

Common Agency, Organizational Design and the Hold-Up Problem*

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Abstract

This paper shows that, in comparison to a single-regulator arrangement, when an agent reports to two regulators, he is confronted with more powerful ex-post incentives. This generates, from an ex-ante perspective, higher incentives for relationship-specific investment.

Keywords: Regulation, Common Agency, Relationship-specific investments

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1 Introduction

Since the seminal works of Williamson (1985), Klein et al (1978), Grossman and Hart (1986), Hart and Moore (1990) and others, economists have devoted considerable attention to the study of inefficiencies in relationship-specific investment decisions in an incomplete contracts world; the hold-up problem.

This paper argues that a well designed organizational structure, through its effect on the design of incentive schemes over contractible variables, is a mean to provide incentives for ex-ante investments.

This point is made in a model in which an agent can exert unobservable effort to reduce the costs of provision of two projects that are subject to mandatory regulation, and is, *a priori*, uncertain about his ability to provide them. A non-contractible ability enhancing investment can be performed. The lack of commitment from a regulator concerned with (informational) rent extraction and who regulates both activities induces the standard underinvestment outcome of hold-up problems.

If each project is assigned to a different regulator, *ex-post* incentives will be more powerful, generating, from an *ex-ante* perspective, higher incentives for ability enhancing investment. The combination of higher ex-ante investments and more powerful ex-post incentives produces an outcome that is superior than a single regulator's outcome in terms of efficiency.

A simple interpretation of these results is that regulators compete for the agent's attention. Such competition transfers some of the ex-post surplus to the agent, who then faces better incentives to invest. Hence,

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a regulatory structure with more than one principal can be seen as a credible way to commit to not extract ex-post surplus from an investing party.

Related Literature. In a model in which an agent has to exert effort toward production, Dixit (1996) shows that principals independently offering (linear) contracts to the agent provide less powerful incentives than if they were to collude. In a model a la Dixit, Costa et al. (2005) allow the principals to have biased preferences toward some activities and establish that, if the biases are large, a two-principal structure might dominate a single-principal arrangement, as competition among principals offsets their individual biases. Laffont and Tirole (1993, Chapter 17) argue that, relatively to public enterprises, the costs of private enterprises stem from managers responding to two sets of principals (shareholders and regulators), which leads to low-powered incentives. Their benefit is that the manager's investment is not expropriated.

This paper focuses on the impact of a two-principal arrangement on the interaction between the power of ex-post incentives and the agent's ex-ante incentives to make a relationship-specific investment. By putting together moral hazard and asymmetric information, I show that (i) fixing the amount of investment made by the agent, a two-principal arrangement always leads to lower cost levels (more powerful incentives ex-post) and (ii) since, ex-post incentives are more powerful in a two-principal structure, incentives for ex-ante investments will be higher than in a single principal arrangement.¹ Thus, in contrast to Laffont and Tirole (1993), a two-principal arrangement is the form by which more powerful ex-post incentives are reconciled with better incentives for ex-ante investment in my model.

Methodologically, I make extensive use of the tools developed by Martimort (1996), Stole (1991), and Martimort and Stole (2002, 2003).

Organization. Section 2 describes the set-up of the model and the timing of events. In Section 3, the model is solved for the single regulator case. In Section 4, the two-regulator case is considered, and the two organizational structures are compared in terms of overall levels of costs and the investment they induce. Results not found in the text lie in the appendix.

2 Model

I consider a variation of the regulatory model of Laffont and Tirole (1993). The players are an agent/firm, who can exert effort to reduce the cost of provision of two indivisible public projects, and two regulators to which this agent reports. Two different regulatory arrangements are analyzed: one in which a single regulator is in charge of both projects, and a second one in which different regulators are in charge of different projects.

Project i , $i = 1, 2$, has value $S_i > 0$ for the consumers. S_i is large so that provision is optimal in all cases studied. The agent can provide the project at a *contractible* cost

$$C_i = \beta - e_i,$$

where β is an efficient parameter (capturing his ability to provide both projects) and $e_i \geq 0$ is the agent's unobservable effort to reduce C_i .

The parameter β is *a priori* unknown to *all* players: it is distributed according to a cdf $F(\cdot)$ over $[\underline{\beta}, \bar{\beta}]$, $\underline{\beta} > \frac{1}{2}$, with density $f(\beta) > 0$. Before building the project, the agent can invest an amount $z \in [0, \bar{z}]$ to

¹The benefits of common agency arrangements in organizations when principals cannot commit to contracts, and, as a consequence, ratchet effects and renegotiation effects ensue have been pointed out by Olsen and Torsvik (1995) and Martimort (1999). In contrast to these papers, the principals can commit to contracts in the model.

enhance his ability to perform such task. The investment is relationship-specific: it does not affect the agent's outside option. Also, although observable, z is not contractible.

A higher z makes it more likely that the realization of the cost parameter β is low: if $z > z' \geq 0$,

$$\frac{f(\beta|z)}{f(\beta|z')}$$

is decreasing in β . Moreover, I assume that $\frac{F(\beta|z)}{f(\beta|z)}$ is increasing in β for all z .²

For simplicity, I assume that, to invest z , the agent incurs a private cost of $\frac{z^2}{2}$, and to exert efforts e_1 and e_2 the agent incurs a cost of $\frac{1}{2}(e_1 + e_2)^2$.

Adopting the standard accounting convention that the costs C_i are paid by the regulator(s), the agent's ex-post utility function is given by

$$t - \frac{(e_1 + e_2)^2}{2} - \frac{z^2}{2} \equiv U(t, e_1, e_2) - \frac{z^2}{2},$$

where t is the total payment the regulators make to him.³

Letting $\lambda > 0$ be the shadow cost of public funds, the net surplus the consumers enjoy if the projects are provided is⁴

$$S_1 + S_2 - (1 + \lambda)(t + C_1 + C_2).$$

As will be shortly seen, at the contracting stage, the investment z is sunk. Hence, an utilitarian single regulator maximizes

$$\begin{aligned} & S_1 + S_2 - (1 + \lambda)(t + C_1 + C_2) + U(t, e_1, e_2) \\ = & S_1 + S_2 - (1 + \lambda) \left[C_1 + C_2 + \frac{1}{2}(e_1 + e_2)^2 \right] - \lambda U(t, e_1, e_2), \end{aligned}$$

whereas, in case there are two regulators, regulator i solely considers the surplus S_i and the cost C_i of by the project he is in charge of.

The timing of events is as follows. At time zero, the agent chooses z . At period 1, the agent privately learns the cost parameter β and, in the one-regulator case, the set of contracts $\{C_1, C_2, t(C_1, C_2)\}_{C_1, C_2}$, specifying costs at which the projects must be provided and payments contingent on those costs, is offered to him. He then selects the contract that fits him better. In case there are two regulators, two sets of contracts, $\{C_1, t_1(C_1)\}_{C_1}$ and $\{C_2, t_2(C_2)\}_{C_2}$ are offered. Regulation is *mandatory*: the agent must contract with both regulators, as in a model of intrinsic common agency (Bernheim and Whinston (1986)). In period 2, he chooses privately the level of efforts, and the costs realize. All payments are made in accordance to the previously signed contract in period 3.

3 The One Regulator Case

At period 1, the investment z made by the agent is sunk. Hence, invoking the Revelation Principle, using $e_i = \beta - C_i$, and normalizing the agent's outside option at the contracting to zero, the regulator's problem

²That is, for all z , $F(\theta|z)$ is log-concave. We also take that $f(\theta|z=0) = f(\theta)$.

³Computations are much easier if one assumes quadratic costs. What is key for the results is that the marginal cost of exerting effort in activity 1 is increasing in the effort exerted in activity 2.

⁴The assumption that there is a positive shadow cost of public funds is standard in regulation models (see Laffont and Tirole, 1993).

is to maximize expected social welfare subject to Participation and Incentive Compatibility constraints:

$$\max_{\{C_1(\beta,z), C_2(\beta,z), t(\beta,z)\}_\beta} E_\beta \left[\left(S_1 + S_2 - (1 + \lambda) \left[C_1(\beta, z) + C_2(\beta, z) + \frac{1}{2} (2\beta - C_1(\beta, z) - C_2(\beta, z))^2 \right] - \lambda U(\beta, z) \right) | z \right]$$

subject to

$$U(\beta, z) \equiv t(\beta, z) - \frac{(2\beta - C_1(\beta, z) - C_2(\beta, z))^2}{2} \geq 0, \quad \forall \beta \quad (\text{IR})$$

and

$$U(\beta, z) \equiv t(\beta, z) - \frac{(2\beta - C_1(\beta, z) - C_2(\beta, z))^2}{2} \geq t(\hat{\beta}, z) - \frac{(2\hat{\beta} - C_1(\hat{\beta}, z) - C_2(\hat{\beta}, z))^2}{2}, \quad \forall \beta, \hat{\beta}. \quad (\text{IC})$$

It is a standard result that the constraints in (IC) are equivalent to

$$U(\beta, z) = U(\bar{\beta}, z) + \int_{\bar{\beta}}^{\beta} 2(2\tau - C_1(\tau, z) - C_2(\tau, z)) d\tau, \quad (\text{ICLocal})$$

and

$$C_1^{1R}(\beta, z) + C_2^{1R}(\beta, z) \text{ non-decreasing in } \beta. \quad (\text{Monotonicity})$$

Substituting the expression for $U(\beta, z)$ implied by (ICLocal) in the objective, integrating by parts, and noting that participation is guaranteed for all types whenever $U(\bar{\beta}, z) \geq 0$, the regulator's program can be written as

$$\max_{U(\bar{\beta}, z), \{C_1(\beta, z), C_2(\beta, z)\}_\beta} E_\beta \left[\left(\begin{array}{c} -(1 + \lambda) [C_1(\beta, z) + C_2(\beta, z) + \frac{1}{2} (2\beta - C_1(\beta, z) - C_2(\beta, z))^2] \\ -\lambda U(\bar{\beta}, z) - 2\lambda \frac{F(\beta|z)}{f(\beta|z)} [2\beta - C_1(\beta, z) - C_2(\beta, z)] \end{array} \right) | z \right]$$

s.t. $U(\bar{\beta}, z) \geq 0$, $C_1^{1R}(\beta, z) + C_2^{1R}(\beta, z)$ non-increasing in β .

Ignoring the monotonicity constraints and maximizing pointwise, one has

$$U(\bar{\beta}, z) = 0, C_1^{1R}(\beta, z) = C_2^{1R}(\beta, z) = \beta - \frac{1}{2} + \frac{\lambda}{(1 + \lambda)} \frac{F(\beta|z)}{f(\beta|z)}.$$

Since $\frac{F(\beta|z)}{f(\beta|z)}$ is increasing in β , the monotonicity constraint is satisfied.

Compared to first best levels, the regulator distorts upward the costs at which the projects are provided. He does so to reduce the rents left to the agent. This is a standard feature of models with asymmetric information. What is non-standard in the model is that such ex-post distortion affects the ex-ante incentives for the agent to invest. More specifically, at period 0, anticipating the regulator's choice of costs and, consequently, his ex-post utility, the agent invests so to maximize

$$E_\theta [U(\beta, z)|z] - \frac{z^2}{2} = \int_{\underline{\beta}}^{\bar{\beta}} \left(2(2\beta - C_1^{1R}(\beta, z) - C_2^{1R}(\beta, z)) \frac{F(\beta|z)}{f(\beta|z)} - \frac{z^2}{2} \right) f(\beta|z) d\beta. \quad (1)$$

Due to the ex-post upward distortion in costs, the agent cannot fully rip the benefits of his ex-ante investment and therefore underinvests.⁵

⁵The efficient level of investment solves

$$\max_z -2E(\beta|z) - \frac{1}{2}z^2,$$

and this amount of investments is attained if the agent is demanded to provide the project at efficient cost levels, $C_1(\beta) = C_2(\beta) = \beta - \frac{1}{2}$.

Proposition 1 *In a single regulator arrangement with asymmetric information, the costs of the public projects will be above first best levels. The upward distortion of costs reduces the agent's ex-post rents, leading to underinvestment.*

4 Two-Regulator Case

It is by now widely known that the Revelation Principle does not apply in common agency settings. However, as shown by Martimort and Stole (2002), an extension of the Taxation Principle (see, Salanie (1997)) – the Delegation Principle – applies and the whole equilibrium set can be computed using a fairly simple methodology.⁶

The main idea is to consider each of the regulator's problem for a fixed set of contracts offered by the other. In such case, under some assumptions that have to be checked in equilibrium, the methodology used in the single regulator case fully applies and the problem reads exactly as a single regulator's one.

Let $\{C_2(z), t_2(C_2(z))\}_{C_2}$ be a fixed set of contracts offered by the second regulator, when agent has invested z . The agent will choose among them the one that maximizes his utility. As a consequence, it is as if regulator 1 had to deal with an agent with utility

$$t_1(C_1) + \phi(C_1, z, \theta), \quad (2)$$

where $\phi(C_1, z, \beta) = \max_{C_2} t_2(C_2(z)) - \frac{1}{2} [2\beta - C_1 - C_2(z)]^2$.

Therefore, for a given $\{C_2(z), t_2(C_2(z))\}_{C_2}$, regulator 1's problem is the same as the one of a single regulator deciding *only* on C_1 and facing an agent with preferences described by (2). In particular, the Revelation Principle applies in such case and attention can be restricted to Direct Mechanisms $\{C_1(\hat{\beta}, z), t_1(\hat{\beta}, z)\}_{\hat{\beta}, z}$.

Defining $\Phi(\beta, z) = \max_{\hat{\beta}} t_1(C_1(\hat{\beta}, z)) + \phi(C_1(\hat{\beta}, z), z, \beta)$, and using the Envelope Theorem, incentive compatibility is, under the assumption that $\phi_{\beta C_1} \geq 0$ (this has to be checked ex-post), now equivalent to

$$\Phi(\beta, z) = \Phi(\bar{\beta}, z) + \int_{\bar{\beta}}^{\beta} 2[2\tau - C_1(\tau, z) - C_2(C_1(\tau, z), \tau, z)] d\tau \quad (3)$$

and $\frac{dC_1(\beta, z)}{d\beta} \geq 0$.

Proceeding as before (i.e., integrating condition (3) by parts, substituting $\Phi(\beta, z)$ in the objective function, and imposing $\Phi(\bar{\beta}, z) = 0$ as it minimizes the payments to the agent and guarantees the satisfaction of the participation constraints), the regulator 1's problem becomes

$$\max_{C_1(\beta, z)} E_{\beta} \left[\left(\begin{array}{c} S_1 - (1 + \lambda) [C_1(\beta, z) - \phi(C_1, z, \beta)] \\ -2\lambda \frac{F(\beta|z)}{f(\beta|z)} [2\beta - C_1(\beta, z) - C_2(C_1(\beta, z), \beta, z)] \end{array} \right) \Big| z \right].$$

The first order necessary condition for optimality is given by

$$-(1 + \lambda) [1 - (2\beta - C_1(\beta, z) - C_2(C_1(\beta, z), \beta, z))] + 2\lambda \frac{F(\beta|z)}{f(\beta|z)} \left[1 + \frac{\partial C_2(C_1(\beta, z), \beta, z)}{\partial C_1} \right] = 0. \quad (4)$$

The problem for regulator 2 is analogous and yields a similar condition.

⁶Throughout, following the literature, I restrict attention to differentiable equilibria.

In a symmetric equilibrium, $C_1 = C_2 = C$. Moreover, in the appendix, it is shown that $\frac{dC_2(C_1(\beta, z), \beta, z)}{dC_1} = \frac{C(\beta, z)}{C(\beta, z) - 2}$. The convexity of the agent's cost in exerting effort implies that costs to provide projects 1 and 2 are *substitutes* in his preferences, i.e., $\frac{dC_2(C_1, \beta, z)}{dC_1} < 0$. As shown by Martimort (1996), this has an important implication

Proposition 2 *For any $z \in [0, \bar{z}]$, there is a unique symmetric equilibrium in the subgame played by the principals. Such equilibrium solves*

$$-(1 + \lambda) [1 - (2\beta - 2C^{2R}(\beta, z))] + 2\lambda \frac{F(\beta|z)}{f(\beta|z)} \left[1 + \frac{C^{2R}(\beta, z)}{C^{2R}(\beta, z) - 2} \right] = 0$$

with boundary condition $C^{2R}(\underline{\beta}, z) = \underline{\beta} - \frac{1}{2}$.

4.1 Investment Decisions in the two-regulator case

For a given z , the negative term $2\lambda \frac{F(\beta|z)}{f(\beta|z)} \frac{C(\beta, z)}{C(\beta, z) - 2}$ reduces, in comparison to a single regulator arrangement, the regulator's perceived cost to demand the provision of the project at lower cost levels, so we have:⁷

Proposition 3 *Fix a level of investment $z \in [0, \bar{z}]$. The costs to provide the projects are lower when the agent reports to two regulators than when he reports to a single regulator: $C_i^{2R}(\beta, z) \leq C_i^{1R}(\beta, z)$, $i = 1, 2$, where the inequality is strict for $\beta \in (\underline{\beta}, \bar{\beta}]$.*

Since the agent's informational rents are decreasing in the costs at which the projects are provided, Proposition 3 implies that, for a *given* z , the agent's ex-post utility will be higher in a two principal arrangement. The agent is then able to recoup, through ex-post rents, a larger fraction of the investment made. This is the first force toward higher investments in a two-regulator setting.

The second force deals with the impact of investments on the costs at which the projects are provided. For any given β , a higher investment increases the likelihood that a regulator faces types lower than β . This raises the cost of leaving informational rents to the agent, which induces the regulator to demand the provision of the projects at higher costs.

This (perverse) effect of a higher investment on the ex-post costs of projects is more critical with a single regulator. This again follows because costs are substitutes in the agent's preferences. In a two-regulator arrangement, say, regulator 1 anticipates that a higher z will lead to an increase in the cost regulator 2 demands from the agent. Regulator 1 then responds demanding lower costs from the project he is in charge of. It follows that in a symmetric equilibrium, in response to an increase in z , the regulators's will increase demanded costs in a less aggressive fashion when compared to what a single regulator would do. It then follows:

Proposition 4 *Investment will be higher in a two-regulator setting. Hence, making the agent report to two regulators strictly improves efficiency by both (i) lowering (pointwise) the costs at which the projects are provided, and (ii) inducing more investment.*

⁷For comparison, the first order condition that defines the cost level in a single-regulator setting is

$$-(1 + \lambda) [1 - (2\beta - 2C^{1R}(\beta, z))] + 2\lambda \frac{F(\beta|z)}{f(\beta|z)} = 0.$$

References

- [1] Bernheim, D., and M. Whinston, 1986. Common Agency, *Econometrica*, 54:923-942.
- [2] Costa, C., Ferreira, D., and H. Moreira, 2005, Biased managers, Organizational Design, and Incentive Provision, *Economics Letters*, v. 86(3), pp. 379-385.
- [3] Grossman, S., and O. Hart, 1986. The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration, *Journal of Political Economy*, 94 August, 691-719.
- [4] Hart, O., and Moore, J., 1990. "Property Rights and the Nature of the Firm", *Journal of Political Economy*, 98(6), December, pp. 1119-58, December.
- [5] Laffont, J. and J. Tirole, 1993, *A Theory of Incentives in Procurement and Regulation*, Cambridge, Massachusetts, The M.I.T Press.
- [6] Klein, B., R. Crawford, and A. Alchian, 1978. Vertical Integration, Appropriable Rents, and the Competitive Process, *Journal of Law and Economics*, 21: 297-326.
- [7] Martimort, D., 1999, Renegotiation Design with Multiple Regulators, *Journal of Economic Theory*, Volume 88, Issue 2, pp. 261-293.
- [8] Martimort, D., 1996. Exclusive Dealing, Common Agency, and Multiprincipal Incentive Theory, *The Rand Journal of Economics*, 27, 1-31.
- [9] Martimort, D., and L. Stole, 2002. The Revelation and Delegation Principles in Common Agency Games, *Econometrica*, 70: 1659-1673.
- [10] Martimort, D., and L. Stole, 2003, Contractual Externalities and Common Agency Equilibria, *Advances in Theoretical Economics: Vol. 3: No. 1, Article 4*. <http://www.bepress.com/bejte/advances/vol3/iss1/art4>.
- [11] Olsen, T, and G. Torsvik, 1995, Intertemporal common agency and organizational design: How much decentralization?, *European Economic Review*, Volume 39, Issue 7, pp.1405-1428.
- [12] Salanie, B., 1997, *The Economics of Contracts*, Cambridge, Massachusetts , The MIT Press.
- [13] Topkis, D., 1998, *Supermodularity and Complementarity*, Princeton, New Jersey, Princeton University Press.
- [14] Williamson, O., 1985. *The Economic Institutions of Capitalism*, New York, Free Press.

5 Appendix:

Lemma 1 *In a symmetric equilibrium,*

$$\frac{dC_2(C_1(\beta), \beta)}{dC_1} = \frac{\dot{C}(\beta)}{C(\beta) - 2}.$$

Proof: The problem faced by the agent when he is offered a menu of contracts $\{C_2, t_2(C_2)\}_{C_2}$ by regulator 2 is:

$$\max_{C_2} t_2(C_2) - \frac{1}{2} [2\beta - C_1 - C_2]^2.$$

The first order condition reads

$$t_2'(C_2) + [2\beta - C_1 - C_2] = 0. \quad (\text{A})$$

By the implicit function theorem,

$$\frac{\partial C_2}{\partial C_1} = \frac{1}{t_2''(C_2) - 1}.$$

Since (A) must hold for all β , one can differentiate it to find, using symmetry,

$$t_2''(C_2) \dot{C}(\beta) + 2 \left[1 - \dot{C}(\beta) \right] = 0,$$

so that $t_2''(C_2) = \frac{2[\dot{C}(\beta)-1]}{\dot{C}(\beta)}$ and $t_2''(C_2) - 1 = \frac{\dot{C}(\beta)-2}{\dot{C}(\beta)}$. Hence,

$$\frac{dC_2(C_1(\beta), \beta)}{dC_1} = \frac{\dot{C}(\beta)}{\dot{C}(\beta) - 2}.$$

■

Proof of Proposition 2: See Martimort (1996, Proposition 4).

Proof of Proposition 3: It follows from noting that, since $\frac{dC_2(C_1, \beta, z)}{dC_1} < 0$, the expression

$$-(1 + \lambda) [1 - (2\beta - 2C^{2R}(\beta, z))] + 2\lambda \frac{F(\beta|z)}{f(\beta|z)} \left[1 + \frac{C^{2R}(\beta, z)}{C^{2R}(\beta, z) - 2} \right]$$

(which is related to the FOC for a regulator in a dual setting) evaluated at $C^{1R}(\beta, z) = \beta - \frac{1}{2} + 2\frac{\lambda}{1+\lambda} \frac{F(\beta|z)}{f(\beta|z)}$ is strictly negative for all $\beta \in (\underline{\beta}, \bar{\beta}]$. Hence, $C^{1R}(\beta, z) > C^{2R}(\beta, z)$ for all $\beta \in (\underline{\beta}, \bar{\beta}]$. For $\underline{\beta}$, the stated result holds as an equality since $\frac{F(\underline{\beta}|z)}{f(\underline{\beta}|z)} = 0$.

To prove Proposition 4, we will use the following result, whose proof can be found in Topkis (1988)

Proposition 5 *Consider the following parameterized maximization problem*

$$\max_{x \in \mathfrak{R}} f(x; \theta).$$

Assume that a solution $x^(\theta)$ exists and that $\frac{\partial f(x, \theta)}{\partial x}$ is (weakly) increasing in θ for all x, θ . Then, $x^*(\theta)$ is (weakly) increasing in θ .*

Proof of Proposition 4: From Proposition 3, we know that, for a fixed z , the projects will be provided at lower costs in a two-principal arrangement. Hence, to prove the result (and, in particular, to establish that a two-principal arrangement leads to more efficient outcomes, i.e., lower costs and higher investments), it suffices to show that investment will be higher.

Consider the following parameterized maximization problem.

$$\max_{z \in [0, \bar{z}]} \Omega(z; x) = \begin{bmatrix} x \int_{\underline{\beta}}^{\bar{\beta}} 2 [2\beta - 2C^{2R}(\beta; z)] F(\beta|z) d\beta \\ + (1-x) \int_{\underline{\beta}}^{\bar{\beta}} 2 [2\beta - 2C^{1R}(\beta; z)] F(\beta|z) d\beta \\ - \frac{1}{2} z^2 \end{bmatrix}.$$

When $x = 0$, the objective function of the above problem coincides with the objective of the agent's problem of choosing how much to invest in a singular regulator arrangement, whereas when $x = 1$, it coincides with the agent's problem in a two-regulator arrangement. Evoking Proposition (5), to establish the result, it suffices to show that

$$\frac{d\Omega(z; 1)}{dz} \geq \frac{d\Omega(z; 0)}{dz}.$$

This amounts to showing that

$$\int_{\underline{\beta}}^{\bar{\beta}} 4 [C^{1R}(\beta; z) - C^{2R}(\beta; z)] F(\beta|z) d\beta$$

is increasing in z . The derivative of this expression with respect to z is

$$\begin{aligned} & \int_{\underline{\beta}}^{\bar{\beta}} 4 [C^{1R}(\beta; z) - C^{2R}(\beta; z)] \frac{\partial F(\beta|z)}{\partial z} d\beta \\ & + \int_{\underline{\beta}}^{\bar{\beta}} 4 \left[\frac{\partial C^{1R}(\beta; z)}{\partial z} - \frac{\partial C^{2R}(\beta; z)}{\partial z} \right] F(\beta|z) d\beta. \end{aligned}$$

The first expression is unambiguously positive as, by Proposition 3, $C^{2R}(\beta; z) < C^{1R}(\beta; z)$ for all $\beta \in [\underline{\beta}, \bar{\beta}]$ and, since for any two levels of investments $z > z' \geq 0$, $\frac{f(\beta|z)}{f(\beta|z')}$ is decreasing in β , $\frac{\partial F(\beta|z)}{\partial z} > 0$.⁸

As for the second term, note first that

$$\frac{\partial C^{1R}(\beta; z)}{\partial z} = \frac{\lambda}{(1+\lambda)} \frac{d \frac{F(\beta|z)}{f(\beta|z)}}{dz} \equiv \frac{\lambda}{(1+\lambda)} H_z(\beta|z).$$

where $\frac{d \frac{F(\beta|z)}{f(\beta|z)}}{dz} \equiv H_z(\beta|z) \geq 0$ since $\frac{f(\beta|z)}{f(\beta|z')}$ is decreasing in β whenever $z > z'$.

For any given β , the output demanded in equilibrium in a two-regulator structure is implicitly defined by

$$F\left(C^{2R}, \beta, \dot{C}^{2R}, z\right) \equiv -(1+\lambda)[1 - (2\beta - 2C)] + 2\lambda \frac{F(\beta|z)}{f(\beta|z)} \left[1 + \frac{\dot{C}}{C-2} \right] = 0.$$

⁸Simple calculations show that, for all $z' \geq z$,

$$F(\beta|z') - F(\beta|z) \geq 0.$$

Hence, $\frac{dF(\beta|z)}{dz} \geq 0$.

By the implicit function theorem, one sees that

$$\frac{\partial C^{2R}(\beta, C^{2R}, z)}{\partial z} = -\frac{\frac{\partial F}{\partial z}}{\frac{\partial F}{\partial C^{2R}}} = \frac{\lambda}{(1+\lambda)} H_z(\beta|z) \left[1 + \frac{\dot{C}(\beta, z)}{C(\beta, z) - 2} \right].$$

Hence,

$$\frac{\partial C^{1R}(\beta; z)}{\partial z} - \frac{\partial C^{2R}(\beta; z)}{\partial z} = -\frac{\lambda}{(1+\lambda)} H_z(\beta|z) \left(\frac{\dot{C}(\beta, z)}{C(\beta, z) - 2} \right) > 0,$$

and the result follows.