to be able to make precise comparisons across some of the arrangements we analyze, we will also consider the following condition on the third derivatives of $g(\cdot, \cdot)$:

$$g_{e_1 e_1 e_1}(e_1, e_2) + g_{e_1 e_2 e_1}(e_1, e_2) \leq 0 \leq g_{e_1 e_1 e_2}(e_1, e_2) + g_{e_1 e_2 e_2}(e_1, e_2). \quad (\text{A1})$$

As an illustration, all these conditions hold for the case of the economy we describe in section 2.2. Nevertheless, as not all results depend on it, whenever condition A1 is used, it will be explicitly stated.

The regulators' payoffs are as follows. In the arrangement in which the firm reports only to regulator i , its utility is

$$v + qe_2 - c - t_i - h(p_i),$$

whereas whenever the firm reports to both regulators, regulator i 's utility is

$$\frac{v + qe_2 - c}{2} - t_i - h(p_i).$$

In words, whenever the agent reports to both regulators, each of them pays for half of the cost, c , to implement the project, and gets half of the total benefits

$$v + qe_2 - c,$$

whereas if the firm reports to only one regulator, the relevant regulator pays for the whole cost of the project c and reap all the benefits.¹⁰

Finally, from the regulators' perspective, the cost parameter θ is distributed over $[\underline{\theta}, \bar{\theta}]$ according to a log-concave distribution $F(\cdot)$, with a continuous density $f(\theta) > 0$.

2.1 Timing of Events and the Contract Space

The timing of the events is as follows. Initially, for any given arrangement, the firm learns privately the cost parameter, θ , of the project in period 1. In period 2, if the firm contracts with a single regulator, say, regulator i , a unique set (menu) of contracts $\{t_i(c, p_i), c, p_i\}_{c \in \mathbb{R}, p_i \in [0, 1]}$, specifying payments, monitoring levels and cost targets, is offered to the firm.

¹⁰An alternative description of the regulators' behavior would have them maximizing the ex-post social welfare taking into account the shadow costs of public funds (as in Laffont and Tirole, 1998). The results we obtain correspond to the case in which the regulators maximize ex-post welfare and face shadow costs of public funds that are prohibitively high. All results we derive would be qualitatively the same in a setting in which regulators maximize ex-post welfare and face positive shadow costs of public funds.

In case the agent reports to two regulators, two sets (menus) of contracts are offered. We consider three possibilities for such case. In the first one, the regulators fully coordinate on the choice of all variables. Hence, it is as if both regulators could merge and offer a single contract $\{t(c, q), c, q\}_{c \in \mathbb{R}, q_i \in [0, 1]}$. In the second case, the regulators coordinate on the choices of monitoring and cost levels but choose payments independently. Hence, regulator i offers to the agent a contract $\{t_i(c, q), c, q\}_{c \in \mathbb{R}, q_i \in [0, 1]}$, $i = 1, 2$. In the third case, the regulators do not coordinate on neither payments, nor monitoring so that regulator i offers to the agent a contract $\{t_i(c, p_i), c, p_i\}_{c \in \mathbb{R}, q_i \in [0, 1]}$.

The firm picks the contracts that fit it better in period 3 – when it interacts with the two regulators, it *must* contract with both of them, i.e., the model is one of intrinsic common agency (Bernheim and Whinston (1986)) –, and then chooses how much to pursue of productive effort and self-dealing. Payments are made in the end of the period, when c is made public. In period 4, the regulators monitor the firm and, with probability q , can recoup the diverted resources.

It is worth noticing that, much as Khalil et al (2007), we allow for contracts where regulators set payments that may be contingent on monitoring levels. With such payments, imposing a given level \tilde{q} of monitoring for, say, the full coordination case is equivalent to making $t(c, q)$ arbitrarily small whenever $q \neq \tilde{q}$. In general, however, it will be optimal for the regulators to allow the firm to select from a range of monitoring levels.

2.2 An Example Economy

Although we are able to derive comparative results across most of the organizational arrangements we consider, for the general model laid down in Section 2, one cannot derive closed form solutions for the variables of interest. However, at the cost of imposing specific functional forms for the monitoring technology and the cost functions, we manage to obtain closed form solutions for most of the cases we consider. Indeed, for the case in which $g(e_1, e_2) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$, $h(p_i) = \frac{1}{4}p_i^4$ and $z(p_1, p_2) = p_1p_2$, we are able to derive explicit solutions for three out of the four organizational arrangements studied in this paper.¹¹

Even for the arrangement for which a full closed form solution is not available (the case in which the regulators do not coordinate on monitoring), with these functional forms we are able to establish the precise cost at which the project will be delivered and place precise bounds on the amount of overall monitoring that will take place in any equilibrium in which positive monitoring takes place. As these solutions add to the understanding of the forces at play in each of such arrangements, on top of the general comparative results we obtain, we also present the results for this particular example economy.

3 The Firm's Induced Cost Function and a Useful Benchmark

To solve the model, it is convenient to start with the firm's problem of choosing the amount of efforts in period 3, once it has committed to implement the project at cost c and knows it will be monitored with intensity q .

The firm chooses both efforts to minimize its total effort costs net of its expected self-dealing benefits, subject to being able to deliver c :

¹¹To guarantee interior solutions throughout the analysis for the example economy, we also assume that $f(\bar{\theta}) \geq 2$.

$$\min_{(e_1, e_2)} g(e_1, e_2) - (1 - q)e_2$$

$$s.t. \ c = \theta - e_1 + e_2.$$

Substituting e_1 from the constraint in the objective function, its induced cost to implement the project when the cost target is c and it is monitored at q is

$$B(c, q, \theta) = \min_{e_2} g(\theta + e_2 - c, e_2) - (1 - q)e_2.$$

It follows that, in period 1, the regulators can choose the incentive instruments treating the firm as if it had utility function $t_1 + t_2 - B(c, q, \theta)$.

Whenever strictly positive, the amount $e_2(q, c, \theta)$ of self-dealing pursued is implicitly defined by

$$g_{e_1}(\theta + e_2(q, c, \theta) - c, e_2(q, c, \theta)) + g_{e_2}(\theta + e_2(q, c, \theta) - c, e_2(q, c, \theta)) = 1 - q$$

The above condition just states that, at an optimal (positive) choice of self-dealing by the agent, the marginal cost of effort toward self-dealing is equal to the benefit, as measured by the amount $1 - q$ of it that the firm can pocket given the overall monitoring level.

If $e_2(q, c, \theta) = 0$, the following condition will hold:

$$g_{e_1}(\theta - c, 0) + g_{e_2}(\theta - c, 0) = g_{e_1}(\theta - c, 0) \geq 1 - q \quad (2)$$

Simple computations show that:

$$e_{2q}(c, q, \theta) \leq 0 \text{ and } e_{2c}(c, q, \theta) \geq 0$$

In words, higher monitoring levels induce lower efforts toward self-dealing, whereas more lenient (i.e., higher) cost targets increases the levels of effort toward self-dealing by the firm.

In our analysis, $e_{2qc}(c, q, \theta)$ – the sensitivity to cost targets of the response of effort toward self-dealing to higher monitoring – will play an important role. Throughout, we assume that higher monitoring is more effective in reducing self-dealing when cost targets are higher, that is:¹²

$$e_{2qc}(c, q, \theta) \leq 0 \quad (A2)$$

Put differently, the condition states that, if the firm is demanded to deliver the project at low (high) costs, the effect of monitoring in reducing self-dealing is small (large).

¹²A sufficient condition on the cost function $g(e_1, e_2)$ for $e_{2qc}(c, q, \theta)$ to be non-positive is

$$\frac{g_{e_1 e_1 e_2} + 2g_{e_1 e_2 e_2} + g_{e_2 e_2 e_2}}{g_{e_2 e_1} + g_{e_2 e_2}} \leq \frac{g_{e_1 e_1 e_1} + 2g_{e_1 e_2 e_1} + g_{e_2 e_2 e_1}}{g_{e_1 e_1} + g_{e_2 e_1}}$$

for all (e_1, e_2) . It is easy to see that this condition holds in the example economy.

3.1 Symmetric Information and The First Best

In order to understand better the forces at play in the model and, in particular, the difficulties faced by the regulators in inducing both high effort toward cost reduction and no self-dealing from the firm, it is worth analyzing the model for the case in which the cost parameter θ is known by the regulators.

In this case, we show that there are always contracts that induce socially efficient amounts of effort toward cost reduction and self-dealing activities, irrespective of whether the firm reports to one or two regulators and irrespective of any assumption regarding the degree of coordination among the regulators. This is so because it turns out that it is not necessary to monitor the agent when θ is known.

To see this, consider first the one-regulator case. Assume the principal that contracts with the agent is regulator 1. When θ is known, normalizing the firm's outside option to zero, the regulator's problem is one of maximizing $v + qe_2 - c - t_1 - h(p_1)$, subject to $t_1 - B(c, q, \theta) \geq 0$, the technological constraint (1), and $p_2 = 0$.

It is optimal to set payments so that the constraint holds with equality. Hence, the problem becomes

$$\max_{(c, p_1)} v + qe_2 - c - B(c, q, \theta) - h(p_1)$$

$$s.t. \quad q = z(p_1, 0) \text{ and } c, p_1 \geq 0.$$

Assume for a moment that the regulator sets $p_1 = 0$. In such case, one has $q = 0$. Plugging this in the objective, we show in the Appendix that the solution to the problem entails demanded cost levels implicitly defined by

$$-1 - B_c(c, 0, \theta) = -1 + g_{e_1}(\theta + e_2(c, 0, \theta) - c, e(c, 0, \theta)) = 0.$$

Noticing that $\theta + e_2(c, 0, \theta) - c = e_1(c, 0, \theta)$, the demanded cost target induces the firm to equate, for a given level of effort toward self-dealing, the marginal social benefit of higher productive effort to its marginal cost

$$g_{e_1}(e_1^*, e_2) = 1.$$

More surprisingly, even without monitoring, the firm exerts no effort toward self-dealing. That is, given the demanded cost targets, the firm will find optimal to choose $e_2^{FB} = 0$.

The interpretation is simple. To induce first best levels of productive effort, the firm must be made residual claimant of any cost reduction it achieves. In such case, any private gains the firm may have pursuing self-dealing will be offset by the increase in the cost of the project. Since it is costly for the firm to exert effort toward self-dealing, it will choose $e_2^{FB} = 0$. In response to the firm's choice of not pursuing self-dealing, the regulator will in fact choose $p_1 = 0$.

A widely spread interpretation of the outcome discussed above is that the project is "sold" to the firm. Being the residual claimant of the project's proceeds, it will decide not to exert inefficient effort toward self-dealing. Intuitively, the possibility of selling the project to the firm does not depend on whether it reports to one or two regulators. In fact, there is also an equilibrium for an arrangement with two regulators. in which

$$g_{e_1}(e_1^{FB}, 0) = 1.$$

and $p_i = 0$, $i = 1, 2$.

Proposition 1 *If the cost parameter is known by the regulators, the chosen contract under a single-regulator arrangement attains socially efficient amounts of effort toward cost reduction and self-dealing, without the need of monitoring the firm. Moreover, for the two-regulator case, there is always an equilibrium that mimics the single-regulator's outcome.*

A setting with private information introduces two new effects. First, as the regulator is forced to rely on the firm's information to set cost targets, to reduce the amount of informational rents to be left to the firm, it is optimal to distort the level of cost targets (by allowing the firm to implement the project at a higher cost) when it reports to a single regulator. This distortion introduces an endogenous motive for monitoring: for the same amount of monitoring exerted by the regulator in the full information case, namely $p_i = 0$, an increase in targeted costs induces positive self-dealing from the firm and, as a consequence, a benefit for monitoring. Put somewhat differently, as cost targets and monitoring are substitute instruments to preclude self-dealing, the reduction of the former introduces the need for the latter. In a single regulator arrangement, monitoring will be beyond what would be feasible in a two regulator arrangement, however, as the complementarities in the monitoring technology cannot be explored. Second, a structure that has two regulators will always introduce a non-trivial strategic interdependence between them.

4 Asymmetric Information: The Single Regulator Case

Under asymmetric information, when the firm reports to a single regulator, matters are relatively simple. Throughout, we assume that the regulator to whom the agent report is regulator 1. By the Revelation Principle, one can restrict attention to direct mechanisms in which the firm reports a cost parameter $\hat{\theta}$ to the regulator and is demanded to implement the project at a cost $c(\hat{\theta})$, is monitored at a level $q(\hat{\theta})$ and is paid $t_1(\hat{\theta})$.

Define $m_1(q)$ as regulator 1's total cost of monitoring the firm at rates that guarantee that the extracted resources can be recouped with at least probability q . Notice that the function $m_1(q)$ is the value of the following program:¹³

$$m_1(q) = \min_{p_1} h(p_1)$$

$$s.t. \quad q \leq z(p_1, 0).$$

The properties of $m_1(\cdot)$ depend on what one assumes about the monitoring technology when just a regulator monitors. If, as in the example economy we consider throughout, $z(p_1, 0) = 0$ for all p_1 , the cost of monitoring is prohibitive ($m_1(q) = \infty$ for all $q > 0$), and no monitoring will ever take place in a single regulator arrangement.

¹³This minimization problem is indeed:

$$m_1(q) = \min_{p_1, p_2} h(p_1) + h(p_2)$$

$$s.t. \quad q \leq z(p_1, p_2) \text{ and } p_2 = 0.$$

So, using the fact that $h(0) = 0$ we get the problem defined in the text. This will be useful when comparing $m_1(q)$ with $m(q)$, to be defined in the next section.

However, except when considering the example economy, we assume through the paper that $z(p_1, 0)$ is strictly increasing in p_1 . For such case, we have:

Lemma 1 *The function $m_1(\cdot)$ is strictly convex, differentiable and so that $m_1'(0) = 0$.*

Using Lemma 1, the regulator's problem can be written as

$$\begin{aligned} \max_{\{c(\theta), q(\theta), t_1(\theta)\}} & v + [E_\theta [q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - t_1(\theta) - m_1(q(\theta))]] \\ \text{s.t. } \theta & = \arg \max_{\hat{\theta}} t_1(\hat{\theta}) - B(c(\hat{\theta}), q(\hat{\theta}), \theta) \text{ for all } \theta \\ U(\theta) & = t_1(\theta) - B(c(\theta), q(\theta), \theta) \geq 0 \text{ for all } \theta. \end{aligned} \quad (3)$$

When compared to the symmetric information case, regulator 1 faces a new set of constraints: not only the firm must be given at least its outside option payoff, but also it must be in the firm's best interest to report the true cost parameter rather than any other.

An application of the Envelope Theorem (Milgrom and Segal, 2002) along with a single crossing condition that the firm's utility satisfies allows one to replace these news constraints by

$$U(\theta) = t_1(\theta) - B(c(\theta), q(\theta), \theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} B_\theta(c(\tau), q(\tau), \tau) d\tau, \quad (4)$$

and $B_\theta(c(\tau), q(\tau), \tau)$ non-increasing in τ .

In the Appendix (see Lemma 3), we show that

$$B_\theta(c(\tau), q(\tau), \tau) = g_{e_1}(\theta + e_2(c(\tau), q(\tau), \theta) - c, e_2(c(\tau), q(\tau), \theta)).$$

Hence, the monotonicity condition calls for the firm's marginal cost to exert productive to decrease (weakly) with the firm's announcement. The interpretation is standard: for given demanded cost targets (and monitoring levels), firms with lower θ have, at the margin, lower costs to deliver such targets in terms of productive effort than firms with higher θ ; in other words, firms with lower θ are more willing to exert productive than firms with higher θ . Hence, in any Incentive Compatible mechanism, firms who are more productive must exert higher productive effort (and, therefore, face higher marginal costs).

Using (4), one can see that

$$t_1(\theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} B_\theta(c(\tau), q(\tau), \tau) d\tau + B(c(\theta), q(\theta), \theta).$$

Substituting this out in the objective, one has, after some integration by parts, that the regulator's problem reads:

$$\max_{\{c(\theta), q(\theta)\}_\theta, U(\bar{\theta})} v + E_\theta \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - B(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta) - m_1(q(\theta)) \right]$$

subject to

$$U(\bar{\theta}) \geq 0 \text{ and } B_{\theta}(c(\tau), q(\tau), \theta) \text{ non-increasing in } \tau.$$

It is optimal to set $U(\bar{\theta})$ equal to zero. Moreover, if the monotonicity constraint can be ignored, an optimal is found by pointwise maximization of the objective. We show in the appendix that the following first order conditions with respect to c and q , respectively, have to hold (we omit the arguments to save on notation):

$$\begin{aligned} 1 &= q^{1R} e_{2c}(c^{1R}, q^{1R}, \theta) + g_{e_1}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) & (\text{FOC1P}c) \\ &+ \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) [1 - e_{2c}(c^{1R}, q^{1R}, \theta)] \\ -g_{e_1 e_2}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) e_{2c}(c^{1R}, q^{1R}, \theta) \end{array} \right] \end{aligned}$$

and

$$e_{2q}(c^{1R}, q^{1R}, \theta) \left[q^{1R} - \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) \\ +g_{e_1 e_2}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) \end{array} \right] \right] = m'_1(q^{1R}) \quad (\text{FOC1P}q)$$

Notice from equation (FOC1Pc) that, comparing with the first best case, the term

$$q^{1R} e_{2c}(c^{1R}, q^{1R}, \theta) + \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) [1 - e_{2c}(c^{1R}, q^{1R}, \theta)] \\ -g_{e_1 e_2}(\theta + e_2(c^{1R}, q^{1R}, \theta) - c^{1R}, e_2(c^{1R}, q^{1R}, \theta)) e_{2c}(c^{1R}, q^{1R}, \theta) \end{array} \right]$$

is added for all $\theta \in (\underline{\theta}, \bar{\theta}]$ to the optimality condition for the cost of the project. While in the appendix we establish it formally (see Lemma 3), we now argue that this term is strictly positive.

In the expression, the first term is related to the fact that, for a given level of monitoring, by being more lenient regarding costs, the regulator will induce more self-dealing. Therefore, whenever the regulator finds hard evidence of self-dealing, the amount recouped will be larger. The second term is standard in a model of regulation with asymmetric information: to induce truthful revelation, the regulator needs to leave informational rents to the firm. The higher the amount of productive effort induced, the larger the informational rents. Hence, such term represents the benefit of being more lenient regarding demanded costs stemming from lower informational rents left to the firm.

From the observations above, it follows that the regulator will find optimal to distort upward the cost to implement any given project. More precisely, one has that

$$c^{1P}(\theta) > c^{FB}(\theta) \text{ for all } \theta \in (\underline{\theta}, \bar{\theta}].$$

is added for all $\theta \in (\underline{\theta}, \bar{\theta}]$ to the cost of inducing productive effort from the firm.

At the level of monitoring that prevails in the complete information case (namely, zero), a reduction in the demanded cost induces strictly positive self-dealing from the firm. Now, whenever the firm exerts effort toward self-dealing activities, the marginal benefit of monitoring – mainly, its effect on reducing the firm's incentives to pursue inefficient self-dealing – is positive. Since the marginal cost of monitoring by regulator 1 is zero at zero efforts, it will be optimal to monitor the firm. More precisely, we will have $q^{1R}(\theta) > 0$ for all types in $(\underline{\theta}, \bar{\theta}]$ (this is formally shown in the appendix).

Since the only instrument to induce productive effort is the demanded cost target, productive effort will be smaller than what prevails under full information. Regarding efforts toward self-dealing, notice that there

are two instruments to preclude it: monitoring levels and demanded cost targets. The larger the demanded cost targets, the higher the incentives for self-dealing. In turn, the amount of monitoring that takes place is not sufficient to preclude self-dealing over $(\underline{\theta}, \bar{\theta}]$.

Proposition 2 *If the monotonicity constraint does not bind, the optimal contract in a single regulator arrangement is implicitly defined by equations (FOC1Pc) and (FOC1Pq). For all types in $(\underline{\theta}, \bar{\theta}]$, monitoring will be positive and demanded cost targets (resp. productive effort levels) will be larger (resp. smaller) than in the full information case. Also, for all such types, positive self-dealing will take place.*

The contract offered by the regulator induces positive self-dealing whenever the cost parameter is not the lowest one. For the example economy we consider throughout, Proposition 2 specializes to:

Corollary 1 *For the example economy, the optimal contract in a single regulator arrangement has*

$$q^{1R}(\theta) = 0; \quad c^{1R}(\theta) = \theta + \frac{F(\theta)}{f(\theta)} - 1$$

with efforts

$$e_1^{1R}(\theta) = 1 - \frac{F(\theta)}{2f(\theta)}; \quad e_2^{1R}(\theta) = \frac{F(\theta)}{2f(\theta)}.$$

5 Two Regulators with Full Coordination

This section analyzes the case in which the two regulators can fully coordinate on all relevant decisions: the payments to be made to the agent, the monitoring levels and the demanded costs. Such an arrangement is akin to one in which the regulators can make side payments to each other or to a setting in which the regulators merge and behave as a single entity. Hence, their problem is to design contracts $\{t(\theta), c(\theta), q(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$ specifying payments to the firm, demanded costs and monitoring levels to maximize the sum of their payoffs subject to Incentive Compatibility and Participation Constraints.¹⁴

As a first step toward solving the problem, we consider how the regulators choose their monitoring levels. As in Khalil et al (2007), this choice is made to minimize the sum of their monitoring costs subject to the monitoring technology (1). Define $m(q)$ as the regulators' minimum total cost of monitoring the firm at rates that guarantee that they can recoup the extracted resources with at least probability q , when they coordinate on monitoring. The function $m(q)$ is the value of of the following program:

$$m(q) = \min_{(p_1, p_2)} h(p_1) + h(p_2)$$

$$s.t. \quad q \leq z(p_1, p_2) \quad \text{and} \quad p_1, p_2 \geq 0.$$

Before proceeding, we pause to discuss some possible interpretations of full coordination in monitoring. The regulators may rely on a common third party, such as an auditor, to watch the firm. In such case, monitoring will necessarily be coordinated. In this example, the ability to pool resources together and

¹⁴Since there is full coordination among the principals, we can, by the Revelation Principle (Myerson, 1981), restrict attention to Direct Mechanisms.

joint efforts to recruit and oversee the auditor would justify the technological advantages of a two-regulator arrangement.

When the regulators themselves are in charge of watching the firm, full coordination would correspond to explicit communication between them, perfect information sharing and transmission, avoidance of redundant efforts, and exploration of comparative advantages when executing the different activities that compose the monitoring process.

Formally, $m(q)$ – the minimum total cost of monitoring the firm at rates that guarantee that they can recoup the extracted resources with at least probability q – would prevail when either the regulators can make side payments to each other (so that “efficient” levels of monitoring efforts) or they behave cooperatively when choosing how much to monitor

The next result establishes a few useful properties of the function $m(\cdot)$:

Lemma 2 *The function $m(\cdot)$ is strictly convex, differentiable and such that $m'(0) = 0$. Also, $m'(q) < m'_i(q)$ for all $q > 0$. For the case in which $h(p_i) = \frac{1}{4}p_i^4$ and $z(p_1, p_2) = p_1p_2$, $m(q) = \frac{1}{2}q^2$.*

A key feature of the above Lemma is the fact that, in a setting in which regulators fully coordinate, the marginal cost of monitoring is smaller than in a single regulator arrangement: $m'(q) < m'_i(q)$. This is an implication of the fact that the monitoring technology exhibits complementarities in the regulators’ efforts. Put differently, there are technological advantages of having (coordinated) monitoring by the regulators.

We are now able to write the regulators’ problem as:

$$\begin{aligned} \max_{(t(\theta), c(\theta), q(\theta))_{\theta \in [\underline{\theta}, \bar{\theta}]}} & v + E_{\theta} [q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - t(\theta) - m(q(\theta))] \\ \text{s.t. } \theta &= \arg \max_{\hat{\theta}} [t(\hat{\theta}) - B(c(\hat{\theta}), q(\hat{\theta}), \theta)], \text{ for all } \theta \\ U(\theta) &= t(\theta) - B(c(\theta), q(\theta), \theta) \geq 0, \text{ for all } \theta \\ \text{and } c(\theta) &\geq 0 \text{ and } q(\theta) \in [0, 1], \text{ for all } \theta. \end{aligned}$$

Solving this problem involves exactly the same steps as the ones used in section 4. Indeed, except for the fact that the cost of monitoring is different, this problem is virtually the same as the one of a single regulator. The question then is to understand what is the impact of a more effective way of monitoring the firm on the whole set of contracts offered.

It is clear that, since the marginal cost of monitoring is smaller when both regulators watch the firm, the firm will be monitored more closely than in a single regulator arrangement whenever θ is in $(\underline{\theta}, \bar{\theta}]$. As it turns out, coordination on higher monitoring levels will induce the regulators to be *less* aggressive regarding output incentives. The reasoning is the following. Since there are technological advantages in having both regulators’ monitoring, the firm will be watched more closely. Also, full coordination among the regulators allows for a more balanced use of incentives. Indeed, in a single regulator arrangement, as monitoring is beyond what can be attained when both regulators watch the firm, demanded cost targets carry the burden of also being an important instrument to reduce the firm’s incentives to self-deal. In contrast, in a two regulator arrangement, the regulators are able to perform more effective monitoring. As a consequence, there is no need to for an overuse of output incentives, which have the cost of raising the amount of informational rents the regulators have to leave to the agent.

Regarding efforts, in comparison to what was obtained in section 4, both types of efforts are reduced. Productive effort is reduced to save on informational rents. The possibility to combine in the best way input-based and output based incentives allows the regulators to induce lower effort toward self-dealing activities from the firm.

Proposition 3 *Assume the relevant monotonicity constraint does not bind. Then, when both regulators can coordinate on the choice of demanded cost targets and monitoring levels, $\{c^{SB}(\theta), q^{SB}(\theta)\}_\theta$ are implicitly defined by*

$$1 = q^{SB}(\theta) e_{2c}(c^{SB}(\theta), q^{SB}(\theta), \theta) + g_{e_1}(\theta + e_2(c^{SB}(\theta), q^{SB}(\theta), \theta) - c^{SB}(\theta), e_2(c^{SB}(\theta), q^{SB}(\theta), \theta)) \\ + \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2^{SB}(\theta) - c^{SB}(\theta), e_2^{SB}(\theta)) [1 - e_{2c}(c^{SB}(\theta), q^{SB}(\theta), \theta)] \\ - g_{e_1 e_2}(\theta + e_2^{SB}(\theta) - c^{SB}(\theta), e_2^{SB}(\theta)) e_{2c}(c^{SB}(\theta), q^{SB}(\theta), \theta) \end{array} \right]$$

and

$$e_{2q}(c^{SB}(\theta), q^{SB}(\theta), \theta) \left[q^{SB}(\theta) - \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2^{SB}(\theta) - c^{SB}(\theta), e_2^{SB}(\theta)) \\ + g_{e_1 e_2}(\theta + e_2^{SB}(\theta) - c^{SB}(\theta), e_2^{SB}(\theta)) \end{array} \right] \right] = m'(q^{SB}(\theta))$$

For all $\theta \in (\underline{\theta}, \bar{\theta}]$, monitoring levels will be larger and than what prevails in a single regulator arrangement. Under (A1) and (A2), demanded cost targets will be weakly larger and both types of efforts will be strictly lower than in a single regulator arrangement.

In the example economy, the form by which the balance of incentives takes place is particularly interesting. The allocation is such that the cost targets demanded from the firm coincide with the ones that prevail in a single-regulator arrangement, and, of course, more monitoring takes place. Hence, under full coordination, when compared to a single regulator arrangement, the regulators proceed by *fixing* the level of the output-based instrument, and then choosing in a coordinated fashion how much to monitor the firm.

Corollary 2 *For the example economy, when both regulators can coordinate on the choice of all variables, the optimal contract sets*

$$q^{SB}(\theta) = \frac{1}{3} \frac{F(\theta)}{f(\theta)}; \quad c^{SB}(\theta) = \theta + \frac{F(\theta)}{f(\theta)} - 1$$

and efforts are

$$e_1^{SB}(\theta) = 1 - \frac{2F(\theta)}{3f(\theta)}; \quad e_2^{SB}(\theta) = \frac{1}{3} \frac{F(\theta)}{f(\theta)}.$$

Full coordination among the regulators allows for a balanced provision of output-based incentives and input-based incentives. As we will see in the next sections, if there is any degree of miscoordination among the regulators, the provision of incentives will be tilted toward more intense use of monitoring and a less intense use of cost targets.

6 Two Regulators Setting Transfers Independently

We now consider the case in which, despite coordinating on monitoring levels, $\{q(\theta)\}_\theta$, and demanded cost targets, $\{c(\theta)\}_\theta$, the regulators make payments to the firm in an *independent* fashion. This arrangement could be descriptive of a situation in which, for example, the regulators hire a common third party – e.g.,

an auditor – to monitor the firm, but try to directly influence the firm’s choice of contracts by setting pay independently. Indeed, in such example, although the regulators are forced to coordinate on the contractible variables *once* the firm has picked a contract, by setting pay independently, regulator i may try to induce the firm to pick the contract that is best for him.¹⁵ The fact that the regulators can set pay independently induces a game among them. We now proceed to compute the Nash Equilibria of such game.

As shown by Martimort and Stole (2002), an extension of the Taxation Principle (see Salanie, 1997) – the Delegation Principle – applies so we can, without any loss, restrict attention to indirect contracts of the form $\{t_i(c, q), c, q\}_{c \geq 0, q \in [0, 1]}$.^{16,17}

Taking as given the payments $\{t_2(c, q)\}_{c \geq 0, q \in [0, 1]}$ made by regulator 2 to the agent, regulator 1’s problem is the following:

$$\max_{(q(\theta), c(\theta), t_1(c(\theta), q(\theta)))_\theta} \frac{1}{2}v + E_\theta \left[\frac{1}{2} [q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - m(q(\theta))] - t_1(c(\theta), q(\theta)) \right] \quad (5)$$

$$s.t. \ U(\theta) = \max_{(c, q)} t_1(c, q) + t_2(c, q) - B(c, q, \theta), \text{ for all } \theta \quad (6)$$

$$U(\theta) \geq 0 \text{ for all } \theta, \quad (\text{IR})$$

and

$$c(\theta) \geq 0, \ q(\theta) \in [0, 1] \text{ for all } \theta. \quad (\text{Feasibility})$$

An application of the Envelope Theorem to (6) allows us to write

$$U(\theta) = U(\bar{\theta}) + \int_{\bar{\theta}}^{\theta} B_\theta(c(\tau), q(\tau), \tau) d\tau$$

which, in turn, implies that

$$t_1(c(\theta), q(\theta)) = -t_2(c(\theta), q(\theta)) + B(c(\theta), q(\theta), \theta) + U(\bar{\theta}) + \int_{\bar{\theta}}^{\theta} B_\theta(c(\tau), q(\tau), \tau) d\tau$$

Plugging the above expression in regulator 1’s objective and noticing that an optimum implied by (6) must be such that $B_\theta(c(\tau), q(\tau), \theta)$ is non-increasing in τ , his problem can be rewritten as:

$$\max_{\{q(\theta), c(\theta)\}_\theta} \frac{1}{2}v - U(\bar{\theta}) + E_\theta \left[\begin{array}{l} \frac{1}{2} [q(\theta) \cdot e_2(c(\theta), q(\theta), \theta) - c(\theta) - m(q(\theta))] \\ + t_2(c(\theta), q(\theta)) - B(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta) \end{array} \right]$$

¹⁵A similar interpretation applies when the regulators, rather than a third-party, are in charge of monitoring: *once* picked the contract, the regulators would – by explicitly communicating with each other, avoiding redundant efforts and so on – coordinate on monitoring but, ex-ante, by setting pay independently, each regulator would try to induce the firm to pick his favorite contract.

¹⁶Since the regulators coordinate on cost targets and monitoring levels, the no-externality condition of Peters (2003) holds, and one could restrict attention to Direct Mechanisms in this section. Also, as we show in Proposition 4, when, following the Delegation Principle, the regulators use menu of contracts, there is a unique equilibrium, which must coincide with the equilibrium that would prevail with Direct Mechanisms.

¹⁷As standard in the common agency literature (see, for instance, Martimort and Stole (2002, 2003)), in this and the next section, we restrict attention to differentiable equilibria.

$$s.t. B_\theta(c(\tau), q(\tau), \theta) \text{ non increasing in } \tau \quad (7)$$

$$U(\bar{\theta}) \geq 0, c(\theta) \geq 0, q(\theta) \in [0, 1] \text{ for all } \theta.$$

Notice that the higher the payment made by regulator 2 to the agent, the higher regulator 1's payoff. This is intuitive: for any given allocation (demanded costs and monitoring levels) the regulators agree upon, regulator 1 will benefit if a larger fraction of the payments needed to induce the firm to pick that allocation is made by regulator 2.

Returning to his problem, it is clear that regulator 1 finds it optimal to set $U(\bar{\theta}) = 0$. Ignoring the monotonicity condition (7) and maximizing the objective pointwise, we obtain the following first order conditions for $c(\theta)$ and $q(\theta)$:

$$\frac{q(\theta)}{2} e_{2c}(c(\theta), q(\theta), \theta) - \frac{1}{2} + \frac{\partial t_2}{\partial c}(c(\theta), q(\theta)) - B_c(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_{\theta c}(c(\theta), q(\theta), \theta) = 0 \quad (\text{FOC2Pc})$$

and

$$\frac{e_2(c(\theta), q(\theta), \theta)}{2} + \frac{q}{2} e_{2q}(c(\theta), q(\theta), \theta) q(\theta) + \frac{\partial t_2}{\partial q}(c(\theta), q(\theta)) - B_q(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_{\theta q}(c(\theta), q(\theta), \theta) = 0. \quad (\text{FOC2q})$$

Since a higher payment made by regulator 2 to the firm allows her to save on the payments she has to make to implement a given allocation, upon deciding the demanded costs and monitoring levels to propose, she considers the effect of such proposal on the payments made by regulator 2. The terms $\frac{\partial t_2}{\partial c}(c(\theta), q(\theta))$ and $\frac{\partial t_2}{\partial q}(c(\theta), q(\theta))$ capture this effect.

Of course, proceeding exactly as above, similar best response functions for regulator 1 can be computed. Indeed, those best response functions are exactly like (FOC2Pc) and (FOC2q), except for the fact that $\frac{\partial t_2}{\partial c}(c(\theta), q(\theta))$ and $\frac{\partial t_2}{\partial q}(c(\theta), q(\theta))$ are replaced by $\frac{\partial t_1}{\partial c}(c(\theta), q(\theta))$ and $\frac{\partial t_1}{\partial q}(c(\theta), q(\theta))$.

To solve for an equilibrium, we need to compute the equilibrium values of $\left\{ \frac{\partial t_i}{\partial c}(c(\theta), q(\theta)) \right\}_{i=1,2}$ and $\left\{ \frac{\partial t_i}{\partial q}(c(\theta), q(\theta)) \right\}_{i=1,2}$. Toward that, assume the payments made by the regulators to the firm are $\{t_i(c, q)\}_{i=1,2,c,q}$. The firm's problem is then

$$\max_{(c,q)} t_1(c, q) + t_2(c, q) - B(c, q, \theta).$$

The first order conditions for an optimum of this program are:

$$\frac{\partial t_1}{\partial c}(c, q) + \frac{\partial t_2}{\partial c}(c, q) - B_c(c, q, \theta) = 0,$$

and

$$\frac{\partial t_1}{\partial q}(c, q) + \frac{\partial t_2}{\partial q}(c, q) - B_q(c, q, \theta) = 0. \quad (\text{ICFOC})$$

Solving for the system implied by (ICFOC) and the regulators' best responses, we obtain the following result:

Proposition 4 *Assume that, given the contract offered by regulator j , the relevant monotonicity constraint does not bind for regulator i . Then, the unique levels of demanded cost targets and monitoring $(c^{2P}(\theta), q^{2P}(\theta))$ that prevail in any equilibrium for the case in which regulators set payments independently solve the following system of equations:*

$$1 = q^{2P} e_{2c}(c^{2P}, q^{2P}, \theta) + g_{e_1}(\theta + e_2(c^{2P}, q^{2P}, \theta) - c^{2P}, e_2(c^{2P}, q^{2P}, \theta)) \\ + 2 \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(c^{2P}, q^{2P}, \theta) - c^{2P}, e_2(c^{2P}, q^{2P}, \theta)) [1 - e_{2c}(c^{2P}, q^{2P}, \theta)] \\ - g_{e_1 e_2}(\theta + e_2(c^{2P}, q^{2P}, \theta) - c^{2P}, e_2(c^{2P}, q^{2P}, \theta)) e_{2c}(c^{2P}, q^{2P}, \theta) \end{array} \right]$$

and

$$e_{2q}(c^{2P}, q^{2P}, \theta) \left[q^{2P} - 2 \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(c^{2P}, q^{2P}, \theta) - c^{2P}, e_2(c^{2P}, q^{2P}, \theta)) \\ + g_{e_1 e_2}(\theta + e_2(c^{2P}, q^{2P}, \theta) - c^{2P}, e_2(c^{2P}, q^{2P}, \theta)) \end{array} \right] \right] = m'(q^{2P})$$

Equilibrium payments are, up to a constant, uniquely defined. Moreover, the payments made by the regulators can only differ by a constant term. Last, under (A1) and (A2), in comparison to the second best (two regulators with full coordination), productive effort is lower and both the amount of monitoring and the demanded cost targets are (weakly) larger when regulators set pay independently.

Proposition 4 states that, except for constant terms in the payments made by them (that do not affect incentives), the equilibrium when the regulators coordinate on costs and monitoring levels but set pay independently is (i) uniquely defined and (ii) so that the regulators behave symmetrically. Alternatively, the only potential (and uninteresting) source of multiplicity and asymmetry that can prevail relates to a constant difference in the payments made by the regulators.

To understand why monitoring and the demanded cost targets are (weakly) larger when regulators set pay independently, it is worth noticing that the equilibrium conditions of Proposition 4 correspond to the first order conditions of the problem of maximizing the expression

$$v + E_\theta \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - B(c(\theta), q(\theta), \theta) - 2 \frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta) - m(q(\theta)) \right],$$

which is equal to the objective function of the full coordination case plus the term $-\frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta)$.

Hence, in terms of the induced equilibrium outcomes, it is as if the regulators were acting in a fully coordinated fashion but, relatively to the full information case, perceiving the informational rents left to the agent as being costlier. As the informational component decreases with q and c , both cost targets and monitoring levels will be larger when compared to the full information case. Therefore, an arrangement in which regulators make payments independently leads to more lenience with respect to the cost at which the project is implemented. Since demanded cost targets is the only instrument capable to induce effort toward cost reduction, an immediate implication of higher demanded cost targets is that the firm will exert less effort toward cost reduction in a two-regulator arrangement with full coordination. Second, the regulators will monitor the firm more closely than when they coordinate on all choices.

An increase in monitoring (i.e. more stringent input-based incentives) and higher costs targets (i.e. less stringent output based incentives) are two implications of any arrangement in which regulators do not fully coordinate.¹⁸ The reason for why this is the case is as follows.

¹⁸As we show in the next section, this also holds when the regulators choose monitoring levels independently. Indeed, the forces that lead to more monitoring and higher costs are magnified in a setting in which the agents do not coordinate on monitoring.

If the firm exerts less effort toward self-dealing, its marginal cost to exert productive effort will be smaller. This, in turn, reduces the informational rents – which, as we argued above, are perceived as being costlier in a setting without coordination – left to the firm. Hence, since more monitoring reduces the firm’s effort toward self-dealing activities (and, as a consequence, informational rents left to the firm), an increase in monitoring is perceived by each regulator as having the additional benefit of (possibly) making room for a higher level of productive effort. This is the source of more monitoring. In equilibrium, to justify the perception of the additional benefit of monitoring, cost targets levels must be below what prevails when regulators fully coordinate.

In terms of efforts, since the firm can deliver the project at a higher cost, it will exert less productive effort than what prevails when the regulators fully coordinate. As for self-dealing, on the one hand, for a fixed level of cost targets, an increase in monitoring reduces the marginal benefit to self-deal. On the other hand, as the cost at which the project is implemented affects the firm’s marginal cost to self-deal, the higher the cost targets, the higher the incentives for the firm to pursue self-dealing, so, in principle, the effect on self-dealing is ambiguous.

For the example economy, we are able to show that the net effect of higher monitoring and higher cost targets on self-dealing is *positive*: even when compared to a single regulator arrangement (in which no monitoring is performed whatsoever), the firm will exert *more* effort toward self-dealing when it reports to two principals who choose payments independently.

Corollary 3 *For the example economy, the solution to problem (5) is:*

$$q^{2P}(\theta) = \frac{2F(\theta)}{3f(\theta)}; \quad c^{2P}(\theta) = \theta + \frac{2F(\theta)}{f(\theta)} - 1$$

with effort levels

$$e_1^{2P}(\theta) = 1 - \frac{4F(\theta)}{3f(\theta)}; \quad e_2^{2P}(\theta) = \frac{2F(\theta)}{3f(\theta)}.$$

7 Two Regulators Setting Transfers *and* Monitoring Levels Independently

In this section, the regulators choose payments *and* monitoring levels p_i , $i = 1, 2$, independently. As in the previous section, we invoke the Delegation Principle and, without loss of generality, restrict attention to indirect contracts of the form $\{t_i(c, p_i), c, p_i\}_{c \geq 0, p_i \geq 0, i=1,2}$.

While solving for the equilibrium was simple for the case in which the regulators coordinate on both cost targets and monitoring, matters are more complicated when they choose monitoring levels independently. Indeed, when choosing the contract to offer, regulator i can affect not only the payments made by regulator j , but also the amount of monitoring that regulator j exerts when facing a firm with a given cost θ . Although finding a profile of mutual best responses becomes harder for such case, the methodology developed by Martimort and Stole (2002, 2003) to characterize the set of equilibria of non-linear pricing games with asymmetric information allows us to derive equilibria in our setting.

While a full derivation can be found in the Appendix, the next section, by considering each of the regulators’ problem, briefly describes Martimort and Stole’s (2002) methodology for the present model.

7.1 The Regulators' Problem

The main idea is to consider individually each of the regulator's problem for a fixed set of contracts offered by the other. In such case, under some assumptions that have to be checked in equilibrium, the methodology used in sections 4 and 5 can be applied.

For simplicity, consider regulator 1's problem (the analysis of the problem faced by regulator 2 is analogous). Letting $\{t_2(c, p_2), c, p_2\}_{c, p_2}$ be a menu of contracts offered by regulator 2, the firm will choose among them the one that maximizes its utility. As a consequence, it is *as if* regulator 1 had to deal with a firm with preferences given by

$$\Phi(t_1, c, p_1, \theta) = t_1 + \phi(c, p_1, \theta), \quad (8)$$

where $\phi(c, p_1, \theta) = \max_{p_2} t_2(c, p_2) - B(c, q(p_1, p_2), \theta)$, and

$$p_2^*(c, p_1, \theta) = \arg \max_{p_2} t_2(c, p_2) - B(c, q(p_1, p_2), \theta)$$

is the solution to the problem that defines $\phi(c, p_1, \theta)$.

Therefore, for a *fixed* set of contracts offered by the other regulator, 1's problem is exactly the same as the one of a single regulator that faces a firm with preferences described by (8). That is, her problem reads (to save on notation, we omit the arguments of the functions):

$$\max_{(p_1(\theta), c(\theta))_\theta} \frac{1}{2}v + E_\theta \left[\frac{1}{2}qe_2 - \frac{1}{2}c - h(p_1) - t_1 \right]$$

$$s.t. \ c(\theta), p_1(\theta) \geq 0 \text{ for all } \theta,$$

$$\theta = \arg \max_{\hat{\theta}} t_1(\hat{\theta}) + \phi(c(\hat{\theta}), p_1(\hat{\theta}), \theta)$$

and

$$t_1(\theta) + \phi(c(\theta), p_1(\theta), \theta) \geq 0, \text{ for all } \theta.$$

The solution to this problem yields regulator 1's best response to regulator 2's contract $\{t_2(c, p_2), c, p_2\}_{c, p_2}$. An equilibrium is then a fixed point of a mapping defined by the regulators' best response functionals.

For general functional forms, it is hard to derive properties of individual monitoring levels and cost targets, as these variables will be implicitly defined by a system of two equations, one of them being an ordinary differential equation. Such ordinary differential equation is non-linear and depends not only on the marginal cost of monitoring, $h'(p_i)$, but also on the first, second and cross derivatives of the monitoring technology $z(p_1, p_2)$.

To make some progress, we focus on the case of the example economy, for which we are able to derive some properties of the equilibrium levels of monitoring and cost targets and compare them with what was obtained in the previous arrangements.

7.2 Equilibrium levels of monitoring and cost targets for the example economy:

After deriving each of the regulator's best response, the main steps to derive an equilibrium are related to (i) finding how the equilibrium choices (payments made by the firm and monitoring levels) of, say, regulator 2, respond to an increase in regulator 1's monitoring level p_1 and proposed cost target c , and (ii) guaranteeing that the assumptions made to apply the single regulator methodology to the problem of finding the regulators' best responses are satisfied.

In the Appendix, we deal with points (i) and (ii), and prove the following.

Proposition 5 *Consider the example economy. Letting $c^{2PD}(\theta)$ and $q^{2PD}(\theta)$ be, respectively, the cost targets and overall monitoring level that prevail in an equilibrium with positive monitoring¹⁹ when the principals do not coordinate, one has that*

$$\begin{aligned} c^{2PD}(\theta) &= \theta + \frac{2F(\theta)}{f(\theta)} - 1, \\ q^{2PD}(\theta) &\in \left(\frac{F(\theta)}{f(\theta)}, 2\frac{F(\theta)}{f(\theta)} \right). \end{aligned}$$

It follows that, for the example economy, for any θ , the cost at which the project is implemented will be the same whether or not the regulators coordinate on monitoring levels. However, as shown in Proposition 4, when there is coordination on the choice of monitoring levels, the overall amount of monitoring is $q^{2P}(\theta) = \frac{2F(\theta)}{3f(\theta)}$, which is smaller than the lower bound established by Proposition 5.

The interpretation is very similar to the one of section 6. An unilateral increase in monitoring is perceived by each regulator as having the additional benefit of reducing the firm's marginal cost to deliver projects more efficiently (i.e., at lower costs). The lack of coordination among the regulators when choosing monitoring levels exacerbates this effect and leads to even more monitoring than what prevails in the case of independent payments. In equilibrium, to justify the perception of the additional benefit of monitoring, cost targets levels must be below what prevails when regulators fully coordinate. As it turns out, for the example economy, cost targets when there is no coordination are the same as when regulators set payments independently.

Hence, somewhat surprisingly, in the example economy, the lack of coordination on monitoring leads to *higher* overall monitoring whenever regulators exert positive effort to monitor the agent. The reason is that, without an explicit coordination on monitoring, the perception that monitoring has the additional benefit of reducing the firm's marginal cost to deliver more efficient project is *magnified*. This effect more than compensates the standard effect of free-riding in teams, which is a force toward less monitoring. Higher monitoring, along with the fact that the demanded cost targets are the same whether regulators coordinate on monitoring or not, implies that self-dealing is smaller when there is no coordination on monitoring.

We now discuss how the outcome of a two-regulator arrangement without coordination on monitoring compares to a single regulator arrangement. The cost at which the project is implemented is always larger in a two-regulator setting (with or without coordination on monitoring) when compared to a single-regulator one.

¹⁹Since monitoring from both regulators is indispensable in the case of the example economy, there always exists an equilibrium in which regulator 1 believes that regulator 2 will not monitor and, therefore, chooses $p_1 = 0$. We focus on equilibria in which positive monitoring takes place.

However, when monitoring levels are chosen independently, the increase in overall monitoring more than offsets the higher costs at which the project is implemented in terms of their effects on self-dealing, that is:

$$e_2^{2PD}(\theta) = \frac{1}{2} \left[\frac{2F(\theta)}{f(\theta)} - q^{2PD}(\theta) \right] < \frac{1}{2} \frac{F(\theta)}{f(\theta)} = e_2^{1P}(\theta),$$

so that the amount of self-dealing will be smaller than what prevails when the firm reports to a single regulator. The following result summarizes this discussion.

Proposition 6 *For the example economy, in a two-regulator arrangement without coordination on monitoring levels, efforts toward both self-dealing and cost reduction are smaller than their counterparts with a single regulator arrangement.*

Proposition 6 establishes a possible trade-off between a single regulator arrangement and a two-regulator arrangement in which monitoring levels are chosen independently in terms of their implications on the firm's choice of how to allocate its efforts.

On the one hand, in a single regulator arrangement, the firm is forced to deliver lower cost targets but is not monitored. Lower cost targets naturally trigger more effort toward cost reduction. The lack of monitoring, however, by increasing the firm's marginal benefit to self-deal, leads to more self-dealing. In a two regulator setting in which monitoring levels are chosen independently, the regulators are more lenient with the firm regarding cost targets – and this induces lower efforts toward cost reduction – but exerts an overall amount of monitoring that leads to lower self-dealing.

8 Conclusions

We have considered how the degree of coordination between two regulators affects the regulatory outcome when incentives can be either provided through direct monitoring or based on an output measure. For the example economy, the following table summarizes the cost, monitoring and effort levels that prevail in each of the four arrangements we have considered:

	Single Regulator	Two Regulators: Full Coordination	Two Regulators: Coordination on Monitoring	Two Regulators: No Coordination
c	$\theta + \frac{F(\theta)}{f(\theta)} - 1$	$\theta + \frac{F(\theta)}{f(\theta)} - 1$	$\theta + \frac{2F(\theta)}{f(\theta)} - 1$	$\theta + \frac{2F(\theta)}{f(\theta)} - 1$
q	0	$\frac{F(\theta)}{3f(\theta)}$	$\frac{2F(\theta)}{3f(\theta)}$	$\left(\frac{F(\theta)}{f(\theta)}, \frac{2F(\theta)}{f(\theta)} \right)$
e_1	$1 - \frac{F(\theta)}{2f(\theta)}$	$1 - \frac{2F(\theta)}{3f(\theta)}$	$1 - \frac{4F(\theta)}{3f(\theta)}$	$\left(1 - \frac{2F(\theta)}{f(\theta)}, 1 - \frac{3F(\theta)}{2f(\theta)} \right)$
e_2	$\frac{F(\theta)}{2f(\theta)}$	$\frac{F(\theta)}{3f(\theta)}$	$\frac{2F(\theta)}{3f(\theta)}$	$\left(0, \frac{F(\theta)}{2f(\theta)} \right)$

The best arrangement is one in which the regulated firm reports to all regulators, who then coordinate on all relevant aspects of the regulatory process. When full coordination among the regulators is not feasible, it may be beneficial to allow the regulators to choose monitoring levels independently as suggested by our

example economy. In fact, in such case, a trade-off between a single regulator arrangement and a two-regulator arrangement arises, as they have different implications for the firm's choice of how to allocate its efforts. On the one hand, in a single regulator arrangement, the firm is forced to deliver lower cost targets but is not monitored. Lower cost targets naturally trigger more effort toward cost reduction. The lack of monitoring, however, by increasing the firm's marginal benefit to self-deal, leads to more self-dealing. In a two regulator setting in which monitoring levels are chosen independently, the regulators are more lenient with the firm regarding cost targets – and this induces lower efforts toward cost reduction – but exerts an overall amount of monitoring that leads to lower self-dealing.

We conclude with a few final words about the model we put forth in this paper. Although applied to a regulatory setting, our "multi-tasking" model with private information can be used to address other relevant questions. Taxation by multiple authorities of an agent whose ability to produce is private information and who can shirk to reduce taxable income and evade resources from the taxing authorities is one example. Another example is the study of the optimal combination of compensation and monitoring of a manager, whose ability to generate profits is private information, and who faces costly effort decisions and can extract resources from the firm. We believe these applications are interesting avenues for future research.

9 Appendix

We start this Appendix by proving Lemmas 1 and 2.

Proof of Lemma 1: $m_1(q)$ is the value of a minimization problem in which the objective is a strictly convex function and the choice set is convex. It follows from standard arguments that $m_1(\cdot)$ is strictly convex. Convexity of $m_1(\cdot)$, along with the fact that it is the value of a minimization problem, imply that $m(\cdot)$ is differentiable (see Corollary 3 of Milgrom and Segal's (2002) Envelope Theorem).

For any q , there is a unique p so that $z(p, 0) = q$. One can then define the inverse of z , $z^{-1}(q, 0)$, which we denote by $\bar{p}(q) = z^{-1}(q, 0)$. Since z is differentiable, so is $\bar{p}(q)$. Now,

$$\frac{m_1(\epsilon) - m(0)}{\epsilon} \equiv \frac{h(\bar{p}(\epsilon)) - h(\bar{p}(0))}{\epsilon}.$$

Taking limits as ϵ goes to zero (the limit exists as the function $h \circ \bar{p} : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ is differentiable), one has that

$$m'_1(0) = h'(0)\bar{p}'(0) = 0,$$

since $h'(0) = 0$ and $\bar{p}'(0)$ is bounded (indeed, it equals $\frac{1}{z_{p_1}(\bar{p}(0), 0)}$ which is bounded, as $z_{p_1}(\bar{p}(0), 0) > 0$). ■

Proof of Lemma 2: Except for $m'(q) < m'_1(q)$ for all $q > 0$, all other properties follow from similar arguments to the ones presented in the proof of Lemma 1.

It is clear that $m'(0) = 0$. Indeed, since $m(q) \leq m_1(q)$ for all q , $m(\cdot)$ is increasing and $m(0) = m_1(0) = 0$,

$$0 \leq \frac{m(q) - m(0)}{q} \leq \frac{m_1(q) - m_1(0)}{q} \text{ for all } q > 0.$$

Taking limits as $q \rightarrow 0$ and using the fact that $m'_1(0) = 0$, one has $m'(0) = 0$.

For $q > 0$, consider the problem

$$\min_{(p_1, p_2)} h(p_1) + h(p_2)$$

subject to

$$z(p_1, p_2) \geq q, \text{ and } p_1, p_2 \geq 0.$$

At an optimum, the constraint will be binding. Let γ be the multiplier on such constraint, and \tilde{p}_1 and \tilde{p}_2 be the optimal amount of effort by the regulators. Standard envelope arguments imply that

$$m'(q) = \gamma.$$

Moreover, the necessary and sufficient conditions for optimality imply that

$$\gamma = \frac{h'(\tilde{p}_1)}{z_{p_1}(\tilde{p}_1, \tilde{p}_2)} = \frac{h'(\tilde{p}_2)}{z_{p_2}(\tilde{p}_1, \tilde{p}_2)}.$$

Letting $\bar{p}(q) = z^{-1}(q, 0)$, one has that

$$m'_1(q) = \frac{dh}{dq}(\bar{p}(q)) = \frac{h'(\bar{p}(q))}{z_{p_1}(\bar{p}(q), 0)}$$

where the second equality uses the Inverse Function Theorem.

Now, $\bar{p}(q) > \tilde{p}_1$. Hence, since $h(\cdot)$ is strictly convex, $h'(\bar{p}(q)) > h'(\tilde{p}_1)$. Moreover, (weak) concavity of $z(\cdot, 0)$ implies that

$$z_{p_1}(\tilde{p}_1, 0) \geq z_{p_1}(\bar{p}(q), 0).$$

Since monitoring efforts are complements, and $\tilde{p}_2 > 0$,

$$z_{p_1}(\tilde{p}_1, \tilde{p}_2) > z_{p_1}(\tilde{p}_1, 0) \geq z_{p_1}(\bar{p}(q), 0).$$

It then follows that

$$m'(q) = \frac{h'(\tilde{p}_1)}{z_{p_1}(\tilde{p}_1, \tilde{p}_2)} < \frac{h'(\bar{p}(q))}{z_{p_1}(\bar{p}(q), 0)} = m'_1(q)$$

as claimed

For the case in which $h(p_i) = \frac{1}{4}p_i^4$ and $z(p_1, p_2) = p_1 \cdot p_2$, consider the Lagrangian

$$L = \frac{1}{4}p_1^4 + \frac{1}{4}p_2^4 + \lambda[q - p_1 \cdot p_2].$$

The minimum of such Lagrangian satisfies the following first order conditions

$$\begin{aligned} p_1^3 - \lambda p_2 &= 0 \\ p_2^3 - \lambda p_1 &= 0 \\ q - p_1 \cdot p_2 &= 0. \end{aligned}$$

The first two equations imply that $p_1 = p_2 = p^*$. Plugging this in the third equation, we obtain $p^* = \sqrt[3]{q}$. It follows that $m(q) = \frac{1}{2}q^2$. ■

We now derive some properties of the firm's induced cost function. As shown in the text, the firm's induced cost function is

$$\begin{aligned} B(c, q, \theta) &= \min_{(e_1, e_2) \in \mathbb{R}_+^2} g(e_1, e_2) - (1 - q)e_2 \\ \text{s.t. } c &= \theta - e_1 + e_2. \end{aligned}$$

Lemma 3 *The partial derivatives of $B(c, q, \theta)$ are*

$$\begin{aligned} B_c(c, q, \theta) &= -g_{e_1}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)) \\ B_q(c, q, \theta) &= e_2(c, q, \theta) \\ B_\theta(c, q, \theta) &= g_{e_1}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)) \end{aligned}$$

Also,

$$B_{\theta c}(c, q, \theta) \leq 0, B_{\theta q} < 0.$$

Proof. Substituting the constraint in the objective function, one can write $B(c, q, \theta)$ as

$$B(c, q, \theta) = \min_{e_2} g(\theta + e_2 - c, e_2) - (1 - q)e_2.$$

By the Envelope Theorem, one has that

$$\begin{aligned} B_c(c, q, \theta) &= -g_{e_1}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)), \\ B_q(c, q, \theta) &= e_2(c, q, \theta), \end{aligned}$$

and

$$B_\theta(c, q, \theta) = g_{e_1}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)).$$

as claimed.

Now,

$$\begin{aligned} B_{\theta c}(c, q, \theta) &= -g_{e_1 e_1}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)) [1 - e_{2c}(c, q, \theta)] \\ &\quad + g_{e_1 e_2}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)) e_{2c}(c, q, \theta). \end{aligned}$$

Whenever strictly positive, $e_2(c, q, \theta)$ is implicitly defined by

$$g_{e_1}(\theta + e_2 - c, e_2) + g_{e_2}(\theta + e_2 - c, e_2) - (1 - q) = 0.$$

Hence, by the Implicit Function Theorem,

$$e_{2c}(c, q, \theta) = \frac{g_{e_1 e_1}(e_1, e_2) + g_{e_2 e_1}(e_1, e_2)}{g_{e_1 e_1}(e_1, e_2) + 2g_{e_1 e_2}(e_1, e_2) + g_{e_2 e_2}(e_1, e_2)} \in (0, 1).$$

(We have omitted the arguments to save on notation). Plugging this in the expression for $B_{\theta c}(c, q, \theta)$ one gets:

$$\frac{g_{e_1 e_2}(g_{e_1 e_1} + g_{e_1 e_2}) - g_{e_1 e_1}(g_{e_1 e_2} + g_{e_2 e_2})}{g_{e_1 e_1} + 2g_{e_1 e_2} + g_{e_2 e_2}} = \frac{(g_{e_1 e_2})^2 - g_{e_1 e_1} g_{e_2 e_2}}{g_{e_1 e_1} + 2g_{e_1 e_2} + g_{e_2 e_2}}$$

Since $g(\cdot, \cdot)$ is strictly convex, its Hessian is positive semi-definite. Hence, its determinant is non-negative. Noticing that the determinant of the Hessian of $g(\cdot, \cdot)$ is

$$g_{e_1 e_1} g_{e_2 e_2} - (g_{e_1 e_2})^2 \geq 0,$$

one has that

$$B_{\theta c}(c, q, \theta) \leq 0.$$

Also,

$$B_{\theta q}(c, q, \theta) = e_{2q}(c, q, \theta) [g_{e_1 e_1}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta)) + g_{e_1 e_2}(\theta + e_2(c, q, \theta) - c, e_2(c, q, \theta))].$$

Using again the expression that implicitly defines $e_2(c, q, \theta)$, one has

$$e_{2q}(c, q, \theta) = -\frac{1}{g_{e_1 e_1}(e_1, e_2) + 2g_{e_1 e_2}(e_1, e_2) + g_{e_2 e_2}(e_1, e_2)} < 0$$

so that $B_{\theta q}(c, q, \theta) < 0$. ■

With Lemma 3 in hands, we now move to prove the following:

Proof of Proposition 1. The problem of a single regulator who knows θ is

$$\begin{aligned} \max_{(c, p_1) \in \mathfrak{R}_+^2} \quad & v + qe_2(c, q, \theta) - c - B(c, q, \theta) - h(p_1) \\ \text{s.t.} \quad & q = z(p_1, 0) \end{aligned}$$

If we set $p_1 = 0$ then the problem simplifies to $\max_c v - c - B(c, 0, \theta)$. So, using

$$B_c(c, 0, \theta) = -g_{e_1}(\theta + e_2(c, 0, \theta) - c, e_2(c, 0, \theta)),$$

the first order condition is the one expressed in the text. Moreover, if one plugs $1 = g_{e_1}(\theta + e_2(c, 0, \theta) - c, e_2(c, 0, \theta))$ in equation (2), we get

$$1 = g_{e_1}(\theta + 0 - c, 0) + g_{e_2}(\theta + 0 - c, 0) > 1 - q$$

which implies $e_2^* = 0$.

Hence, the contract that has $p_1 = 0$ and $c = g_{e_1}^{-1}(e_1^*, 0) + \theta$ allows the regulator to extract all the surplus from the firm – which implies that the regulator’s payoff will coincide with total surplus – implementing first best levels of efforts without incurring monitoring costs. Hence, it is optimal.

For the two-principal case, consider the following payments by the principals: $t_i = \frac{1}{2}B(c, q, \theta)$.

In such case, regulator 1 solves:

$$\begin{aligned} \max_{(c, p_1) \in \mathfrak{R}_+^2} \quad & \frac{1}{2}(v + qe_2(c, q, \theta) - c - B(c, q, \theta)) - h(p_1) \\ \text{s.t.} \quad & q = z(p_1, p_2) \end{aligned}$$

Then, the first order conditions are:

$$z(p_1, p_2)e_{2c}(c, z(p_1, p_2), \theta) - 1 - B_c(c, z(p_1, p_2), \theta) = 0$$

and

$$z_{p_1}(p_1, p_2)e_2(c, z(p_1, p_2), \theta) + z(p_1, p_2)e_{2q}(c, q, \theta) - B_q(c, q, \theta)z_{p_1}(p_1, p_2) - h'(p_1) = 0$$

Assume that $p_1 = p_2 = 0$. Then, from the first order conditions with respect to c , we get that both efforts equal the levels attained at the First Best. Moreover, at this levels, $z(0, 0) = e_2(c^*, 0, \theta) = B_q(c^*, 0, \theta) = 0$, which implies that the first order conditions with respect to p_1 is satisfied.

Therefore there is one equilibrium which has the same outcome as the First Best allocations. ■

In the next result, we provide a standard characterization of the Incentive Compatibility constraints for the single-regulator case.

Lemma 4 *The Incentive Compatibility constraints in (3) are equivalent to*

$$U(\theta) = t(\theta) - B(c(\theta), q(\theta), \theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} B_{\theta}(c(\tau), q(\tau), \tau) d\tau \quad (9)$$

and

$$B_{\theta}(c(\tau), q(\tau), \theta) = g_{e_1}(\theta + e_2(c(\tau), q(\tau), \theta) - c(\tau), e_2(c(\tau), q(\tau), \theta)) \text{ decreasing in } \tau \quad (10)$$

Proof. The proof is standard (see, for example, Guesnerie and Laffont (1984)) and is omitted. ■

Proof of Proposition 2. From the previous Lemma, the firm's payoff can be written as:

$$t(\theta) - B(c(\theta), q(\theta), \theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} B_{\theta}(c(\tau), (\tau), \tau) d\tau.$$

Using this expression, one can see that the participation constraints can be replaced by $U(\bar{\theta}) \geq 0$. Taking the expected value of $t(\theta)$, we obtain:

$$E_{\theta} [t(\theta)] = E_{\theta} \left[B(c(\theta), (\theta), \theta) + \int_{\theta}^{\bar{\theta}} B_{\theta}(c(\tau), (\tau), \tau) d\tau \right] + U(\bar{\theta}). \quad (11)$$

Solving the expression for B_{θ} , we have that:

$$E_{\theta} \left[\int_{\theta}^{\bar{\theta}} B_{\theta}(c(\tau), q(\tau), \tau) d\tau \right] = \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\theta}^{\bar{\theta}} B_{\theta}(c(\tau), q(\tau), \tau) d\tau \right] f(\theta) d\theta$$

Integrating by parts and using the fact that $F(\underline{\theta}) = 0$, the expression in (11) equals:

$$E_{\theta} [t(\theta)] = E_{\theta} \left[B(c(\theta), q(\theta), \theta) + \frac{F(\theta)}{f(\theta)} B_{\theta}(c(\theta), q(\theta), \theta) \right] + U(\bar{\theta})$$

Plugging this in the objective function, we get, after some integration by parts, the principal's problem

$$\max_{(c(\theta), q(\theta))_{\theta}, U(\bar{\theta})} v + E_{\theta} \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - B(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_{\theta}(c(\theta), q(\theta), \theta) - m_1(q(\theta)) \right]$$

subject to

$$\begin{aligned} U(\bar{\theta}) &\geq 0, \quad B_{\theta}(c(\tau), q(\tau), \theta) \text{ non-increasing in } \tau \\ \text{and } c(\theta) &\geq 0, \quad q(\theta) \in [0, 1] \text{ for all } \theta. \end{aligned}$$

as stated on the text.

Therefore, if the monotonicity constraints is not binding, the optimality conditions for the regulator are obtained by maximizing the objective pointwise, which leads to the following conditions:

$$\begin{aligned} 1 &= q(\theta) e_{2c}(c(\theta), q(\theta), \theta) + g_{e_1}(\theta + e_2(c(\theta), q(\theta), \theta) - c(\theta), e_2(c(\theta), q(\theta), \theta)) \\ &+ \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(\theta) - c(\theta), e_2(\theta)) [1 - e_{2c}(c(\theta), q(\theta), \theta)] \\ -g_{e_1 e_2}(\theta + e_2(\theta) - c(\theta), e_2(\theta)) e_{2c}(c(\theta), q(\theta), \theta) \end{array} \right] \end{aligned}$$

and

$$e_{2q}(c(\theta), q(\theta), \theta) \left[q(\theta) - \frac{F(\theta)}{f(\theta)} \left[\begin{array}{l} g_{e_1 e_1}(\theta + e_2(\theta) - c(\theta), e_2(\theta)) \\ + g_{e_1 e_2}(\theta + e_2(\theta) - c(\theta), e_2(\theta)) \end{array} \right] \right] = m'(q(\theta))$$

as claimed.

We now argue that $e_2(c, q, \theta)$ is strictly positive over $(\underline{\theta}, \bar{\theta}]$. Toward a contradiction, assume there is an interval $(\theta', \theta'') \subset (\underline{\theta}, \bar{\theta}]$ so that $e_2(c, q, \theta) = 0$ over (θ', θ'') . Then

$$e_{2q}(c, q, \theta) = e_{2c}(c, q, \theta) = 0$$

over such set. Plugging this in (FOC1P q), one gets $q(\theta) = 0$.

Substituting $e_2(c(\theta), q(\theta), \theta) = q(\theta) = 0$ out in (FOC1P c), we obtain:

$$g_{e_1}(\theta - c(\theta), 0) + \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1}(\theta - c(\theta), 0)] = 1.$$

Since $\frac{F(\theta)}{f(\theta)} g_{e_1 e_1}(\theta - c(\theta), 0) > 0$,

$$g_{e_1}(\theta - c(\theta), 0) < 1.$$

Now, if, anticipating that $q(\theta) = 0$, the firm chooses $e_2(c(\theta), q(\theta), \theta) = 0$, it must be the case that (as we argued in section 3)

$$g_{e_1}(\theta - c(\theta), 0) + g_{e_2}(\theta - c(\theta), 0) = g_{e_1}(\theta - c(\theta), 0) \geq 1$$

which yields the desired contradiction.

Now, since $e_2(c(\theta), q(\theta), \theta) > 0$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$,

$$e_{2q}(c(\theta), q(\theta), \theta) < 0.$$

Hence, (FOC1P q) can only hold if $q(\theta) > 0$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$.

Last, the fact that productive effort will be smaller than what prevails under symmetric information follows directly from (FOC1P c), which says that

$$\begin{aligned} & q(\theta) e_{2c}(c(\theta), q(\theta), \theta) + g_{e_1}(e_1, e_2) + \\ & \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1}(e_1, e_2) [1 - e_{2c}(c(\theta), q(\theta), \theta)] + g_{e_1 e_2}(e_1, e_2) e_{2c}(c(\theta), q(\theta), \theta)] \\ & = 1. \end{aligned}$$

Since both $q(\theta) e_{2c}(c(\theta), q(\theta), \theta)$ and $\frac{F(\theta)}{f(\theta)} [g_{e_1 e_1}(e_1, e_2) [1 - e_{2c}(c(\theta), q(\theta), \theta)] + g_{e_1 e_2}(e_1, e_2) e_{2c}(c(\theta), q(\theta), \theta)]$ are strictly positive,

$$g_{e_1}(e_1, e_2) < 1.$$

Now, from the fact that the first best level of productive effort (which is the amount of productive effort prevailing under symmetric information) is characterized by

$$g_{e_1}(e_1^*, 0) = 1,$$

it follows that productive effort will be smaller. Indeed, if e_1 were weakly larger than e_1^* , one would have

$$1 = g_{e_1}(e_1^*, 0) \leq g_{e_1}(e_1, 0) \leq g_{e_1}(e_1, e_2)$$

(the first equality follows the condition that defines the first best level of productive effort, the first inequality follows from the assumption that $e_1 \geq e_1^*$ and the fact that $g_{e_1 e_1} > 0$, and the last inequality follows from the fact that $g_{e_1 e_2} \geq 0$) which would contradict

$$g_{e_1}(e_1, e_2) < 1.$$

■

Proof of Corollary 1. Simply replace all the functional forms on the first order conditions. Notice that, for such case, the monotonicity constraint is indeed not binding. ■

In Propositions 3 and 4, we compare monitoring levels and demanded cost targets across different regulatory arrangements. To establish these comparisons, we make use of a more general result that we now prove.

Consider the following maximization problems:

$$\max_{(x,y) \in \mathbb{R}_+^2} f(x,y) \text{ and } \max_{(x,y) \in \mathbb{R}_+^2} g(x,y),$$

where both $f(.,.)$ and $g(.,.)$ are continuously differentiable and quasiconcave, and, for each problem, it is assumed that there is a unique solution.

Denote the solutions to these problems by, respectively, (x_f^*, y_f^*) and (x_g^*, y_g^*) and assume they are characterized by the following first order conditions:

$$\begin{aligned} f_x(x_f^*, y_f^*) &= f_y(x_f^*, y_f^*) = 0 \\ g_x(x_g^*, y_g^*) &= g_y(x_g^*, y_g^*) = 0. \end{aligned}$$

We then have the following:

Lemma 5 *Assume that*

$$f_x(x_g^*, y_g^*) > 0, f_y(x_g^*, y_g^*) \geq 0$$

and

$$f_{xy}(x,y) \geq 0 \text{ for all } (x,y) \geq (x_g^*, y_g^*)$$

Then $x_f^* > x_g^*$ and $y_f^* \geq y_g^*$.

Proof. Since f is quasiconcave and (i) $f_x(x_g^*, y_g^*) > 0$, (ii) $f_y(x_g^*, y_g^*) \geq 0$, and (iii) $f_{xy}(x,y) \geq 0$ for all $(x,y) \geq (x_g^*, y_g^*)$, one must have that either $x_f^* \geq x_g^*$ or $y_f^* \geq y_g^*$. Consider the case in which $x_f^* \geq x_g^*$ (the other one is analogous). Now, notice that (x_f^*, y_f^*) solves

$$\max_{y \geq 0} \max_{x \geq 0} f(x,y)$$

so that

$$y_f^* = \arg \max_{y \geq 0} f(x_f^*, y)$$

Now, the first order condition of the problem $\max_y f(x_f^*, y)$ evaluated at y_g^* is

$$f_y(x_f^*, y_g^*) \geq f_y(x_g^*, y_g^*) \geq 0,$$

where the first inequality uses the fact that $f_{xy}(x, y) \geq 0$ for all $(x, y) \geq (x_g^*, y_g^*)$ (and that $x_f^* \geq x_g^*$), and the second uses the fact that, by assumption, $f_y(x_g^*, y_g^*) \geq 0$.

Since

$$f_y(x_f^*, y_g^*) \geq 0,$$

$$y_f^* \geq y_g^*.$$

Hence, one must have that $x_f^* \geq x_g^*$ and $y_f^* \geq y_g^*$. Since $f_x(x_g^*, y_g^*) > 0$, one must indeed have $x_f^* > x_g^*$. ■

The interpretation of the above result is simple. When evaluated at (x_g^*, y_g^*) – the point that maximizes g –, for the problem of maximizing f , the marginal net benefit of y is non-negative and the marginal net benefit of x is strictly positive. Therefore, starting at (x_g^*, y_g^*) , if one increases x slightly, the objective f will be improved. Now, since $f_{xy}(x, y) \geq 0$ for all $(x, y) \geq (x_g^*, y_g^*)$, by increasing x , one also increases the marginal benefit of y , which, at (x_g^*, y_g^*) , was non-negative. Hence, if one increases y , the objective f will (weakly) improve. But, again, since $f_{xy}(x, y) \geq 0$ for all $(x, y) \geq (x_g^*, y_g^*)$, by increasing y , one also increases the marginal benefit of x , and the whole process resumes. In other words, since at (x_g^*, y_g^*) , the marginal benefits of x and y are, in the problem of maximizing f , non-negative and these variable are "complements" in the relevant range, the maximum of the function f is no smaller than the maximum for the function g .²⁰

It is also worth pointing out that, in the result above, if either $f_y(x_g^*, y_g^*) > 0$ or $f_{xy}(x, y) > 0$ for all $(x, y) \geq (x_g^*, y_g^*)$, then $y_f^* > y_g^*$.

With Lemma 5 in hands, we are able to prove Propositions 3 and 4.

Proof of Proposition 3. Proceeding exactly as in the proof of proposition 2, we can rewrite the incentive compatibility constraints as an integral equation and a monotonicity constraint. If one substitutes the payment induced by the integral representation of the firm's payoff in the objective, the regulators' problem reads:

$$\max_{(c(\theta), q(\theta))_{\theta}, U(\bar{\theta})} v - U(\bar{\theta}) + E \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - B(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_{\theta}(c(\theta), q(\theta), \theta) - m(q(\theta)) \right]$$

$$s.t. U(\bar{\theta}) \geq 0, B_{\theta}(c(\tau), q(\tau), \theta) \text{ non increasing in } \tau$$

$$\text{and } c(\theta) \geq 0, q(\theta) \in [0, 1] \text{ for all } \theta.$$

Clearly, it is optimal to set $U(\bar{\theta}) = 0$.

If the monotonicity constraint is not binding, an optimal is found by pointwise maximization. The first order conditions for such problem are (to save on notation, throughout the proof, we omit the arguments of the function $g(\cdot)$ and its derivatives):

$$1 = q(\theta) e_{2c}(c(\theta), q(\theta), \theta) + g_{e_1} + \frac{F(\theta)}{f(\theta)} \begin{bmatrix} g_{e_1 e_1} [1 - e_{2c}(c(\theta), q(\theta), \theta)] \\ -g_{e_1 e_2} e_{2c}(c(\theta), q(\theta), \theta) \end{bmatrix} \quad (\text{FOCSBCost})$$

and

$$e_{2q}(c(\theta), q(\theta), \theta) \left[q(\theta) - \frac{F(\theta)}{f(\theta)} (g_{e_1 e_1} + g_{e_1 e_2}) \right] = m'(q(\theta)) \quad (12)$$

²⁰The logic of the proof of Lemma 5 can be interpreted as a "local" counterpart of the arguments that are used in the Monotone Comparative Statics literature (see, Topkis (1998)). For an application of such arguments to the problem of comparing optima in different maximization problems, see, for example, Segal (1999).

as stated in the Proposition.

We now argue that, in comparison to a single regulator arrangement, monitoring levels and demanded cost targets will be larger. To do so, we will use Lemma 5.

Toward that, notice that, if one evaluates equations (FOCSBCost) and (12) at the levels $(c^{1R}(\theta), q^{1R}(\theta))$ that prevail in a single regulator, one has the following (we omit the arguments to save on notation. The expressions below should be read as being evaluated at $(c^{1R}(\theta), q^{1R}(\theta))$):

$$1 = q^{1R} e_{2c}(c^{1R}, q^{1R}, \theta) + g_{e_1} + \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} [1 - e_{2c}(c^{1R}, q^{1R}, \theta)] - g_{e_1 e_2} e_{2c}(c^{1R}, q^{1R}, \theta)] = 0$$

and

$$e_{2q}(c^{1R}, q^{1R}, \theta) \left[q^{1R} - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] - m'(q^{1R}) > 0,$$

where the inequality above follows from the fact that

$$e_{2q}(c^{1R}, q^{1R}, \theta) \left[q^{1R} - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] = m'_1(q^{1R}) > m'(q^{1R})$$

for all $q^{1R} > 0$.

Moreover,

$$\begin{aligned} & \frac{\partial}{\partial c} \left[e_{2q}(c^{1R}, q^{1R}, \theta) \left[q^{1R} - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] - m'_1(q^{1R}) \right] \\ &= e_{2qc}(c^{1R}, q^{1R}, \theta) \left[q^{1R} - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] \\ & \quad + \frac{F(\theta)}{f(\theta)} e_{2q}(c^{1R}, q^{1R}, \theta) \left[\begin{array}{l} [1 - e_{2c}] [g_{e_1 e_1 e_1} + g_{e_1 e_2 e_1}] \\ - e_{2c} [g_{e_1 e_1 e_2} + g_{e_1 e_2 e_2}] \end{array} \right] \end{aligned}$$

which is (weakly) larger than zero when evaluated at (c^{1R}, q^{1R}) , as

$$e_{2qc} \left[q - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] \geq 0$$

under condition A2 and, under condition A1,

$$\frac{F(\theta)}{f(\theta)} e_{2q}(c^{1R}, q^{1R}, \theta) \left[\begin{array}{l} [1 - e_{2c}(c^{1R}, q^{1R}, \theta)] [g_{e_1 e_1 e_1} + g_{e_1 e_2 e_1}] \\ - e_{2c}(c^{1R}, q^{1R}, \theta) [g_{e_1 e_1 e_2} + g_{e_1 e_2 e_2}] \end{array} \right] \geq 0.$$

Noticing that

$$\left[q - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] \leq 0$$

at any candidate at an optimum – so that one can restrict attention to (q, c) that satisfy this inequality,

$$\begin{aligned} 0 &\leq e_{2qc}(c^{1R}, q^{1R}, \theta) \left[q^{1R} - \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} + g_{e_1 e_2}] \right] \\ & \quad + \frac{F(\theta)}{f(\theta)} e_{2q}(c^{1R}, q^{1R}, \theta) \left[\begin{array}{l} [1 - e_{2c}(c^{1R}, q^{1R}, \theta)] [g_{e_1 e_1 e_1} + g_{e_1 e_2 e_1}] \\ - e_{2c}(c^{1R}, q^{1R}, \theta) [g_{e_1 e_1 e_2} + g_{e_1 e_2 e_2}] \end{array} \right] \end{aligned}$$

for all $(q, c) \geq (q^{1R}, c^{1R})$ over the relevant range. Invoking Lemma 5, the result follows.

Now, notice that

$$e_1(c, q, \theta) = \theta + e_2(c, q, \theta) - c$$

and that

$$e_{2q}(c, q, \theta) < 0 \text{ and } 1 - e_{2c}(c, q, \theta) > 0.$$

Hence, when compared to the single regulator arrangement, the regulators will induce a lower level of productive effort.

Now, using

$$B(c, q, \theta) = g(\theta + e_2 - c, e_2) - (1 - q) e_2,$$

and

$$c = \theta + e_2 - e_1.$$

one can write the objective function of the regulators in terms of both types of efforts and monitoring levels as:

$$v - \theta + e_1 - g(e_1, e_2) - \frac{F(\theta)}{f(\theta)} g_{e_1}(e_1, e_2) - m(q).$$

Since both $g(e_1, e_2)$ and $g_{e_1}(e_1, e_2)$ are increasing in e_2 , one sees that self-dealing only harms the objective.

Now, since $q^{2R} > q^{1R}$, the level of demanded cost targets \bar{c} so that $e_2(q^{2R}, \bar{c}, \theta) = e_2(q^{1R}, c^{1R}, \theta) \equiv \bar{e}_2$ must be larger c^{1R} . Hence, when the amount self-dealt is the same across arrangements, productive effort will be strictly smaller with two regulators as

$$e_1^{1R}(\bar{e}_2, c^{1R}) = \theta + \bar{e}_2 - c^{1R} > e_1^{2R}(\bar{e}_2, \bar{c}) = \theta + \bar{e}_2 - \bar{c}$$

Hence, given q^{2R} , starting at \bar{e}_2 , the marginal net gain of increasing productive effort in a two regulator arrangement is larger than in a single regulator arrangement. Noticing that, by the definition of \bar{e}_2 and e_1^{1R} , such marginal net gain is zero in a single regulator arrangement, the productive effort that will prevail in a two-regulator arrangement is larger than $e_1^{2R}(\bar{e}_2, \bar{c})$. Therefore, $\bar{c} > c^{2R}$ which implies that

$$e_2(q^{2R}, c^{2R}, \theta) < e_2(q^{2R}, \bar{c}, \theta) = e_2(q^{1R}, c^{1R}, \theta).$$

■

Proof of Corollary 2. In the example, $m(q) = \frac{1}{2}q^2$, so using Lemma 3 we have:

$$\max_{U(\bar{\theta}), (c(\theta), q(\theta))_\theta} v - U(\bar{\theta}) + E_\theta \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - B(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} e_1(c(\theta), q(\theta), \theta) - \frac{1}{2}q(\theta)^2 \right]$$

$$s.t. \ U(\bar{\theta}) \geq 0$$

$$\frac{d}{d\theta} (q(\theta) + c(\theta)) \geq 0, \ c(\theta) \geq 0, \ q(\theta) \in [0, 1] \text{ for all } \theta.$$

Ignoring the monotonicity constraint (the result will be such that it will be satisfied), one has that, at an optimum, $U(\bar{\theta}) = 0$. Maximizing pointwise, we get (we omit the arguments to save on notation):

$$\begin{aligned} qe_{2q} + e_2 - \frac{\partial B}{\partial q} - \frac{F}{f} e_{1q} - q &= 0 \\ qe_{2c} - 1 - \frac{\partial B}{\partial c} - \frac{F}{f} e_{1c} &= 0. \end{aligned}$$

Using Lemma 3 again, and solving the above system, we get the expressions in the text. ■

Proof of Proposition 4. As derived in the text, regulator i 's ($i = 1, 2$) best responses read for, respectively, c and q :

$$\frac{q(\theta)}{2} e_{2c}(c(\theta), q(\theta), \theta) - \frac{1}{2} + \frac{\partial t_i}{\partial c}(c(\theta), q(\theta)) - B_c(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_{\theta c}(c(\theta), q(\theta), \theta) = 0$$

and

$$\frac{e_2(c(\theta), q(\theta), \theta)}{2} + \frac{q}{2} e_{2q}(c(\theta), q(\theta), \theta) q(\theta) + \frac{\partial t_i}{\partial q}(c(\theta), q(\theta)) - B_q(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} B_{\theta q}(c(\theta), q(\theta), \theta) = 0.$$

Summing each of the above equations over i and using (ICFOC), one gets the two equations that (implicitly) define the unique cost targets and monitoring levels that prevail in equilibrium.

Uniqueness, up to a constant, of the equilibrium payments and the fact that the regulators' payments can only differ by a constant term follows because the allocation $\{c(\theta), q(\theta)\}_\theta$ is uniquely determined and the regulators' best response functions are symmetric.

Indeed, symmetry of the regulators' best response functions, along with (ICFOC), implies that

$$\frac{\partial t_1}{\partial c}(c(\theta), q(\theta)) = \frac{\partial t_2}{\partial c}(c(\theta), q(\theta)) = \frac{1}{2} B_c(c(\theta), q(\theta), \theta)$$

and

$$\frac{\partial t_1}{\partial q}(c(\theta), q(\theta)) = \frac{\partial t_2}{\partial q}(c(\theta), q(\theta)) = \frac{1}{2} B_q(c(\theta), q(\theta), \theta)$$

Hence, the payments made by the regulators can only differ by a constant term and are uniquely defined up to a constant .

As for the comparison result, notice that, although the two principals are involved in strategic interaction, the equations that characterize the Nash Equilibrium are the same as the (pointwise) first order conditions of the following maximization problem:

$$\max_{(c(\theta), q(\theta))_{\theta \in [\underline{\theta}, \bar{\theta}]}} v + E_\theta \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - B(c(\theta), q(\theta), \theta) - 2 \frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta) - m(q(\theta)) \right].$$

The objective of this problem differs from the one in the full coordination case by the added term $-\frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta)$.

It then follows that the first order conditions to the above problem evaluated at the levels (q^{SB}, c^{SB}) that prevail under full coordination are (we omit the arguments to save on notation. The expressions below should be read as being evaluated at $(c^{SB}(\theta), q^{SB}(\theta))$)

$$-\frac{F(\theta)}{f(\theta)} B_{\theta c} = \frac{F(\theta)}{f(\theta)} [g_{e_1 e_1} [1 - e_{2c}(c^{SB}(\theta), q^{SB}(\theta), \theta)] - g_{e_1 e_2} e_{2c}(c^{SB}(\theta), q^{SB}(\theta), \theta)] > 0$$

and

$$-\frac{F(\theta)}{f(\theta)} B_{\theta q} = -\frac{F(\theta)}{f(\theta)} e_{2q}(c^{SB}(\theta), q^{SB}(\theta), \theta) [g_{e_1 e_1} + g_{e_1 e_2}] > 0.$$

Now, using exactly the same steps as the ones used in the proof of Proposition 3, one has that, under conditions A1 and A2,

$$\frac{\partial^2}{\partial q \partial c} \left[v + E_\theta \left[q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - B(c(\theta), q(\theta), \theta) - 2 \frac{F(\theta)}{f(\theta)} B_\theta(c(\theta), q(\theta), \theta) - m(q(\theta)) \right] \right] \geq 0$$

for all $(q, c) \geq (q^{SB}, c^{SB})$ over the relevant range. Invoking Lemma 5, the result follows.

Now, since productive effort decreases with demanded costs and monitoring levels, it follows that productive effort will be smaller when regulators set pay independently when compared to what happens in the full coordination case. ■

Proof of Corollary 3. In the example economy, using Lemma 3, one can write the firm's utility in any Incentive Compatible mechanism as:

$$t_1(c(\theta), q(\theta)) + t_2(c(\theta), q(\theta)) - B(c(\theta), q(\theta), \theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} e_1(c(\tau), q(\tau), \tau) d\tau$$

From that, one can write the payment made by the regulators to the agent as

$$t_1(c(\theta), q(\theta)) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} e_1(c(\tau), q(\tau), \tau) d\tau + B(c(\theta), q(\theta), \theta) - t_2(c(\theta), q(\theta)).$$

Plugging this expression in the objective function, one has, after some integration by parts, that regulator 1's problem reads

$$\max_{U(\bar{\theta}), (c(\theta), q(\theta))_{\theta}} \frac{1}{2}v - U(\bar{\theta}) + E_{\theta} \left[\begin{array}{l} \frac{1}{2} \left(q(\theta) e_2(c(\theta), q(\theta), \theta) - c(\theta) - \frac{q(\theta)^2}{2} \right) + t_2(c(\theta), q) \\ - B(c(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} e_1(c(\theta), q(\theta), \theta) \end{array} \right]$$

subject to

$$\frac{d}{d\theta} (c(\theta) + q(\theta)) \geq 0.$$

and

$$c(\theta) \geq 0, q(\theta) \in [0, 1] \text{ for all } \theta.$$

It is optimal to set $U(\bar{\theta}) = 0$. Moreover, ignoring the monotonicity constraint and maximizing pointwise, we obtain the first order conditions in the text, (FOC2Pc) and (FOC2q)

From Proposition 4, one has that:

$$\begin{aligned} \frac{\partial t_i}{\partial c}(c(\theta), q(\theta)) &= \frac{1}{2} B_c(c(\theta), q(\theta), \theta), \quad i = 1, 2 \\ \frac{\partial t_i}{\partial q}(c(\theta), q(\theta)) &= \frac{1}{2} B_q(c(\theta), q(\theta), \theta), \quad i = 1, 2. \end{aligned}$$

Plugging these expressions in the first order conditions, we obtain

$$\frac{q(\theta)}{2} e_{2c}(c(\theta), q(\theta), \theta) = \frac{1}{2} + \frac{1}{2} B_c(c(\theta), q(\theta), \theta) + \frac{F(\theta)}{f(\theta)} B_{\theta c}(c(\theta), q(\theta), \theta)$$

and

$$e_2(c(\theta), q(\theta), \theta) + q e_{2q}(c(\theta), q(\theta), \theta) = q(\theta) + B_q(c(\theta), q(\theta), \theta) + 2 \frac{F(\theta)}{f(\theta)} B_{\theta q}(c(\theta), q(\theta), \theta)$$

Using Lemma 3, and solving both equations, we obtain

$$q^{2P}(\theta) = \frac{2F(\theta)}{3f(\theta)}; \quad c^{2P}(\theta) = \theta + \frac{2F(\theta)}{f(\theta)} - 1$$

as claimed.

Substituting these values on

$$e_1(c, q, \theta) = \frac{1}{2} [\theta - c + (1 - q)]$$

and

$$e_2(c, q, \theta) = \frac{1}{2} [c - \theta + (1 - q)],$$

we obtain the effort levels that prevail in equilibrium. ■

Proof of Proposition 5. We prove the result in a series of steps. In Step 1, we establish the cost targets that prevail when the regulators do not coordinate on monitoring. Through STEPS 2 to 6, we derive the equilibrium levels of overall monitoring and the bounds the overall monitoring has to satisfy when they do not coordinate on monitoring.

STEP 1: Cost Targets:

We fix the payments $\{t_2(c, p_2)\}_{c, p_2}$ offered by regulator 2 to the firm, and consider regulator 1's problem. Define

$$\phi(c, p_1, \theta) = \max_{p_2} t_2(c, p_2) - B(c, z(p_1, p_2), \theta),$$

and let $p_2^*(c, p_1, \theta)$ be the solution to this problem. Clearly, given $\{t_2(c, p_2)\}_{c, p_2}$, it is as if regulator 1 is dealing with an a firm with utility

$$t_1 + \phi(c, p_1, \theta),$$

Fixing regulator 2's payments, regulator 1 can restrict attention to direct mechanisms of the form $\{t_1(\theta), p_1(\theta)\}_\theta$. Her problem then reads:

$$\max_{(c(\theta), p_1(\theta), t_1(\theta))_\theta} \frac{1}{2} v + E_\theta \left[\begin{array}{l} \frac{1}{2} [z(p_1(\theta), p_2^*(c(\theta), p_1(\theta), \theta)) \cdot e_2(c(\theta), z(p_1(\theta), p_2^*(c(\theta), p_1(\theta), \theta)), \theta) - c(\theta))] \\ -h(p_1(\theta)) - t_1(\theta) \end{array} \right]$$

$$s.t. \ U(\theta) = t_1(\theta) + \phi(c(\theta), p_1(\theta), \theta) \geq 0 \text{ for all } \theta \quad (13)$$

$$\theta = \arg \max_{\hat{\theta}} t_1(\hat{\theta}) + \phi(c(\hat{\theta}), p_1(\hat{\theta}), \theta) \quad (14)$$

and the following non-negativity constraints: $c(\theta), p_1(\theta) \geq 0$ for all θ .

Much as in the single-regular case, we need a single-crossing condition to ensure that the constraints in (14) can be replaced by the following Envelope condition

$$U(\theta) = t_1(\theta) + \phi(p_1(\theta), c(\theta), \theta) = U(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} \phi_\theta(c(\tau), p_1(\tau), \tau) d\tau,$$

along with the monotonicity constraint: $\phi_\theta(c(\tau), p_1(\tau), \theta)$ non-decreasing in τ . While this condition is endogenous (it depends on the contract offered by regulator 2 in equilibrium), in STEP 6, we show that, in equilibrium, this is satisfied.

From the previous condition we get that,

$$t_1(\theta) = U(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} \phi_{\theta}(c(\tau), p_1(\tau), \tau) d\tau - \phi(c(\theta), p_1(\theta), \theta).$$

Substituting this in the objective, ignoring the monotonicity constraint, and noting that setting $U(\bar{\theta}) = 0$ is optimal, we get the following representation of regulator 1's problem (we ignore constant terms):

$$\max_{(c(\theta), p_1(\theta))_{\theta}} E_{\theta} \left[\begin{array}{l} \frac{1}{2} (z(p_1(\theta), p_2^*(c(\theta), p_1(\theta), \theta)), e_2(c(\theta), z(p_1(\theta), p_2^*(c(\theta), p_1(\theta), \theta)), \theta) - c(\theta)) \\ -h(p_1(\theta)) + \phi(c(\theta), p_1(\theta), \theta) + \frac{F(\theta)}{f(\theta)} \phi_{\theta}(c(\theta), p_1(\theta), \theta) \end{array} \right] \quad (15)$$

subject to the non-negativity constraints.

The first order conditions with respect to $c(\theta)$ is (to save on notation, we have omitted the arguments)

$$\left[\frac{1}{2} \left(e_2 + q \frac{\partial e_2}{\partial q} \right) - \frac{F}{f} e_{1q} \right] p_1 \frac{\partial p_2^*}{\partial c} + \frac{1}{2} (q e_{2c} - 1) + \frac{\partial t_2}{\partial c} - \frac{\partial B}{\partial c} - \frac{F}{f} e_{1c} = 0 \quad (16)$$

where we have used the fact that

$$\phi_c(c(\theta), p_1(\theta), \theta) = \frac{\partial}{\partial c} t_2(c(\theta), p_2^*(c(\theta), p_1(\theta), \theta)) - B_c(c(\theta), z(p_1(\theta), p_2^*(c(\theta), p_1(\theta), \theta)), \theta),$$

and

$$\phi_{\theta}(c(\theta), p_1(\theta), \theta) = -B_{\theta}(c(\theta), z(p_1(\theta), p_2^*(c(\theta), p_1(\theta), \theta)), \theta).$$

Moreover, any (necessarily) *coordinated*, incentive compatible, choice of c must satisfy the following first order condition for optimality of the firm's problem:

$$\frac{\partial t_1}{\partial c}(c, p_1) + \frac{\partial t_2}{\partial c}(c, p_2) - B_c(c, z(p_1, p_2), \theta) = 0.$$

Hence, in a symmetric equilibrium,

$$\frac{\partial t_1}{\partial c}(c, p_1) = \frac{\partial t_2}{\partial c}(c, p_2) = \frac{1}{2} B_c(c, z(p_1, p_2), \theta) = \frac{1}{4} [c - \theta - (1 - q)].$$

Also, we prove in STEP 4 below that, in equilibrium,

$$p_{2c}^*(p_1(\theta), c(\theta), \theta) = 0$$

Plugging these conditions back in (16), we get

$$\frac{1}{2} \left[\frac{1}{2} q - 1 \right] - \frac{1}{4} [c - \theta - (1 - q)] - \frac{1}{2} \frac{F}{f} = 0,$$

which yields

$$c(\theta) = \theta + \frac{2F(\theta)}{f(\theta)} - 1,$$

as claimed.

STEP 2: Deriving the first order conditions for Individual monitoring levels:

From (15), given the contract offered by regulator 2, the first order condition for an optimal amount of monitoring exerted by regulator 1 is (we again omit the arguments to save on notation)

$$\frac{1}{2} \left[e_2 \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right) + q \frac{\partial e_2}{\partial q} \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right) \right] - p_1^3 - B_q q p_1 + \frac{F}{f} \phi_{p_1} = 0$$

Using

$$\begin{aligned} B_q(c, q, \theta) &= e_2(c, q, \theta), \\ \phi_\theta(c, p_1, \theta) &= -B_\theta(c, q, \theta) = -e_1(c, q, \theta), \end{aligned}$$

and

$$\phi_{\theta p_1}(c, p_1, \theta) = \frac{1}{2} z_{p_1}(p_1, p_2^*(c, p_1, \theta)) = \frac{1}{2} \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right),$$

we can re-write the above condition as

$$\frac{1}{2} \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right) (e_2 + qe_{2q}) - p_1^3 - \left(\frac{c - \theta + 1 - q}{2} \right) p_2^* + \frac{1}{2} \frac{F}{f} \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right) = 0,$$

or

$$\frac{1}{2} \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right) \left(\frac{c - \theta + 1 - 2q}{2} + \frac{F}{f} \right) = p_1^3 + \left(\frac{c - \theta + 1 - q}{2} \right) p_2^*$$

STEP 3: Computing $\frac{\partial p_2^*}{\partial p_1}$ and $\frac{\partial p_2^*}{\partial c}$

We have that $p_2^*(p_1, c, \theta)$ solves

$$\max_{p_2} t_2(c, p_2) - B(c, z(p_1, p_2), \theta).$$

The first order condition for this program is (in STEP 6, we show that the second order conditions for $p_2^*(p_1, c, \theta)$ being optimal for the firm are satisfied in equilibrium):

$$\frac{\partial t_2}{\partial p_2}(c, p_2^*(p_1, c, \theta)) - B_{p_2}(c, z(p_1, p_2^*(p_1, c, \theta)), \theta) = 0,$$

or, using the fact that $q = z(p_1, p_2) = p_1 p_2$ and

$$\begin{aligned} B_{p_2}(c, z(p_1, p_2^*(p_1, c, \theta)), \theta) &= B_q(c, z(p_1, p_2^*(p_1, c, \theta)), \theta) z_{p_2}(p_1, p_2^*(p_1, c, \theta)) \\ &= \frac{1}{2} (c - \theta + 1 - z(p_1, p_2^*(p_1, c, \theta))) p_1, \end{aligned}$$

we have that:

$$\frac{\partial t_2}{\partial p_2}(c, p_2^*(p_1, c, \theta)) - \left(\frac{c - \theta + 1 - z(p_1, p_2^*(p_1, c, \theta))}{2} \right) p_1 = 0$$

Therefore, from the Implicit Function Theorem, it follows that

$$\frac{\partial p_2^*}{\partial p_1} = \frac{\frac{1}{2} (c - \theta + 1 - z(p_1, p_2^*)) + \frac{1}{2} p_1 p_2^*}{\frac{\partial^2 t_2}{\partial p_2^2}(c, p_2^*) + \frac{p_1^2}{2}}.$$

And,

$$\frac{\partial p_2^*}{\partial c} = - \frac{\frac{\partial^2 t_2}{\partial p_2 \partial c}(c, p_2^*) - \frac{1}{2} p_1}{\frac{\partial^2 t_2}{\partial p_2^2}(c, p_2^*) + \frac{p_1^2}{2}}.$$

In a symmetric equilibrium, $p_1 = p_2 = p$, and

$$t_1(c, p) = t_2(c, p) \equiv T(c, p), \text{ for all } c, p.$$

Hence, in a symmetric equilibrium,

$$\frac{\partial p_2^*}{\partial p_1} = \frac{\frac{1}{2}(c - \theta + 1 - q) - \frac{1}{2}q}{T_{pp} + \frac{q}{2}}, \quad \frac{\partial p_2^*}{\partial c} = \frac{\frac{1}{2}p - T_{pc}}{T_{pp} + \frac{q}{2}},$$

where

$$T_{pp} = \frac{\partial^2 T(c, p)}{\partial p^2} \text{ and } T_{pc} = \frac{\partial^2 T(c, p)}{\partial p \partial c}.$$

STEP 4: Computing T_{pp} , T_{cc} and T_{pc} , and showing that $\frac{\partial p_2^*}{\partial c} = 0$ in equilibrium

We first compute the value of T_{pc} . As argued in STEP 2, the coordinated choice of c must satisfy:

$$\frac{\partial t_1}{\partial c}(c, p_1) + \frac{\partial t_2}{\partial c}(c, p_2^*(p_1, c, \theta)) - B_c(c, z(p_1, p_2^*(p_1, c, \theta)), \theta) = 0, \text{ for all } p_1, \theta.$$

Differentiating with respect to p_1 , we get

$$\frac{\partial^2 t_1}{\partial c \partial p_1} + \frac{\partial^2 t_2}{\partial c \partial p_2} \frac{\partial p_2^*}{\partial p_1} - \frac{1}{2} \left(z_{p_1} + z_{p_2} \frac{\partial p_2^*}{\partial p_1} \right) = 0$$

Hence, in a symmetric equilibrium,

$$\left(T_{cp} - \frac{p}{2} \right) \left(1 + \frac{\partial p_2^*}{\partial p_1} \right) = 0$$

From STEP 2, we know that the first order condition for an optimal choice of monitoring by regulator 1 evaluated at equilibrium levels yield:

$$0 = \frac{1}{2} \left(p + p \frac{\partial p_2^*}{\partial p_1} \right) \left(\frac{c - \theta + 1 - 2q}{2} + \frac{F(\theta)}{f(\theta)} \right) - p^3 - \left(\frac{c - \theta + 1 - q}{2} \right) p.$$

Using this expression to solve for $\frac{\partial p_2^*}{\partial p_1}$, we get

$$\frac{\partial p_2}{\partial p_1} + 1 = \frac{2(q + e_2)}{e_2 - \frac{q}{2} + \frac{F(\theta)}{2f(\theta)}} \neq 0.$$

This is different from zero; otherwise we would have $q + e_2 = 0$. This, however, would imply $q(\theta) = -2\frac{F(\theta)}{f(\theta)}$ which is negative, and consequently cannot be optimal.

Since

$$\left(T_{cp} - \frac{p}{2} \right) \left(1 + \frac{\partial p_2}{\partial p_1} \right) = 0,$$

and $\frac{\partial p_2}{\partial p_1} + 1 \neq 0$, one must have

$$T_{cp} = \frac{p}{2}.$$

Now, in STEP 3, we have shown that

$$\frac{\partial p_2^*}{\partial c} = -\frac{T_{cp} - \frac{1}{2}p}{T_{pp} + \frac{p^2}{2}}.$$

Using $T_{cp} = \frac{p}{2}$, we have that $\frac{\partial p_2^*}{\partial c} = 0$ as we claimed in STEP 1.

As seen in STEP 1, we have that $p_2^*(p_1, c, \theta)$ solves

$$\max_{p_2} t_2(c, p_2) - B(c, z(p_1, p_2), \theta).$$

The first order condition for this program is;

$$\begin{aligned}\frac{\partial t_2}{\partial p_2}(c, p_2^*(c, p_1, \theta)) - B_{p_2}(c, z(p_1, p_2^*(c, p_1, \theta)), \theta) &= 0, \text{ or} \\ \frac{\partial t_2}{\partial p_2}(c, p_2^*(c, p_1, \theta)) - e_2(c, z(p_1, p_2^*(c, p_1, \theta)), \theta) p_1 &= 0.\end{aligned}$$

Therefore, in a symmetric equilibrium, it must be the case that

$$T_p(c(\theta), p(\theta)) - e_2(c(\theta), z(p(\theta), p(\theta)), \theta) p(\theta) = 0, \text{ for all } \theta.$$

Differentiating with respect to θ , we have (defining $\dot{p}(\theta) \equiv \frac{dp(\theta)}{d\theta}$ and $\dot{q}(\theta) \equiv \frac{dq(\theta)}{d\theta}$)

$$0 = T_{pp}(c(\theta), p(\theta)) \dot{p}(\theta) - e_2(c(\theta), z(p(\theta), p(\theta)), \theta) \dot{p}(\theta) + \frac{1}{2} (1 + \dot{q}(\theta)) p(\theta), \quad (17)$$

where we have used $T_{cp} = \frac{p}{2}$.

Also, a coordinated choice of c must satisfy

$$\frac{\partial t_1}{\partial c}(c, p_1) + \frac{\partial t_2}{\partial c}(c, p_2^*(c, p_1, \theta)) - B_c(c, z(p_1, p_2^*(c, p_1, \theta)), \theta) = 0, \text{ for all } \theta.$$

In a symmetric equilibrium

$$\begin{aligned}0 &= 2T_c(c(\theta), p(\theta)) - B_c(c(\theta), q(\theta), \theta) \\ &= 2T_c(c(\theta), p(\theta)) - \frac{1}{2} [c(\theta) - \theta - (1 - q(\theta))], \text{ for all } \theta.\end{aligned}$$

Totally differentiating the above expression with respect to θ , we get, using $T_{cp} = \frac{p}{2}$ and defining $\dot{c}(\theta) \equiv \frac{dc(\theta)}{d\theta}$,

$$0 = 2T_{cc}(c(\theta), p(\theta)) \dot{c}(\theta) + p(\theta) \dot{p}(\theta) - \frac{1}{2} [\dot{c}(\theta) - 1 + \dot{q}(\theta)] \quad (18)$$

Solving for equations (17) and (18), we get, after a few algebraic manipulations:

$$T_{cc} = -\frac{\dot{p}(\theta)}{2\dot{c}(\theta)} p(\theta) + \frac{1}{4} \left(1 + \frac{(\dot{q}(\theta) - 1)}{\dot{c}(\theta)} \right)$$

and

$$T_{pp} = \frac{e_2(c(\theta), z(p(\theta), p(\theta)), \theta) \dot{p}(\theta) - \frac{1}{2} (1 + \dot{q}(\theta)) p(\theta)}{\dot{p}(\theta)}$$

We know from STEP 1 that

$$c(\theta) = \theta + \frac{2F(\theta)}{f(\theta)} - 1,$$

so that

$$\dot{c}(\theta) = 1 + 2 \frac{d}{d\theta} \left(\frac{F(\theta)}{f(\theta)} \right) \equiv 1 + 2\eta(\theta).$$

Substituting this back in the expressions for T_{cc} and T_{pp} , we get

$$T_{cc} = \frac{2\eta(\theta) + \dot{q}(\theta) - 2\dot{p}(\theta) p(\theta)}{4(1 + 2\eta(\theta))} = \frac{\eta(\theta)}{(2 + 4\eta(\theta))}.$$

(where we have used the fact that $\dot{q}(\theta) = \frac{dp(\theta)^2}{d\theta} = 2\dot{p}(\theta)p(\theta)$), and

$$\begin{aligned} T_{pp} &= \frac{\left(\frac{2F(\theta)}{f(\theta)} - q(\theta)\right)\dot{p}(\theta) - \left(1 + 2\dot{p}(\theta)p(\theta)\right)p(\theta)}{2\dot{p}(\theta)} \\ &= \frac{\frac{2F(\theta)}{f(\theta)}\dot{p}(\theta) - q(\theta)\dot{p}(\theta) - p(\theta) - 2\dot{p}(\theta)q(\theta)}{2\dot{p}(\theta)} \\ &= \frac{\dot{p}(\theta)\left[\frac{2F(\theta)}{f(\theta)} - 3q(\theta)\right] - p(\theta)}{2\dot{p}(\theta)}. \end{aligned}$$

STEP 5: The Ordinary Differential Equation that characterizes the equilibrium values of $q(\theta)$ and the bounds for $q(\theta)$:

The first order condition for the individual monitoring level of regulator 1 evaluated at equilibrium levels of p and c is:

$$0 = \frac{1}{2} \left(p + p \frac{\partial p_2^*}{\partial p_1} \right) \left(\frac{c - \theta + 1 - 2q}{2} + \frac{F(\theta)}{f(\theta)} \right) - p^3 - \left(\frac{c - \theta + 1 - q}{2} \right) p.$$

Substituting

$$c(\theta) = \theta + \frac{2F(\theta)}{f(\theta)} - 1$$

in the above expression, we get:

$$\frac{q(\theta)}{\frac{F(\theta)}{f(\theta)} - \frac{q(\theta)}{2}} = \frac{\partial p_2^*}{\partial p_1}.$$

From STEP 2,

$$\frac{\partial p_2^*}{\partial p_1} = \frac{\frac{F(\theta)}{f(\theta)} - q(\theta)}{T_{pp} + \frac{q(\theta)}{2}} = \frac{\frac{F(\theta)}{f(\theta)} - q(\theta)}{\frac{e_2(c(\theta), z(p(\theta), p(\theta)), \theta)\dot{p}(\theta) - \frac{1}{2}(1+q(\theta))p(\theta)}{\dot{p}(\theta)} + \frac{q(\theta)}{2}},$$

where we have used the expression for T_{pp} we derived in STEP 4.

Hence, the following has to hold in equilibrium:

$$\frac{\frac{F(\theta)}{f(\theta)} - q(\theta)}{\frac{e_2(c(\theta), z(p(\theta), p(\theta)), \theta)\dot{p}(\theta) - \frac{1}{2}(1+q(\theta))p(\theta)}{\dot{p}(\theta)} + \frac{q(\theta)}{2}} = \frac{q(\theta)}{\frac{F(\theta)}{f(\theta)} - \frac{q(\theta)}{2}}. \quad (19)$$

Now, notice that, in equilibrium, since $p_1(\theta) = p_2(\theta) = p(\theta)$, $q(\theta) = p^2(\theta)$. Totally differentiating this expression, we obtain

$$\dot{q}(\theta) = 2p(\theta)\dot{p}(\theta)$$

Substituting the above equation in (19), we get that the following ordinary differential equation has to hold in equilibrium::

$$\dot{p}(\theta) = -\frac{f(\theta)p(\theta)^3(1-p(\theta))}{2F(\theta)^2 - 4F(\theta)f(\theta)p(\theta)^2 + 4f(\theta)^2p(\theta)^4} \quad ((ODE'))$$

The denominator would only be zero when p is a complex number. Therefore, since both F and f are continuous, the right hand side of the above equation is continuous in both, $p > 0$ and θ . Therefore, by Caratheodory's existence theorem (see Coddington and Levinson (1995, page 43)), there is a solution to (ODE').

To establish the bounds, first notice that $q(\theta) \geq 0$. Moreover, the second order condition for the agent's problem calls for $T_{pp} + \frac{q}{2}$ being negative, so that

$$\frac{e_2(c(\theta), q(p(\theta), p(\theta)), \theta) \dot{p}(\theta) - \frac{1}{2} (1 + \dot{q}(\theta)) p(\theta)}{\dot{p}(\theta)} + \frac{q(\theta)}{2} < 0.$$

Hence, (19) can only hold if

$$q(\theta) \in \left(\frac{F(\theta)}{f(\theta)}, 2 \frac{F(\theta)}{f(\theta)} \right).$$

STEP 6: Checking the Agent's Second Order and Single Crossing Conditions

The first order conditions for the firm's problem are

$$\frac{\partial t_2(c, p_2^*(c, p_1, \theta))}{\partial p_2} - p_1 \left(\frac{c - \theta + 1 - z(p_1, p_2^*(c, p_1, \theta))}{2} \right) = 0$$

and

$$\frac{\partial t_1}{\partial c}(c, p_1) + \frac{\partial t_2}{\partial c}(c, p_2) - B_c(c, z(p_1, p_2), \theta) = 0.$$

In equilibrium, the derivative of the left hand side of the first expression with respect to p is

$$\begin{aligned} T_{pp} + \frac{q}{2} &= \frac{\frac{2F(\theta)}{f(\theta)} - 3q(\theta)}{2} - \frac{p(\theta)}{\dot{p}(\theta)} + \frac{q}{2} \\ &= \frac{F(\theta)}{f(\theta)} - q(\theta) - \frac{p(\theta)}{\dot{p}(\theta)} < 0. \end{aligned}$$

The derivative of the left hand side of the second expression with respect to c evaluated at the equilibrium values of c and p is

$$\begin{aligned} 2T_{cc} - \frac{1}{2} \\ = 2 \left(\frac{\eta(\theta)}{(2 + 4\eta(\theta))} \right) - \frac{1}{2} < 0. \end{aligned}$$

Now, the derivative of the left hand side of the first equation with respect to c evaluated at equilibrium values of c and p is

$$T_{cp} - \frac{1}{2}p = 0.$$

It follows that the second order conditions for the firm's problem are satisfied (the Hessian of his objective function evaluated at the optimum is semi negative definite).

We now verify that the (a priori assumed) single crossing condition indeed holds. Note that

$$\phi_{\theta p_1}(p_1, c, \theta) = \frac{1}{2} z_{p_1} = \frac{1}{2} \left(p_2^* + p_1 \frac{\partial p_2^*}{\partial p_1} \right).$$

Evaluating this expression at the equilibrium values of c and p , one has

$$\frac{p}{2} \left(1 + \frac{\frac{F(\theta)}{f(\theta)} - q}{T_{pp} + \frac{q}{2}} \right) \geq 0$$

since $T_{pp} + \frac{q}{2} < 0$ and $q(\theta) > \frac{F(\theta)}{f(\theta)}$. ■

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